

## 2. ASTROPHYSICAL CONSTANTS AND PARAMETERS

**Table 2.1.** Revised May 2010 by E. Bergren and D.E. Groom (LBNL). The figures in parentheses after some values give the one standard deviation uncertainties in the last digit(s). Physical constants are from Ref. 1. While every effort has been made to obtain the most accurate current values of the listed quantities, the table does not represent a critical review or adjustment of the constants, and is not intended as a primary reference.

The values and uncertainties for the cosmological parameters depend on the exact data sets, priors, and basis parameters used in the fit. Many of the parameters reported in this table are derived parameters or have non-Gaussian likelihoods. The quoted errors may be highly correlated with those of other parameters, so care must be taken in propagating them. Unless otherwise specified, cosmological parameters are best fits of a spatially-flat  $\Lambda$ CDM cosmology with a power-law initial spectrum to 5-year WMAP data alone [2]. For more information see Ref. 3 and the original papers.

Quantity	Symbol, equation	Value	Reference, footnote
speed of light	$c$	299 792 458 m s <sup>-1</sup>	exact[4]
Newtonian gravitational constant	$G_N$	6.674 3(7) $\times 10^{-11}$ m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>	[1]
Planck mass	$\sqrt{\hbar c/G_N}$	1.220 89(6) $\times 10^{19}$ GeV/c <sup>2</sup> = 2.176 44(11) $\times 10^{-8}$ kg	[1]
Planck length	$\sqrt{\hbar G_N/c^3}$	1.616 25(8) $\times 10^{-35}$ m	[1]
standard gravitational acceleration	$g_N$	9.806 65 m s <sup>-2</sup> $\approx \pi^2$	exact[1]
jansky (flux density)	Jy	10 <sup>-26</sup> W m <sup>-2</sup> Hz <sup>-1</sup>	definition
tropical year (equinox to equinox) (2011)	yr	31 556 925.2 s $\approx \pi \times 10^7$ s	[5]
sidereal year (fixed star to fixed star) (2011)		31 558 149.8 s $\approx \pi \times 10^7$ s	[5]
mean sidereal day (2011) (time between vernal equinox transits)		23 <sup>h</sup> 56 <sup>m</sup> 04 <sup>s</sup> .090 53	[5]
astronomical unit	$au, A$	149 597 870 700(3) m	[6]
parsec (1 $au/1$ arc sec)	pc	3.085 677 6 $\times 10^{16}$ m = 3.262 ... ly	[7]
light year (deprecated unit)	ly	0.306 6 ... pc = 0.946 053 ... $\times 10^{16}$ m	
Schwarzschild radius of the Sun	$2G_N M_\odot/c^2$	2.953 250 077 0(2) km	[8]
Solar mass	$M_\odot$	1.988 4(2) $\times 10^{30}$ kg	[9]
Solar equatorial radius	$R_\odot$	6.9551(4) $\times 10^8$ m	[10]
Solar luminosity	$L_\odot$	3.842 7(1 4) $\times 10^{26}$ W	[11]
Schwarzschild radius of the Earth	$2G_N M_\oplus/c^2$	8.870 055 94(2) mm	[12]
Earth mass	$M_\oplus$	5.972 2(6) $\times 10^{24}$ kg	[13]
Earth mean equatorial radius	$R_\oplus$	6.378 137 $\times 10^6$ m	[5]
luminosity conversion (deprecated)	$L$	3.02 $\times 10^{28} \times 10^{-0.4 M_{\text{bol}}}$ W	[14]
flux conversion (deprecated)	$\mathcal{F}$	( $M_{\text{bol}}$ = absolute bolometric magnitude = bolometric magnitude at 10 pc) 2.52 $\times 10^{-8} \times 10^{-0.4 m_{\text{bol}}}$ W m <sup>-2</sup>	from above
ABsolute monochromatic magnitude	AB	( $m_{\text{bol}}$ = apparent bolometric magnitude) -2.5 log <sub>10</sub> $f_\nu$ - 56.10 (for $f_\nu$ in W m <sup>-2</sup> Hz <sup>-1</sup> ) = -2.5 log <sub>10</sub> $f_\nu$ + 8.90 (for $f_\nu$ in Jy)	[15]
Solar distance from Galactic center	$R_0$	8.4(4) kpc	[16]
[Solar circular velocity at $R_0$ ]/ $R_0$	$v_\odot/R_0$	30.2 $\pm$ 0.2 km s <sup>-1</sup> kpc <sup>-1</sup>	[17]
circular velocity at $R_0$	$\Theta_0$	240(10) km s <sup>-1</sup>	[18]
local disk density	$\rho_{\text{disk}}$	3-12 $\times 10^{-24}$ g cm <sup>-3</sup> $\approx$ 2-7 GeV/c <sup>2</sup> cm <sup>-3</sup>	[19]
local dark matter density	$\rho_\chi$	canonical value 0.3 GeV/c <sup>2</sup> cm <sup>-3</sup> within factor 2-3	[20]
escape velocity from Galaxy	$v_{\text{esc}}$	498 km/s < $v_{\text{esc}}$ < 608 km/s	[21]
present day CMB temperature	$T_0$	2.725(1) K	[22]
present day CMB dipole amplitude		3.355(8) mK	[2]
Solar velocity with respect to CMB		369(1) km/s towards $(\ell, b) = (263.99(14)^\circ, 48.26(3)^\circ)$	[2]
Local Group velocity with respect to CMB	$v_{\text{LG}}$	627(22) km/s towards $(\ell, b) = (276(3)^\circ, 30(3)^\circ)$	[23]
entropy density/Boltzmann constant	$s/k$	2 889.2 $(T/2.725)^3$ cm <sup>-3</sup>	[14]
number density of CMB photons	$n_\gamma$	410.5 $(T/2.725)^3$ cm <sup>-3</sup>	[24]
baryon-to-photon ratio <sup>†</sup>	$\eta = n_b/n_\gamma$	6.23(17) $\times 10^{-10}$ 5.1 $\times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}$ (95% CL)	[2] [25]
number density of baryons <sup>†</sup>	$n_b$	(2.56 $\pm$ 0.07) $\times 10^{-7}$ cm <sup>-3</sup> (2.1 $\times 10^{-7} < n_b < 2.7 \times 10^{-7}$ ) cm <sup>-3</sup> (95% CL)	from $\eta$ in [2] from $\eta$ in [25]
present day Hubble expansion rate	$H_0$	100 h km s <sup>-1</sup> Mpc <sup>-1</sup> = $h \times (9.777 752 \text{ Gyr})^{-1}$	[26]
present day normalized Hubble expansion rate <sup>†</sup>	$h$	0.72(3)	[2,3]
Hubble length	$c/H_0$	0.925 063 $\times 10^{26} h^{-1}$ m = 1.28(5) $\times 10^{26}$ m	
scale factor for cosmological constant	$c^2/3H_0^2$	2.852 $\times 10^{51} h^{-2}$ m <sup>2</sup> = 5.5(5) $\times 10^{51}$ m <sup>2</sup>	
critical density of the Universe	$\rho_c = 3H_0^2/8\pi G_N$	2.775 366 27 $\times 10^{11} h^2 M_\odot \text{Mpc}^{-3}$ = 1.878 35(19) $\times 10^{-29} h^2$ g cm <sup>-3</sup> = 1.053 68(11) $\times 10^{-5} h^2$ (GeV/c <sup>2</sup> ) cm <sup>-3</sup>	
pressureless matter density of the Universe <sup>†</sup>	$\Omega_m = \rho_m/\rho_c$	0.133(6) $h^{-2} = 0.26(2)$	[2,3]
baryon density of the Universe <sup>†</sup>	$\Omega_b = \rho_b/\rho_c$	0.0227(6) $h^{-2} = 0.044(4)$	[2,3]
dark matter density of the universe <sup>†</sup>	$\Omega_{\text{cdm}} = \Omega_m - \Omega_b - \Omega_\nu$	0.110(6) $h^{-2} = 0.21(2)$	[2,3]
dark energy density of the $\Lambda$ CDM Universe <sup>†</sup>	$\Omega_\Lambda$	0.74(3)	[2,3]
dark energy equation of state parameter	$w$	-1.04 <sup>+0.09</sup> <sub>-0.10</sub>	[27]
CMB radiation density of the Universe	$\Omega_\gamma = \rho_\gamma/\rho_c$	2.471 $\times 10^{-5} (T/2.725)^4 h^{-2} = 4.8(4) \times 10^{-5}$	[24]
neutrino density of the Universe <sup>†</sup>	$\Omega_\nu$	0.0005 < $\Omega_\nu h^2$ < 0.025 $\Rightarrow$ 0.0009 < $\Omega_\nu$ < 0.048	[28]
total energy density of the Universe <sup>†</sup>	$\Omega_{\text{tot}} = \Omega_m + \dots + \Omega_\Lambda$	1.006(6)	[2,3]

Quantity	Symbol, equation	Value	Reference, footnote
fluctuation amplitude at $8h^{-1}$ Mpc scale <sup>‡</sup>	$\sigma_8$	0.80(4)	[2,3]
curvature fluctuation amplitude, $k_0 = 0.002$ Mpc <sup>-1</sup> <sup>‡</sup>	$\Delta_{\mathcal{R}}^2$	$2.41(11) \times 10^{-9}$	[2,3]
scalar spectral index <sup>‡</sup>	$n_s$	0.96(1)	[2,3]
running spectral index slope, $k_0 = 0.002$ Mpc <sup>-1</sup> <sup>‡</sup>	$dn_s/d \ln k$	-0.04(3)	[2]
tensor-to-scalar field perturbations ratio, $k_0 = 0.002$ Mpc <sup>-1</sup> <sup>‡</sup>	$r = T/S$	< 0.43 at 95% C.L.	[2,3]
redshift at decoupling <sup>‡</sup>	$z_*$	1090(1)	[2]
age at decoupling <sup>‡</sup>	$t_*$	$3.80(6) \times 10^5$ yr	[2]
sound horizon at decoupling <sup>‡</sup>	$r_s(z_*)$	147(2) Mpc	[2]
redshift of matter-radiation equality <sup>‡</sup>	$z_{\text{eq}}$	$3180 \pm 150$	[2]
redshift of reionization <sup>‡</sup>	$z_{\text{reion}}$	$11.0 \pm 1.4$	[2]
age at reionization <sup>‡</sup>	$t_{\text{reion}}$	$430^{+90}_{-70}$ Myr	[2,29]
reionization optical depth <sup>‡</sup>	$\tau$	0.09(2)	[2,3]
age of the Universe <sup>‡</sup>	$t_0$	$13.69 \pm 0.13$ Gyr	[2]

<sup>‡</sup> Best fit of a spatially-flat  $\Lambda$ CDM cosmology with a power-law initial spectrum to 5-year WMAP data alone [2].

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- G. Hinshaw *et al.*, *Astrophys. J. Supp.* **180**, 225 (2009); J. Dunkley *et al.*, *Astrophys. J. Supp.* **180**, 306 (2009); E. Komatsu *et al.*, *Astrophys. J. Supp.* **180**, 330 (2009). Post-deadline 7-year WMAP values\* have not been used. In any case, they vary by less than  $1\sigma$  from the 5-year WMAP values.
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- While  $A$  is approximately equal to the semi-major axis of the Earth’s orbit, it is not exactly so. Nor is it exactly the mean Earth-Sun distance. There are a number of reasons: a) the Earth’s orbit is not exactly Keplerian due to relativity and to perturbations from other planets; b) the adopted value for the Gaussian gravitational constant  $k$  is not exactly equal to the Earth’s mean motion; and c) the mean distance in a Keplerian orbit is not equal to the semi-major axis  $a$ :  $\langle r \rangle = a(1 + e^2/2)$ , where  $e$  is the eccentricity. (Discussion courtesy of Myles Standish, JPL).
- The distance at which 1  $A$  subtends 1 arc sec: 1  $A$  divided by  $\pi/648000$ .
- Product of  $2/c^2$  and the heliocentric gravitational constant  $G_N M_\odot = A^3 k^2 / 86400^2$ , where  $k$  is the Gaussian gravitational constant, 0.01720209895 (exact) [5]. The value and error for  $A$  given in this table are used.
- Obtained from the heliocentric gravitational constant [5] and  $G_N$  [1]. The error is the 100 ppm standard deviation of  $G_N$ .
- T. M. Brown & J. Christensen-Dalsgaard, *Astrophys. J.* **500**, L195 (1998) Many values for the Solar radius have been published, most of which are consistent with this result.
- $4\pi A^2 \times (1366.4 \pm 0.5) \text{ W m}^{-2}$  [30]. Assumes isotropic irradiance.
- Schwarzschild radius of the Sun (above) scaled by the Earth/Sun mass ratio given in Ref. 5.
- Obtained from the geocentric gravitational constant [5] and  $G_N$  [1]. The error is the 100 ppm standard deviation of  $G_N$ .
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- C. McCabe, [arXiv:1005.0579](https://arxiv.org/abs/1005.0579) Other papers report values closer to 220(20) km s<sup>-1</sup>; S.E. Kojosov, H.-W. Rix, & D.W. Hogg (2009), [arXiv:0907.1085](https://arxiv.org/abs/0907.1085); P.J. McMillan & J.J. Binney, [arXiv:0907.4685](https://arxiv.org/abs/0907.4685).
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- Sampling of many references: M. Mori *et al.*, *Phys. Lett.* **B289**, 463 (1992); E.I. Gates *et al.*, *Astrophys. J.* **449**, L133 (1995); M. Kamionkowski & A. Kinkhabwala, *Phys. Rev.* **D57**, 325 (1998); M. Weber and W. de Boer (2009), [arXiv:0910.4272](https://arxiv.org/abs/0910.4272); P. Salucci *et al.*, (2010), [arXiv:1003.3101](https://arxiv.org/abs/1003.3101).
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- D. Scott & G.F. Smoot, “Cosmic Microwave Background,” in this *Review*.
- $n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT}{hc}\right)^3$  and  $\rho_\gamma = \frac{\pi^2 (kT)^4}{15 (hc)^3 c^2}$ ;  $\frac{kT_0}{hc} = 11.900(4)/\text{cm}$ .
- B.D. Fields & S. Sarkar, “Big-Bang Nucleosynthesis,” in this *Review*.
- Conversion using length of sidereal year.
- R. Amanullah *et al.*, *Astrophys. J.* **716**, 712 (2010). Fit with curvature unconstrained. For a flat Universe,  $w = -1.00 \pm 0.08$ .
- $\Omega_\nu h^2 = \sum m_{\nu_j} / 93 \text{ eV}$ , where the sum is over all neutrino mass eigenstates. The lower limit follows from neutrino mixing results reported in this *Review* combined with the assumptions that there are three light neutrinos ( $m_\nu < 45 \text{ GeV}/c^2$ ) and that the lightest neutrino is substantially less massive than the others:  $\Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2$ , so  $\sum m_{\nu_j} \geq m_{\nu_3} \approx \sqrt{\Delta m_{32}^2} = 0.05 \text{ eV}$ . (This becomes 0.10 eV if the mass hierarchy is inverted, with  $m_{\nu_1} \approx m_{\nu_2} \gg m_{\nu_3}$ .) Astrophysical determinations of  $\sum m_{\nu_j}$ , reported in the Full Listings of this *Review* under “Sum of the neutrino masses,” range from  $< 0.17 \text{ eV}$  to  $< 2.3 \text{ eV}$  in papers published since 2002. Alternatively, if the limit obtained from tritium decay experiments ( $m_\nu < 2 \text{ eV}$ ) is used for the upper limit, then  $\Omega_\nu < 0.04$ .
- If the Universe were reionized instantaneously at  $z_{\text{reion}}$ .
- R.C. Willson & A.V. Mordvinov, *Geophys. Res. Lett.* **30**, 1119 (2003); C. Frölich, *Space Sci. Rev.* **125**, 53–65 (2006).

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