

## RARE KAON DECAYS

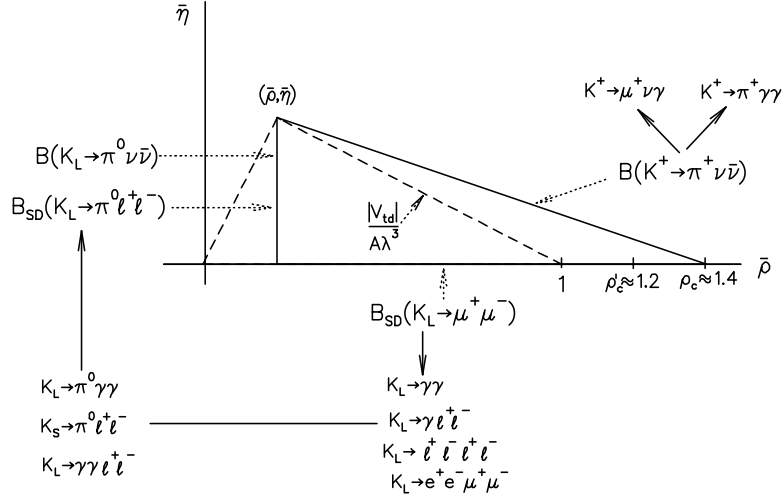
Revised November 2009 by L. Littenberg (BNL) and G. Valencia (Iowa State University).

**A. Introduction:** There are several useful reviews on rare kaon decays and related topics [1–14]. Activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for  $CP$  violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay  $K_L \rightarrow \mu e$ . Category 2 includes processes such as  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , which is sensitive to  $|V_{td}|$ . Much of the interest in Category 3 is focused on the decays  $K_L \rightarrow \pi^0 \ell \bar{\ell}$ , where  $\ell \equiv e, \mu, \nu$ . Category 4 includes reactions like  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$  which constitute a testing ground for the ideas of chiral perturbation theory. Category 4 also includes  $K_L \rightarrow \pi^0 \gamma \gamma$  and  $K_L \rightarrow \ell^+ \ell^- \gamma$ . The former is important in understanding a  $CP$ -conserving contribution to  $K_L \rightarrow \pi^0 \ell^+ \ell^-$ , whereas the latter could shed light on long distance contributions to  $K_L \rightarrow \mu^+ \mu^-$ .

The interplay between Categories 2-4 can be illustrated in Fig. 1. The modes  $K \rightarrow \pi \nu \bar{\nu}$  are the cleanest ones theoretically. They can provide accurate determinations of certain CKM parameters (shown in the figure). In combination with alternate determinations of these parameters, they also constrain new interactions. The modes  $K_L \rightarrow \pi^0 e^+ e^-$  and  $K_L \rightarrow \mu^+ \mu^-$  are also sensitive to CKM parameters. However, they suffer from a series of hadronic uncertainties that can be addressed, at least in part, through a systematic study of the additional modes indicated in the figure.



**Figure 1:** Role of rare kaon decays in determining the unitarity triangle. The solid arrows point to auxiliary modes needed to interpret the main results, or potential backgrounds to them.

**B. Explicit violations of the Standard Model:** Much activity has focussed on searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass  $M_X$  that couples to left-handed fermions with electroweak strength and without mixing angles yields  $B(K_L \rightarrow \mu e) = 4.7 \times 10^{-12} (148 \text{ TeV}/M_X)^4$  [4]. This simple dimensional analysis may be used to read from Table 1 that the reaction  $K_L \rightarrow \mu e$  is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV. The decays  $K_L \rightarrow \mu^\pm e^\mp$  and  $K^+ \rightarrow \pi^+ e^\mp \mu^\pm$  (or  $K_L \rightarrow \pi^0 e^\mp \mu^\pm$ ) provide complementary information on potential family number violating interactions, since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings. Limits on certain lepton-number violating kaon decays [15,16] also exist. Related searches in  $\mu$  and  $\tau$  processes are discussed in our section “Tests of Conservation Laws.”

**Table 1:** Searches for lepton flavor violation in  $K$  decay

Mode	90% CL	Exp't	Yr./Ref.
	upper limit		
$K^+ \rightarrow \pi^+ e^- \mu^+$	$1.2 \times 10^{-11}$	BNL-865	2005/Ref. 17
$K^+ \rightarrow \pi^+ e^+ \mu^-$	$5.2 \times 10^{-10}$	BNL-865	2000/Ref. 15
$K_L \rightarrow \mu e$	$4.7 \times 10^{-12}$	BNL-871	1998/Ref. 18
$K_L \rightarrow \pi^0 e \mu$	$7.6 \times 10^{-11}$	KTeV	2008/Ref. 19
$K_L \rightarrow \pi^0 \pi^0 e \mu$	$1.7 \times 10^{-10}$	KTeV	2008/Ref. 19

Physics beyond the SM is also pursued through the search for  $K^+ \rightarrow \pi^+ X^0$ , where  $X^0$  is a very light, long-lived particle (*e.g.*, hyperphoton, axion, familon, *etc.*). The 90% CL upper limit on this process is  $7.3 \times 10^{-11}$  [20].

***C. Measurements of Standard Model parameters:***

In the SM, the decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  is dominated by one-loop diagrams with top-quark intermediate states and long-distance contributions are known to be quite small [2,21,22]. This permits a precise calculation [23] of this rate in terms of SM parameters. Studies of this process are thus motivated by the possibility of detecting non-SM physics when comparing with the results of global fits [24,25].

BNL-787 observed two candidate events [26,27] in the clean high  $\pi^+$  momentum and one event [28] in the low-momentum region. The successor experiment BNL-949 observed one more in the high-momentum region [20] and three more in the low-momentum region [29] yielding a branching ratio of  $(1.73_{-1.05}^{+1.15}) \times 10^{-10}$  [30]. A new experiment, NA62, with a sensitivity goal of  $\sim 10^{-12}$ /event was proposed [31] at CERN in 2005. It has been approved and is scheduled to run in 2011. In the future, this mode may provide grounds for precision tests of flavor dynamics [32]. The branching ratio can be written in terms of the very well-measured  $K_{e3}$  rate as [2]:

$$\begin{aligned}
 \text{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) &= \frac{\alpha^2 \text{B}(K^+ \rightarrow \pi^0 e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4 \theta_W} \\
 &\times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^\ell + V_{ts}^* V_{td} X(m_t)|^2 \quad (1)
 \end{aligned}$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [33,23]. In Eq. (1), the Inami-Lim function  $X(m_t)$  is of order 1 [34], and  $X_{NL}^\ell$  is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on  $|V_{td}|$ . QCD corrections, which mainly affect  $X_{NL}^\ell$ , lead to a residual error of  $< 5\%$  for the decay amplitude [11,22,35,36]. Evaluating the constants in Eq. (1), one can cast this result in terms of the CKM parameters  $\lambda$ ,  $V_{cb}$ ,  $\bar{\rho}$  and  $\bar{\eta}$  (see our Section on “The Cabibbo-Kobayashi-Maskawa mixing matrix”) [11]:

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \approx 1.6 \times 10^{-5} |V_{cb}|^4 [\sigma \bar{\eta}^2 + (\rho_c - \bar{\rho})^2], \quad (2)$$

where  $\rho_c \equiv 1 + (\frac{2}{3}X_{NL}^e + \frac{1}{3}X_{NL}^\tau)/(|V_{cb}|^2 X(m_t)) \approx 1.4$  and  $\sigma \equiv 1/(1 - \frac{1}{2}\lambda^2)^2$ . Thus,  $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  determines an ellipse in the  $\bar{\rho}$ ,  $\bar{\eta}$  plane with center  $(\rho_c, 0)$  and semiaxes  $\approx \frac{1}{|V_{cb}|^2} \sqrt{\frac{B(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{1.6 \times 10^{-5}}}$  and  $\frac{1}{\sigma |V_{cb}|^2} \sqrt{\frac{B(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{1.6 \times 10^{-5}}}$ . Current constraints on the CKM parameters lead to a predicted branching ratio  $(8.5 \pm 0.7) \times 10^{-11}$  [36], near the lower end of the measurement of BNL-787 and 949.

The decay  $K_L \rightarrow \mu^+ \mu^-$  also has a short distance contribution sensitive to the CKM parameter  $\bar{\rho}$ , given by [11]:

$$B_{SD}(K_L \rightarrow \mu^+ \mu^-) \approx 2.7 \times 10^{-4} |V_{cb}|^4 (\rho'_c - \bar{\rho})^2 \quad (3)$$

where  $\rho'_c$  depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is determined by the measured rate for  $K_L \rightarrow \gamma\gamma$  to be  $B_{\text{abs}}(K_L \rightarrow \mu^+ \mu^-) = (6.64 \pm 0.07) \times 10^{-9}$ ; and it almost completely saturates the observed rate  $B(K_L \rightarrow \mu^+ \mu^-) = (6.84 \pm 0.11) \times 10^{-9}$  [37]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. The latter cannot be derived directly from experiment [38], but can be estimated with certain assumptions [39,40]. The decay  $K_L \rightarrow e^+ e^-$  is completely dominated by long distance physics and is easier to estimate. The result,

$B(K_L \rightarrow e^+e^-) \sim 9 \times 10^{-12}$  [38,41], is in good agreement with the BNL-871 measurement,  $(8.7_{-4.1}^{+5.7}) \times 10^{-12}$  [42].

**D. Searches for direct  $CP$  violation:** The mode  $K_L \rightarrow \pi^0\nu\bar{\nu}$  is dominantly  $CP$ -violating and free of hadronic uncertainties [2,43,44]. In the Standard Model, this mode is dominated by an intermediate top-quark state and does not suffer from the small uncertainty associated with the charm-quark intermediate state that affects the mode  $K^+ \rightarrow \pi^+\nu\bar{\nu}$ . The branching ratio is given approximately by Ref. 11:

$$B(K_L \rightarrow \pi^0\nu\bar{\nu}) \approx 7.6 \times 10^{-5} |V_{cb}|^4 \bar{\eta}^2 . \quad (4)$$

With current constraints on the CKM parameters this leads to a predicted branching ratio  $(2.6 \pm 0.3) \times 10^{-11}$  [14]. The 90% CL bound on  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  provides a nearly model-independent bound  $B(K_L \rightarrow \pi^0\nu\bar{\nu}) < 1.46 \times 10^{-9}$  [45]. KEK-391a, which began data-taking in early 2004, has published a result of  $B(K_L \rightarrow \pi^0\nu\bar{\nu}) \leq 6.7 \times 10^{-8}$  [46] based on partial data. Very recently a preliminary result on the remainder of the data yielded  $B(K_L \rightarrow \pi^0\nu\bar{\nu}) \leq 6.8 \times 10^{-8}$  [47]. The KOTO experiment, whose initial goal is to reach the  $10^{-11}$ /event level, is under construction at J-PARC [48].

There has been much theoretical work on possible contributions to rare  $K$  decays beyond the SM. A comprehensive discussion of these can be found in Refs. [14] and [49].

The decay  $K_L \rightarrow \pi^0 e^+ e^-$  also has sensitivity to the CKM parameter  $\eta$  through its  $CP$ -violating component. There are both direct and indirect  $CP$ -violating amplitudes which can interfere. The direct  $CP$ -violating amplitude is short distance dominated and has been calculated in detail within the SM [8]. The indirect  $CP$ -violating amplitude can be inferred from a measurement of  $K_S \rightarrow \pi^0 e^+ e^-$ . The complete  $CP$ -violating contribution to the rate can be written as [50]:

$$B_{CPV} \approx 10^{-12} \left[ 15.7 |a_S|^2 \pm 1.45 \left( \frac{|V_{cb}|^2 \bar{\eta}}{10^{-4}} \right) |a_S| + 0.129 \left( \frac{|V_{cb}|^2 \bar{\eta}}{10^{-4}} \right)^2 \right] \quad (5)$$

where the three terms correspond to the indirect  $CP$  violation, the interference, and the direct  $CP$  violation respectively. The parameter  $a_S$  has been extracted by NA48 from a measurement of the decay  $K_S \rightarrow \pi^0 e^+ e^-$  with the result  $|a_S| = 1.06_{-0.21}^{+0.26} \pm 0.07$  [51]. With current constraints on the CKM parameters this implies that

$$B_{\text{CPV}} \approx (17.2 \pm 9.4 + 4.7) \times 10^{-12}. \quad (6)$$

The indirect  $CP$  violation is larger than the direct  $CP$  violation. While the sign of the interference is *a priori* unknown, arguments in favor of a positive sign have been put forward in Ref. 52 and Ref. 53. NA48 has also obtained the value  $a_s = 1.54_{-0.32}^{+0.40} \pm 0.06$  [54] from a measurement of the  $K_S \rightarrow \pi^0 \mu^+ \mu^-$  rate, in agreement with the value extracted from  $K_S \rightarrow \pi^0 e^+ e^-$

$K_L \rightarrow \pi^0 \gamma \gamma$  also has a  $CP$ -conserving component dominated by a two-photon intermediate state. This component can be decomposed into an absorptive and a dispersive part. The absorptive part can be extracted from the measurement of the low  $m_{\gamma\gamma}$  region of the  $K_L \rightarrow \pi^0 \gamma \gamma$  spectrum. The rate and the shape of the distribution  $d\Gamma/dm_{\gamma\gamma}$  in  $K_L \rightarrow \pi^0 \gamma \gamma$  are well described in chiral perturbation theory in terms of three (*a priori*) unknown parameters [55,56].

Both KTeV and NA48 have studied the mode  $K_L \rightarrow \pi^0 \gamma \gamma$ , reporting similar results. KTeV finds  $B(K_L \rightarrow \pi^0 \gamma \gamma) = (1.29 \pm 0.03_{\text{stat}} \pm 0.05_{\text{sys}}) \times 10^{-6}$  [57], while NA48 finds  $B(K_L \rightarrow \pi^0 \gamma \gamma) = (1.36 \pm 0.03_{\text{stat}} \pm 0.03_{\text{sys}} \pm 0.03_{\text{norm}}) \times 10^{-6}$  [58]. Both experiments are consistent with a negligible rate in the low  $m_{\gamma\gamma}$  region, suggesting a very small  $CP$ -conserving component  $B_{\text{CP}}(K_L \rightarrow \pi^0 e^+ e^-) \sim \mathcal{O}(10^{-13})$  [52,56,58]. There remains some model dependence in the estimate of the dispersive part of the  $CP$ -conserving  $K_L \rightarrow \pi^0 e^+ e^-$  [52].

The related process,  $K_L \rightarrow \pi^0 \gamma e^+ e^-$ , is potentially an additional background in some region of phase space [59]. This process has been observed with a branching ratio of  $(1.62 \pm 0.14_{\text{stat}} \pm 0.09_{\text{sys}}) \times 10^{-8}$  [60].

The decay  $K_L \rightarrow \gamma \gamma e^+ e^-$  constitutes the dominant background to  $K_L \rightarrow \pi^0 e^+ e^-$ . It was first observed by BNL-845 [61],

and subsequently confirmed with a much larger sample by FNAL-799 [62]. It has been estimated that this background will enter at about the  $10^{-10}$  level [63,64], comparable to or larger than the signal level. Because of this, the observation of  $K_L \rightarrow \pi^0 e^+ e^-$  at the SM level will depend on background subtraction with good statistics. Possible alternative strategies are discussed in Ref. 52 and references cited therein.

The 90% CL upper bound for the process  $K_L \rightarrow \pi^0 e^+ e^-$  is  $2.8 \times 10^{-10}$  [64]. For the closely related muonic process, the published upper bound is  $B(K_L \rightarrow \pi^0 \mu^+ \mu^-) \leq 3.8 \times 10^{-10}$  [65], compared with the SM prediction of  $(1.5 \pm 0.3) \times 10^{-11}$  [66] (assuming positive interference between the direct- and indirect-CP violating components). KTeV has additional data corresponding to about a factor 1.3 in sensitivity for the latter reaction that is under analysis.

A study of  $K_L \rightarrow \pi^0 \mu^+ \mu^-$  has indicated that it might be possible to extract the direct  $CP$ -violating contribution by a joint study of the Dalitz plot variables and the components of the  $\mu^+$  polarization [67]. The latter tends to be quite substantial so that large statistics may not be necessary.

Combined information from the two  $K_L \rightarrow \pi^0 \ell^+ \ell^-$  modes complements the  $K \rightarrow \pi \nu \bar{\nu}$  measurements in constraining physics beyond the SM [68].

### ***E. Other long distance dominated modes:***

The decays  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$  ( $\ell = e$  or  $\mu$ ) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [69,70]. Ref. 50 has proposed a parametrization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections.

The related rare hyperon decay mode  $\Sigma^+ \rightarrow p \mu^+ \mu^-$ , has been recently observed by the HyperCP collaboration [71]. The observed rate is in rough agreement with the standard model expectation [72] but the three events that have been observed cluster at a common  $m_{\mu\mu}$  invariant mass and this can be interpreted as evidence for a new particle  $X^0$ . The existing

measurements of  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  rule out a scalar  $X^0$  and the new bounds,  $B(K_L \rightarrow \pi^0 \pi^0 \mu^+ \mu^-) < 9.44 \times 10^{-11}$  [73] and  $B(K_L \rightarrow \pi^0 \pi^0 \gamma \gamma) < 2.4 \times 10^{-7}$  [74] aim at closing the possibility of a pseudoscalar  $X^0$  [75].

Much information has been recorded by KTeV and NA48 on the rates and spectrum for the Dalitz pair conversion modes  $K_L \rightarrow \ell^+ \ell^- \gamma$  [76,77], and  $K_L \rightarrow \ell^+ \ell^- \ell'^+ \ell'^-$  for  $\ell, \ell' = e$  or  $\mu$  [16,78–80]. All these results are used to test hadronic models and could further our understanding of the long distance component in  $K_L \rightarrow \mu^+ \mu^-$ .

## References

1. D. Bryman, *Int. J. Mod. Phys.* **A4**, 79 (1989).
2. J. Hagelin and L. Littenberg, *Prog. in Part. Nucl. Phys.* **23**, 1 (1989).
3. L. Littenberg and G. Valencia, *Ann. Rev. Nucl. and Part. Sci.* **43**, 729 (1993).
4. J. Ritchie and S. Wojcicki, *Rev. Mod. Phys.* **65**, 1149 (1993).
5. B. Winstein and L. Wolfenstein, *Rev. Mod. Phys.* **65**, 1113 (1993).
6. G. D’Ambrosio *et al.*, *Radiative Non-Leptonic Kaon Decays*, in *The DAΦNE Physics Handbook* (second edition), eds. L. Maiani, G. Pancheri, and N. Paver (Frascati), Vol. I, 265 (1995).
7. A. Pich, *Rept. on Prog. in Phys.* **58**, 563 (1995).
8. G. Buchalla, A.J. Buras, and M.E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).
9. G. D’Ambrosio and G. Isidori, *Int. J. Mod. Phys.* **A13**, 1 (1996).
10. P. Buchholz and B. Renk *Prog. in Part. Nucl. Phys.* **39**, 253 (1997).
11. A.J. Buras and R. Fleischer, TUM-HEP-275-97, [hep-ph/9704376](#), *Heavy Flavours II*, World Scientific, eds. A.J. Buras and M. Lindner (1997), 65–238.
12. A.J. Buras, TUM-HEP-349-99, Lectures given at Lake Louise Winter Institute: Electroweak Physics, Lake Louise, Alberta, Canada, 14–20 Feb. 1999.
13. A.R. Barker and S.H. Kettell, *Ann. Rev. Nucl. and Part. Sci.* **50**, 249 (2000).
14. A.J. Buras, F. Schwab, and S. Uhlig, *Rev. Mod. Phys.* **80**, 965 (2008).



15. R. Appel *et al.*, Phys. Rev. Lett. **85**, 2877 (2000).
16. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **90**, 141801 (2003).
17. A. Sher *et al.*, Phys. Rev. **D72**, 012005 (2005).
18. D. Ambrose *et al.*, Phys. Rev. Lett. **81**, 5734 (1998).
19. E. Abouzaid *et al.*, Phys. Rev. Lett. **100**, 131803 (2008).
20. V.V. Anisimovsky *et al.*, Phys. Rev. Lett. **93**, 031801 (2004).
21. M. Lu and M.B. Wise, Phys. Lett. **B324**, 461 (1994);  
A.F. Falk, A. Lewandowski, and A.A. Petrov, Phys. Lett. **B505**, 107 (2001).
22. G. Isidori, F. Mescia, and C. Smith, Nucl. Phys. **B718**, 319 (2005).
23. F. Mescia and C. Smith, Phys. Rev. **D76**, 034017 (2007).
24. CKMfitter Group (J. Charles *et al.*), Eur. Phys. J. **C41**, 1 (2005), [[hep-ph/0406184](http://hep-ph/0406184)], updated results and plots available at: <http://ckmfitter.in2p3.fr>.
25. M. Bona *et al.*, [UTfit Collaboration] “Model-independent constraints on Delta F=2 operators and the scale of New Physics,” [arXiv:0707.0636](https://arxiv.org/abs/0707.0636) [hep-ph].
26. S. Adler *et al.*, Phys. Rev. Lett. **88**, 041803 (2002).
27. S. Adler *et al.*, Phys. Rev. Lett. **84**, 3768 (2000).
28. S. Adler *et al.*, Phys. Lett. **B537**, 237 (2002).
29. A.V. Artamonov *et al.*, Phys. Rev. Lett. **101**, 191802 (2008).
30. A.V. Artamonov *et al.*, Phys. Rev. **D79**, 092004 (2009).
31. G. Anelli *et al.*, CERN-SPSC-2005-013, 11 June 2005.
32. G. D’Ambrosio and G. Isidori, Phys. Lett. **B530**, 108 (2002).
33. W. Marciano and Z. Parsa, Phys. Rev. **D53**, 1 (1996).
34. T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, 297 (1981);  
Erratum Prog. Theor. Phys. **65**, 172 (1981).
35. G. Buchalla and A.J. Buras, Nucl. Phys. **B548**, 309 (1999);  
M. Misiak and J. Urban, Phys. Lett. **B451**, 161 (1999);  
A.J. Buras *et al.*, Phys. Rev. Lett. **95**, 261805 (2005).
36. J. Brod and M. Gorbahn, Phys. Rev. **D78**, 034006 (2008).
37. D. Ambrose *et al.*, Phys. Rev. Lett. **84**, 1389 (2000).
38. G. Valencia, Nucl. Phys. **B517**, 339 (1998).
39. G. D’Ambrosio, G. Isidori, and J. Portoles, Phys. Lett. **B423**, 385 (1998).
40. G. Isidori and R. Unterdorfer, JHEP **0401**, 009 (2004).

41. D. Gomez-Dumm and A. Pich, Phys. Rev. Lett. **80**, 4633 (1998).
42. D. Ambrose *et al.*, Phys. Rev. Lett. **81**, 4309 (1998).
43. L. Littenberg, Phys. Rev. **D39**, 3322 (1989).
44. G. Buchalla and G. Isidori, Phys. Lett. **B440**, 170 (1998).
45. Y. Grossman and Y. Nir, Phys. Lett. **B398**, 163 (1997).
46. J.K. Ahn *et al.*, Phys. Rev. Lett. **100**, 201802 (2008).
47. H. Morii, “Search for the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  at E391a Experiment,” KAON 09, 12 June 2009.
48. J. Comfort *et al.*, “Proposal for  $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$  Experiment at J-Parc,” J-PARC Proposal 14 (2006).
49. D. Bryman *et al.*, Int. J. Mod. Phys. **A21**, 487 (2006).
50. G. D’Ambrosio *et al.*, JHEP **9808**, 004 (1998);  
C.O. Dib, I. Dunietz, and F.J. Gilman, Phys. Rev. **D39**, 2639 (1989).
51. J.R. Batley *et al.*, Phys. Lett. **B576**, 43 (2003).
52. G. Buchalla, G. D’Ambrosio, and G. Isidori, Nucl. Phys. **B672**, 387 (2003).
53. S. Friot, D. Greynat, and E. de Rafael, Phys. Lett. **B595**, 301 (2004).
54. J.R. Batley *et al.*, Phys. Lett. **B599**, 197 (2004).
55. G. Ecker, A. Pich, and E. de Rafael, Phys. Lett. **237B**, 481 (1990);  
L. Cappiello, G. D’Ambrosio, and M. Miragliuolo, Phys. Lett. **B298**, 423 (1993);  
A. Cohen, G. Ecker, and A. Pich, Phys. Lett. **B304**, 347 (1993).
56. F. Gabbiani and G. Valencia, Phys. Rev. **D66**, 074006 (2002).
57. E. Abouzaid *et al.*, Phys. Rev. **D77**, 112004 (2008).
58. A. Lai *et al.*, Phys. Lett. **B536**, 229 (2002).
59. J. Donoghue and F. Gabbiani, Phys. Rev. **D56**, 1605 (1997).
60. E. Abouzaid *et al.*, Phys. Rev. **D76**, 052001 (2007).
61. W.M. Morse *et al.*, Phys. Rev. **D45**, 36 (1992).
62. A. Alavi-Harati *et al.*, Phys. Rev. **D64**, 012003 (2001).
63. H.B. Greenlee, Phys. Rev. **D42**, 3724 (1990).
64. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **93**, 021805 (2004).
65. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **84**, 5279 (2000).
66. G. Isidori, C. Smith, and R. Unterdorfer, Eur. Phys. J. **C36**, 57 (2004).

67. M.V. Diwan, H. Ma, and T.L. Trueman, Phys. Rev. **D65**, 054020 (2002).
68. F. Mescia, C. Smith, and S. Trine, JHEP **0608**, 088 (2006).
69. R. Appel *et al.*, Phys. Rev. Lett. **83**, 4482 (1999);  
J.R. Batley *et al.*, Phys. Lett. **B677**, 246 (2009).
70. S.C. Adler *et al.*, Phys. Rev. Lett. **79**, 4756 (1997);  
R. Appel *et al.*, Phys. Rev. Lett. **84**, 2580 (2000);  
H.K. Park *et al.*, Phys. Rev. Lett. **88**, 111801 (2002).
71. H. Park *et al.*, Phys. Rev. Lett. **94**, 021801 (2005).
72. X.G. He, J. Tandean, and G. Valencia, Phys. Rev. **D72**, 074003 (2005).
73. D. Phillips “Search for the Rare Decay  $K_L \rightarrow \pi^0 \pi^0 \mu^+ \mu^-$ ,”  
KAON 09, 11 June 2009.
74. Y.C. Tung *et al.*, Phys. Rev. Lett. **102**, 051802 (2009).
75. X.G. He, J. Tandean, and G. Valencia, JHEP **0806**, 002 (2008) and references therein.
76. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **87**, 071801 (2001).
77. A. Abouzaid *et al.*, Phys. Rev. Lett. **99**, 051804 (2007).
78. J.R. LaDue “Understanding Dalitz Decays of the  $K_L$  in particular the decays of  $K_L \rightarrow e^+ e^- \gamma$  and  $K_L \rightarrow e^+ e^- e^+ e^-$ ” University of Colorado Thesis, May 2003. The preliminary result for  $K_L \rightarrow e^+ e^- \gamma$  in this thesis has been superseded by the final result in [77].
79. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **86**, 5425 (2001).
80. V. Fanti *et al.*, Phys. Lett. **B458**, 458 (1999).