

## 2. ASTROPHYSICAL CONSTANTS AND PARAMETERS

**Table 2.1.** Revised February 2012 by E. Bergren and D.E. Groom (LBNL). The figures in parentheses after some values give the 1- $\sigma$  uncertainties in the last digit(s). Physical constants are from Ref. 1. While every effort has been made to obtain the most accurate current values of the listed quantities, the table does not represent a critical review or adjustment of the constants, and is not intended as a primary reference.

The values and uncertainties for the cosmological parameters depend on the exact data sets, priors, and basis parameters used in the fit. Many of the derived parameters reported in this table have non-Gaussian likelihoods. Parameters may be highly correlated, so care must be taken in propagating errors. Unless otherwise specified, cosmological parameters are from six-parameter fits to a flat  $\Lambda$ CDM cosmology using 7-year WMAP data alone [2]. For more information see Ref. 3 and the original papers.

Quantity	Symbol, equation	Value	Reference, footnote
speed of light	$c$	299 792 458 m s <sup>-1</sup>	exact[4]
Newtonian gravitational constant	$G_N$	6.673 8(8) $\times 10^{-11}$ m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>	[1]
Planck mass	$\sqrt{\hbar c/G_N}$	1.220 93(7) $\times 10^{19}$ GeV/ $c^2$ = 2.176 51(13) $\times 10^{-8}$ kg	[1]
Planck length	$\sqrt{\hbar G_N/c^3}$	1.616 20(10) $\times 10^{-35}$ m	[1]
standard gravitational acceleration	$g_N$	9.806 65 m s <sup>-2</sup> $\approx \pi^2$	exact[1]
jansky (flux density)	Jy	10 <sup>-26</sup> W m <sup>-2</sup> Hz <sup>-1</sup>	definition
tropical year (equinox to equinox) (2011)	yr	31 556 925.2 s $\approx \pi \times 10^7$ s	[5]
sidereal year (fixed star to fixed star) (2011)		31 558 149.8 s $\approx \pi \times 10^7$ s	[5]
mean sidereal day (2011) (time between vernal equinox transits)		23 <sup>h</sup> 56 <sup>m</sup> 04 <sup>s</sup> .9090 53	[5]
astronomical unit	$au, A$	149 597 870 700(3) m	[6]
parsec (1 $au/1$ arc sec)	pc	3.085 677 6 $\times 10^{16}$ m = 3.262 ... ly	[7]
light year (deprecated unit)	ly	0.306 6 ... pc = 0.946 053 ... $\times 10^{16}$ m	
Schwarzschild radius of the Sun	$2G_N M_\odot/c^2$	2.953 250 077 0(2) km	[8]
Solar mass	$M_\odot$	1.988 5(2) $\times 10^{30}$ kg	[9]
Solar equatorial radius	$R_\odot$	6.9551(4) $\times 10^8$ m	[10]
Solar luminosity	$L_\odot$	3.828 $\times 10^{26}$ W	[11]
Schwarzschild radius of the Earth	$2G_N M_\oplus/c^2$	8.870 055 94(2) mm	[12]
Earth mass	$M_\oplus$	5.972 6(7) $\times 10^{24}$ kg	[13]
Earth mean equatorial radius	$R_\oplus$	6.378 137 $\times 10^6$ m	[5]
luminosity conversion (deprecated)	$L$	3.02 $\times 10^{28} \times 10^{-0.4 M_{\text{bol}}}$ W ( $M_{\text{bol}}$ = absolute bolometric magnitude = bolometric magnitude at 10 pc)	[14]
flux conversion (deprecated)	$\mathcal{F}$	2.52 $\times 10^{-8} \times 10^{-0.4 m_{\text{bol}}}$ W m <sup>-2</sup> ( $m_{\text{bol}}$ = apparent bolometric magnitude)	from above
ABsolute monochromatic magnitude	AB	-2.5 log <sub>10</sub> $f_\nu$ - 56.10 (for $f_\nu$ in W m <sup>-2</sup> Hz <sup>-1</sup> ) = -2.5 log <sub>10</sub> $f_\nu$ + 8.90 (for $f_\nu$ in Jy)	[15]
Solar circular velocity $v_0$ at $R_0$ from Galactic center	$v_0/R_0$	30.2 $\pm$ 0.2 km s <sup>-1</sup> kpc <sup>-1</sup>	[16]
Solar distance from Galactic center	$R_0$	8.4(4) kpc	[17]
circular velocity at $R_0$	$v_0$ or $\Theta_0$	240(10) km s <sup>-1</sup>	[18]
local disk density	$\rho_{\text{disk}}$	3-12 $\times 10^{-24}$ g cm <sup>-3</sup> $\approx$ 2-7 GeV/ $c^2$ cm <sup>-3</sup>	[19]
local dark matter density	$\rho_\chi$	canonical value 0.3 GeV/ $c^2$ cm <sup>-3</sup> within factor 2-3	[20]
escape velocity from Galaxy	$v_{\text{esc}}$	498 km/s $< v_{\text{esc}} < 608$ km/s	[21]
present day CMB temperature	$T_0$	2.7255(6) K	[22]
present day CMB dipole amplitude		3.355(8) mK	[2]
Solar velocity with respect to CMB		369(1) km/s towards $(\ell, b) = (263.99(14)^\circ, 48.26(3)^\circ)$	[2]
Local Group velocity with respect to CMB	$v_{\text{LG}}$	627(22) km/s towards $(\ell, b) = (276(3)^\circ, 30(3)^\circ)$	[23]
entropy density/Boltzmann constant	$s/k$	2 889.2 $(T/2.725)^3$ cm <sup>-3</sup>	[14]
number density of CMB photons	$n_\gamma$	410.5 $(T/2.725)^3$ cm <sup>-3</sup>	[24]
baryon-to-photon ratio	$\eta = n_b/n_\gamma$	6.19(15) $\times 10^{-10}$ 5.1 $\times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}$ (95% CL)	[2]
number density of baryons	$n_b$	(2.54 $\pm$ 0.06) $\times 10^{-7}$ cm <sup>-3</sup> (2.1 $\times 10^{-7} < n_b < 2.7 \times 10^{-7}$ ) cm <sup>-3</sup> (95% CL)	from $\eta$ in [2] from $\eta$ in [25]
present day Hubble expansion rate	$H_0$	100 $h$ km s <sup>-1</sup> Mpc <sup>-1</sup> = $h \times (9.777 752 \text{ Gyr})^{-1}$	[26]
scale factor for Hubble expansion rate	$h$	0.710(25) WMAP7; WMAP7 $\oplus$ Cepheids=0.721(17)	[2,27]
Hubble length	$c/H_0$	0.925 063 $\times 10^{26} h^{-1}$ m = 1.28(5) $\times 10^{26}$ m	
scale factor for cosmological constant	$c^2/3H_0^2$	2.852 $\times 10^{51} h^{-2}$ m <sup>2</sup> = 5.5(5) $\times 10^{51}$ m <sup>2</sup>	
critical density of the Universe	$\rho_c = 3H_0^2/8\pi G_N$	2.775 366 27 $\times 10^{11} h^2 M_\odot \text{Mpc}^{-3}$ = 1.878 47(23) $\times 10^{-29} h^2$ g cm <sup>-3</sup> = 1.053 75(13) $\times 10^{-5} h^2$ (GeV/ $c^2$ ) cm <sup>-3</sup>	
baryon density of the Universe	$\Omega_b = \rho_b/\rho_c$	$\dagger$ 0.0226(6) $h^{-2} = \dagger$ 0.045(3)	[2,3]
cold dark matter density of the universe	$\Omega_{\text{cdm}} = \rho_{\text{cdm}}/\rho_c$	$\dagger$ 0.111(6) $h^{-2} = \dagger$ 0.22(3)	[2,3]
dark energy density of the $\Lambda$ CDM Universe	$\Omega_\Lambda$	$\dagger$ 0.73(3)	[2,3]
pressureless matter density of the Universe	$\Omega_m = \Omega_{\text{cdm}} + \Omega_b$	0.27 $\pm$ 0.03 (From $\Omega_\Lambda$ and flatness constraint)	[2,3]
dark energy equation of state parameter	$w$	$\#$ -0.98 $\pm$ 0.05 (WMAP7+BAO+ $H_0$ )	[28]
CMB radiation density of the Universe	$\Omega_\gamma = \rho_\gamma/\rho_c$	2.471 $\times 10^{-5} (T/2.725)^4 h^{-2} = 4.75(23) \times 10^{-5}$	[24]
neutrino density of the Universe	$\Omega_\nu$	0.0005 $< \Omega_\nu h^2 < 0.025 \Rightarrow 0.0009 < \Omega_\nu < 0.048$	[29]
total energy density of the Universe (curvature)	$\Omega_{\text{tot}} = \Omega_m + \dots + \Omega_\Lambda$	$\#$ 1.002 $\pm$ 0.011 (WMAP7+BAO+ $H_0$ )	[2,3]

Quantity	Symbol, equation	Value	Reference, footnote
fluctuation amplitude at $8 h^{-1}$ Mpc scale	$\sigma_8$	$\dagger 0.80(3)$	[2,3]
curvature fluct. amplitude at $k_0 = 0.002$ Mpc $^{-1}$	$\Delta_{\mathcal{R}}^2$	$\ddagger 2.43(11) \times 10^{-9}$	[2,3]
scalar spectral index	$n_s$	$\ddagger 0.963(14)$	[2,3]
running spectral index slope, $k_0 = 0.002$ Mpc $^{-1}$	$dn_s/d \ln k$	$\# -0.03(3)$	[2]
tensor-to-scalar field perturbations ratio, $k_0 = 0.002$ Mpc $^{-1}$	$r = T/S$	$\# < 0.36$ at 95% CL	[2,3]
redshift at decoupling	$z_{\text{dec}}$	$\dagger 1091(1)$	[2]
age at decoupling	$t_*$	$\dagger 3.79(5) \times 10^5$ yr	[2]
sound horizon at decoupling	$r_s(z_*)$	$\dagger 147(2)$ Mpc	[2]
redshift of matter-radiation equality	$z_{\text{eq}}$	$\dagger 3200 \pm 130$	[2]
redshift of reionization	$z_{\text{reion}}$	$\dagger 10.5 \pm 1.2$	[2]
age at reionization	$t_{\text{reion}}$	$430^{+90}_{-70}$ Myr	[2,30]
reionization optical depth	$\tau$	$\ddagger 0.088(15)$	[2,3]
age of the Universe	$t_0$	$\dagger 13.75 \pm 0.13$ Gyr	[2]

$\ddagger$  Parameter in six-parameter  $\Lambda$ CDM fit [2].

$\dagger$  Derived parameter in six-parameter  $\Lambda$ CDM fit [2].

$\#$  Extended model parameter [2].

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- B.W. Petley, *Nature* **303**, 373 (1983).
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- While  $A$  is approximately equal to the semi-major axis of the Earth’s orbit, it is not exactly so. Nor is it exactly the mean Earth-Sun distance. There are a number of reasons: a) the Earth’s orbit is not exactly Keplerian due to relativity and to perturbations from other planets; b) the adopted value for the Gaussian gravitational constant  $k$  is not exactly equal to the Earth’s mean motion; and c) the mean distance in a Keplerian orbit is not equal to the semi-major axis  $a$ :  $\langle r \rangle = a(1 + e^2/2)$ , where  $e$  is the eccentricity. (Discussion courtesy of Myles Standish, JPL).
- The distance at which 1  $A$  subtends 1 arc sec: 1  $A$  divided by  $\pi/648000$ .
- Product of  $2/c^2$  and the heliocentric gravitational constant  $G_N M_\odot = A^3 k^2 / 86400^2$ , where  $k$  is the Gaussian gravitational constant, 0.017 202 098 95 (exact) [5]. The value and error for  $A$  given in this table are used.
- Obtained from the  $G_N M_\odot$  product [5] and  $G_N$  [1].
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- $4\pi A^2 \times (1361 \text{ W m}^{-2})$  [31]. Assumes isotropic irradiance.
- Schwarzschild radius of the Sun (above) scaled by the Earth/Sun mass ratio given in Ref. 5.
- Obtained from the  $G_N M_\oplus$  product [5] and  $G_N$  [1].
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The IAU (Commission 36) has recommended  $3.055 \times 10^{28}$  W for the zero point. Based on newer Solar measurements, the value and significance given in the table seems more appropriate.
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- Sampling of many references:  
M. Mori *et al.*, *Phys. Lett.* **B289**, 463 (1992); E.I. Gates *et al.*, *Astrophys. J.* **449**, L133 (1995); M. Kamionkowski, A. Kinkhabwala, *Phys. Rev.* **D57**, 325 (1998); M. Weber, W. de Boer, *Astron. & Astrophys.* **509**, A25 (2010); P. Salucci *et al.*, *Astron. & Astrophys.* **523**, A83 (2010).
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- D. Scott & G.F. Smoot, “Cosmic Microwave Background,” in this *Review*.
- $n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT}{hc}\right)^3$  and  $\rho_\gamma = \frac{\pi^2}{15} \frac{(kT)^4}{(hc)^3 c^2}$ ;  $\frac{kT_0}{hc} = 11.900(4)/\text{cm}$ .
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- Conversion using length of sidereal year.
- Average of WMAP7 [2] and independent Cepheid-based measurement by A.G. Reiss *et al.*, *Astrophys. J.* **730**, 119 (2011). Other high-quality measurements could have been included.
- R. Amanullah *et al.*, *Astrophys. J.* **716**, 712 (2010). Fit with curvature unconstrained. For a flat Universe,  $w = -1.00 \pm 0.08$ .
- $\Omega_\nu h^2 = \sum m_{\nu_j} / 93 \text{ eV}$ , where the sum is over all neutrino mass eigenstates. The lower limit follows from neutrino mixing results reported in this *Review* combined with the assumptions that there are three light neutrinos ( $m_\nu < 45 \text{ GeV}/c^2$ ) and that the lightest neutrino is substantially less massive than the others:  $\Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2$ , so  $\sum m_{\nu_j} \geq m_{\nu_3} \approx \sqrt{\Delta m_{32}^2} = 0.05 \text{ eV}$ . (This becomes 0.10 eV if the mass hierarchy is inverted, with  $m_{\nu_1} \approx m_{\nu_2} \gg m_{\nu_3}$ .) Astrophysical determinations of  $\sum m_{\nu_j}$ , reported in the Full Listings of this *Review* under “Sum of the neutrino masses,” range from  $< 0.17 \text{ eV}$  to  $< 2.3 \text{ eV}$  in papers published since 2003. Alternatively, if the limit obtained from tritium decay experiments ( $m_\nu < 2 \text{ eV}$ ) is used for the upper limit, then  $\Omega_\nu < 0.04$ .
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