## 73. Production and Decay of $b$-flavored Hadrons

Revised September 2023 by P. Eerola (Research Council of Finland), M. Kreps (Warwick U.) and Y. Kwon (Yonsei U., Seoul).

The $b$ quark belongs to the third generation of quarks and is the weak-doublet partner of the $t$ quark. The existence of the third-generation quark doublet was proposed in 1973 by Kobayashi and Maskawa [1] in their model of the quark mixing matrix ("CKM" matrix), and confirmed four years later by the first observation of a $b \bar{b}$ meson [2]. In the KM model, $C P$ violation is explained within the Standard Model (SM) by an irreducible phase of the $3 \times 3$ unitary matrix. The regular pattern of the three lepton and quark families is one of the most intriguing puzzles in particle physics. The existence of families gives rise to many of the free parameters in the SM, including the fermion masses, and the elements of the CKM matrix.

Since the $b$ quark is the lighter element of the third-generation quark doublet, the decays of $b$-flavored hadrons occur via generation-changing processes through CKM matrix. Because of this, and the fact that the CKM matrix is close to a $3 \times 3$ unit matrix, many interesting features such as loop and box diagrams, flavor oscillations, as well as large $C P$ asymmetries, can be observed in the weak decays of $b$-flavored hadrons.

The CKM matrix is parameterized by three real parameters and one complex phase. This complex phase is the source of $C P$ violation in $B$ meson decays in the Standard Model. A crucial milestone was the first observation of $C P$ violation in the $B$ meson system in 2001, by the BaBar [3] and Belle [4] collaborations. They measured a large value for the parameter $\sin 2 \beta\left(=\sin 2 \phi_{1}\right)$ [5], almost four decades after the discovery of a small $C P$ asymmetry in neutral kaons. A more detailed discussion of the CKM matrix and $C P$ violation can be found elsewhere in this Review $[6,7]$.

The structure of this mini-review is organized as follows. After a discussion of $b$-quark production and current results on spectroscopy, we discuss lifetimes of $b$-flavored hadrons. We then discuss some basic properties of $B$-meson decays, followed by summaries of dominant hadronic, rare hadronic, and electroweak penguin decays of $B$-mesons. There are separate mini-reviews for $B^{0}-\bar{B}^{0}$ mixing [8] and the extraction of the CKM matrix elements $V_{c b}$ and $V_{u b}$ from $B$-meson decays [9] in this Review.

### 73.1 Production and spectroscopy

The bound states of a $\bar{b}$ antiquark and a $u, d, s$, or $c$ quark are referred to as the $B_{u}\left(B^{+}\right)$, $B_{d}\left(B^{0}\right), B_{s}\left(B_{s}^{0}\right)$, and $B_{c}\left(B_{c}^{+}\right)$mesons, respectively. The $B_{c}^{+}$is the heaviest of the ground-state $b$-flavored mesons, and the most difficult to produce: it was observed for the first time in the semileptonic mode by CDF in 1998 [10], but its mass was accurately determined only in 2006, from the fully reconstructed mode $B_{c}^{+} \rightarrow J / \psi \pi^{+}$[11]. Many exclusive decay channels can now be used for the accurate mass measurements, given the large statistics available at the LHC. Currently the most precise measurement is made by LHCb and yields $m\left(B_{c}^{+}\right)=6274.47 \pm 0.27 \pm 0.17 \mathrm{MeV} / c^{2}[12]$, combining $B_{c}^{+} \rightarrow J / \psi \pi^{+}, B_{c}^{+} \rightarrow J / \psi \pi^{+} \pi^{-} \pi^{+}, B_{c}^{+} \rightarrow J / \psi p \bar{p} \pi^{+}, B_{c}^{+} \rightarrow J / \psi D_{s}^{+}, B_{c}^{+} \rightarrow J / \psi D^{0} K^{+}$ and $B_{c}^{+} \rightarrow B_{s}^{0} \pi^{+}$decay modes.

The first excited meson is called the $B^{*}$ meson, while $B^{* *}$ is the generic name for the four orbitally excited $(L=1) B$-meson states that correspond to the $P$-wave mesons in the charm system, $D^{* *}$. Excited states of the $B_{s}^{0}$ meson are similarly named $B_{s}^{*}$ and $B_{s}^{* *}$.

Of the possible bound $\bar{b} b$ states, the $\Upsilon(n S)$ and $\chi_{b J}(n P)$ states are well studied. The pseudoscalar ground state $\eta_{b}$ has been observed for the first time by BaBar [13] indirectly through the decay $\Upsilon(3 S) \rightarrow \gamma \eta_{b}$, and then confirmed by Babar in $\Upsilon(2 S)$ decays [14] and CLEO in $\Upsilon(3 S)$ decays [15]. The most accurate mass and width measurements come now from Belle, using de-
cays $\Upsilon(5 S) \rightarrow h_{b}(1 P) \pi^{+} \pi^{-}, h_{b}(1 P) \rightarrow \gamma \eta_{b}(1 S)[16], \Upsilon(4 S) \rightarrow \eta h_{b}(1 P), h_{b}(1 P) \rightarrow \gamma \eta_{b}(1 S)$ [17], and $\Upsilon(2 S) \rightarrow \gamma \eta_{b}(1 S)$ [18]. Belle has also reported first evidence for the $\eta_{b}(2 S)$ in the $h_{b}(2 P) \rightarrow \eta_{b}(2 S) \gamma$ transition [16]. In addition, Belle has observed $T_{b \bar{b} 1}(10610)$ (was $Z_{b}(10610)$ ) and $T_{b \bar{b} 1}(10650)$ (was $Z_{b}(10650)$ ) states in the processes $\Upsilon(5 S) \rightarrow \Upsilon(n S) \pi^{+} \pi^{-}(n=1,2,3)$ and $\Upsilon(5 S) \rightarrow h_{b}(m P) \pi^{+} \pi^{-}(m=1,2)[19]$. These $T_{b \bar{b} 1}$ states are observed to decay to $\Upsilon(n S) \pi^{ \pm}$and $h_{b}(m P) \pi^{ \pm}$, hence electrically charged and do not belong to ordinary $q \bar{q}$ mesons. For classification and naming of these and other states, see Ref. [20].

Experimental studies of $b$ decays have been performed in $e^{+} e^{-}$collisions at the $\Upsilon(4 S)$ (ARGUS, CLEO, Belle, BaBar) and $\Upsilon(5 S)$ (CLEO, Belle) resonances. The $e^{+} e^{-} \rightarrow b \bar{b}$ production crosssection at the $\Upsilon(4 S)(\Upsilon(5 S))$ resonance is about 1.1 nb ( 0.3 nb ). The full data samples of BaBar and Belle are $560 \mathrm{fb}^{-1}$ and $1020 \mathrm{fb}^{-1}$, respectively, of which $433 \mathrm{fb}^{-1}$ and $710 \mathrm{fb}^{-1}$ are at the $\Upsilon(4 S)$ resonance. Since the $\Upsilon(4 S)$ decays dominantly to a pair of $B$ mesons (either $B^{+} B^{-}$or $B^{0} \bar{B}^{0}$ ), a precise knowledge of the energy-momentum of one $B$ meson (the 'tagging $B^{\prime}$ ) enables deducing the properties of the other $B$ (the 'signal $B$ '). This property has been exploited by both BaBar and Belle, in particular, to measure inclusive decay modes as well as final states with missing neutrinos. The Belle II experiment at SuperKEKB has started recording data in 2019, and the experiment has so far collected about $428 \mathrm{fb}^{-1}$ of data by summer 2022, when their long shutdown 1 started. Of collected data, $363 \mathrm{fb}^{-1}$ is at the $\Upsilon(4 S)$ resonance. At the $Z$ resonance (SLC, LEP) all species of $b$-flavored hadrons could be studied for the first time. The $e^{+} e^{-} \rightarrow b \bar{b}$ production cross-section at the $Z$ resonance is about 6.6 nb .

High-energy $p \bar{p}$ (Tevatron) and $p p$ collisions (LHC) produce $b$-flavored hadrons of all species with large cross-sections. At the Tevatron $(\sqrt{s}=1.96 \mathrm{TeV})$ the visible cross section $\sigma(p \bar{p} \rightarrow b X,|\eta|<1)$ is about $30 \mu \mathrm{~b}$. CDF and D0 experiments at the Tevatron have accumulated by the end of their running about $10 \mathrm{fb}^{-1}$ each.

At the LHC $p p$ collider at $\sqrt{s}=7-13 \mathrm{TeV}$, the visible $b$-hadron cross section at the LHCb experiment with pseudorapidity acceptance $2<\eta<5$ has been measured to be $\sim 72 \mu \mathrm{~b}$ at 7 TeV and $\sim 144 \mu \mathrm{~b}$ at 13 TeV [21] (cross section at 13 TeV corrected in Erratum). LHCb has collected about $1 \mathrm{fb}^{-1}$ at $7 \mathrm{TeV}, 2 \mathrm{fb}^{-1}$ at 8 TeV , and close to $5.9 \mathrm{fb}^{-1}$ at 13 TeV during LHC Runs 1 and 2. CMS and ATLAS have collected each about $5 \mathrm{fb}^{-1}$ of data at $\sqrt{s}=7,20 \mathrm{fb}^{-1}$ at 8 TeV and about $150 \mathrm{fb}^{-1}$ at 13 TeV during LHC Runs 1 and 2. The latest LHC Run 3 at 13.6 TeV started in summer 2022, with upgraded detectors. By the time of the writing (March 2024), LHC has delivered about $70 \mathrm{fb}^{-1}$ to ATLAS and CMS experiments, and about $300 \mathrm{pb}^{-1}$ to the LHCb experiment with major detector upgrades.

In hadron collisions, production happens as $b \bar{b}$ pairs via leading order flavor creation or higher order processes such as gluon-splitting. Single $b$-quarks can be produced by flavor excitation. The total $b$-production cross section is an interesting test of our understanding of leading and higher order QCD processes. With a wealth of measurements at LHC and at Tevatron (see Ref. [21] and references therein), and improved calculations [22], there is a reasonable agreement between measurements and predictions.

Each quark of a $b \bar{b}$ pair produced in hadron collisions hadronizes separately and incoherently from the other, but it is still possible to obtain a statistical indication of the charge of a produced $b / \bar{b}$ quark ("flavor tag" or "charge tag") from the accompanying particles produced in the hadronization process, or from the decay products of the other quark. The momentum spectrum of produced $b$-quarks typically peaks near the $b$-quark mass, and extends to much higher momenta, dropping by about a decade for every ten GeV. Typical decay lengths are of the order of a centimeter at 13 $\mathrm{TeV} p p$ collisions; the resolution for the decay vertex must be more precise than this to resolve the fast oscillations of $B_{s}^{0}$ mesons.

In $e^{+} e^{-}$colliders, since the $B$ mesons are very slow in the $\Upsilon(4 S)$ rest frame, asymmetric beam energies are used to boost the decay products to allow time-dependent measurements that are crucial for the study of $C P$ violation. At KEKB, the boost was $\beta \gamma=0.43$, while PEP-II used a slightly larger boost, $\beta \gamma=0.55$. The typical $B$-meson decay length is dilated from $\approx 20 \mu \mathrm{~m}$ to $\approx 200 \mu \mathrm{~m}$. At SuperKEKB the boost is lower, $\beta \gamma=0.28$, which puts more demanding requirements on the track reconstruction precision at Belle II to reach a resolution in decay time measurements similar to Belle. The two $B$ mesons produced in $\Upsilon(4 S)$ decay are in a coherent quantum state, which makes it easier than in hadron collisions to infer the charge state of one $B$ meson from observation of the other; however, the coherence also requires determination of the decay time of both mesons, rather than just one, in order to perform time-dependent $C P$-violation measurements. For $B_{s}^{0}$, which can be produced at $\Upsilon(5 S)$ the situation is less favourable, as boost is not high enough to provide sufficient time resolution to resolve the fast $B_{s}^{0}$ oscillations.

For the measurement of branching fractions, the initial composition of the data sample must be known. The $\Upsilon(4 S)$ resonance decays predominantly to $B^{0} \bar{B}^{0}$ and $B^{+} B^{-}$; the current experimental upper limit for non- $B \bar{B}$ decays of the $\Upsilon(4 S)$ is less than $4 \%$ at the $95 \%$ confidence level (CL) [23]. The observed modes of this category are decays to lower $\Upsilon$ states and a pion pair, $\eta$, or $\eta^{\prime}$, measured branching fractions being of order $10^{-4}-10^{-5}$ [24], and decays to $h_{b}(1 P) \eta$ with branching fraction of order $10^{-3}[17]$.

The ratio $f_{+} / f_{0}$ of the fractions of charged to neutral $B$ productions from $\Upsilon(4 S)$ decays has been measured by CLEO, BaBar, and Belle in various ways. They typically use pairs of isospin-related decays of $B^{+}$and $B^{0}$, such that it can be assumed that $\Gamma\left(B^{+} \rightarrow x^{+}\right)=\Gamma\left(B^{0} \rightarrow x^{0}\right)$. In this way, the ratio of the number of events observed in these modes is proportional to $\left(f_{+} \tau_{+}\right) /\left(f_{0} \tau_{0}\right)$ [25]. BaBar has also performed an independent measurement of $f_{0}$ with a different method that does not require isospin symmetry or the value of the lifetime ratio, based on the number of events with one or two reconstructed $B^{0} \rightarrow D^{*-} \ell^{+} \nu$ decays [26]. The combined result, from the current average of $\tau_{+} / \tau_{0}$, is $f_{+} / f_{0}=1.058 \pm 0.024$ [27]. The result is consistent within $2.4 \sigma$ with equal production of $B^{+} B^{-}$and $B^{0} \bar{B}^{0}$ pairs, and we assume $f_{+} / f_{0}=1$ in this mini-review except where explicitly stated otherwise. This assumption is also supported by the near equality of the $B^{+}$and $B^{0}$ masses: our fit yields $m\left(B^{0}\right)=5279.72 \pm 0.08 \mathrm{MeV} / c^{2}, m\left(B^{+}\right)=5279.41 \pm 0.07 \mathrm{MeV} / c^{2}$, and $m\left(B^{0}\right)-m\left(B^{+}\right)=0.31 \pm 0.05 \mathrm{MeV} / c^{2}$.

Data collected at the $\Upsilon(5 S)$ resonance gave CLEO, Belle and BaBar access to $B_{s}^{0}$ decays. In $\Upsilon(5 S)$ decays there are seven possible final states including a pair of non-strange $B$ mesons and 0 , 1 or 2 pions, and three final states with a pair of strange $B$ mesons ( $B_{s}^{* 0} \bar{B}_{s}^{* 0}, B_{s}^{* 0} \bar{B}_{s}^{0}$, and $B_{s}^{0} \bar{B}_{s}^{0}$ ). The fraction of events with a pair of $B_{s}^{0}$ mesons over the total number of events with a pair of $b$-flavored hadrons has been measured to be $f_{s}[\Upsilon(5 S)]=0.199_{-0.029}^{+0.030}[28]$, of which $88 \%$ is $B_{s}^{* 0} \bar{B}_{s}^{* 0}$ events. However, the small boost of $B_{s}^{0}$ mesons produced in this way prevents resolution of their fast oscillations for time-dependent measurements; these are only accessible in hadron collisions (or at the $Z$ peak).

In high-energy collisions, the produced $b$ or $\bar{b}$ quarks can hadronize with different probabilities into the full spectrum of $b$-hadrons, either in their ground or excited states. The hadronization does not have to be identical in $p \bar{p}$ or $p p$ collisions and in $Z$ decay, because of the different momentum distributions of the $b$-quark in these processes; the sample used in the $p \bar{p}$ measurements has momenta close to the $b$ mass, rather than $m_{Z} / 2$ in $Z$ decay. The available data from Tevatron and LHC show that the production fractions $f_{d}, f_{u}, f_{s}$, and $f_{\text {baryon }}$ of $B^{0}, B^{+}, B_{s}^{0}$, and $b$ baryons, respectively, of weakly decaying $b$ hadrons depend on the kinematics of the produced $b$ hadron. Recently LHCb experiment found evidence for dependence of $b$-quark hadronization on multiplicity in $p p$ collisions [29]. The production fractions of $b$ hadrons are discussed in more detail in the $B^{0}-\bar{B}^{0}$
mixing section in this Review [8].
Excited $B$-meson states have been thoroughly studied by CLEO, LEP, CUSB, D0 and CDF (an admixture of $B$ mesons) and LHCb ( $B^{*+}$-meson). The current world average of the $B^{*}-B$ mass difference is $45.21 \pm 0.21 \mathrm{MeV} / c^{2}$. Excited $B_{s}^{*}$-meson states have observed in $\Upsilon(5 S)$ decays by CUSB, CLEO and Belle.

For orbitally excited $B_{(s)}$ meson states, with relative angular momentum $\mathrm{L}=1$ of the two quarks, there exist four states $\left(J, j_{q}\right)=(0,1 / 2),(1,1 / 2),(1,3 / 2),(2,3 / 2)$, where $j_{q}$ is the total angular momentum of the light $u, d$ or $s$ quark and $J$ is the total angular momentum of the $B$ meson. These states are collectively called as $B_{(s)}^{* *}$ mesons. The $j_{q}=1 / 2$ states are named $B_{(s) 0}^{*}(J=0)$ and $B_{(s) 1}$ $(J=1)$ mesons, while the states with $j_{q}=3 / 2$ are named $B_{(s) 1}(J=1)$ and $B_{(s) 2}^{*}(J=2)$ mesons. The states with $j_{q}=1 / 2$ can decay through an $S$-wave transition and are expected to have a large width, but the $j_{q}=3 / 2$ states are narrow $D$-wave decays. Evidence for $B^{* *}$ production has been initially obtained at LEP as a broad $B \pi$ resonance [30] or a $B^{+} K^{-}$enhancement [31]. Detailed results have been obtained for the narrow states $B_{1}(5721)^{0,+}$ and $B_{2}(5747)^{0,+}$ at the Tevatron and by LHCb, and clear enhancements compatible with the higher mass states $B_{J}(5840)^{0,+}$ and $B_{J}(5970)^{0,+}$ have been observed [32]. Also the narrow $B_{s}^{* *}$ states $B_{s 1}(5830)^{0}$ and $B_{s 2}(5840)^{0}$ have been measured at the CDF [32], LHCb [33], and CMS [34].

Excited states of $B_{c}^{+}$mesons will provide important information about the strong potential. A $B_{c}^{+} \pi^{+} \pi^{-}$resonance has been observed for the first time by ATLAS [35]. The mass of the resonance has been measured precisely by CMS and LHCb as $6871.2 \pm 1.0 \mathrm{MeV} / \mathrm{c}^{2}$ [36]. The resonance may be interpreted as the second $S$-wave state of the $B_{c}^{+}$meson, $B_{c}^{+}(2 S)$, but the quantum numbers are to be confirmed.

Baryon states containing a $b$ quark are labeled according to the same scheme used for non- $b$ baryons, with the addition of a $b$ subscript [20]. The first observed $b$ baryon was the $\Lambda_{b}^{0}$ (quark composition $u d b$ ). Thanks to the large samples accumulated at the Tevatron and specially at the LHC many new $b$ baryons have been found. The masses of all these new baryons have been measured to a precision of a few $\mathrm{MeV} / c^{2}$, and found to be in agreement with predictions from Heavy Quark Effective Theory (HQET).

Clear signals of four strongly-decaying baryon states, $\Sigma_{b}^{+}, \Sigma_{b}^{*+}(u u b), \Sigma_{b}^{-}, \Sigma_{b}^{*-}(d d b)$ have been obtained by CDF [37] and LHCb [38]. LHCb has also observed two new mass peaks in the $\Lambda_{b}^{0} \pi^{ \pm}$systems, consistent with single resonances and named as $\Sigma_{b}^{ \pm}(6097)$ [38]. The nature of these resonances is, however, not yet clear. The isodublet of strange $b$ baryons $\Xi_{b}^{0}$ (usb) and $\Xi_{b}^{ \pm}$(dsb) has been observed by CDF and D0 [39]. Masses, lifetimes, and branching ratios have been accurately measured by LHCb [40-42] and CDF [43]. LHCb has also measured several parameters sensitive to $P$ and $C P$ violation [42,44]. Other observed $\Xi_{b}$ baryons are spin-3/2 states $\Xi_{b}(5945)^{0}\left(\Xi_{b}^{* 0}\right)[45,46]$, $\Xi_{b}^{*}(5955)^{-}[46,47]$ and $\Xi_{b}(6100)^{-}[46,48]$, a spin- $1 / 2$ state $\Xi_{b}^{\prime}(5935)^{-}[46,47]$, and resonance states $\Xi_{b}^{-}(6227)[41,49], \Xi_{b}^{0}(6327)[50]$ and $\Xi_{b}^{0}(6333)$ [50]. The doubly-strange bottom baryon $\Omega_{b}^{-}$has been observed first by D0 and CDF [51]. Mass and mean life have been measured precisely by LHCb [52] and CDF [43]. LHCb has also observed four excited $\Omega_{b}^{-}$states $\Omega_{b}^{-}(6316), \Omega_{b}^{-}(6330)$, $\Omega_{b}^{-}$(6340) and $\Omega_{b}^{-}$(6350) [53].

The so-called exotic states have raised a lot of interest recently. While many exotic states were seen in the charm sector, in bottom sector there are fewer seen. The D0 Collaboration claimed a narrow state $X(5568)$ decaying into a $B_{s}^{0} \pi^{ \pm}$final state [54]. While this would be an interesting addition to the observed states as the first exotic state with constituent quarks with four different flavours ( $b, s, u, d$ ), analysis by LHCb yields negative result [55]. Also CMS finds no such a state [56].

### 73.2 Lifetimes

Precise lifetimes are key in extracting the weak parameters that are important for understanding the role of the CKM matrix in $C P$ violation, such as the determination of $V_{c b}$ and $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing parameters. In the naive spectator model, the heavy-flavored hadrons can decay only via the external spectator mechanism, and thus, the lifetimes of all mesons and baryons containing $b$ quarks would be equal. Non-spectator effects, such as the interference between contributing amplitudes, modify this simple picture and give rise to a lifetime hierarchy for $b$-flavored hadrons similar to the one in the charm sector. However, since the lifetime differences are expected to scale as $1 / m_{Q}^{2}$, where $m_{Q}$ is the mass of the heavy quark, the variations in the $b$ system are expected to be only $10 \%$ or less $[57,58]$. We expect:

$$
\begin{equation*}
\tau\left(B^{+}\right) \geq \tau\left(B^{0}\right) \approx \tau\left(B_{s}^{0}\right)>\tau\left(\Lambda_{b}^{0}\right) \gg \tau\left(B_{c}^{+}\right) . \tag{73.1}
\end{equation*}
$$

For the $B_{c}^{+}$, both quarks decay weakly, so the lifetime is much shorter.
Measurements of the lifetimes of the different $b$-flavored hadrons thus provide a means to determine the importance of non-spectator mechanisms in the $b$ sector. Availability of large samples of fully-reconstructed decays of different $b$-hadron species has resulted in precise measurements with small statistical and systematic uncertainties ( $\sim 1 \%$ ). The world averages given in Table 73.1 have been determined by the Heavy Flavor Averaging Group (HFLAV) [59].

Table 73.1: Summary of the world-average $b$-hadron lifetime measurements. For the $B_{s}^{0}$ lifetimes, see text below.

| Particle | Lifetime $[\mathrm{ps}]$ |
| :--- | :---: |
| $B^{+}$ | $1.638 \pm 0.004$ |
| $B^{0}$ | $1.517 \pm 0.004$ |
| $B_{s}^{0}$ | $1.520 \pm 0.005$ |
| $B_{s L}^{0}$ | $1.429 \pm 0.006$ |
| $B_{s H}^{0}$ | $1.623 \pm 0.008$ |
| $B_{c}^{+}$ | $0.510 \pm 0.009$ |
| $\Lambda_{b}^{0}$ | $1.471 \pm 0.009$ |
| $\Xi_{b}^{-}$ | $1.572 \pm 0.040$ |
| $\Xi_{b}^{0}$ | $1.480 \pm 0.030$ |
| $\Omega_{b}^{-}$ | $1.64_{-0.17}^{+0.18}$ |

The $B_{s}^{0}$ lifetime in Table 73.1 is defined as $1 / \Gamma_{s}$, where $\Gamma_{s}$ is the average width of the light ( L ) and heavy (H) mass eigenstates, $\left(\Gamma_{L}+\Gamma_{H}\right) / 2$. In the absence of $C P$ violation, the light (heavy) $B_{s}^{0}$ mass eigenstate is the $C P$-even ( $C P$-odd) eigenstate. Thus, the lifetime of the light (heavy) mass eigenstate can be measured from $C P$-even (odd) final states. The lifetimes can also be obtained from time-dependent angular analysis of $B_{s}^{0} \rightarrow J / \psi \phi$ decays.

The short $B_{c}^{+}$lifetime is in good agreement with predictions [60]. With large samples of $B_{c}^{+}$ mesons at the LHC precision on the lifetimes can still improve. The measurement using semileptonic decays gives $\tau_{B_{c}^{+}}=0.509 \pm 0.008 \pm 0.012 \mathrm{ps}$ [61] while using decays $B_{c}^{+} \rightarrow J / \psi \pi^{+}$yields $\tau_{B_{c}^{+}}=$ $0.5134 \pm 0.0110 \pm 0.0057 \mathrm{ps}[62]$. Each of these is more precise than the combination of all previous experiments.

The recent $\Lambda_{b}^{0}$ lifetime measurements from LHC experiments and CDF are precise and favour lifetime close to the lifetime of $B^{0}$ meson, in agreement with theory.

For precision comparisons with theory, lifetime ratios are more sensitive. Experimentally it is
found [59]:

$$
\begin{gathered}
\frac{\tau_{B^{+}}}{\tau_{B^{0}}}=1.076 \pm 0.004, \frac{\tau_{B_{s}^{0}}}{\tau_{B^{0}}}=1.002 \pm 0.004 \\
\frac{\tau_{\Lambda_{b}^{0}}}{\tau_{B^{0}}}=0.969 \pm 0.006
\end{gathered}
$$

while recent Heavy Quark Expansion (HQE) predictions give [58]:

$$
\begin{gathered}
\frac{\tau_{B^{+}}}{\tau_{B^{0}}}=1.04_{-0.01}^{+0.05} \pm 0.02 \pm 0.01, \frac{\tau_{B_{s}^{0}}}{\tau_{B^{0}}}=1.001 \pm 0.002 \\
\frac{\tau_{\Lambda_{b}^{0}}}{\tau_{B^{0}}}=0.935 \pm 0.054
\end{gathered}
$$

The ratio of $B^{+}$to $B^{0}$ lifetimes has a precision of better than $1 \%$, and is significantly different from 1.0, in agreement with predictions [57]. The ratio of $B_{s}^{0}$ to $B^{0}$ lifetimes is expected to be very close to 1.0 .

For a detailed discussion on neutral $B^{0}$ and $B_{s}^{0}$ oscillation and relevant $C P$ violation measurements see Ref. [8].

### 73.3 Features of decays

The ground states of $b$-flavored hadrons decay via weak interactions. In most decays of the $b$-flavored hadrons, where the $b$-quark is accompanied by lighter partner quarks ( $d, u, s$, or $c$ ), the decay modes are well described by the decay of the $b$ quark (spectator model) [63]. The dominant decay mode of a $b$ quark is $b \rightarrow c W^{*-}$ (referred to as a "tree" or "spectator" decay), where the virtual $W$ materializes either into a pair of leptons $\ell \bar{\nu}$ ("semileptonic decay"), or into a pair of quarks which then hadronizes. The transition $b \rightarrow u$ is suppressed by $\left|V_{u b} / V_{c b}\right|^{2} \sim(0.1)^{2}$ relative to $b \rightarrow c$ transitions. The decays in which the spectator quark combines with one of the quarks from $W^{*}$ to form one of the final state hadrons are suppressed by a factor $\sim(1 / 3)^{2}$, because the colors of the two quarks from different sources must match ("color-suppression").

Semileptonic $B$ decays $B \rightarrow X_{c} \ell \nu$ and $B \rightarrow X_{u} \ell \nu$ provide an excellent way to measure the magnitude of the CKM elements $\left|V_{c b}\right|$ and $\left|V_{u b}\right|$ respectively, because the strong interaction effects are much simplified due to the two leptons in the final state. Both exclusive and inclusive decays can be used with dominant uncertainties being complementary. For exclusive decay analysis, knowledge of the form factors for the exclusive hadronic system $X_{c(u)}$ is required. For inclusive analysis, it is usually necessary to restrict the available phase-space of the decay products to suppress backgrounds; subsequently uncertainties are introduced in the extrapolation to the full phase-space. Moreover, restriction to a small corner of the phase-space may result in breakdown of the operator-product expansion scheme, thus making theoretical calculations unreliable. A more detailed discussion of $B$ semileptonic decays and the extraction of $\left|V_{c b}\right|$ and $\left|V_{u b}\right|$ is given elsewhere in this Review [9]. While traditionally $B^{0}$ and $B^{+}$decays were used, over time also other $B$ hadron studies became available. Most notably, determination of of $\left|V_{u b}\right|$ using $\Lambda_{b}^{0} \rightarrow p \mu^{-} \bar{\nu}_{\mu}$ decays by LHCb [64] was more precise than expected. Besides, there have been measurements of inclusive semileptonic decay rates of $B_{s}^{0}$ [65] and $B_{c}^{+}[66]$ mesons. One of the latest additions in this area is the observation of the $B_{s}^{0} \rightarrow K^{-} \mu^{+} \nu_{\mu}$ decays by LHCb using only a fraction of their available data [67].

On the other hand, hadronic $B$ decays are complicated because of strong interaction effects caused by the surrounding cloud of light quarks and gluons. While this complicates the extraction of CKM matrix elements, it also provides a great opportunity to study perturbative and nonperturbative QCD, hadronization, and Final State Interaction (FSI) effects.

Many aspects of $B$ decays can be understood through the Heavy Quark Effective Theory (HQET) [68]. This has been particularly successful for semileptonic decays. For further discussion
of HQET, see for instance Ref. [69]. For hadronic decays, one typically uses effective Hamiltonian calculations that rely on a perturbative expansion with Wilson coefficients. In addition, some form of the factorization hypothesis is commonly used, where, in analogy with semileptonic decays, two-body hadronic decays of $B$ mesons are expressed as the product of two independent hadronic currents, one describing the formation of a charm meson (in case of the dominant $b \rightarrow c W^{*-}$ decays), and the other describing the hadronization of the remaining $\bar{u} d$ (or $\bar{c} s$ ) system from the virtual $W^{-}$. Qualitatively, for $B$ decays with a large energy release, e.g. $b \rightarrow u W^{*-}$ transitions, the $\bar{u} d$ pair (produced as a color singlet) travels fast enough to leave the interaction region without influencing the meson containing the spectator quark. This is known to work well for the dominant spectator decays [70]. There are several common implementations of these ideas for hadronic $B$ decays, the most common of which are QCD factorization (QCDF) [71], perturbative QCD (pQCD) [72], and soft collinear effective theory (SCET) [73].

The transitions $b \rightarrow s$ and $b \rightarrow d$ are flavor-changing neutral-current (FCNC) processes. Although they are not allowed in the SM as a tree-process, they can occur via loop diagrams (denoted "penguin" decays). The rates for $b \rightarrow s$ penguin decays are comparable to the CKMsuppressed $b \rightarrow u$ tree processes. Pure-penguin decays were first established by the observation of $B \rightarrow K^{*}(892) \gamma$ [74]. Penguin processes involving $b \rightarrow d$ transitions are further suppressed by CKM, and have been observed for $B \rightarrow(\rho / \omega) \gamma$ decays [75, 76]. LHCb has observed a $b \rightarrow d$ penguin transition in the $B^{+} \rightarrow \pi^{+} \mu^{+} \mu^{-}$mode and measured its branching fraction to be $(1.83 \pm 0.24 \pm 0.05) \times 10^{-8}[77]$.

Other decay processes discussed in this Review include $W$-exchange (a $W$ is exchanged between initial-state quarks), penguin annihilation (the gluon from a penguin loop attaches to the spectator quark, similar to an exchange diagram), and pure-annihilation (the initial quarks annihilate to a virtual $W$, which then decays). Some observed decay modes such as $B^{0} \rightarrow D_{s}^{-} K^{+}$, may be interpreted as evidence of a $W$-exchange process [78]. The evidence for the purely leptonic decay $B^{+} \rightarrow \tau^{+} \nu$ from Belle [79] and BaBar [80] is the first sign of a pure annihilation decay. The average branching fraction is $(1.09 \pm 0.24) \times 10^{-4}$, which is somewhat larger than, though consistent with, the value expected in the SM. A substantial region of parameter space of charged Higgs mass vs. $\tan \beta$ is excluded by the measurements of this mode. A dedicated discussion of purely leptonic decays of charged pseudoscalar mesons is given elsewhere in this Review [81].

### 73.4 Dominant hadronic decays

Most of the hadronic $B$ decays involve $b \rightarrow c$ transition at the quark level, resulting in a charmed hadron or charmonium in the final state. Other types of hadronic decays are very rare and will be discussed separately in the next section. The experimental results on hadronic $B$ decays have steadily improved over the years, and the measurements have reached sufficient precision to challenge our understanding of the dynamics of these decays. With good particle detection and hadron identification capabilities of $B$-factory detectors, a substantial fraction (roughly on the order of a few per mill) of hadronic $B$ decay events can be fully reconstructed. In particular, good performances for detecting $\pi^{0}$ and other neutral particles helped Belle and BaBar to make comprehensive measurements of the decays $\bar{B}^{0} \rightarrow D^{(*) 0} h^{0}$ [82], where $h^{0}$ stands for light neutral mesons such as $\pi^{0}, \eta^{\left({ }^{( }\right)}, \rho^{0}, \omega$. The measurements are being complemented by LHCb, in decays like $\overline{B^{0}} \rightarrow D^{0} \pi^{+} \pi^{-}$[83], where no neutral particles reconstruction is needed. These decays proceed through color-suppressed diagrams, hence they provide useful tests on the factorization models.

Because of the kinematic constraint of $\Upsilon(4 S) \rightarrow B \bar{B}$, the energy sum of the final-state particles of a $B$ meson decay is always equal to one half of the total energy in the center of mass frame. As a result, the two variables, $\Delta E$ (energy difference) and $M_{B}$ ( $B$ candidate mass with a beamenergy constraint) are very effective for reducing combinatorial background both from $\Upsilon(4 S)$ and
$e^{+} e^{-} \rightarrow q \bar{q}$ continuum events. In particular, the energy-constraint in $M_{B}$ improves the signal resolution by almost an order of magnitude.

The kinematically clean environment of $B$ meson decays provides an excellent opportunity to search for new states. For instance, quark-level $b \rightarrow c \bar{c} s$ decays have been used to search for new charmonium and charm-strange mesons and study their properties in detail. While narrow charmstrange states $D_{s 0}^{*}(2317)$ [84] and $D_{s 1}(2460)$ [85] were discovered by BaBar and CLEO, respectively, the properties of these new states were revealed by studying the $B$ meson decays, $B \rightarrow D D_{s 0}^{*}(2317)$ and $B \rightarrow D D_{s 1}(2460)$ by Belle [86] and BaBar [87]. Another example is Dalitz plot analysis of decay $B_{s}^{0} \rightarrow \bar{D}^{0} K^{-} \pi^{+}$in which the decay to spin-3 resonance was observed for the first time [88]. One of the most significant improvements in the past decade is establishment of decays $B^{+} \rightarrow D^{0} K^{+}$ and $B^{+} \rightarrow D^{0} K^{*}(892)^{+}$with $D^{0}$ decaying to final state common to $D^{0}$ and $\bar{D}^{0}$ [89]. This allows direct determination of CKM angle $\gamma\left(=\phi_{3}\right)$ to be $65.9_{-3.5}^{+3.3}$.

Information on $B_{s}^{0}, B_{c}^{+}$and $\Lambda_{b}^{0}$ decays have been remarkably improved with recent studies of large samples from LHCb. Noticeable additions in $B_{s}$ include decay modes to $D_{s}^{(*)+} D_{s}^{(*)-}, \bar{D}^{0} \bar{K}^{0}$, and $J / \psi \bar{K}^{*}(892)^{0}$. The $B_{s}^{0} \rightarrow D_{s}^{(*)+} D_{s}^{(*)-}$ decays were first observed by CDF [90], followed by Belle [91]. LHCb has improved the precision with $\mathcal{B}\left(B_{s}^{0} \rightarrow D_{s}^{(*)+} D_{s}^{(*)-}\right)=(3.07 \pm 0.22 \pm 0.33) \%$ [92], which suggests that $B_{s}^{0} \rightarrow D_{s}^{(*)+} D_{s}^{(*)-}$ decays do not saturate the $C P$-even modes of the $B_{s}$ decays. The $B_{s}^{0} \rightarrow \bar{D}^{0} \bar{K}^{0}$ decay occurs mostly via a color-suppressed tree diagram, and has a small theoretical uncertainty in the SM, thus this mode can significantly improve the determination of the $C P-$ violation angle $\phi_{s}$. LHCb has observed this decay and the branching fraction is $(4.3 \pm 0.5 \pm 0.7) \times 10^{-4}$ [93]. The $B_{s}^{0} \rightarrow J / \psi \bar{K}^{*}(892)^{0}$ decay can be used to constrain the penguin pollution in determining $\phi_{s}$. LHCb has updated the branching fraction and measured the $C P$ asymmetries of this decay, thereby constraining the penguin pollution in $\phi_{s}$ [94], although a much more stringent constraint on penguin pollution can come from $B^{0} \rightarrow J / \psi \rho^{0}$ which has been observed by BaBar [95] and LHCb [96]. The $B_{c}^{+} \rightarrow B_{s}^{0} \pi^{+}$decay is unique as the only observed mode of $b$-flavored hadron decays where the partner quark decays ( $c$ in this case) while the $b$ quark remains a spectator. LHCb has observed this mode [97] and measured $\mathcal{B}\left(B_{c}^{+} \rightarrow B_{s}^{0} \pi^{+}\right) / \mathcal{B}\left(B_{c}^{+} \rightarrow J / \psi \pi^{+}\right)=(91 \pm 10 \pm 8 \pm 3)$ [98]. In addition, LHCb [99] and ATLAS [100] have measured $B_{c}^{+} \rightarrow J / \psi D_{s}^{(*)+}$, which, by comparing with $B_{c}^{+} \rightarrow B_{s}^{0} \pi^{+}$, provides a ratio of exclusive $b \rightarrow c$ and $c \rightarrow s$ decays of $B_{c}^{+}$. For $\Lambda_{b}^{0} \rightarrow \Lambda_{c}^{+} \pi^{+} \pi^{-} \pi^{-}$ [101], not only the total rate is measured, but also structure involving decays through excited $\Lambda_{c}$ and $\Sigma_{c}$ baryons.

In addition, a variety of exotic particles that do not fit the conventional meson spectroscopy have been discovered in $B$ decays. Belle found the $X(3872)$ state by studying $B^{+} \rightarrow J / \psi \pi^{+} \pi^{-} K^{+}$[102], which was confirmed by CDF [103], D0 [104] and BaBar [105]. Production of $X(3872)$ has been studied by the LHC experiments, LHCb [106], CMS [107] and ATLAS [108].

A charged charmonium-like state $X(4430)^{ \pm}$that decays to $\psi(2 S) \pi^{ \pm}$was observed by Belle in $B \rightarrow \psi(2 S) K \pi^{ \pm}[109]$. Since it is charged, it could not be an ordinary charmonium state. A highstatistics study by LHCb confirmed the existence of the $X(4430)^{ \pm}$in decays $B \rightarrow \psi(2 S) K \pi^{ \pm}$[110], demonstrated its resonance character by studying the phase motion, unambiguously determined its spin-parity, and saw evidence for another state. In a Dalitz plot analysis of $\bar{B}^{0} \rightarrow J / \psi K^{-} \pi^{+}$ [111], Belle has found another state, labeled as $X(4200)^{+}$in this Review, adding to the list of exotic charged charmonium-like states. In an amplitude analysis of the decay $\Lambda_{b}^{0} \rightarrow J / \psi p K^{-}$, LHCb observed exotic structures, labeled as $P_{c}(4380)^{+}$and $P_{c}(4450)^{+}$in this Review, in the $J / \psi p$ channel [112]. The subsequent analysis with significantly increased statistics observed additional state and resolved the peak at $4450 \mathrm{MeV} / c^{2}$ as being due to the two states close in the mass [113]. The structure in the $J / \psi p$ channel was also seen in the $B_{s}^{0} \rightarrow J / \psi p \bar{p}$ decays [114] and in the $J / \psi \Lambda$ channel in $B^{+} \rightarrow J / \psi \Lambda p$ and $\Xi_{b}^{-} \rightarrow J / \psi \Lambda K^{-}$decays [115]. They are referred to as charmonium-
pentaquark states. More detailed discussions of exotic meson-like states and pentaquarks are given elsewhere in this Review [116].

### 73.5 Rare hadronic decays

All $B$-meson decays that do not occur through the $b \rightarrow c$ transition are usually called rare $B$ decays. These include both semileptonic and hadronic $b \rightarrow u$ decays that are suppressed at leading order by the small CKM matrix element $V_{u b}$, as well as higher-order $b \rightarrow s(d)$ processes such as electroweak and gluonic penguin decays. In this section, we review hadronic rare $B$ decays, while electroweak penguin decays and others are discussed in the next.

Charmless $B$ meson decays into two-body hadronic final states such as $B \rightarrow \pi \pi$ and $K \pi$ are experimentally clean, and provide good opportunities to probe new physics and search for indirect and direct $C P$ violations. Since the final state particles in these decays tend to have larger momenta than average $B$ decay products, the event environment is cleaner than for $b \rightarrow c$ decays. Branching fractions are typically around $10^{-5}$. Over the past decade, many such modes have been observed not only by $e^{+} e^{-}$collider experiments such as BaBar and Belle, but also by hadron collider experiments such as $\mathrm{CDF}(p \bar{p})$ and $\mathrm{LHCb}(p p)$. In the latter cases, huge data samples of the modes with all charged final-state particles have been reconstructed by triggering on the impact parameter of the charged tracks. This has also allowed observation of charmless decays of the $B_{s}$, in final states such as $\phi \phi[117], K^{+} K^{-}[118,119]$, and $K^{-} \pi^{+}[119,120]$, and of charmless decays of the $\Lambda_{b}^{0}$ baryon [120]. The large samples available at LHCb experiment allow to perform also timedependent $C P$ violation measurements [121]. Charmless $B_{s}$ modes are related to corresponding $B^{0}$ modes by U-spin symmetry, and are determined by similar amplitudes. Combining the observables from $B_{s}^{0}$ and $B^{0}$ modes is a further way of eliminating hadronic uncertainties and extracting relevant CKM information [122].

Because of relatively high-momenta for final state particles, the dominant source of background in $e^{+} e^{-}$collisions is $q \bar{q}$ continuum events; sophisticated background suppression techniques exploiting event shape variables are essential for these analyses. In hadron collisions, the dominant background comes from QCD or partially reconstructed heavy flavors, and is similarly suppressed by a combination of kinematic and isolation requirements. The results are in general consistent among the experiments.

Most rare decay modes including $B^{0} \rightarrow K^{+} \pi^{-}$have contributions from both $b \rightarrow u$ tree and $b \rightarrow s g$ penguin processes. If the size of the two contributions are comparable, the interference between them may result in direct $C P$ violation, seen experimentally as a charge asymmetry in the decay rate measurement. BaBar [123], Belle [124], CDF [118], and LHCb [121, 125] have measured the direct $C P$ violating asymmetry in $B^{0} \rightarrow K^{+} \pi^{-}$decays. Direct $C P$ violation has been observed in this decay with a significance of more than $5 \sigma$. The world average value of the asymmetry is now rather precise, $A_{C P}\left(K^{+} \pi^{-}\right)=-0.0834 \pm 0.0032$. The $C P$ asymmetry in $B^{+} \rightarrow K^{+} \pi^{0}$ mode has been measured by BaBar [126], *Belle [124] and LHCb [127] with the average value $A_{C P}\left(K^{+} \pi^{0}\right)=0.030 \pm 0.013$. These two asymmetries differ significantly, in contrast to a naive expectation based on simplified picture in the SM. For more detailed tests, there are sum rules [128] that relate the decay rates and decay-rate asymmetries between the four $K \pi$ charge states. A crucial ingredient of the sum rule test is $A_{C P}\left(K^{0} \pi^{0}\right)$. Currently, measured values are reported by both BaBar [129] and Belle [130]. Using the $A_{C P}\left(K^{0} \pi^{0}\right)$ value of Ref. [130], Belle reports the sum rule test result that is consistent with zero within $1.9 \sigma$ [124]. With the future improvements via Belle II and upgraded LHCb , the measurements are expected to become precise enough to shape a definite conclusion. The $C P$ asymmetry in the $\pi^{+} K^{-}$mode has also been measured in $B_{s}^{0}$ decays, by CDF [118] and LHCb [121,125]. The combined value is $A_{C P}\left(B_{s}^{0} \rightarrow \pi^{+} K^{-}\right)=0.224 \pm 0.012$.

In addition to $B_{(s)} \rightarrow K \pi$ modes, significant $(>3 \sigma)$ non-zero $C P$ asymmetries have been
measured in several other rare decay modes: $A_{C P}\left(B^{+} \rightarrow \rho^{0} K^{+}\right)=0.37 \pm 0.10$ [131], $A_{C P}\left(B^{+} \rightarrow\right.$ $\left.\eta K^{+}\right)=-0.37 \pm 0.08[132], A_{C P}\left(B^{0} \rightarrow \eta K^{* 0}\right)=0.19 \pm 0.05[133]$, and $A_{C P}\left(B^{+} \rightarrow f_{2}(1270) K^{+}\right)=$ $-0.68_{-0.17}^{+0.19}$ [131]. In at least the first two cases, a large direct $C P$ violation might be expected since the penguin amplitude is suppressed so the tree and penguin amplitudes may have comparable magnitudes. There are also measurements by LHCb of $C P$ asymmetries in several 3-body modes: $A_{C P}\left(B^{+} \rightarrow \pi^{+} \pi^{-} \pi^{+}\right)=0.057 \pm 0.013, A_{C P}\left(B^{+} \rightarrow K^{+} \pi^{-} \pi^{+}\right)=0.027 \pm 0.008, A_{C P}\left(B^{+} \rightarrow\right.$ $\left.K^{+} K^{-} \pi^{+}\right)=-0.122 \pm 0.0021$, and $A_{C P}\left(B^{+} \rightarrow K^{+} K^{-} K^{+}\right)=-0.033 \pm 0.008$ [134]. Many of these analyses now include Dalitz plot treatments with many intermediate resonances.

BaBar [135] and Belle $[124,136]$ have observed the decays $B^{+} \rightarrow \bar{K}^{0} K^{+}$and $B^{0} \rightarrow K^{0} \bar{K}^{0}$. The world-average branching fractions are $\mathcal{B}\left(B^{0} \rightarrow K^{0} \bar{K}^{0}\right)=(1.21 \pm 0.16) \times 10^{-6}$ and $\mathcal{B}\left(B^{+} \rightarrow\right.$ $\left.\bar{K}^{0} K^{+}\right)=(1.31 \pm 0.17) \times 10^{-6}$. These are the first observations of hadronic $b \rightarrow d$ transitions, with significance bigger than $5 \sigma$ for all four measurements. $C P$ asymmetries have been measured for these modes, but with large errors. LHCb has observed $B^{0} \rightarrow K^{+} K^{-}$mode which occurs via a weak-annihilation process and is the rarest hadronic $B$-meson decay thus far observed, with $\mathcal{B}\left(B^{0} \rightarrow K^{+} K^{-}\right)=(7.80 \pm 1.52) \times 10^{-8}[137] . B_{s}^{0} \rightarrow K^{+} K^{-}$decay mode, which occurs mostly via $b \rightarrow s$ penguin process, has been observed by Belle [138], CDF [139] and LHCb [119]. The average branching fraction is $\mathcal{B}\left(B_{s}^{0} \rightarrow K^{+} K^{-}\right)=(26.6 \pm 2.2) \times 10^{-6}$. Belle has also observed $B_{s}^{0} \rightarrow K^{0} \bar{K}^{0}[140]$ which also occurs via $b \rightarrow s$ penguin transition in the SM. This was recently confirmed by LHCb [141]. The average branching fraction is $(1.76 \pm 0.31) \times 10^{-5}$.

The decay $B^{0} \rightarrow \pi^{+} \pi^{-}$can be used to extract the CKM angle $\alpha$ (for details see elsewhere in this Review [142]). This is complicated by the presence of significant contributions from penguin diagrams. An isospin analysis [143] can be used to untangle the penguin complications. The decay $B^{0} \rightarrow \pi^{0} \pi^{0}$ is crucial in this analysis. Both BaBar and Belle have observed $B^{0} \rightarrow \pi^{0} \pi^{0}$, with a mild tension in the measured branching fractions: $(1.83 \pm 0.25) \times 10^{-6}$ for BaBar [123] and $(1.31 \pm 0.26) \times 10^{-6}$ for Belle [144]. It turns out that the amount of penguin pollution in the $B \rightarrow \pi \pi$ system is rather large. In the past few years, measurements in the $B^{0} \rightarrow \rho \rho$ system have produced more precise values of $\alpha$, since penguin amplitudes are generally smaller for decays with vector mesons. An important ingredient in the analysis is the $B^{0} \rightarrow \rho^{0} \rho^{0}$ branching fraction. The average of measurements from BaBar [145] and Belle [146] yields a branching fraction of ( $0.96 \pm 0.15$ ) $\times 10^{-6}$. This is only $3 \%$ of the $\rho^{+} \rho^{-}$branching fraction, much smaller than the corresponding ratio ( $\gtrsim 20 \%$ ) in the $\pi \pi$ system.

Since $B \rightarrow \rho \rho$ has two vector mesons in the final state, the $C P$ eigenvalue of the final state depends on the longitudinal polarization fraction $f_{L}$ for the decay. Therefore, a measurement of $f_{L}$ is needed to extract the CKM angle $\alpha$. Both BaBar and Belle have measured $f_{L}$ for the decays $\rho^{+} \rho^{-}[147]$ and $\rho^{+} \rho^{0}$ [148] and in both cases the measurements show $f_{L}>0.9$, making a complete angular analysis unnecessary. In $B^{0} \rightarrow \rho^{0} \rho^{0}, f_{L}$ is measured by BaBar [145], Belle [146] and LHCb [149], with the average value being $0.71_{-0.09}^{+0.08}$.

By analyzing the angular distributions of the $B$ decays to two vector mesons, we can learn a lot about both weak- and strong-interaction dynamics in $B$ decays. Decays that are penguindominated surprisingly have values of $f_{L}$ near 0.5 . The list of such decays has now grown to include $B \rightarrow \phi K^{*}(892), B \rightarrow \rho K^{*}(892)$, and $B \rightarrow \omega K^{*}(892)$. The reasons for this "polarization puzzle" are not fully understood. A detailed description of the angular analysis of $B$ decays to two vector mesons can be found in a separate mini-review [150] in this Review .

### 73.6 Electroweak penguin decays

Electroweak penguin decays are one-loop FCNC decays proceeding through penguin or box Feynman diagrams with final state including real photon or pair of leptons. Such decays were first observed by CLEO experiment when it observed decay $B \rightarrow K^{*}(892) \gamma$ [74]. Since then significant
amount of experimental information was obtained. Branching fractions for these decays are $10^{-5}$ or less, which makes them excellent candidates for searches for new physics beyond SM. Often several observables are available, which allows for stringent tests of the SM.

Starting with radiative decays, experimentally easiest to study are exclusive decays with a fully reconstructed final state. The best studied decay in this class is $B \rightarrow K^{*}(892) \gamma$ seen by CLEO, Belle, BaBar experiments [151,152] with world average branching fraction $\mathcal{B}\left(B^{0} \rightarrow K^{*}(892)^{0} \gamma\right)=$ $(41.8 \pm 2.5) \times 10^{-6}$. Decays through several other kaon resonances such as $B \rightarrow K_{1}(1270) \gamma$, $K_{2}^{*}(1430) \gamma$, etc. were studied at B-factories $[153,154]$. It is worth to mention decay $B^{+} \rightarrow$ $K^{+} \pi^{+} \pi^{-} \gamma$ for which besides measurements of the branching fraction [154,155] one can also use the angular distribution to access photon polarization. Such a measurement was done by the LHCb experiment, which was able to clearly demonstrate that the photon in $B^{+} \rightarrow K^{+} \pi^{+} \pi^{-} \gamma$ decay is polarized [156]. Unfortunately given non-trivial hadronic structure, more work is needed before turning this into test of the SM. The exclusive radiative decays $B_{s}^{0} \rightarrow \phi \gamma$ was seen by the Belle and LHCb experiments [157] with an average branching fraction of $(3.4 \pm 0.4) \times 10^{-5}$ and more recently also decay $\Lambda_{b} \rightarrow \Lambda \gamma$ was observed by LHCb [158].

Compared to $b \rightarrow s \gamma$, the $b \rightarrow d \gamma$ transitions such as $B \rightarrow \rho \gamma$, are suppressed by the CKM elements ratio $\left|V_{t d} / V_{t s}\right|^{2}$. Both Belle and BaBar have observed these decays [75, 76]. The world average $\mathcal{B}(B \rightarrow(\rho, \omega) \gamma)=(1.30 \pm 0.23) \times 10^{-6}$. This can be used to calculate $\left|V_{t d} / V_{t s}\right|$ [159]; the measured values are $0.195_{-0.024}^{+0.025}$ from Belle [75] and $0.233_{-0.032}^{+0.033}$ from BaBar [76].

The observed radiative penguin branching fractions can constrain a large class of SM extensions [160]. However, due to the uncertainties in the hadronization, only the inclusive $b \rightarrow s \gamma$ rate can be reliably compared with theoretical calculations. This rate can be measured from the endpoint of the inclusive photon spectrum in $B$ decay. By combining the measurements of $B \rightarrow X_{s} \gamma$ from the CLEO, BaBar, and Belle experiments [161,162], HFLAV obtains the average: $\mathcal{B}(B \rightarrow$ $\left.X_{s} \gamma\right)=(3.49 \pm 0.19) \times 10^{-4}[59]$ for $E_{\gamma} \geq 1.6 \mathrm{GeV}$, averaging over $B^{+}$and $B^{0}$. Consistent but less precise results have been reported by ALEPH for inclusive $b$-hadrons produced at the $Z$, which includes also contribution from $B_{s}^{0}$ and $\Lambda_{b}^{0}$ hadrons. Using the sum of seven exclusive final states, the BaBar experiment measured the branching fraction of inclusive $b \rightarrow d \gamma$ decays to be $(9.2 \pm 2.0 \pm 2.3) \times 10^{-6}[163]$. The measured branching fraction can be compared to theoretical calculations. Recent calculations of $\mathcal{B}(b \rightarrow s \gamma)$ at NNLO level predict for the $E_{\gamma} \geq 1.6 \mathrm{GeV}$ values of $(3.36 \pm 0.23) \times 10^{-4}$ for $b \rightarrow s \gamma$ and $(1.68 \pm 0.17) \times 10^{-5}$ for $b \rightarrow d \gamma$ decays [164].

The $C P$ asymmetry in $b \rightarrow s \gamma$ is extensively studied theoretically both in the SM and beyond [165]. According to the SM, the $C P$ asymmetry in $b \rightarrow s \gamma$ is smaller than $1 \%$, but some non-SM models allow significantly larger $C P$ asymmetry ( $\sim 10 \%$ ) without altering the branching fraction. The current world average is $A_{C P}=0.015 \pm 0.011$, again dominated by BaBar and Belle [166, 167]. In addition to the $C P$ asymmetry, BaBar and Belle also measured the isospin asymmetry $\Delta_{0-}=$ $-0.005 \pm 0.020$ in $b \rightarrow s \gamma$ measured using sum of exclusive decays [166, 168]. An alternative measurement using full reconstruction of the companion $B$ in the hadronic decay modes yields a consistent, but less precise result [169]. Both Belle and BaBar experiments measured the isospin asymmetry in exclusive $B \rightarrow K^{*}(892) \gamma$ decay with average of $6.3 \pm 1.7 \%$ [152] and therefore providing evidence for the non-zero isospin asymmetry.

In addition, experiments have measured the inclusive photon energy spectrum for $b \rightarrow s \gamma$, and by analyzing the shape of the spectrum they obtain the first and second moments for photon energies. Belle has measured these moments covering the widest range in the photon energy ( $1.7<$ $\left.E_{\gamma}<2.8 \mathrm{GeV}\right)$ [162]. The measurement by BaBar has slightly smaller range with lower limit at 1.8 GeV [170]. These results can be used to extract non-perturbative HQET parameters that are needed for precise determination of the CKM matrix element $V_{u b}$ (see further discussion elsewhere in this Review [171]).

Additional information on FCNC processes can be obtained from $b \rightarrow s \ell^{+} \ell^{-}$decays. These processes are studied as a function of dilepton invariant mass squared, $q^{2}$. Different $q^{2}$ regions are sensitive to different physics. Starting at the very low $q^{2}$ decays exhibit sensitivity to the same physics as the radiative decays. Then for the $q^{2}$ in region 1.1 to $6.0 \mathrm{GeV}^{2} / c^{4}$ the SM and new physics have best chance to compete. At the high $q^{2}$ above the $\psi(2 S)$ mass, the interference of SM and new physics is to some extend complementary to that in lower $q^{2}$. Regions around $J / \psi$ and $\psi(2 S)$ is normally excluded from measurements as these are dominated by the $b \rightarrow c$ transitions to charmonia. For exclusive decays, theory predictions require calculations of hadronic form factors. With current theory predictions, the most useful are measurements within the $q^{2}$ regions 1.1 to $6.0 \mathrm{GeV}^{2} / c^{4}$ and from $16.0 \mathrm{GeV}^{2} / c^{4}$ up to the kinematic limit. From this reason in the listing we provide results mainly in those two regions.

Similar as for radiative decays, also for the $b \rightarrow s \ell^{+} \ell^{-}$decays the inclusive measurements provide some benefits. Both Belle and BaBar performed such measurement without reconstructing hadronic part exclusively and measure a branching fraction of $(5.8 \pm 1.3) \times 10^{-6}$ [172]. Unfortunately this measurement is not trivially possible at hadron colliders and also does not easily allow the angular distributions of the decay products to be exploited. One alternative is to extract information on the inclusive decay as sum of exclusive decays. Such a measurement was performed by Belle [173], but in this case the difficulty lies in extrapolation for the missing hadronic states.

Turning to the exclusive decays, the initial measurements performed by B-factories typically averaged between charged and neutral $B$ mesons as well as between $e^{+} e^{-}$and $\mu^{+} \mu^{-}$finals states. The experiments CDF, LHCb, ATLAS and CMS are much better suited for the $\mu^{+} \mu^{-}$finals states compared to the $e^{+} e^{-}$final states. As such most measurements at hadron colliders are done only with $\mu^{+} \mu^{-}$pairs and by separating charged and neutral $B$ mesons. Recently, however, with much increased statistics, LHCb measured several final states with $e^{+} e^{-}$, to make a series of tests of $e / \mu$ universality in $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$decays. At hadron colliders other $b$ hadrons are produced and as such CDF and LHCb experiments did observe also $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}[174,175], \Lambda_{b}^{0} \rightarrow \Lambda \mu^{+} \mu^{-}[174,176]$ and $\Lambda_{b}^{0} \rightarrow p K^{-} \mu^{+} \mu^{-}$decays [177]. The averages of the total branching fractions integrated over whole $q^{2}$ regions are $(5.6 \pm 0.6) \times 10^{-7}$ for $B^{+} \rightarrow K^{+} e^{+} e^{-},(4.53 \pm 0.35) \times 10^{-7}$ for $B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}$, $\left(1.03_{-0.17}^{+0.19}\right) \times 10^{-6}$ for $B^{0} \rightarrow K^{*}(892)^{0} e^{+} e^{-}$and $(0.94 \pm 0.05) \times 10^{-6}$ for $B^{0} \rightarrow K^{*}(892)^{0} \mu^{+} \mu^{-}$ decays [178-182]. The total branching fractions for $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$and $\Lambda_{b}^{0} \rightarrow \Lambda \mu^{+} \mu^{-}$decays are $(8.4 \pm 0.4) \times 10^{-7}[174,175,183]$ and $(1.08 \pm 0.28) \times 10^{-6}[174,176]$ respectively. With increased precision of $B^{0} \rightarrow K^{*}(892)^{0} \ell^{+} \ell^{-}$decay, there is a question on what fraction of the seen branching fraction is due to the $K^{*}(892)^{0}$ resonance and what fraction is due to the $K \pi$ in s-wave. This has been studied by LHCb which found that the $K \pi$ in s-wave fraction varies between $1 \%$ and about $10 \%$ depending on the $q^{2}$ region [182]. It should be noted, that for all relevant $B$ meson decays the branching fractions so far studied are consistently below the SM expectation.

In the $b \rightarrow s \ell^{+} \ell^{-}$decays angular distributions offer rich source of information. The full angular analysis was performed for decays $B^{+} \rightarrow K^{+} \ell^{+} \ell^{-}, B^{0} \rightarrow K^{0} \ell^{+} \ell^{-}, B^{+} \rightarrow K^{*}(892)^{+} \ell^{+} \ell^{-}$, $B^{0} \rightarrow K^{*}(892)^{0} \ell^{+} \ell^{-}, B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$and $\Lambda_{b}^{0} \rightarrow \Lambda \mu^{+} \mu^{-}$decays [184-192]. An attempt to increase sensitivity to the NP was made by constructing observables, which have reduced theory uncertainties and measurements of these are done. Most notably the observable called $P_{5}^{\prime}$ [193] shows a discrepancy with the SM in the $q^{2}$ region which is highly sensitive to new physics [186-188]. Measurements of the $C P$ asymmetries [177,178,181, 194,195] and the isospin asymmetry [179-181,194] were also performed. All these measurements are well consistent with the small $A_{C P}$ and small isospin asymmetry expected in the SM [196]. With statistics available at the LHC, the measurement of phase difference between long- and short-distance contribution in $B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}$decays became possible [197].

With the data samples available at LHC, the lepton universality in $b \rightarrow s \ell^{+} \ell^{-}$can be tested.

While in the SM decays to electron-positron and muon pairs are expected to be same up to small corrections due to the different masses of leptons, in extensions of the SM this does not have to hold. The angular analysis of $B^{0} \rightarrow K^{*}(892)^{0} e^{+} e^{-}$decays was performed by LHCb at low dilepton invariant masses [198] and Belle in several regions over whole $q^{2}$ range [188]. The LHCb measurement yields the most stringent constraint on the photon polarization. The result on lepton universality test which over past few years attracted most attention is the ratio of branching fractions between $B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}$and $B^{+} \rightarrow K^{+} e^{+} e^{-}$and between $B^{0} \rightarrow K^{*}(892)^{0} \mu^{+} \mu^{-}$and $B^{0} \rightarrow K^{*}(892)^{0} e^{+} e^{-}$ decays [179,199]. The measurements by LHCb showed mild discrepancy from the SM, with significance of $3.1 \sigma$ for $B^{+} \rightarrow K^{+} \ell^{+} \ell^{-}$and about $2.4 \sigma$ for $B^{0} \rightarrow K^{*}(892)^{0} \ell^{+} \ell^{-}$. The latest analysis in Ref. [200] identified previously missed hadronic misidentification background. With better handling of such background the results are now consistent with the SM. LHCb experiment performed similar test with $\Lambda_{b}^{0} \rightarrow p K^{-} \ell^{+} \ell^{-}$decays [201], $B^{0} \rightarrow K_{S}^{0} \ell^{+} \ell^{-}$and $B^{+} \rightarrow K^{*}(892)^{+} \ell^{+} \ell^{-}$[202].

While $b \rightarrow d \ell^{+} \ell^{-}$decays are further suppressed, they recently became accessible. Signals were observed for $B^{+} \rightarrow \pi^{+} \mu^{+} \mu^{-}$[77], $B^{0} \rightarrow \pi^{+} \pi^{-} \mu^{+} \mu^{-}$[203] and $\Lambda_{b}^{0} \rightarrow p \pi^{-} \mu^{+} \mu^{-}$[204] decays. Search for the $B_{s}^{0} \rightarrow \bar{K}^{*}(892)^{0} \mu^{+} \mu^{-}$at LHCb is complicated by large background from $B^{0} \rightarrow$ $K^{*}(892)^{0} \mu^{+} \mu^{-}$and current significance for the decay is $3.4 \sigma$ [205]. The total branching fractions are only quantities measured and these are about $2 \times 10^{-8}$ for the meson decays and about $7 \times 10^{-8}$ for the $\Lambda_{b}^{0}$ decay.

A closely related process is $B \rightarrow X_{s} \nu \bar{\nu}$. Since the neutrinos are not detected, the final state is a strange hadron system $X_{s}$ plus missing energy-momentum. Depending on $X_{s}$, the SM branching fraction is $\mathcal{O}\left(10^{-6}\right)$. New physics effects beyond SM, e.g. those from dark sector models can greatly enhance the yield of $X_{s}$ plus missing energy. BaBar [206], Belle [207], and Belle II [208] have searched for these decays and determined the upper limits in the range $\mathcal{O}\left(10^{-5}\right)$.

Finally the decays $B_{(s)}^{0} \rightarrow e^{+} e^{-}$and $\mu^{+} \mu^{-}$are interesting since they only proceed at second order in weak interactions in the SM, but may have large contributions from supersymmetric loops, proportional to $(\tan \beta)^{6}$. First limits were published more than 30 years ago and since then experiments at Tevatron, $B$-factories and LHC gradually improved those and effectively excluded whole models of new physics and significantly constrained allowed parameter space of others. For the decays to $\mu^{+} \mu^{-}$, Tevatron experiments pushed the limits down to roughly factor of $5-10$ above the SM expectation [209]. The long journey in the search for these decays culminated in 2012, when first evidence for $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$decay was seen [210]. Subsequently, LHC experiments ATLAS [211], CMS [212] and LHCb [213] observed statistically significant signal for $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$decay. The average branching fraction is found to be $(3.01 \pm 0.435) \times 10^{-9}$. In experiments at hadron colliders searches for $B^{0} \rightarrow \mu^{+} \mu^{-}$decays are performed at the same time. The $B\left(B^{0} \rightarrow \mu^{+} \mu^{-}\right)$is extracted in simultaneously with $B\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)$and is found to be $\left(0.07_{-0.11}^{+0.13}\right) \times 10^{-9}$. The limits for the $e^{+} e^{-}$modes are: $<9.4 \times 10^{-9}$ and $<2.5 \times 10^{-9}$, respectively, for $B_{s}^{0}$ and $B^{0}$ [214]. The searches for decays to $\tau^{+} \tau^{-}$are more challenging with current best limits of $B\left(B^{0} \rightarrow \tau^{+} \tau^{-}\right)<2.1 \times 10^{-3}$ and $B\left(B_{s}^{0} \rightarrow \tau^{+} \tau^{-}\right)<6.8 \times 10^{-3}$ at $95 \%$ C.L. [215]. All existing measurements of $B^{0}$ and $B_{s}^{0}$ decays to same flavour dilepton pair is consistent with SM expectation [216]. With $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$ decay observed, it was suggested that the effective lifetime is useful further test of the decay [217]. Attempt was made by LHCb and CMS experiments, but its precision is not yet sufficient to provide test of the SM $[212,213]$. It will take couple of years until interesting precision is reached. The searches were also performed for lepton flavour violating decays to two leptons with best limits in $e^{ \pm} \mu^{\mp}$ channel, where limits are $<1.3 \times 10^{-9}$ for $B^{0}$ and $<6.3 \times 10^{-9}$ for $B_{s}^{0}$, at $95 \%$ confidence level [218].

Several theory groups performed global analysis of electroweak decays concluding that significant tension between data and SM is present [219]. The tension can be relieved by new physics beyond SM. For more detailed reviews see e.g. Ref. [220].

### 73.7 Summary and Outlook

The study of $B$ mesons continues to be one of the most productive fields in particle physics. With the two asymmetric $B$-factory experiments Belle and BaBar, we now have a combined data sample of well over $1 \mathrm{ab}^{-1}$. $C P$ violation has been firmly established in many decays of $B$ mesons. Evidence for direct $C P$ violation has been observed. Many rare decays resulting from hadronic $b \rightarrow u$ transitions and $b \rightarrow s(d)$ penguin decays have been observed, and the emerging pattern is still full of surprises. Despite the remarkable successes of the $B$-factory experiments, many fundamental questions in the flavor sector remain unanswered.

At Fermilab, CDF and D0 each has accumulated about $10 \mathrm{fb}^{-1}$, which is the equivalent of about $10^{12} b$-hadrons produced. In spite of the low trigger efficiency of hadronic experiments, a selection of modes have been reconstructed in large quantities, giving a start to a program of studies on $B_{s}$ and $b$-flavored baryons, in which a first major step has been the determination of the $B_{s}$ oscillation frequency.

As Tevatron and $B$-factories finished their data taking few year ago, the experiments at the LHC have become very active. LHCb has collected about $1 \mathrm{fb}^{-1}$ at $7 \mathrm{TeV}, 2 \mathrm{fb}^{-1}$ at 8 TeV , and close to $5.9 \mathrm{fb}^{-1}$ at 13 TeV during LHC Runs 1 and 2. CMS and ATLAS have collected each about $5 \mathrm{fb}^{-1}$ of data at $\sqrt{s}=7 \mathrm{TeV}, 20 \mathrm{fb}^{-1}$ at 8 TeV and about $150 \mathrm{fb}^{-1}$ at 13 TeV during LHC Runs 1 and 2. The latest LHC Run 3 at 13.6 TeV started in summer 2022, with upgraded detectors. By the time of the writing (summer 2023), LHC has delivered about $70 \mathrm{fb}^{-1}$ to ATLAS and CMS experiments. LHCb, which is dedicated to the studies of $b$ - and $c$-hadrons, has a data sample that is for many decays larger than the sum of all previous experiments. With it, we are entering to regime of precision physics even for many rare decays, which allows much more detailed measurements.

The Belle II experiment at the SuperKEKB has started recording data in 2019 and has by summer 2022 collected about $428 \mathrm{fb}^{-1}$ of data, when their long shutdown 1 started. The aim to increase sample to $\sim 50 \mathrm{ab}^{-1}$ will make it possible to explore the indirect evidence of new physics beyond the SM in the heavy-flavor particles ( $b, c$, and $\tau$ ), in a way that is complementary to the LHC. The LHCb Collaboration is commissioning upgrade of its detector and is planning on nominal running from 2024. The aim of the upgrade was to increase flexibility of the trigger, which will allow about a factor of five increase in instantaneous luminosity and of about a factor of two in efficiencies on triggering on purely hadronic decays. The plan is to integrate about $50 \mathrm{fb}^{-1}$ of data during LHC runs 3 and 4.

These experiments promise a rich spectrum of rare and precise measurements that have the potential to fundamentally affecting our understanding of the SM and $C P$-violating phenomena.

## References

[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[2] S. W. Herb et al., Phys. Rev. Lett. 39, 252 (1977).
[3] B. Aubert et al. (BaBar), Phys. Rev. Lett. 87, 091801 (2001), [hep-ex/0107013].
[4] K. Abe et al. (Belle), Phys. Rev. Lett. 87, 091802 (2001), [hep-ex/0107061].
[5] Currently two different notations ( $\phi_{1}, \phi_{2}, \phi_{3}$ ) and ( $\alpha, \beta, \gamma$ ) are used in the literature for CKM unitarity angles. In this mini-review, we use the latter notation following the other minireviews in this Review. The two notations are related by $\phi_{1}=\beta, \phi_{2}=\alpha$ and $\phi_{3}=\gamma$.
[6] See the "CP Violation in Meson Decays" by D. Kirkby and Y. Nir in this Review.
[7] See the "CKM Quark Mixing Matrix," by A. Cecucci, Z. Ligeti, and Y. Sakai, in this Review.
[8] See the note on " $B^{0}-\bar{B}^{0}$ mixing," by O. Schneider in this Review.
[9] See the "Determination of $\left|V_{c b}\right|$ and $\left|V_{u b}\right|$," by R. Kowalewski and T. Mannel in this Review.
[10] F. Abe et al. (CDF), Phys. Rev. Lett. 81, 2432 (1998), [hep-ex/9805034]; F. Abe et al. (CDF), Phys. Rev. D58, 112004 (1998), [hep-ex/9804014].
[11] A. Abulencia et al. (CDF), Phys. Rev. Lett. 96, 082002 (2006), [hep-ex/0505076].
[12] R. Aaij et al. (LHCb), JHEP 07, 123 (2020), [arXiv:2004.08163].
[13] B. Aubert et al. (BaBar), Phys. Rev. Lett. 101, 071801 (2008), [Erratum: Phys. Rev. Lett. 102, 029901 (2009)], [arXiv:0807.1086].
[14] B. Aubert et al. (BaBar), Phys. Rev. Lett. 103, 161801 (2009), [arXiv:0903.1124].
[15] G. Bonvicini et al. (CLEO), Phys. Rev. D81, 031104 (2010), [arXiv:0909.5474].
[16] R. Mizuk et al. (Belle), Phys. Rev. Lett. 109, 232002 (2012), [arXiv:1205.6351].
[17] U. Tamponi et al. (Belle), Phys. Rev. Lett. 115, 14, 142001 (2015), [arXiv:1506.08914].
[18] B. Fulsom et al. (Belle), Phys. Rev. Lett. 121, 232001 (2018), [arXiv:1807.01201].
[19] A. Bondar et al. (Belle), Phys. Rev. Lett. 108, 122001 (2012), [arXiv:1110.2251].
[20] See the note on "Naming scheme for hadrons," by M. Roos and C.G. Wohl in this Review.
[21] R. Aaij et al. (LHCb), Phys. Rev. Lett. 118, 5, 052002 (2017), [Erratum: Phys. Rev. Lett. 119, 169901 (2017)], [arXiv:1612.05140].
[22] M. Cacciari et al., JHEP 10, 137 (2012), [arXiv:1205.6344]; B. A. Kniehl et al., Phys. Rev. D84, 094026 (2011), [arXiv:1109.2472]; M. Cacciari, M. L. Mangano and P. Nason, Eur. Phys. J. C75, 12, 610 (2015), [arXiv:1507.06197].
[23] B. Barish et al. (CLEO), Phys. Rev. Lett. 76, 1570 (1996).
[24] E. Guido et al. (Belle), Phys. Rev. Lett. 121, 062001 (2018), [arXiv:1803.10303]; E. Guido et al. (Belle), Phys. Rev. D96, 052005 (2017), [arXiv:1707.04973]; A. Sokolov et al. (Belle), Phys. Rev. D79, 051103 (2009), [arXiv:0901.1431]; B. Aubert et al. (BaBar), Phys. Rev. D78, 112002 (2008), [arXiv:0807.2014].
[25] J. P. Alexander et al. (CLEO), Phys. Rev. Lett. 86, 2737 (2001), [hep-ex/0006002]; S. B. Athar et al. (CLEO), Phys. Rev. D66, 052003 (2002), [hep-ex/0202033]; N. C. Hastings et al. (Belle), Phys. Rev. D67, 052004 (2003), [hep-ex/0212033].
[26] B. Aubert et al. (BaBar), Phys. Rev. Lett. 95, 042001 (2005), [hep-ex/0504001].
[27] Y. Amhis et al. (HFLAV), Eur. Phys. J. C77, 12, 895 (2017), [arXiv:1612.07233].
[28] Y. S. Amhis et al. (HFLAV), Eur. Phys. J. C 81, 3, 226 (2021), [arXiv:1909.12524].
[29] R. Aaij et al. (LHCb) (2022), [arXiv:2204.13042].
[30] P. Abreu et al. (DELPHI), Phys. Lett. B345, 598 (1995).
[31] R. Akers et al. (OPAL), Z. Phys. C66, 19 (1995).
[32] T. A. Aaltonen et al. (CDF), Phys. Rev. D90, 1, 012013 (2014), [arXiv:1309.5961]; R. Aaij et al. (LHCb), JHEP 04, 024 (2015), [arXiv:1502.02638].
[33] R. Aaij et al. (LHCb), Phys. Rev. Lett. 110, 15, 151803 (2013), [arXiv:1211.5994].
[34] A. M. Sirunyan et al. (CMS), Eur. Phys. J. C78, 939 (2018), [arXiv:1809.03578].
[35] G. Aad et al. (ATLAS), Phys. Rev. Lett. 113, 21, 212004 (2014), [arXiv:1407.1032].
[36] A. Sirunyan et al. (CMS), Phys. Rev. Lett. 122, 132001 (2019), [arXiv:1902.00571]; R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 232001 (2019), [arXiv:1904.00081].
[37] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 99, 202001 (2007), [arXiv:0706.3868]; T. Aaltonen et al. (CDF), Phys. Rev. D85, 092011 (2012), [arXiv:1112.2808].
[38] R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 012001 (2019), [arXiv:1809.07752].
[39] V. M. Abazov et al. (D0), Phys. Rev. Lett. 99, 052001 (2007), [arXiv:0706.1690]; T. Aaltonen et al. (CDF), Phys. Rev. Lett. 99, 052002 (2007), [arXiv:0707.0589].
[40] R. Aaij et al. (LHCb), Phys. Rev. Lett. 113, 032001 (2014), [arXiv:1405.7223]; R. Aaij et al. (LHCb), Phys. Lett. B736, 154 (2014), [arXiv:1405.1543]; R. Aaij et al. (LHCb), Phys. Rev. D89, 3, 032001 (2014), [arXiv:1311.4823]; R. Aaij et al. (LHCb), Phys. Rev. Lett. 113, 24, 242002 (2014), [arXiv:1409.8568]; R. Aaij et al. (LHCb), Phys. Rev. Lett. 115, 24, 241801 (2015), [arXiv:1510.03829]; R. Aaij et al. (LHCb), Phys. Rev. Lett. 118, 7, 071801 (2017), [arXiv:1612.02244]; R. Aaij et al. (LHCb), Phys. Lett. B722, 265 (2017), [arXiv:1701.05274]; R. Aaij et al. (LHCb), JHEP 02, 98 (2018), [arXiv:1711.05490]; R. Aaij et al. (LHCb), Phys. Rev. D99, 052006 (2019), [arXiv:1901.07075]; R. Aaij et al. (LHCb), Phys. Rev. Lett. 124, 11, 111802 (2020), [arXiv:1912.02110].
[41] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 103, 012004 (2021), [arXiv:2010.14485], URL https://link.aps.org/doi/10.1103/PhysRevD.103.012004.
[42] R. Aaij et al. (LHCb), Phys. Rev. D104, 052010 (2021), [arXiv:2104.15074].
[43] T. A. Aaltonen et al. (CDF), Phys. Rev. D89, 7, 072014 (2014), [arXiv:1403.8126].
[44] R. Aaij et al. (LHCb), JHEP 08, 039 (2018), [arXiv:1805.03941]; R. Aaij et al. (LHCb), Eur. Phys. J. C 79, 9, 745 (2019), [arXiv:1903.06792]; R. Aaij et al. (LHCb), Eur. Phys. J. C79, 9, 745 (2019), [arXiv:1903.06792].
[45] S. Chatrchyan et al. (CMS), Phys. Rev. Lett. 108, 252002 (2012), [arXiv:1204.5955]; R. Aaij et al. (LHCb), JHEP 1605, 161 (2016), [arXiv:1604.03896].
[46] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 131, 171901 (2023), [arXiv:2307.13399], URL https://link.aps.org/doi/10.1103/PhysRevLett.131.171901.
[47] R. Aaij et al. (LHCb), Phys. Rev. Lett. 114, 062004 (2015), [arXiv:1411.4849].
[48] A. M. Sirunyan et al. (CMS Collaboration), Phys. Rev. Lett. 126, 252003 (2021), [arXiv:2102.04524], URL https://link.aps.org/doi/10.1103/PhysRevLett.126.252003.
[49] R. Aaij et al. (LHCb), Phys. Rev. Lett. 121, 072002 (2018), [arXiv:1805.09418].
[50] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 128, 162001 (2022), [arXiv:2110.04497], URL https://link.aps.org/doi/10.1103/PhysRevLett.128.162001.
[51] V. M. Abazov et al. (D0), Phys. Rev. Lett. 101, 232002 (2008), [arXiv:0808.4142]; T. Aaltonen et al. (CDF), Phys. Rev. D80, 072003 (2009), [arXiv:0905.3123].
[52] R. Aaij et al. (LHCb), Phys. Rev. Lett. 110, 18, 182001 (2013), [arXiv:1302.1072]; R. Aaij et al. (LHCb), Phys. Lett. B736, 154 (2014), [arXiv:1405.1543]; R. Aaij et al. (LHCb), Phys. Rev. D93, 9, 092007 (2016), [arXiv:1604.01412]; R. Aaij et al. (LHCb), Phys. Rev. D104, L091102 (2021), [arXiv:2107.03419].
[53] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 124, 082002 (2020), [arXiv:2001.00851], URL https://link.aps.org/doi/10.1103/PhysRevLett.124.082002.
[54] V. M. Abazov et al. (D0), Phys. Rev. Lett. 117, 2, 022003 (2016), [arXiv:1602.07588]; V. M. Abazov et al. (D0), Phys. Rev. D97, 092004 (2018), [arXiv:1712.10176].
[55] R. Aaij et al. (LHCb), Phys. Rev. Lett. 117, 15, 152003 (2016), [Addendum: Phys. Rev. Lett.118,no.10,109904(2017)], [arXiv:1608.00435].
[56] A. Sirunyan et al. (CMS), Phys. Rev. Lett. 120, 202005 (2018), [arXiv:1712.06144].
[57] C. Tarantino, Eur. Phys. J. C33, S895 (2004), [hep-ph/0310241]; F. Gabbiani, A. I. Onishchenko and A. A. Petrov, Phys. Rev. D70, 094031 (2004), [hep-ph/0407004]; F. Gabbiani, A. I. Onishchenko and A. A. Petrov, Phys. Rev. D68, 114006 (2003), [hep-ph/0303235].
[58] A. Lenz, Int. J. Mod. Phys. A30, 10, 1543005 (2015), [arXiv:1405.3601].
[59] Y. S. Amhis et al. (Heavy Flavor Averaging Group, HFLAV), Phys. Rev. D 107, 5, 052008 (2023), [arXiv:2206.07501].
[60] C.-H. Chang et al., Phys. Rev. D64, 014003 (2001), [hep-ph/0007162]; V. V. Kiselev, A. E. Kovalsky and A. K. Likhoded, Nucl. Phys. B585, 353 (2000), [hep-ph/0002127]; A. Yu. Anisimov et al., Phys. Lett. B452, 129 (1999), [hep-ph/9812514]; M. Beneke and G. Buchalla, Phys. Rev. D53, 4991 (1996), [hep-ph/9601249].
[61] R. Aaij et al. (LHCb), Eur. Phys. J. C74, 5, 2839 (2014), [arXiv:1401.6932].
[62] R. Aaij et al. (LHCb), Phys. Lett. B742, 29 (2015), [arXiv:1411.6899].
[63] The $B_{c}$ is a special case, where a weak decay of the $c$ quark is also possible, but the spectator model still applies.
[64] R. Aaij et al. (LHCb), Nature Phys. 11, 743 (2015), [arXiv:1504.01568].
[65] J. P. Lees et al. (BaBar), Phys. Rev. D85, 011101 (2012), [arXiv:1110.5600]; C. Oswald et al. (Belle), Phys. Rev. D87, 7, 072008 (2013), [Erratum: Phys. Rev. D90, 119901 (2014)], [arXiv:1212.6400]; C. Oswald et al. (Belle), Phys. Rev. D92, 7, 072013 (2015), [arXiv:1504.02004].
[66] T. A. Aaltonen et al. (CDF), Phys. Rev. D93, 5, 052001 (2016), [arXiv:1601.03819].
[67] R. Aaij et al. (LHCb), Phys. Rev. Lett. 126, 8, 081804 (2021), [arXiv:2012.05143].
[68] B. Grinstein, Nucl. Phys. B339, 253 (1990); H. Georgi, Phys. Lett. B240, 447 (1990); A. F. Falk et al., Nucl. Phys. B343, 1 (1990); E. Eichten and B. R. Hill, Phys. Lett. B234, 511 (1990).
[69] "Heavy-Quark and Soft-Collinear Effective Theory" by C.W. Bauer and M. Neubert in this Review.
[70] M. Neubert, "Aspects of QCD Factorization," hep-ph/ 0110093, Proceedings of HF9, Pasadena (2001) and references therein; Z. Ligeti, M. E. Luke and M. B. Wise, Phys. Lett. B507, 142 (2001), [hep-ph/0103020].
[71] M. Beneke et al., Phys. Rev. Lett. 83, 1914 (1999), [hep-ph/9905312]; M. Beneke et al., Nucl. Phys. B591, 313 (2000), [hep-ph/0006124]; M. Beneke et al., Nucl. Phys. B606, 245 (2001), [hep-ph/0104110]; M. Beneke and M. Neubert, Nucl. Phys. B675, 333 (2003), [hepph/0308039].
[72] Y.-Y. Keum, H.-n. Li and A. I. Sanda, Phys. Lett. B504, 6 (2001), [hep-ph/0004004]; Y. Y. Keum, H.-N. Li and A. I. Sanda, Phys. Rev. D63, 054008 (2001), [hep-ph/0004173]; Y.-Y. Keum and H.-n. Li, Phys. Rev. D63, 074006 (2001), [hep-ph/0006001]; C.-D. Lu, K. Ukai and M.-Z. Yang, Phys. Rev. D63, 074009 (2001), [hep-ph/0004213]; C.-D. Lu and M.-Z. Yang, Eur. Phys. J. C23, 275 (2002), [hep-ph/0011238].
[73] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D63, 014006 (2000), [hep-ph/0005275]; C. W. Bauer et al., Phys. Rev. D63, 114020 (2001), [hep-ph/0011336]; C. W. Bauer and I. W. Stewart, Phys. Lett. B516, 134 (2001), [hep-ph/0107001].
[74] R. Ammar et al. (CLEO), Phys. Rev. Lett. 71, 674 (1993).
[75] N. Taniguchi et al. (Belle), Phys. Rev. Lett. 101, 111801 (2008), [Erratum: Phys. Rev. Lett. 101, 129904 (2008)], [arXiv:0804.4770].
[76] B. Aubert et al. (BaBar), Phys. Rev. D78, 112001 (2008), [arXiv:0808.1379].
[77] R. Aaij et al. (LHCb), JHEP 10, 034 (2015), [arXiv:1509.00414].
[78] P. Krokovny et al. (Belle), Phys. Rev. Lett. 89, 231804 (2002), [hep-ex/0207077]; B. Aubert et al. (BaBar), Phys. Rev. Lett. 98, 081801 (2007), [hep-ex/0604012].
[79] B. Kronenbitter et al. (Belle), Phys. Rev. D92, 5, 051102 (2015), [arXiv:1503.05613]; I. Adachi et al. (Belle), Phys. Rev. Lett. 110, 13, 131801 (2013), [arXiv:1208.4678].
[80] J. P. Lees et al. (BaBar), Phys. Rev. D88, 3, 031102 (2013), [arXiv:1207.0698]; B. Aubert et al. (BaBar), Phys. Rev. D81, 051101 (2010), [arXiv:0912.2453].
[81] See the "Leptonic decays of charged pseudoscalar mesons," by R. Briere, J. Rosner, S. Stone, and R. Van de Water, in this Review.
[82] J. P. Lees et al. (BaBar), Phys. Rev. D84, 112007 (2011), [Erratum: Phys. Rev. D87, 039901 (2013)], [arXiv:1107.5751]; S. Blyth et al. (Belle), Phys. Rev. D74, 092002 (2006), [hepex/0607029].
[83] R. Aaij et al. (LHCb), Phys. Rev. D 92, 3, 032002 (2015), [arXiv:1505.01710].
[84] B. Aubert et al. (BaBar), Phys. Rev. Lett. 90, 242001 (2003), [hep-ex/0304021].
[85] D. Besson et al. (CLEO), Phys. Rev. D68, 032002 (2003), [Erratum: Phys. Rev.D75,119908(2007)], [hep-ex/0305100].
[86] P. Krokovny et al. (Belle), Phys. Rev. Lett. 91, 262002 (2003), [hep-ex/0308019]; Y. Mikami et al. (Belle), Phys. Rev. Lett. 92, 012002 (2004), [hep-ex/0307052].
[87] B. Aubert et al. (BaBar), Phys. Rev. Lett. 93, 181801 (2004), [hep-ex/0408041].
[88] R. Aaij et al. (LHCb), Phys. Rev. Lett. 113, 162001 (2014), [arXiv:1407.7574].
[89] R. Aaij et al. (LHCb), JHEP 12, 141 (2021), [arXiv:2110.02350]; F. Abudinén et al. (Belle, Belle-II), JHEP 02, 063 (2022), [Erratum: JHEP 12, 034 (2022)], [arXiv:2110.12125].
[90] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 108, 201801 (2012), [arXiv:1204.0536].
[91] S. Esen et al. (Belle), Phys. Rev. D87, 3, 031101 (2013), [arXiv:1208.0323].
[92] R. Aaij et al. (LHCb), Phys. Rev. D93, 9, 092008 (2016), [arXiv:1602.07543].
[93] R. Aaij et al. (LHCb), Phys. Rev. Lett. 116, 16, 161802 (2016), [arXiv:1603.02408].
[94] R. Aaij et al. (LHCb), JHEP 11, 082 (2015), [arXiv:1509.00400].
[95] B. Aubert et al. (BaBar), Phys. Rev. D76, 031101 (2007), [arXiv:0704.1266].
[96] R. Aaij et al. (LHCb), Phys. Rev. D90, 1, 012003 (2014), [arXiv:1404.5673].
[97] R. Aaij et al. (LHCb), Phys. Rev. Lett. 111, 18, 181801 (2013), [arXiv:1308.4544].
[98] R. Aaij et al. (LHCb), JHEP 07, 066 (2023), [arXiv:2210.12000].
[99] R. Aaij et al. (LHCb), Phys. Rev. D87, 11, 112012 (2013), [Addendum: Phys. Rev. D89, 019901 (2014)], [arXiv:1304.4530].
[100] G. Aad et al. (ATLAS), Eur. Phys. J. C76, 1, 4 (2016), [arXiv:1507.07099]; G. Aad et al. (ATLAS), JHEP 08, 087 (2022), [arXiv:2203.01808].
[101] R. Aaij et al. (LHCb), Phys. Rev. D84, 092001 (2011), [Erratum: Phys. Rev.D85,039904(2012)], [arXiv:1109.6831]; T. Aaltonen et al. (CDF), Phys. Rev. D85, 032003 (2012), [arXiv:1112.3334].
[102] S. K. Choi et al. (Belle), Phys. Rev. Lett. 91, 262001 (2003), [hep-ex/0309032].
[103] D. Acosta et al. (CDF), Phys. Rev. Lett. 93, 072001 (2004), [hep-ex/0312021].
[104] V. M. Abazov et al. (D0), Phys. Rev. Lett. 93, 162002 (2004), [hep-ex/0405004].
[105] B. Aubert et al. (BaBar), Phys. Rev. D71, 071103 (2005), [hep-ex/0406022].
[106] R. Aaij et al. (LHCb), Eur. Phys. J. C72, 1972 (2012), [arXiv:1112.5310].
[107] S. Chatrchyan et al. (CMS), JHEP 04, 154 (2013), [arXiv:1302.3968].
[108] M. Aaboud et al. (ATLAS), JHEP 01, 117 (2017), [arXiv:1610.09303].
[109] S. K. Choi et al. (Belle), Phys. Rev. Lett. 100, 142001 (2008), [arXiv:0708.1790]; R. Mizuk et al. (Belle), Phys. Rev. D80, 031104 (2009), [arXiv:0905.2869].
[110] R. Aaij et al. (LHCb), Phys. Rev. Lett. 112, 22, 222002 (2014), [arXiv:1404.1903]; R. Aaij et al. (LHCb), Phys. Rev. D92, 11, 112009 (2015), [arXiv:1510.01951].
[111] K. Chilikin et al. (Belle), Phys. Rev. D90, 11, 112009 (2014), [arXiv:1408.6457].
[112] R. Aaij et al. (LHCb), Phys. Rev. Lett. 115, 072001 (2015), [arXiv:1507.03414].
[113] R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 22, 222001 (2019), [arXiv:1904.03947].
[114] R. Aaij et al. (LHCb), Phys. Rev. Lett. 128, 6, 062001 (2022), [arXiv:2108.04720].
[115] R. Aaij et al. (LHCb), Phys. Rev. Lett. 131, 3, 031901 (2023), [arXiv:2210.10346]; R. Aaij et al. (LHCb), Sci. Bull. 66, 1278 (2021), [arXiv:2012.10380].
[116] See the "Non- $q \bar{q}$ mesons," by C. Amsler and C. Hanhart, and "Pentaquarks," by M. Karliner and T. Skwarnicki, in this Review.
[117] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 107, 261802 (2011), [arXiv:1107.4999]; R. Aaij et al. (LHCb), JHEP 10, 053 (2015), [arXiv:1508.00788].
[118] T. A. Aaltonen et al. (CDF), Phys. Rev. Lett. 113, 24, 242001 (2014), [arXiv:1403.5586].
[119] R. Aaij et al. (LHCb), JHEP 10, 037 (2012), [arXiv:1206.2794].
[120] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 103, 031801 (2009), [arXiv:0812.4271].
[121] R. Aaij et al. (LHCb), Phys. Rev. D 98, 3, 032004 (2018), [arXiv:1805.06759]; R. Aaij et al. (LHCb), JHEP 12, 155 (2019), [arXiv:1907.10003]; R. Aaij et al. (LHCb) (2023), [arXiv:2304.06198].
[122] R. Fleischer, Phys. Lett. B459, 306 (1999), [hep-ph/9903456]; D. London and J. Matias, Phys. Rev. D70, 031502 (2004), [hep-ph/0404009].
[123] J. P. Lees et al. (BaBar), Phys. Rev. D87, 5, 052009 (2013), [arXiv:1206.3525].
[124] Y. T. Duh et al. (Belle), Phys. Rev. D87, 3, 031103 (2013), [arXiv:1210.1348].
[125] R. Aaij et al. (LHCb), JHEP 03, 075 (2021), [arXiv:2012.05319].
[126] B. Aubert et al. (BaBar), Phys. Rev. D76, 091102 (2007), [arXiv:0707.2798].
[127] R. Aaij et al. (LHCb), Phys. Lett. B 726, 646 (2013), [arXiv:1308.1277]; R. Aaij et al. (LHCb), Phys. Rev. Lett. 126, 9, 091802 (2021), [arXiv:2012.12789].
[128] M. Gronau and J. L. Rosner, Phys. Rev. D71, 074019 (2005), [hep-ph/0503131]; M. Gronau, Phys. Lett. B627, 82 (2005), [hep-ph/0508047].
[129] B. Aubert et al. (BaBar), Phys. Rev. D 79, 052003 (2009), [arXiv:0809.1174].
[130] M. Fujikawa et al. (Belle), Phys. Rev. D 81, 011101 (2010), [arXiv:0809.4366].
[131] B. Aubert et al. (BaBar), Phys. Rev. D78, 012004 (2008), [arXiv:0803.4451]; A. Garmash et al. (Belle), Phys. Rev. Lett. 96, 251803 (2006), [hep-ex/0512066].
[132] C. T. Hoi et al. (Belle), Phys. Rev. Lett. 108, 031801 (2012), [arXiv:1110.2000]; B. Aubert et al. (BaBar), Phys. Rev. D80, 112002 (2009), [arXiv:0907.1743].
[133] B. Aubert et al. (BaBar), Phys. Rev. Lett. 97, 201802 (2006), [hep-ex/0608005]; C. H. Wang et al. (Belle), Phys. Rev. D75, 092005 (2007), [hep-ex/0701057].
[134] R. Aaij et al. (LHCb), Phys. Rev. D90, 11, 112004 (2014), [arXiv:1408.5373]; C.-L. Hsu et al. (Belle), Phys. Rev. D 96, 3, 031101 (2017), [arXiv:1705.02640].
[135] B. Aubert et al. (BaBar), Phys. Rev. Lett. 97, 171805 (2006), [hep-ex/0608036].
[136] K. Abe et al. (Belle), Phys. Rev. Lett. 98, 181804 (2007), [hep-ex/0608049].
[137] R. Aaij et al. (LHCb), Phys. Rev. Lett. 118, 8, 081801 (2017), [arXiv:1610.08288].
[138] C. C. Peng et al. (Belle), Phys. Rev. D82, 072007 (2010), [arXiv:1006.5115].
[139] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 106, 181802 (2011), [arXiv:1103.5762].
[140] B. Pal et al. (Belle), Phys. Rev. Lett. 116, 16, 161801 (2016), [arXiv:1512.02145].
[141] R. Aaij et al. (LHCb), Phys. Rev. D 102, 1, 012011 (2020), [arXiv:2002.08229].
[142] See the "Determination of CKM angles from B hadrons," by T. Gershon, M. Kenzie, and K. Trabelsi, in this Review.
[143] M. Gronau and D. London, Phys. Rev. Lett. 65, 3381 (1990).
[144] T. Julius et al. (Belle), Phys. Rev. D96, 3, 032007 (2017), [arXiv:1705.02083].
[145] B. Aubert et al. (BaBar), Phys. Rev. D78, 071104 (2008), [arXiv:0807.4977].
[146] P. Vanhoefer et al. (Belle), Phys. Rev. D89, 072008 (2014), [Addendum: Phys. Rev. D89 119903 (2014)], [arXiv:1212.4015].
[147] B. Aubert et al. (BaBar), Phys. Rev. D76, 052007 (2007), [arXiv:0705.2157]; P. Vanhoefer et al. (Belle), Phys. Rev. D 93, 3, 032010 (2016), [Addendum: Phys.Rev.D 94, 099903 (2016)], [arXiv:1510.01245].
[148] B. Aubert et al. (BaBar), Phys. Rev. Lett. 102, 141802 (2009), [arXiv:0901.3522]; J. Zhang et al. (Belle), Phys. Rev. Lett. 91, 221801 (2003), [hep-ex/0306007].
[149] R. Aaij et al. (LHCb), Phys. Lett. B747, 468 (2015), [arXiv:1503.07770].
[150] See the "Polarization in B Decays," by A. Gritsan in this Review.
[151] T. E. Coan et al. (CLEO), Phys. Rev. Lett. 84, 5283 (2000), [hep-ex/9912057].
[152] B. Aubert et al. (BaBar), Phys. Rev. Lett. 103, 211802 (2009), [arXiv:0906.2177]; T. Horiguchi et al. (Belle), Phys. Rev. Lett. 119, 19, 191802 (2017), [arXiv:1707.00394].
[153] B. Aubert et al. (BaBar), Phys. Rev. D70, 091105 (2004), [hep-ex/0409035]; S. Nishida et al. (Belle), Phys. Lett. B610, 23 (2005), [hep-ex/0411065]; B. Aubert et al. (BaBar), Phys. Rev. D74, 031102 (2006), [hep-ex/0603054].
[154] H. Yang et al. (Belle), Phys. Rev. Lett. 94, 111802 (2005), [hep-ex/0412039].
[155] B. Aubert et al. (BaBar), Phys. Rev. Lett. 98, 211804 (2007), [Erratum: Phys. Rev. Lett.100,199905(2008)], [hep-ex/0507031]; P. del Amo Sanchez et al. (BaBar), Phys. Rev. D93, 5, 052013 (2016), [arXiv:1512.03579].
[156] R. Aaij et al. (LHCb), Phys. Rev. Lett. 112, 16, 161801 (2014), [arXiv:1402.6852].
[157] J. Wicht et al. (Belle), Phys. Rev. Lett. 100, 121801 (2008), [arXiv:0712.2659]; D. Dutta et al. (Belle), Phys. Rev. D91, 1, 011101 (2015), [arXiv:1411.7771]; R. Aaij et al. (LHCb), Nucl. Phys. B867, 1 (2013), [arXiv:1209.0313].
[158] R. Aaij et al. (LHCb), Phys. Rev. Lett. 123, 3, 031801 (2019), [arXiv:1904.06697].
[159] A. Ali, E. Lunghi and A. Ya. Parkhomenko, Phys. Lett. B595, 323 (2004), [hep-ph/0405075];
P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D75, 054004 (2007), [hep-ph/0612081].
[160] J. L. Hewett, Phys. Rev. Lett. 70, 1045 (1993), [hep-ph/9211256].
[161] S. Chen et al. (CLEO), Phys. Rev. Lett. 87, 251807 (2001), [hep-ex/0108032]; J. P. Lees et al. (BaBar), Phys. Rev. D86, 112008 (2012), [arXiv:1207.5772].
[162] A. Limosani et al. (Belle), Phys. Rev. Lett. 103, 241801 (2009), [arXiv:0907.1384]; T. Saito et al. (Belle), Phys. Rev. D91, 5, 052004 (2015), [arXiv:1411.7198].
[163] P. del Amo Sanchez et al. (BaBar), Phys. Rev. D82, 051101 (2010), [arXiv:1005.4087].
[164] M. Misiak et al., Phys. Rev. Lett. 114, 22, 221801 (2015), [arXiv:1503.01789]; M. Czakon et al., JHEP 04, 168 (2015), [arXiv:1503.01791]; R. Bause et al., Eur. Phys. J. C 83, 5, 419 (2023), [arXiv:2209.04457].
[165] L. Wolfenstein and Y. L. Wu, Phys. Rev. Lett. 73, 2809 (1994), [hep-ph/9410253]; G. M. Asatrian and A. Ioannisian, Phys. Rev. D54, 5642 (1996), [hep-ph/9603318]; M. Ciuchini, E. Gabrielli and G. F. Giudice, Phys. Lett. B388, 353 (1996), [Erratum: Phys. Lett.B393,489(1997)], [hep-ph/9604438]; S. Baek and P. Ko, Phys. Rev. Lett. 83, 488 (1999), [hep-ph/9812229]; A. L. Kagan and M. Neubert, Phys. Rev. D58, 094012 (1998), [hepph/9803368]; K. Kiers, A. Soni and G.-H. Wu, Phys. Rev. D62, 116004 (2000), [hepph/0006280].
[166] S. Watanuki et al. (Belle), Phys. Rev. D 99, 3, 032012 (2019), [arXiv:1807.04236].
[167] J. P. Lees et al. (BaBar), Phys. Rev. D90, 9, 092001 (2014), [arXiv:1406.0534].
[168] B. Aubert et al. (BaBar), Phys. Rev. D72, 052004 (2005), [hep-ex/0508004].
[169] B. Aubert et al. (BaBar), Phys. Rev. D77, 051103 (2008), [arXiv:0711.4889].
[170] J. P. Lees et al. (BaBar), Phys. Rev. Lett. 109, 191801 (2012), [arXiv:1207.2690].
[171] See the "Semileptonic $b$-Hadron Decays, Determination of $V_{c b}, V_{u b}$," by T. Mannel and P. Urquijo in this Review.
[172] M. Iwasaki et al. (Belle), Phys. Rev. D72, 092005 (2005), [hep-ex/0503044]; J. P. Lees et al. (BaBar), Phys. Rev. Lett. 112, 211802 (2014), [arXiv:1312.5364].
[173] Y. Sato et al. (Belle), Phys. Rev. D93, 3, 032008 (2016), [Addendum: Phys. Rev.D93,no.5,059901(2016)], [arXiv:1402.7134].
[174] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 107, 201802 (2011), [arXiv:1107.3753].
[175] R. Aaij et al. (LHCb), JHEP 07, 084 (2013), [arXiv:1305.2168]; R. Aaij et al. (LHCb), JHEP 09, 179 (2015), [arXiv:1506.08777].
[176] R. Aaij et al. (LHCb), Phys. Lett. B725, 25 (2013), [arXiv:1306.2577].
[177] R. Aaij et al. (LHCb), JHEP 06, 108 (2017), [arXiv:1703.00256].
[178] B. Aubert et al. (BaBar), Phys. Rev. Lett. 102, 091803 (2009), [arXiv:0807.4119].
[179] S. Choudhury et al. (BELLE), JHEP 03, 105 (2021), [arXiv:1908.01848].
[180] R. Aaij et al. (LHCb), JHEP 06, 133 (2014), [arXiv:1403.8044].
[181] J. T. Wei et al. (Belle), Phys. Rev. Lett. 103, 171801 (2009), [arXiv:0904.0770].
[182] R. Aaij et al. (LHCb), JHEP 11, 047 (2016), [Erratum: JHEP 04, 142 (2017)], [arXiv:1606.04731].
[183] R. Aaij et al. (LHCb), Phys. Rev. Lett. 127, 15, 151801 (2021), [arXiv:2105.14007].
[184] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 108, 081807 (2012), [arXiv:1108.0695].
[185] R. Aaij et al. (LHCb), JHEP 05, 082 (2014), [arXiv:1403.8045].
[186] A. M. Sirunyan et al. (CMS), Phys. Lett. B 781, 517 (2018), [arXiv:1710.02846].
[187] R. Aaij et al. (LHCb), Phys. Rev. Lett. 125, 1, 011802 (2020), [arXiv:2003.04831].
[188] S. Wehle et al. (Belle), Phys. Rev. Lett. 118, 11, 111801 (2017), [arXiv:1612.05014].
[189] M. Aaboud et al. (ATLAS), JHEP 10, 047 (2018), [arXiv:1805.04000].
[190] R. Aaij et al. (LHCb), Phys. Rev. Lett. 126, 16, 161802 (2021), [arXiv:2012.13241].
[191] R. Aaij et al. (LHCb), JHEP 11, 043 (2021), [arXiv:2107.13428].
[192] R. Aaij et al. (LHCb), JHEP 09, 146 (2018), [arXiv:1808.00264].
[193] S. Descotes-Genon et al., JHEP 01, 048 (2013), [arXiv:1207.2753].
[194] J. P. Lees et al. (BaBar), Phys. Rev. D86, 032012 (2012), [arXiv:1204.3933].
[195] R. Aaij et al. (LHCb), JHEP 09, 177 (2014), [arXiv:1408.0978].
[196] J. Lyon and R. Zwicky, Phys. Rev. D88, 9, 094004 (2013), [arXiv:1305.4797].
[197] R. Aaij et al. (LHCb), Eur. Phys. J. C77, 3, 161 (2017), [arXiv:1612.06764].
[198] R. Aaij et al. (LHCb), JHEP 12, 081 (2020), [arXiv:2010.06011].
[199] R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 19, 191801 (2019), [arXiv:1903.09252]; R. Aaij et al. (LHCb), JHEP 08, 055 (2017), [arXiv:1705.05802]; R. Aaij et al. (LHCb), Nature Phys. 18, 3, 277 (2022), [arXiv:2103.11769]; A. Abdesselam et al. (Belle), Phys. Rev. Lett. 126, 16, 161801 (2021), [arXiv:1904.02440].
[200] R. Aaij et al. (LHCb), Phys. Rev. Lett. 131, 5, 051803 (2023), [arXiv:2212.09152].
[201] R. Aaij et al. (LHCb), JHEP 05, 040 (2020), [arXiv:1912.08139].
[202] R. Aaij et al. (LHCb), Phys. Rev. Lett. 128, 19, 191802 (2022), [arXiv:2110.09501].
[203] R. Aaij et al. (LHCb), Phys. Lett. B743, 46 (2015), [arXiv:1412.6433].
[204] R. Aaij et al. (LHCb), JHEP 04, 029 (2017), [arXiv:1701.08705].
[205] R. Aaij et al. (LHCb), JHEP 07, 020 (2018), [arXiv:1804.07167].
[206] J. P. Lees et al. (BaBar), Phys. Rev. D 87, 11, 112005 (2013), [arXiv:1303.7465].
[207] J. Grygier et al. (Belle), Phys. Rev. D 96, 9, 091101 (2017), [Addendum: Phys. Rev. D97, 099902 (2018)], [arXiv:1702.03224].
[208] F. Abudinén et al. (Belle-II), Phys. Rev. Lett. 127, 18, 181802 (2021), [arXiv:2104.12624].
[209] T. Aaltonen et al. (CDF), Phys. Rev. Lett. 107, 191801 (2011), [Addendum: Phys. Rev. Lett. 107, 239903 (2011)], [arXiv:1107.2304]; V. M. Abazov et al. (D0), Phys. Rev. D87, 7, 072006 (2013), [arXiv:1301.4507].
[210] R. Aaij et al. (LHCb), Phys. Rev. Lett. 110, 2, 021801 (2013), [arXiv:1211.2674].
[211] M. Aaboud et al. (ATLAS), JHEP 04, 098 (2019), [arXiv:1812.03017].
[212] A. M. Sirunyan et al. (CMS), JHEP 04, 188 (2020), [arXiv:1910.12127].
[213] R. Aaij et al. (LHCb), Phys. Rev. Lett. 118, 19, 191801 (2017), [arXiv:1703.05747]; R. Aaij et al. (LHCb), Phys. Rev. Lett. 128, 4, 041801 (2022), [arXiv:2108.09284].
[214] R. Aaij et al. (LHCb), Phys. Rev. Lett. 124, 21, 211802 (2020), [arXiv:2003.03999].
[215] R. Aaij et al. (LHCb), Phys. Rev. Lett. 118, 25, 251802 (2017), [arXiv:1703.02508].
[216] C. Bobeth et al., Phys. Rev. Lett. 112, 101801 (2014), [arXiv:1311.0903].
[217] K. De Bruyn et al., Phys. Rev. Lett. 109, 041801 (2012), [arXiv:1204.1737]; A. J. Buras et al., JHEP 07, 77 (2013), [arXiv:1303.3820].
[218] R. Aaij et al. (LHCb), JHEP 03, 078 (2018), [arXiv:1710.04111].
[219] J. Aebischer et al., Eur. Phys. J. C 80, 3, 252 (2020), [arXiv:1903.10434]; F. Beaujean, C. Bobeth and D. van Dyk, Eur. Phys. J. C74, 2897 (2014), [Erratum: Eur. Phys. J. C74, 3179 (2014)], [arXiv:1310.2478]; M. Algueró et al., JHEP 07, 096 (2019), [arXiv:1902.04900]; M. Algueró et al., Eur. Phys. J. C 79, 8, 714 (2019), [Addendum: Eur.Phys.J.C 80,

511 (2020)], [arXiv:1903.09578]; M. Algueró et al., Eur. Phys. J. C 82, 4, 326 (2022), [arXiv:2104.08921]; A. Arbey et al., Phys. Rev. D 100, 1, 015045 (2019), [arXiv:1904.08399]; W. Altmannshofer and P. Stangl, Eur. Phys. J. C 81, 10, 952 (2021), [arXiv:2103.13370]; T. Hurth et al., Phys. Lett. B 824, 136838 (2022), [arXiv:2104.10058].
[220] T. Blake, G. Lanfranchi and D. M. Straub, Prog. Part. Nucl. Phys. 92, 50 (2017), [arXiv:1606.00916]; J. Albrecht, S. Reichert and D. van Dyk, Int. J. Mod. Phys. A 33, 1830016 (2018), [arXiv:1806.05010]; J. Albrecht, D. van Dyk and C. Langenbruch, Prog. Part. Nucl. Phys. 120, 103885 (2021), [arXiv:2107.04822].

