FORM FACTORS FOR RADIATIVE PION AND KAON DECAYS

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The radiative decays, $\pi^{\pm} \rightarrow l^{\pm}\nu\gamma$ and $K^{\pm} \rightarrow l^{\pm}\nu\gamma$, with l standing for an e or a μ, and γ for a real or virtual photon ($e^{+}e^{-}$ pair), provide a powerful tool to investigate the hadronic structure of pions and kaons. The structure-dependent part SD_i of the amplitude describes the emission of photons from virtual hadronic states, and is parametrized in terms of form factors F_i, with $i = V, A$ (vector, axial vector), in the standard description [1,2]. Exotic, non-standard contributions like $i = T, S$ (tensor, scalar) have also been considered, and we shall discuss them below. Apart from the SD terms, the decay amplitude depends also on Inner Bremsstrahlung IB from the weak decay $\pi^{\pm}(K^{\pm}) \rightarrow l^{\pm}\nu$ accompanied by the photon radiated from the external charged particles. Naturally, experiments try to optimize their kinematics so as to minimize the "trivial" IB part of the amplitude.

The SD amplitude in its standard form is given as

$$M(\mathrm{SD}_V) = \frac{-eG_F V_{qq'}}{\sqrt{2}m_P}\epsilon^{\mu} l^{\nu} F_V^P \epsilon_{\mu\nu\sigma\tau} k^{\sigma} q^{\tau} \tag{1}$$

$$M(\mathrm{SD}_A) = \frac{-ieG_F V_{qq'}}{\sqrt{2}m_P}\epsilon^{\mu} l^{\nu} \{F_A^P[(qk - k^2)g_{\mu\nu} - q_{\mu}k_{\nu}] + R^P k^2 g_{\mu\nu}\} \ , \tag{2}$$

which contains an additional axial form factor R^P which only can be accessed if the photon remains virtual. $V_{qq'}$ is the Cabibbo-Kobayashi-Maskawa mixing-matrix element; ϵ^{μ} is the polarization vector of the photon (or the effective vertex, $\epsilon^{\mu} = (e/k^2)\overline{u}(p_-)\gamma^{\mu}v(p_+)$, of the $e^{+}e^{-}$ pair); $\ell^{\nu} = \overline{u}(p_{\nu})\gamma^{\nu}(1-\gamma_5)v(p_{\ell})$ is the lepton-neutrino current; q and k are the meson and photon four-momenta ($k = p_+ + p_-$ for virtual photons); and P stands for π or K.

The pion vector form factor, F_V^{π}, is related via CVC (Conserved Vector Current) to the $\pi^0 \rightarrow \gamma\gamma$ decay width by $|F_V^{\pi}| = (1/\alpha)\sqrt{2\Gamma_{\pi^0\rightarrow\gamma\gamma}/\pi m_{\pi^0}}$ [3]. The resulting value, $F_V^{\pi}(0) = 0.0259(9)$, has been confirmed by calculations based on chiral perturbation theory (χPT) [4], and by two experiments given in the Listings below. A recent experiment by the PIBETA collaboration [5] obtained an F_V that is in excellent agreement with the CVC hypothesis. It also measured the slope parameter a in $F_V^\pi(s) = F_V^\pi(0)(1 + a \cdot s)$, where $s = (1 - 2E_\gamma/m_\pi)$, and E_γ is the gamma energy in the pion rest frame: $a = 0.095 \pm 0.058$. A functional dependence on s is expected for all form factors. It becomes non-negligible in the case of $F_V^\pi(s)$ when a wide range of photon momenta is recorded; proper treatment in the analysis of K decays is mandatory.

The form factor, R^P, can be related to the electromagnetic radius, r_P, of the meson [2]: $R^P = \frac{1}{3} m_P f_P \langle r_P^2 \rangle$ using PCAC (Partial Conserved Axial vector Current; f_P is the meson decay constant). In lowest order χPT, the ratio F_A/F_V is related to the pion electric polarizability $\alpha_E = [\alpha/(8\pi^2 m_\pi f_\pi^2)] \times F_A/F_V$ [6]. The calculation of the other form factors, F_A^π, F_V^K, and F_A^K, is model-dependent [1,2,4].

For decay processes where the photon is real, the partial decay width can be written in analytical form as a sum of IB, SD, and IB/SD interference terms INT [1,4]:

$$\begin{aligned}
\frac{d^2\Gamma_{P\to\ell\nu\gamma}}{dxdy} &= \frac{d^2\left(\Gamma_{\rm IB} + \Gamma_{\rm SD} + \Gamma_{\rm INT}\right)}{dxdy} \\
&= \frac{\alpha}{2\pi}\Gamma_{P\to\ell\nu}\frac{1}{(1-r)^2}\Bigg\{\mathrm{IB}(x,y) \\
&+\frac{1}{r}\left(\frac{m_P}{2f_P}\right)^2\Big[(F_V+F_A)^2\mathrm{SD}^+(x,y) + (F_V-F_A)^2\mathrm{SD}^-(x,y)\Big] \\
&+\frac{m_P}{f_P}\Big[(F_V+F_A)\mathrm{S}^+_{\rm INT}(x,y) + (F_V-F_A)\mathrm{S}^-_{\rm INT}(x,y)\Big]\Bigg\}. \qquad (3)
\end{aligned}$$

Here

$$\mathrm{IB}(x,y) = \left[\frac{1-y+r}{x^2(x+y-1-r)}\right] \left[x^2 + 2(1-x)(1-r) - \frac{2xr(1-r)}{x+y-1-r}\right]$$

$$\mathrm{SD}^+(x,y) = (x+y-1-r)\Big[(x+y-1)(1-x) - r\Big]$$

$$\begin{aligned}
\mathrm{SD}^{-}(x,y) &= (1-y+r)\Big[(1-x)(1-y)+r\Big] \\
\mathrm{S}^{+}_{\mathrm{INT}}(x,y) &= \left[\frac{1-y+r}{x(x+y-1-r)}\right]\Big[(1-x)(1-x-y)+r\Big] \\
\mathrm{S}^{-}_{\mathrm{INT}}(x,y) &= \left[\frac{1-y+r}{x(x+y-1-r)}\right]\Big[x^2-(1-x)(1-x-y)-r\Big]
\end{aligned} \tag{4}$$

where $x = 2E_\gamma/m_P$, $y = 2E_\ell/m_P$, and $r = (m_\ell/m_P)^2$. Recently, formulas (3) and (4) have been extended to describe polarized distributions in radiative meson and muon decays [7].

The "helicity" factor r is responsible for the enhancement of the SD over the IB amplitude in the decays $\pi^\pm \to e^\pm\nu\gamma$, while $\pi^\pm \to \mu^\pm\nu\gamma$ is dominated by IB. Interference terms are important for the decay $K^\pm \to \mu^\pm\nu\gamma$ [8], but contribute only a few percent correction to pion decays. However, they provide the basis for determining the signs of F_V and F_A. Radiative corrections to the decay $\pi^+ \to e^+\nu\gamma$ have to be taken into account in the analysis of the precision experiments. They make up to 4% corrections in the total decay rate [9]. In $\pi^\pm \to e^\pm\nu e^+e^-$ and $K^\pm \to \ell^\pm\nu e^+e^-$ decays, all three form factors, F_V^P, F_A^P, and R^P, can be determined [10,11].

We give the experimental $\pi^\pm$ form factors F_V^π, F_A^π, and R^π in the Listings below. In the $K^\pm$ Listings, we give the extracted sum $F_A^K + F_V^K$ and difference $F_A^K - F_V^K$, as well as F_V^K, F_A^K and R^K.

Several searches for the exotic form factors F_T^π, F_T^K (tensor), and F_S^K (scalar) have been pursued in the past, some of them claiming non-zero results [12,13]. In particular, F_T^π has been brought into focus by experimental as well as theoretical work. It was shown that a tensor contribution could destructively interfere with the inner bremsstrahlung amplitude, leading to a substantial reduction of the branching ratio as compared with standard V$-$A calculations [14]. In addition, a tensor contribution as large as $F_T = -(5.6\pm1.7)\times10^{-3}$ could not be completely ruled out by constraints from other measurements [15]. New high-statistics data from the PIBETA collaboration have been re-analyzed together with an additional data set optimized for low backgrounds in the radiative pion decay. In particular, lower beam rates have been used in order to reduce the accidental background, thereby making the treatment of systematic uncertainties easier and more reliable. The PIBETA analysis now restricts F_T to the range $-5.2 \times 10^{-4} < F_T < 4.0 \times 10^{-4}$ at a 90% confidence limit [5]. This result is in excellent agreement with the most recent theoretical work [4].

Precision measurements of radiative pion and kaon decays are effective tools to study QCD in the non-perturbative region. The structure-dependent form factors have direct relations to (renormalized) coupling constants of chiral perturbation theories. Therefore, they are of interest beyond the scope of radiative decays. On the other hand, the interest in searching for new physics manifesting in exotic form factors F_T or F_S has weakened over the last years mainly for two reasons: (i) on the experimental side, the lack of results confirming the non-zero findings; (ii) on the theoretical side, numerical uncertainties are still too large to allow a clear distinction of exotic and standard contributions at the currently required level. Likely this will change in the future, but meanwhile other processes such as $\pi^+ \to e^+\nu$ seem to be better suited to search for new physics at the precision frontier, because of the very accurate and reliable theoretical predictions and the more straightforward experimental analysis.

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