EXTRA DIMENSIONS

Updated Sept. 2007 by G.F. Giudice (CERN) and J.D. Wells (MCTP/Michigan).

I Introduction

The idea of using extra spatial dimensions to unify different forces started in 1914 with Nordstöm, who proposed a 5-dimensional vector theory to simultaneously describe electromagnetism and a scalar version of gravity. After the invention of general relativity, in 1919 Kaluza noticed that the 5-dimensional generalization of Einstein theory can simultaneously describe gravitational and electromagnetic interactions. The role of gauge invariance and the physical meaning of the compactification of extra dimensions was elucidated by Klein. However, the Kaluza-Klein (KK) theory failed in its original purpose because of internal inconsistencies and was essentially abandoned until the advent of supergravity in the late 1970's. Higher-dimensional theories were reintroduced in physics to exploit the special properties that supergravity and superstring theories possess for particular values of spacetime dimensions. More recently it was realized [1,2] that extra dimensions with a fundamental scale of order TeV^{-1} could address the M_{W}–M_{Pl} hierarchy problem and therefore have direct implications for collider experiments. Here we will review [3] the proposed scenarios with experimentally accessible extra dimensions.

II Gravity in Flat Extra Dimensions

II.1 Theoretical Setup

Following Ref. 1, let us consider a D-dimensional spacetime with $D = 4 + \delta$, where δ is the number of extra spatial dimensions. The space is factorized into $R^4 \times M_\delta$ (meaning that the 4-dimensional part of the metric does not depend on extra-dimensional coordinates), where M_δ is a δ-dimensional compact space with finite volume V_δ. For concreteness, we will consider a δ-dimensional torus of radius R, for which $V_\delta = (2\pi R)^\delta$. Standard Model (SM) fields are assumed to be localized on a $(3+1)$-dimensional subspace. This assumption can be realized in field theory, but it is most natural [4] in the setting of string theory, where gauge and matter fields can be confined to live on "branes" (for a review see Ref. 5). On the other hand, gravity, which according to general relativity is described by the spacetime geometry, extends to all D dimensions. The Einstein action takes the form

$$S_E = \frac{\bar{M}_D^{2+\delta}}{2} \int d^4x \ d^\delta y \ \sqrt{-\det\ g}\ \mathcal{R}(g), \tag{1}$$

where x and y describe ordinary and extra coordinates, respectively. The metric g, the scalar curvature $\mathcal{R}$, and the reduced Planck mass $\bar{M}_D$ refer to the D-dimensional theory. The effective action for the 4-dimensional graviton is obtained by restricting the metric indices to 4 dimensions and by performing the integral in y. Because of the above-mentioned factorization hypothesis, the integral in y reduces to the volume V_δ, and therefore the 4-dimensional reduced Planck mass is given by

$$\overline{M}_{\rm Pl}^2 = \bar{M}_D^{2+\delta}\, V_\delta = \bar{M}_D^{2+\delta}\, (2\pi R)^\delta\,, \tag{2}$$

where $\overline{M}_{\rm Pl} = M_{\rm Pl}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV. The same formula can be obtained from Gauss's law in extra dimensions [6]. Following ref. [7], we will consider $M_D = (2\pi)^{\delta/(2+\delta)} \bar{M}_D$ as the fundamental D-dimensional Planck mass.

The key assumption of Ref. 1 is that the hierarchy problem is solved because the truly fundamental scale of gravity M_D (and therefore the ultraviolet cut-off of field theory) lies around the TeV region. From Eq. (2) it follows that the correct value of $\overline{M}_{\rm Pl}$ can be obtained with a large value of RM_D. The inverse compactification radius is therefore given by

$$R^{-1} = M_D \left(M_D/\overline{M}_{\rm Pl}\right)^{2/\delta}, \tag{3}$$

which corresponds to 4×10^{-4} eV, 20 keV, 7 MeV for $M_D =$ 1 TeV and $\delta = 2, 4, 6$, respectively. In this framework, gravity is weak because it is diluted in a large space ($R \gg M_D^{-1}$). Of course, a complete solution of the hierarchy problem would require a dynamical explanation for the radius stabilization at a large value.

A D-dimensional bosonic field can be expanded in Fourier modes in the extra coordinates

$$\phi(x,y) = \sum_{\vec{n}} \frac{\varphi^{(\vec{n})}(x)}{\sqrt{V_\delta}} \exp\left(i\frac{\vec{n}\cdot\vec{y}}{R}\right). \tag{4}$$

The sum is discrete because of the finite size of the compactified space. The fields $\varphi^{(\vec{n})}$ are called the n^{th} KK excitations (or modes) of ϕ, and correspond to particles propagating in 4 dimensions with masses $m^2_{(\vec{n})} = |\vec{n}|^2/R^2 + m_0^2$, where m_0 is the mass of the zero mode. The D-dimensional graviton can then be recast as a tower of KK states with increasing mass. However, since R^{-1} in Eq. (3) is smaller than the typical energy resolution in collider experiments, the mass distribution of KK gravitons is practically continuous.

Although each KK graviton has a purely gravitational coupling suppressed by $\overline{M}_{\mathrm{Pl}}^{-1}$, inclusive processes in which we sum over the large number of available gravitons have cross sections suppressed only by powers of M_D. Indeed, for scatterings with typical energy E, we expect $\sigma \sim E^\delta/M_D^{2+\delta}$, as evident from power-counting in D dimensions. Processes involving gravitons are therefore detectable in collider experiments if M_D is in the TeV region.

The astrophysical considerations described in Sec. II.6 set very stringent bounds on M_D for $\delta < 4$, in some cases even ruling out the possibility of observing any signal at the LHC. However, these bounds disappear if there are no KK gravitons lighter than about 100 MeV. Variations of the original model exist [8,9] in which the light KK gravitons receive small extra contributions to their masses, sufficient to evade the astrophysical bounds. Notice that collider experiments are nearly insensitive to such modifications of the infrared part of the KK graviton spectrum, since they mostly probe the heavy graviton modes. Therefore, in the context of these variations, it is important to test at colliders extra-dimensional gravity also for low values of δ, and even for $\delta = 1$ [9]. In addition to these direct experimental constraints, the proposal of gravity in flat extra dimensions has dramatic cosmological consequences and requires a rethinking of the thermal history of the universe for temperatures as low as the MeV scale.

II.2 Collider Signals in Linearized Gravity

By making a derivative expansion of Einstein gravity, one can construct an effective theory describing KK graviton interactions, which is valid for energies much smaller than M_D [7,10,11]. With the aid of this effective theory, it is possible to make predictions for graviton-emission processes at colliders. Since the produced gravitons interact with matter only with rates suppressed by inverse powers of $\overline{M}_{\rm Pl}$, they will remain undetected leaving a "missing-energy" signature. Extra-dimensional gravitons have been searched for in the processes $e^+e^- \to \gamma + \not{E}$ and $e^+e^- \to Z + \not{E}$ at LEP, and $p\bar{p} \to \text{jet} + \not{E}_T$ and $p\bar{p} \to \gamma + \not{E}_T$ at the Tevatron. The combined LEP 95% CL limits are [12] $M_D >$ 1.60, 1.20, 0.94, 0.77, 0.66 TeV for $\delta = 2, \ldots, 6$ respectively. Experiments at the LHC will improve the sensitivity. However, the theoretical predictions for the graviton-emission rates should be applied with care to hadron machines. The effective theory results are valid only for center-of-mass energy of the parton collision much smaller than M_D.

The effective theory under consideration also contains the full set of higher-dimensional operators, whose coefficients are, however, not calculable, because they depend on the ultraviolet properties of gravity. This is in contrast with graviton emission, which is a calculable process within the effective theory because it is linked to the infrared properties of gravity. The higher-dimensional operators are the analogue of the contact interactions described in ref. [13]. Of particular interest is the dimension-8 operator mediated by tree-level graviton exchange [7,11,14]

$$\mathcal{L}_{\rm int} = \pm \frac{4\pi}{\Lambda_T^4}\ \mathcal{T}, \qquad \mathcal{T} = \frac{1}{2}\left(T_{\mu\nu}T^{\mu\nu} - \frac{1}{\delta+2} T^\mu_\mu T^\nu_\nu\right), \qquad (5)$$

where $T_{\mu\nu}$ is the energy momentum tensor. (There exist several alternate definitions in the literature for the cutoff in Eq. (5) including M_{TT} used in the Listings, where $M_{TT}^4 = (2/\pi)\Lambda_T^4$.)

This operator gives anomalous contributions to many high-energy processes. The 95% CL limit from Bhabha scattering and diphoton production at LEP is [15] $\Lambda_T > 1.29\ (1.12)$ TeV for constructive (destructive) interference, corresponding to the $\pm$ signs in Eq. (5). The analogous limit from Drell-Yan and diphotons at Tevatron is [16] $\Lambda_T > 1.43\ (1.27)$ TeV.

Graviton loops can be even more important than tree-level exchange, because they can generate operators of dimension lower than 8. For simple graviton loops, there is only one dimension-6 operator that can be generated (excluding Higgs fields in the external legs) [18,19],

$$\mathcal{L}_{\rm int} = \pm\frac{4\pi}{\Lambda_\Upsilon^2}\ \Upsilon, \qquad \Upsilon = \frac{1}{2}\left(\sum_{f=q,\ell}\bar{f}\gamma_\mu\gamma_5 f\right)^2 . \tag{6}$$

Here the sum extends over all quarks and leptons in the theory. The 95% CL combined LEP limit [20] from lepton-pair processes is $\Lambda_\Upsilon > 17.2\ (15.1)$ TeV for constructive (destructive) interference, and $\Lambda_\Upsilon > 15.3\ (11.5)$ TeV is obtained from $\bar{b}b$ production. Limits from graviton emission and effective operators cannot be compared in a model-independent way, unless one introduces some well-defined cutoff procedure (see, *e.g.,* Ref. 19).

II.3 The Transplanckian Regime

The use of linearized Einstein gravity, discussed in Sec. II.2, is valid for processes with typical center-of-mass energy $\sqrt{s} \ll M_D$. The physics at $\sqrt{s} \sim M_D$ can be described only with knowledge of the underlying quantum-gravity theory. Toy models have been used to mimic possible effects of string theory at colliders [21]. Once we access the transplanckian region $\sqrt{s} \gg M_D$, a semiclassical description of the scattering process becomes adequate. Indeed, in the transplanckian limit, the Schwarzschild radius for a colliding system with center-of-mass energy $\sqrt{s}$ in $D = 4 + \delta$ dimensions,

$$R_S = \left[\frac{2^\delta \pi^{(\delta-3)/2}}{\delta+2}\Gamma\left(\frac{\delta+3}{2}\right)\frac{\sqrt{s}}{M_D^{\delta+2}}\right]^{1/(\delta+1)} , \tag{7}$$

is larger than the D-dimensional Planck length M_D^{-1}. Therefore, quantum-gravity effects are subleading with respect to classical gravitational effects (described by R_S).

If the impact parameter b of the process satisfies $b \gg R_S$, the transplanckian collision is determined by linear semiclassical gravitational scattering. The corresponding cross sections have been computed [22] in the eikonal approximation, valid in the limit of small deflection angle. The collider signal at the LHC is a dijet final state, with features characteristic of gravity in extra dimensions.

When $b < R_S$, we expect gravitational collapse and black-hole formation [23,24] (see Ref. 25 and references therein). The black-hole production cross section is estimated to be of order the geometric area $\sigma \sim \pi R_S^2$. This estimate has large uncertainties due, for instance, to the unknown amount of gravitational radiation emitted during collapse. Nevertheless, for M_D close to the weak scale, the black-hole production rate at the LHC is large. For example, the production cross section of 6 TeV black holes is about 10 pb, for $M_D = 1.5$ TeV. The produced black-hole emits thermal radiation with Hawking temperature $T_H = (\delta+1)/(4\pi R_S)$ until it reaches the Planck phase (where quantum-gravity effects become important). A black hole of initial mass M_{BH} completely evaporates with lifetime $\tau \sim M_{BH}^{(\delta+3)/(\delta+1)}/M_D^{2(\delta+2)/(\delta+1)}$, which typically is 10^{-26}–10^{-27} s for $M_D = 1$ TeV. The black hole can be easily detected because it emits a significant fraction of visible (*i.e.,* non-gravitational) radiation, although the precise amount is not known in the general case of D dimensions. Computations exist [26] for the grey-body factors, which describe the distortion of the emitted radiation from pure black-body caused by the strong gravitational background field.

To trust the semiclassical approximation, the typical energy of the process has to be much larger than M_D. Given the present constraints on extra-dimensional gravity, it is clear that the maximum energy available at the LHC allows, at best, to only marginally access the transplanckian region. If gravitational scattering and black-hole production are observed at the LHC, it is likely that significant quantum-gravity (or string-theory) corrections will affect the semiclassical calculations or estimates. In the context of string theory, it is possible that the production of string-balls [27] dominates over black holes.

If M_D is around the TeV scale, transplanckian collisions would regularly occur in the interaction of high-energy cosmic rays with the earth's atmosphere and could be observed in present and future cosmic ray experiments [28,29].

II.4 Graviscalars

After compactification, the D-dimensional graviton contains KK towers of spin-2 gravitational states (as discussed above), of spin-1 "graviphoton" states, and of spin-0 "graviscalar" states. In most processes, the graviphotons and graviscalars are much less important than their spin-2 counterparts. A single graviscalar tower is coupled to SM fields through the trace of the energy momentum tensor. The resulting coupling is, however, very weak for SM particles with small masses.

Perhaps the most accessible probe of the graviscalars would be through their allowed mixing with the Higgs boson [30] in the induced curvature-Higgs term of the 4-dimensional action. This can be recast as a contribution to the decay width of the SM Higgs boson into an invisible channel. Although the invisible branching fraction is a free parameter of the theory, it is more likely to be important when the SM Higgs boson width is particularly narrow ($m_H \lesssim 140\,\mathrm{GeV}$). The collider phenomenology of invisibly decaying Higgs bosons investigated in the literature is applicable here (see Ref. 31 and references therein).

II.5 Tests of the Gravitational Force Law

The theoretical developments in gravity with large extra dimensions have further stimulated interest in experiments looking for possible deviations from the gravitational inverse-square law (for a review, see Ref. 32). Such deviations are usually parametrized by a modified Newtonian potential of the form

$$V(r) = -G_N \frac{m_1 m_2}{r} \left[1 + \alpha \exp\left(-r/\lambda\right)\right] . \qquad (8)$$

The experimental limits on the parameters α and λ are summarized in Fig. 1, taken from Ref. [33].

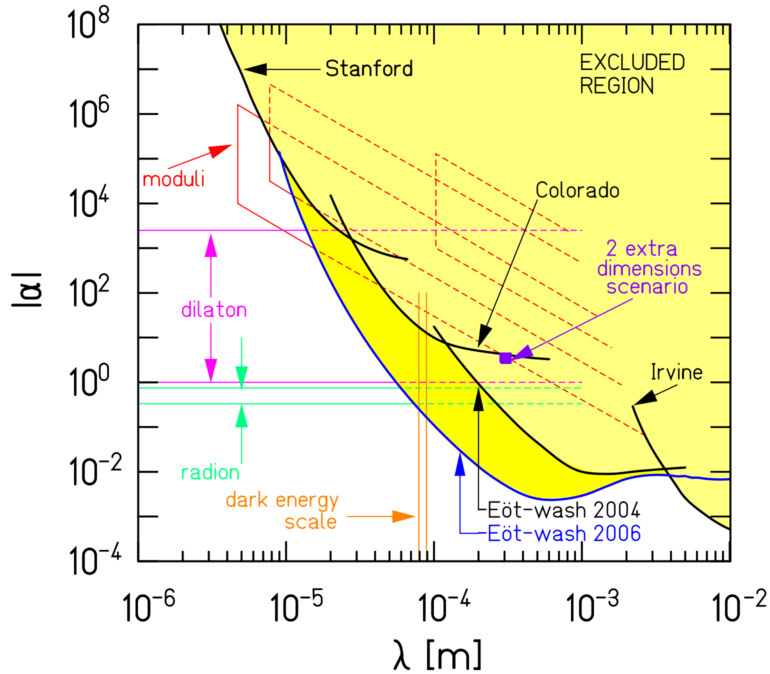

Figure 1: Experimental limits on α and λ of Eq. (8), which parametrize deviations from Newton's law. From Ref. [33].

For gravity with δ extra dimensions, in the case of toroidal compactifications, the parameter α is given by $\alpha = 8\ \delta/3$ and λ is the Compton wavelength of the first graviton Kaluza-Klein mode, equal to the radius R. From the results shown in Fig. 1, one finds $R < 37$ (44) μm at 95% CL for $\delta = 2$ (1) which, using Eq. (3), becomes $M_D > 3.6$ TeV for $\delta = 2$. This bound is weaker than the astrophysical bounds discussed in Sec. II.6, which actually exclude the occurence of any visible signal in planned tests of Newton's law. However, in the context of higher-dimensional theories, other particles like light gauge bosons, moduli or radions could mediate detectable modifications of Newton's law, without running up against the astrophysical limits.

II.6 Astrophysical Bounds

Because of the existence of the light and weakly-coupled KK gravitons, gravity in extra dimensions is strongly constrained by several astrophysical considerations (see Ref. [34] and references therein). The requirement that KK gravitons do not carry away more than half of the energy emitted by the supernova SN1987A gives the bounds [35] $M_D > 14$ (1.6) TeV for $\delta = 2$ (3). KK gravitons produced by all supernovæ in the universe lead to a diffuse γ ray background generated by graviton decays into photons. Measurements by the EGRET satellite imply [36] $M_D > 38$ (4.1) TeV for $\delta = 2$ (3). Most of the KK gravitons emitted by supernova remnants and neutron stars are gravitationally trapped. The gravitons forming this halo occasionally decay, emitting photons. Limits on γ rays from neutron-star sources imply [34] $M_D > 200$ (16) TeV for $\delta = 2$ (3). The decay products of the gravitons forming the halo can hit the surface of the neutron star, providing a heat source. The low measured luminosities of some pulsars imply [34] $M_D > 750$ (35) TeV for $\delta = 2$ (3). These bounds are valid only if the graviton KK mass spectrum below about 100 MeV is not modified by distortions of the compactification space (see Sec. II.1).

III Gravity in Warped Extra Dimensions

III.1 Theoretical Setup

In the proposal of Ref. 2, the $M_{\rm W}$–$M_{\rm Pl}$ hierarchy is explained using an extra-dimensional analogy of the classical gravitational redshift in curved space, as we illustrate below. The setup consists of a 5-dimensional space in which the fifth dimension is compactified on S^1/Z_2, *i.e.,* a circle projected into a segment by identifying points of the circle opposite with respect to a given diameter. Each end-point of the segment (the "fixed-points" of the orbifold projection) is the location of a 3-dimensional brane. The two branes have equal but opposite tensions. We will refer to the negative-tension brane as the infrared (IR) brane, where SM fields are assumed to be localized, and the positive-tension brane as the ultraviolet (UV) brane. The bulk cosmological constant is fine-tuned such that the effective cosmological constant in the 3-dimensional space exactly cancels.

The solution of the Einstein equation in vacuum gives the metric corresponding to the line element

$$ds^2 = \exp\left(-2k|y|\right)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2. \tag{9}$$

Here y is the $5^{\rm th}$ coordinate, with the UV and IR branes located at $y = 0$ and $y = \pi R$, respectively; R is the compactification radius and k is the AdS curvature. The 4-dimensional metric in Eq. (9) is modified with respect to the flat Minkowski metric $\eta_{\mu\nu}$ by the factor $\exp(-2k|y|)$. This shows that the 5-dimensional space is not factorized, meaning that the 4-dimensional metric depends on the extra-dimensional coordinate y. This feature is key to the desired effect.

As is known from general relativity, the energy of a particle travelling through a gravitational field is redshifted by an amount proportional to $|g_{00}|^{-1/2}$, where g_{00} is the time-component of the metric. Analogously, energies (or masses) viewed on the IR brane ($y = \pi R$) are red-shifted with respect to their values at the UV brane ($y = 0$) by an amount equal to the warp factor $\exp(-\pi kR)$, as shown by Eq. (9):

$$m_{IR} = m_{UV}\exp\left(-\pi kR\right). \tag{10}$$

A mass $m_{UV} \sim \mathcal{O}(\overline{M}_{\rm Pl})$ on the UV brane corresponds to a mass on the IR brane with a value $m_{IR} \sim \mathcal{O}(M_{\rm W})$, if $R \simeq 12k^{-1}$. A radius moderately larger than the fundamental scale k is therefore sufficient to reproduce the large hierarchy between the Planck and Fermi scales. A simple and elegant mechanism to stabilize the radius exists [38], by adding a scalar particle with a bulk mass and different potential terms on the two branes.

The effective theory describing the interaction of the KK modes of the graviton is characterized by two mass parameters, which we take to be m_1 and Λ_π. Both are a warp-factor smaller than the UV scale, and therefore they are naturally of order the weak scale. The parameter m_1 is the mass of the first KK graviton mode, from which the mass m_n of the generic $n^{\rm th}$ mode is determined,

$$m_n = \frac{x_n}{x_1}m_1. \tag{11}$$

Here x_n is the $n^{\rm th}$ root of the Bessel function J_1 ($x_1 = 3.83$, $x_2 = 7.02$ and, for large n, $x_n = (n + 1/4)\pi$). The parameter Λ_π determines the strength of the coupling of the KK gravitons $h^{(n)}_{\mu\nu}$ with the energy momentum tensor $T_{\mu\nu}$,

$$\mathcal{L} = -\frac{T^{\mu\nu}}{\overline{M}_{\rm Pl}} h^{(0)}_{\mu\nu} - \frac{T^{\mu\nu}}{\Lambda_\pi} \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}. \tag{12}$$

In the approach discussed in Sec. II.1, $M_{\rm Pl}$ appears to us much larger than the weak scale because gravity is diluted in a large space. In the approach described in this section, the explanation lies instead in the non-trivial configuration of the gravitational field: the zero-mode graviton wavefunction is peaked around the UV brane and it has an exponentially small overlap with the IR brane where we live. The extra dimensions discussed in Sec. II.1 are large and "nearly flat;" the graviton excitations are very weakly coupled and have a mass gap that is negligibly small in collider experiments. Here, instead, the gravitons have a mass gap of $\sim$ TeV size and become strongly-coupled at the weak scale.

III.2 Collider Signals

The KK excitations of the graviton, possibly being of order the TeV scale, are subject to experimental discovery at high-energy colliders. As discussed above, KK graviton production cross-sections and decay widths are set by the first KK mass m_1 and the graviton-matter interaction scale Λ_π. Some studies use m_1 and k as the independent parameters, and so it is helpful to keep in mind that the relationships between all of these parameters are

$$\frac{m_n}{\Lambda_\pi} = \frac{k x_n}{\overline{M}_{\rm Pl}}, \qquad \Lambda_\pi = \overline{M}_{\rm Pl} \exp(-\pi k R), \tag{13}$$

where again x_n are the zeros of the J_1 Bessel function. Resonant and on-shell production of the $n^{\rm th}$ KK gravitons leads to characteristic peaks in the dilepton and diphoton invariant-mass spectra and it is probed at colliders for $\sqrt{s} \geq m_n$. Current limits from dimuon, dielectron, and diphoton channels at CDF and DØ give the 95% CL limits $\Lambda_\pi > 4.3(2.6)$ TeV for $m_1 = 500(700)$ GeV [16,17].

Contact interactions arising from integrating out heavy KK modes of the graviton generate the dimension-8 operator $\mathcal{T}$, analogous to the one in Eq. (5) in the flat extra dimensions case. Although searches for effects of these non-renormalizable operators cannot confirm directly the existence of a heavy spin-2 state, they nevertheless provide a good probe of the model [39,40].

Searches for direct production of KK excitations of the graviton and contact interactions induced by gravity in compact extra-dimensional warped space will continue at the LHC. With the large increase in energy, one expects prime regions of the parameter space up to $m_n, \Lambda_\pi \sim 10\,\mathrm{TeV}$ [39] to be probed.

If SM states are in the AdS bulk, KK graviton phenomenology becomes much more model dependent. Present limits and future collider probes of the masses and interaction strengths of the KK gravitons to matter fields are significantly reduced [41] in some circumstances, and each specific model of SM fields in the AdS bulk should be analyzed on a case-by-case basis.

For warped metrics, black-hole production is analogous to the case discussed in Sec. II.3, as long the radius of the black hole is smaller than the AdS radius $1/k$, when the space is effectively flat. For heavier black holes, the production cross section is expected to grow with energy only as $\log^2 E$, saturating the Froissart bound [37].

III.3 The Radion

The size of the warped extra-dimensional space is controlled by the value of the radion, a scalar field corresponding to an overall dilatation of the extra coordinates. Stabilizing the radion is required for a viable theory, and known stabilization mechanisms often imply that the radion is less massive than the KK excitations of the graviton [38], thus making it perhaps the lightest beyond-the-SM particle in this scenario.

The coupling of the radion r to matter is $\mathcal{L} = -rT/\Lambda_\varphi$, where T is the trace of the energy momentum tensor and $\Lambda_\varphi = \sqrt{24}\Lambda_\pi$ is expected to be near the weak scale. The relative couplings of r to the SM fields are similar to, but not exactly the same as, those of the Higgs boson. The partial widths are generally smaller by a factor of v/Λ_φ compared to

SM Higgs decay widths, where $v = 246\,\mathrm{GeV}$ is the vacuum expectation value of the SM Higgs doublet. On the other hand, the trace anomaly that arises in the SM gauge groups by virtue of quantum effects enhances the couplings of the radion to gluons and photons over the naive v/Λ_φ rescaling of the Higgs couplings to these same particles. Thus, for example, one finds that the radion's large coupling to gluons [30,43] enables a sizeable $gg \to r$ cross section even for Λ_φ large compared to m_W.

Another subtlety of the radion is its ability to mix with the Higgs boson through the curvature-scalar interaction [30],

$$S_{mix} = -\xi \int d^4x \sqrt{-\det g_{\rm ind}} R(g_{\rm ind}) H^+ H \ , \qquad (14)$$

where $g_{\rm ind}$ is the four-dimensional induced metric. With $\xi \neq 0$, there is neither a pure Higgs boson nor pure radion mass eigenstate. Mixing between states enables decays of the heavier eigenstate into lighter eigenstates if kinematically allowed. Overall, the production cross sections, widths and relative branching fractions can all be affected significantly by the value of the mixing parameter ξ [30,42,43,44]. Despite the various permutations of couplings and branching fractions that the radion and the Higgs-radion mixed states can have into SM particles, the search strategies for these particles at high-energy colliders are similar to those of the SM Higgs boson.

IV Standard Model Fields in Flat Extra Dimensions

IV.1 TeV-Scale Compactification

Not only gravity, but also SM fields could live in an experimentally accessible higher-dimensional space [45]. This hypothesis could lead to unification of gauge couplings at a low scale [46]. In contrast with gravity, these extra dimensions must be at least as small as about TeV^{-1} in order to avoid incompatibility with experiment. The canonical extra-dimensional space of this type is a $5^{\rm th}$ dimension compactified on the interval S^1/Z_2, where again the radius of the S^1 is denoted R, and the Z_2 symmetry identifies $y \leftrightarrow -y$ of the extra-dimensional coordinate. The two fixed points $y = 0$ and $y = \pi R$ define the end-points of the compactification interval.

Let us first consider the case in which gauge fields live in extra dimensions, while matter and Higgs fields are confined to a 3–brane. The masses M_n of the gauge-boson KK excitations are related to the masses M_0 of the zero-mode normal gauge bosons by

$$M_n^2 = M_0^2 + \frac{n^2}{R^2}. \tag{15}$$

The KK excitations of the vector bosons have couplings to matter a factor of $\sqrt{2}$ larger than the zero modes ($g_n = \sqrt{2}g$). Therefore, if the first KK excitation is $\sim$ TeV, tree-level virtual effects of the KK gauge bosons can have a significant effect on precision electroweak observables and high-energy processes such as $e^+e^- \to f\bar{f}$. In this theory one expects that observables will be shifted with respect to their SM value by a fractional amount proportional to [47]

$$V = 2\sum_n \left(\frac{g_n^2}{g^2}\right) \frac{M_Z^2 R^2}{n^2} \sim \frac{2}{3}\pi^2 M_Z^2 R^2 \ . \tag{16}$$

More complicated compactifications lead to more complicated representations of V. A global fit to all relevant observables, including precision electroweak data, Tevatron, HERA and LEP2 results, shows that $R^{-1} \gtrsim 6.8\,\mathrm{TeV}$ is required [48,49]. The LHC with $100\,\mathrm{fb}^{-1}$ integrated luminosity would be able to search nearly as high as $R^{-1} \sim 16\,\mathrm{TeV}$ [48].

Fermions can also be promoted to live in the extra dimensions. Although fermions are vector-like in 5-dimension, chiral states in 4-dimensions can be obtained by using the Z_2 symmetry of the orbifold. An interesting possibility to explain the observed spectrum of quark and lepton masses is to assume that different fermions are localized in different points of the extra dimension. Their different overlap with the Higgs wavefunction can generate a hierarchical structure of Yukawa couplings [50], although there are strong bounds on the non-universal couplings of fermions to the KK gauge bosons from flavor-violating processes [51].

The case in which all SM particles uniformly propagate in the bulk of an extra-dimensional space is referred to as Universal Extra Dimensions (UED) [52]. The absence of a reference brane that breaks translation invariance in the extra dimensional direction implies extra-dimensional momentum conservation. After compactification and after inclusion of boundary terms at the fixed points, the conservation law preserves only a discrete Z_2 parity (called KK-parity). The KK-parity of the n^{th} KK mode of each particle is $(-1)^n$. Thus, in UED, the first KK excitations can only be pair-produced and their virtual effect comes only from loop corrections. Therefore the ability to search for and constrain parameter space is diminished. The result is that for one extra dimension the limit on R^{-1} is between 300 and 500 GeV depending on the Higgs mass [53].

Because of KK-parity conservation, the lightest KK state is stable. Thus, one interesting consequence of UED is the possibility of the lightest KK state comprising the dark matter. After including radiative corrections [54], it is found that the lightest KK state is the first excitation of the hypercharge gauge boson $B^{(1)}$. It can constitute the cold dark matter of the universe if its mass is approximately 600 GeV [55], well above current collider limits. The LHC should be able to probe UED up to $R^{-1} \sim 1.5\,\text{TeV}$ [56], and thus possibly confirm the UED dark matter scenario.

An interesting and ambitious approach is to use extra dimensions to explain the hierarchy problem through Higgs-gauge unification [57]. The SM Higgs doublet is interpreted as the extra-dimensional component of an extended gauge symmetry acting in more than four dimensions, and the weak scale is protected by the extra-dimensional gauge symmetry. There are several obstacles to make this proposal fully realistic, but ongoing research is trying to overcome them.

IV.2 Grand Unification in Extra Dimensions

Extra dimensions offer a simple and elegant way to break GUT symmetries [58] by appropriate field boundary conditions in compactifications on orbifolds. In this case the size of the relevant extra dimensions is much smaller than what has been considered so far, with compactification radii that are typically $\mathcal{O}(M_{\text{GUT}})$. This approach has several attractive features (for a review, see Ref. 59). The doublet-triplet splitting problem [60] is solved by projecting out the unwanted light Higgs triplet in the compactification. In the same way, one can eliminate the dangerous supersymmetric $d = 5$ proton-decay operators, or even forbid proton decay [61]. However, the prospects for proton-decay searches are not necessarily bleak. Because of the effect of the KK modes, the unification scale can be lowered to 10^{14}–10^{15} GeV, enhancing the effect of $d = 6$ operators. The prediction for the proton lifetime is model-dependent.

V Standard Model Fields in Warped Extra Dimensions

V.1 Extra Dimensions and Strong Dynamics at the Weak Scale

In the original warped model of Ref. 2, all SM fields are confined on the IR brane, although to solve the hierarchy problem it is sufficient that only the Higgs field lives on the brane. The variation in which SM fermions and gauge bosons are bulk fields is interesting because it links warped extra dimensions to technicolor-like models with strong dynamics at the weak scale. This connection comes from the AdS/CFT correspondence [62], which relates the properties of AdS_5, 5-dimensional gravity with negative cosmological constant, to a strongly-coupled 4-dimensional conformal field theory (CFT). In the correspondence, the motion along the 5^{th} dimension is interpreted as the renormalization-group flow of the 4-dimensional theory, with the UV brane playing the role of the Planck-mass cutoff and the IR brane as the breaking of the conformal invariance. Local gauge symmetries acting on the bulk of AdS_5 correspond to global symmetries of the 4-dimensional theory. The original warped model of Ref. 2 is then reinterpreted as an "almost CFT," whose couplings run very slowly with the renormalization scale until the TeV scale is reached, where the theory develops a mass gap. In the variation in which SM fields, other than the Higgs, are promoted to the bulk, these fields correspond to elementary particles coupled to the CFT. Around the TeV scale the theory becomes strongly-interacting, producing a composite Higgs, which breaks electroweak symmetry. Notice the similarity with walking technicolor [63].

The most basic version of this theory is in conflict with electroweak precision measurements. To reduce the contribution to the ρ parameter, it is necessary to introduce an approximate global symmetry, a custodial $SU(2)$ under which the generators of $SU(2)_L$ transform as a triplet. Using the AdS/CFT correspondence, this requires the extension of the electroweak gauge symmetry to $SU(2)_L \times SU(2)_R \times U(1)$ in the bulk of the 5-dimensional theory [64]. Models along these lines have been constructed. The composite Higgs can be lighter than the strongly-interacting scale in models in which it is a pseudo-Goldstone boson [65]. Nevertheless, electroweak data provide strong constraints on such models.

When SM fermions are promoted to 5 dimensions, they become non-chiral and can acquire a bulk mass. The fermions are localized in different positions along the 5^{th} dimension, with an exponential dependence on the value of the bulk mass (in units of the AdS curvature). Since the masses of the ordinary zero-mode SM fermions depend on their wavefunction overlap with the Higgs (localized on the IR brane), large hierarchies in the mass spectrum of quarks and leptons can be obtained from order-unity variations of the bulk masses [66]. This mechanism can potentially explain the fermion mass pattern, and it can lead to new effects in flavor-changing processes, especially those involving the third-generation quarks [67]. The smallness of neutrino masses can also be explained, if right-handed neutrinos propagate in the bulk [68].

V.2 Higgsless Models

Extra dimensions offer new possibilities for breaking gauge symmetries. Even in the absence of physical scalars, electroweak symmetry can be broken by field boundary conditions on compactified spaces. The lightest KK modes of the gauge bosons corresponding to broken generators acquire masses equal to R^{-1}, the inverse of the compactification radius, now to be identified with $M_{\rm W}$. In the ordinary 4-dimensional case, the SM without a Higgs boson violates unitarity at energies $E \sim 4\pi M_W/g \sim 1$ TeV. On the other hand, in extra dimensions, the breaking of unitarity in the longitudinal-W scattering amplitudes is delayed because of the contribution of the heavy KK gauge-boson modes [69]. The largest effect is obtained for one extra dimension, where the violation of unitarity occurs around $E \sim 12\pi^2 M_W/g \sim 10$ TeV. This is conceivably a large enough scale to render the strong dynamics, which is eventually responsible for unitarization, invisible to the processes measured by LEP experiments.

These Higgsless models, in their minimal version, are inconsistent with observations, because they predict new W gauge bosons with masses nM_{W} (with $n \geq 2$ integers) [70]. However, warping the 5^{th} dimension has a double advantage [71]. The excited KK modes of the gauge bosons can all have masses in the TeV range, making them compatible with present collider limits. Also, by enlarging the bulk gauge symmetry to $SU(2)_L \times SU(2)_R \times U(1)$, one can obtain an approximate custodial symmetry, as described above, to tame tree-level corrections to ρ. If quarks and leptons are extended to the bulk, they can obtain masses through the electroweak-breaking effect on the boundaries. However at present, there is no model that reproduces the top quark mass and is totally consistent with electroweak data [72].

VI. Supersymmetry in Extra Dimensions

Extra dimensions have a natural home within string theory. Similarly, string theory and supersymmetry are closely connected, as the latter is implied by the former in most constructions. Coexistence between extra dimensions and supersymmetry is often considered a starting point for string model building. From a low-energy model-building point of view, perhaps the most compelling reason to introduce extra dimensions with supersymmetry lies in the mechanism of supersymmetry breaking.

When the field periodic boundary conditions on the compactified space are twisted using an R-symmetry, different zero modes for bosons and fermions are projected out and supersymmetry is broken. This is known as the Scherk-Schwarz mechanism of supersymmetry breaking [73]. In the simplest approach [74], a 5^{th} dimension with $R^{-1} \sim 1\,\mathrm{TeV}$ is introduced in which the non-chiral matter (gauge and Higgs multiplets) live. The chiral matter (quark and lepton multiplets) live on the three-dimensional spatial boundary. S^1/Z_2 compactification of the 5^{th} dimension, which simultaneously employs the Scherk-Schwarz mechanism, generates masses for the bulk fields (gauginos and higgsinos) of order R^{-1}. Boundary states (squarks and sleptons) get mass from loop corrections, and are parametrically smaller in value. The right-handed slepton is expected to be the lightest supersymmetric particle (LSP), which being charged is not a good dark matter candidate. Thus, this theory likely requires R-parity violation in order to allow this charged LSP to decay and not cause cosmological problems.

By allowing all supersymmetric fields to propagate in the bulk of a $S_1/Z_2 \times Z_2'$ compactified space, it is possible to construct a model [75] taking advantage of the nonlocal breaking of supersymmetry. In this case, there are no quadratic divergences (except for a Fayet-Iliopoulos term [76]) and the Higgs mass is calculable. In the low-energy effective theory there is a single Higgs doublet, two superpartners for each SM particle, and the stop is the LSP, requiring a small amount of R-parity breaking.

Supersymmetry in warped space is also an interesting possibility. Again, one can consider [77] the case of chiral fields confined to our ordinary 3+1 dimensions, and gravity and gauge fields living in the 5-dimensional bulk space. Rather than being TeV^{-1} size, the 5^{th} dimension is strongly warped to generate the supersymmetry-breaking scale. In this case, the tree-level mass of the gravitino is $\sim 10^{-3}\,\text{eV}$ and the masses of the gauginos are $\sim$ TeV. The sleptons and squarks get mass at one loop from gauge interactions and thus are diagonal in flavor space, creating no additional FCNC problems. It has also been proposed [78] that an approximately supersymmetric Higgs sector confined on the IR brane could coexist with non-supersymmetric SM fields propagating in the bulk of the warped space.

In conclusion, we should reiterate that an important general consequence of extra dimensional theories is retained in supersymmetric extensions: KK excitations of the graviton and/or gauge fields are likely to be accessible at the LHC if the scale of compactification is directly related to solving the hierarchy problem. Any given extra-dimensional theory has many aspects to it, but the KK excitation spectrum is the most generic and most robust aspect of the idea to test in experiments.

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