9. QUANTUM CHROMODYNAMICS AND ITS COUPLING

9.1. The QCD Lagrangian

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Quantum Chromodynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons, is one of the components of the SU(3)×SU(2)×U(1) Standard Model. A quark of specific flavor (such as a charm quark) comes in 3 colors; gluons come in eight colors; hadrons are color-singlet combinations of quarks, anti-quarks, and gluons. The Lagrangian describing the interactions of quarks and gluons is (up to gauge-fixing terms)

$$\begin{aligned} L_{\mathrm{QCD}} = -\frac{1}{4} F^{(a)}_{\mu\nu}\, F^{(a)\mu\nu} + i \sum_q \overline{\psi}^i_q\, \gamma^\mu\, (D_\mu)_{ij}\, \psi^j_q \\ - \sum_q m_q\, \overline{\psi}^i_q\, \psi_{qi}\ , \end{aligned} \tag{9.1}$$

$$F^{(a)}_{\mu\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu - g_s\, f_{abc}\, A^b_\mu\, A^c_\nu\ , \tag{9.2}$$

$$(D_\mu)_{ij} = \delta_{ij}\, \partial_\mu + i g_s\, \sum_a \frac{\lambda^a_{i,j}}{2} A^a_\mu\ , \tag{9.3}$$

where g_s is the QCD coupling constant, and the f_{abc} are the structure constants of the SU(3) algebra (the λ matrices and values for f_{abc} can be found in "SU(3) Isoscalar Factors and Representation Matrices," Sec. 36 of this *Review*). The $\psi^i_q(x)$ are the 4-component Dirac spinors associated with each quark field of (3) color i and flavor q, and the $A^a_\mu(x)$ are the (8) Yang-Mills (gluon) fields. A complete list of the Feynman rules which derive from this Lagrangian, together with some useful color-algebra identities, can be found in Ref. 1.

The principle of "asymptotic freedom" determines that the renormalized QCD coupling is small only at high energies, and it is only in this domain that high-precision tests—similar to those in QED—can be performed using perturbation theory. Nonetheless, there has been in recent years much progress in understanding and quantifying the predictions of QCD in the nonperturbative domain, for example, in soft hadronic processes and on the lattice [2]. This short review will concentrate on QCD at short distances (large momentum transfers), where perturbation theory is the standard tool. It will discuss the processes that are used to determine the coupling constant of QCD. Other recent reviews of the coupling constant measurements may be consulted for a different perspective [3–6].

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9.2. The QCD coupling and renormalization scheme

The renormalization scale dependence of the effective QCD coupling $\alpha_s = g_s^2/4\pi$ is controlled by the β-function:

$$\mu\frac{\partial\alpha_s}{\partial\mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 - \frac{\beta_2}{64\pi^3}\alpha_s^4 - \cdots , \tag{9.4a}$$

$$\beta_0 = 11 - \frac{2}{3}n_f , \tag{9.4b}$$

$$\beta_1 = 51 - \frac{19}{3}n_f , \tag{9.4c}$$

$$\beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2 , \tag{9.4d}$$

where n_f is the number of quarks with mass less than the energy scale μ. The expression for the next term in this series (β_3) can be found in Ref. 8. In solving this differential equation for α_s, a constant of integration is introduced. This constant is the fundamental constant of QCD that must be determined from experiment in addition to the quark masses. The most sensible choice for this constant is the value of α_s at a fixed-reference scale μ_0. It has become standard to choose $\mu_0 = M_Z$. The value at other values of μ can be obtained from $\log(\mu^2/\mu_0^2) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}$. It is also convenient to introduce the dimensional parameter Λ, since this provides a parameterization of the μ dependence of α_s. The definition of Λ is arbitrary. One way to define it (adopted here) is to write a solution of Eq. (9.4) as an expansion in inverse powers of $\ln(\mu^2)$:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0\ \ln(\mu^2/\Lambda^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\ \frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4\ln^2(\mu^2/\Lambda^2)}\right.$$
$$\left.\times\left(\left(\ln\left[\ln(\mu^2/\Lambda^2)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right] . \tag{9.5}$$

This solution illustrates the *asymptotic freedom* property: $\alpha_s \to 0$ as $\mu \to \infty$ and shows that QCD becomes strongly coupled at $\mu \sim \Lambda$.

Consider a "typical" QCD cross section which, when calculated perturbatively [7], starts at $\mathcal{O}(\alpha_s)$:

$$\sigma = A_1\ \alpha_s + A_2\ \alpha_s^2 + \ \cdots\ . \tag{9.6}$$

The coefficients A_1, A_2 come from calculating the appropriate Feynman diagrams. In performing such calculations, various divergences arise, and these must be regulated in a consistent way. This requires a particular renormalization scheme (RS). The most commonly used one is the modified minimal subtraction ($\overline{\text{MS}}$) scheme [9]. This involves continuing momentum integrals from 4 to 4–2ϵ dimensions, and then subtracting off the resulting $1/\epsilon$ poles and also ($\ln 4\pi - \gamma_E$), which is an artifact of continuing the dimension. (Here γ_E is the Euler-Mascheroni constant.) To preserve the dimensionless nature of the coupling, a mass scale μ must also be introduced: $g \to \mu^\epsilon g$. The finite coefficients

A_i $(i \geq 2)$ thus obtained depend implicitly on the renormalization convention used and explicitly on the scale μ.

The first two coefficients (β_0, β_1) in Eq. (9.4) are independent of the choice of RS. In contrast, the coefficients of terms proportional to α_s^n for $n > 3$ *are* RS-dependent. The form given above for β_2 is in the $\overline{\text{MS}}$ scheme.

The fundamental theorem of RS dependence is straightforward. Physical quantities, such as the cross section calculated to all orders in perturbation theory, do not depend on the RS. It follows that a truncated series *does* exhibit RS dependence. In practice, QCD cross sections are known to leading order (LO), or to next-to-leading order (NLO), or in some cases, to next-to-next-to-leading order (NNLO); and it is only the latter two cases, which have reduced RS dependence, that are useful for precision tests. At NLO the RS dependence is completely given by one condition which can be taken to be the value of the renormalization scale μ. At NNLO this is not sufficient, and μ is no longer equivalent to a choice of scheme; both must now be specified. One, therefore, has to address the question of what is the "best" choice for μ within a given scheme, usually $\overline{\text{MS}}$. There is no definite answer to this question—higher-order corrections do not "fix" the scale, rather they render the theoretical predictions less sensitive to its variation.

One should expect that choosing a scale μ characteristic of the typical energy scale (E) in the process would be most appropriate. In general, a poor choice of scale generates terms of order $\ln(E/\mu)$ in the A_i's. Various methods have been proposed including choosing the scale for which the next-to-leading-order correction vanishes ("Fastest Apparent Convergence [10]"); the scale for which the next-to-leading-order prediction is stationary [11], (*i.e.*, the value of μ where $d\sigma/d\mu = 0$); or the scale dictated by the effective charge scheme [12] or by the BLM scheme [13]. By comparing the values of α_s that different reasonable schemes give, an estimate of theoretical errors can be obtained. It has also been suggested to replace the perturbation series by its Padé approximant [14]. Results obtained using this method have, in certain cases, a reduced scale dependence [15,16]. One can also attempt to determine the scale from data by allowing it to vary and using a fit to determine it. This method can allow a determination of the error due to the scale choice and can give more confidence in the end result [17]. In many of the cases discussed below this scale uncertainty is the dominant error.

An important corollary is that if the higher-order corrections are naturally small, then the additional uncertainties introduced by the μ dependence are likely to be small. There are some processes, however, for which the choice of scheme *can* influence the extracted value of $\alpha_s(M_Z)$. There is no resolution to this problem other than to try to calculate even more terms in the perturbation series. It is important to note that, since the perturbation series is an asymptotic expansion, there is a limit to the precision with which any theoretical quantity can be calculated. In some processes, the highest-order perturbative terms may be comparable in size to nonperturbative corrections (sometimes called higher-twist or renormalon effects, for a discussion see Ref. 18); an estimate of these terms and their uncertainties is required if a value of α_s is to be extracted.

Cases occur where there is more than one large scale, say μ_1 and μ_2. In these cases, terms appear of the form $\log(\mu_1/\mu_2)$. If the ratio μ_1/μ_2 is large, these logarithms can render naive perturbation theory unreliable and a modified perturbation expansion that takes these terms into account must be used. A few examples are discussed below.

In the cases where the higher-order corrections to a process are known and are large, some caution should be exercised when quoting the value of α_s. In what follows, we will attempt to indicate the size of the theoretical uncertainties on the extracted value of α_s. There are two simple ways to determine this error. First, we can estimate it by comparing the value of $\alpha_s(\mu)$ obtained by fitting data using the QCD formula to highest known order in α_s, and then comparing it with the value obtained using the next-to-highest-order formula (μ is chosen as the typical energy scale in the process). The corresponding Λ's are then obtained by evolving $\alpha_s(\mu)$ to $\mu = M_Z$ using Eq. (9.4) to the same order in α_s as the fit. Alternatively, we can vary the value of μ over a reasonable range, extracting a value of Λ for each choice of μ. This method is by its nature imprecise, since "reasonable" involves a subjective judgment. In either case, if the perturbation series is well behaved, the resulting error on $\alpha_s(M_Z)$ will be small.

In the above discussion we have ignored quark-mass effects, *i.e.*, we have assumed an idealized situation where quarks of mass greater than μ are neglected completely. In this picture, the β-function coefficients change by discrete amounts as flavor thresholds (a quark of mass M) are crossed when integrating the differential equation for α_s. Now imagine an experiment at energy scale μ; for example, this could be $e^+e^- \to$ hadrons at center-of-mass energy μ. If $\mu \gg M$, the mass M is negligible and the process is well described by QCD with n_f massless flavors and its parameter $\alpha_{(n_f)}$ up to terms of order M^2/μ^2. Conversely if $\mu \ll M$, the heavy quark plays no role and the process is well described by QCD with $n_f - 1$ massless flavors and its parameter $\alpha_{(n_f-1)}$ up to terms of order μ^2/M^2. If $\mu \sim M$, the effects of the quark mass are process-dependent and cannot be absorbed into the running coupling. The values of $\alpha_{(n_f)}$ and $\alpha_{(n_f-1)}$ are related so that a physical quantity calculated in both "theories" gives the same result [19]. This implies, for $\mu = M$

$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2}\alpha^3_{(n_f-1)}(M) + \mathcal{O}\left(\alpha^4_{(n_f-1)}\right) \tag{9.7}$$

which is almost identical to the naive result $\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M)$. Here M is the mass of the value of the running quark mass defined in the $\overline{\mathrm{MS}}$ scheme (see the note on "Quark Masses" in the Particle Listings for more details), *i.e.*, where $M_{\overline{\mathrm{MS}}}(M) = M$.

It also follows that, for a relationship such as Eq. (9.5) to remain valid for all values of μ, Λ must also change as flavor thresholds are crossed, the value corresponds to an effective number of massless quarks: $\Lambda \to \Lambda^{(n_f)}$ [19,20]. The formulae are given in the 1998 edition of this review.

Experiments such as those from deep-inelastic scattering involve a range of energies. After fitting to their measurements to QCD, the resulting fit can be expressed as a value of $\alpha_s(M_Z)$.

Determinations of α_s result from fits to data using NLO and NNLO: LO fits are not useful and their values will not be used in the following. Care must be exercised when comparing results from NLO and NNLO. In order to compare the values of α_s from various experiments, they must be evolved using the renormalization group to a common scale. For convenience, this is taken to be the mass of the Z boson. The extrapolation is performed using same order in perturbation theory as was used in the analysis. This evolution uses third-order perturbation theory and can introduce additional errors particularly if extrapolation from very small scales is used. The variation in the charm and bottom quark masses ($M_b = 4.3 \pm 0.2$ GeV and $M_c = 1.3 \pm 0.3$ GeV are used [21]) can also introduce errors. These result in a fixed value of $\alpha_s(2\ \text{GeV})$ giving an uncertainty in $\alpha_s(M_Z) = \pm 0.001$ if only perturbative evolution is used. There could be additional errors from nonperturbative effects that enter at low energy.

9.3. QCD in deep-inelastic scattering

The original and still one of the most powerful quantitative tests of perturbative QCD is the breaking of Bjorken scaling in deep-inelastic lepton-hadron scattering. The review 'Structure Functions," (Sec. 16 of this *Review*) describes the basic formalism and reviews the data. α_s is obtained together with the structure functions. The global fit from MRST04 [13] of (Sec. 16) gives $\alpha_s(M_Z) = 0.1205 \pm 0.004$ from NLO and $\alpha_s(M_Z) = 0.1167 \pm 0.004$ from NNLO. Other fits are consistent with these values but cannot be averaged as they use overlapping data sets. The good agreement between the NLO and NNLO fits indicates that the theoretical uncertainties are under control.

Nonsinglet structure functions offer in principle the most precise test of the theory and cleanest way to extract α_s, since the Q^2 evolution is independent of the gluon distribution, which is much more poorly constrained. The CCFR collaboration fit to the Gross-Llewellyn Smith sum rule [23] whose value at leading order is determined by baryon number and which is known to order α_s^3 [24,25] (NNLO); estimates of the order α_s^4 term are available [26].

$$\int_0^1 dx \left(F_3^{\overline{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2) \right) = \\ 3 \left[1 - \frac{\alpha_s}{\pi}(1 + 3.58\frac{\alpha_s}{\pi} + 19.0 \left(\frac{\alpha_s}{\pi})^2\right) - \Delta HT \right] \quad , \tag{9.8}$$

where the higher-twist contribution ΔHT is estimated to be $(0.09 \pm 0.045)/Q^2$ in [24,27] and to be somewhat smaller by [28]. The CCFR collaboration [29], combines their data with that from other experiments [30] and gives $\alpha_s\ (\sqrt{3}\ \text{GeV}) = 0.28 \pm 0.035$ (expt.) $\pm\, 0.05$ (sys) ${}^{+0.035}_{-0.03}$ (theory). The error from higher-twist terms (assumed to be $\Delta HT = 0.05 \pm 0.05$) dominates the theoretical error. If the higher twist result of [28] is used, the central value increases to 0.31 in agreement with the fit of [31]. This value corresponds to $\alpha_s(M_Z) = 0.118 \pm 0.011$. Fits of the Q^2 evolution [32] of xF_3 using the CCFR data using NNLO and estimates of NNNLO QCD and higher twist terms enables the effect of these terms to be studied.

The spin-dependent structure functions, measured in polarized lepton-nucleon scattering, can also be used to determine α_s. Note that these experiments measure asymmetries and rely on measurements of unpolarized data to extract the spin-dependent structure functions. Here the values of $Q^2 \sim 2.5\ \text{GeV}^2$ are small, particularly for the

E143 data [33], and higher-twist corrections are important. A fit [34] by an experimental group using the measured spin dependent structure functions for several experiments [33,35] as well as their own data has been made. When data from HERMES [36] and SMC are included [37] $\alpha_s(M_Z) = 0.120 \pm 0.009$ is obtained: this is used in the final average.

α_s can also be determined from the Bjorken spin sum rule [38]; a fit gives $\alpha_s(M_Z) = 0.118^{+0.010}_{-0.024}$ [39]; consistent with an earlier determination [40], the larger error being due to the extrapolation into the (unmeasured) small x region. Theoretically, the sum rule is preferable as the perturbative QCD result is known to higher order and these terms are important at the low Q^2 involved. It has been shown that the theoretical errors associated with the choice of scale are considerably reduced by the use of Padé approximants [15], which results in $\alpha_s(1.7\ \mathrm{GeV}) = 0.328 \pm 0.03$ (expt.) $\pm\, 0.025$ (theory) corresponding to $\alpha_s(M_Z) = 0.116\ ^{+0.003}_{-0.005}$ (expt.) $\pm\, 0.003$ (theory). No error is included from the extrapolation into the region of x that is unmeasured. Should data become available at smaller values of x so that this extrapolation could be more tightly constrained. This result is not used in the final average.

9.4. QCD in decays of the τ lepton

The semi-leptonic branching ratio of the tau ($\tau \to \nu_\tau +$ hadrons, R_τ) is an inclusive quantity. It is related to the contribution of hadrons to the imaginary part of the W self energy $\big(\Pi(s)\big)$. It is sensitive to a range of energies since it involves an integral

$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}(1 - \frac{s}{m_\tau^2})^2 \Big((1 + 2s/m_\tau^2) Im\Pi(1) + Im\Pi(0)\Big) \tag{9.9}$$

where $Im\Pi(1)$ denotes the vector part and $Im\Pi(0)$ the scalar part. Since the scale involved is low, one must take into account nonperturbative (higher-twist) contributions which are suppressed by powers of the τ mass.

$$\begin{aligned} R_\tau =& 3.058\Big[1 + \delta_{EW} + \frac{\alpha_s(m_\tau)}{\pi} + 5.2\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^2 + 26.4\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^3 \\ &+ a\frac{m^2}{m_\tau^2} + b\frac{m\psi\overline{\psi}}{m_\tau^4} + c\frac{\psi\overline{\psi}\psi\overline{\psi}}{m_\tau^6} + \cdots\Big] \ . \end{aligned} \tag{9.10}$$

$\delta_{EW} = 0.0010$ is the electroweak correction. Here a, b, and c are dimensionless constants and m is a light quark mass. The term of order $1/m_\tau^2$ is a kinematical effect due to the light quark masses and is consequently very small. The nonperturbative terms are estimated using sum rules [41]. In total, they are estimated to be -0.014 ± 0.005 [42,43]. This estimate relies on there being no term of order Λ^2/m_τ^2 $\left(\text{note that } \frac{\alpha_s(m_\tau)}{\pi} \sim (\frac{0.5\ \mathrm{GeV}}{m_\tau})^2\right)$. The a, b, and c can be determined from the data [44] by fitting to moments of the $\Pi(s)$ and separately to the final states accessed by the vector and axial parts of the W coupling. The values so extracted [45,46] are

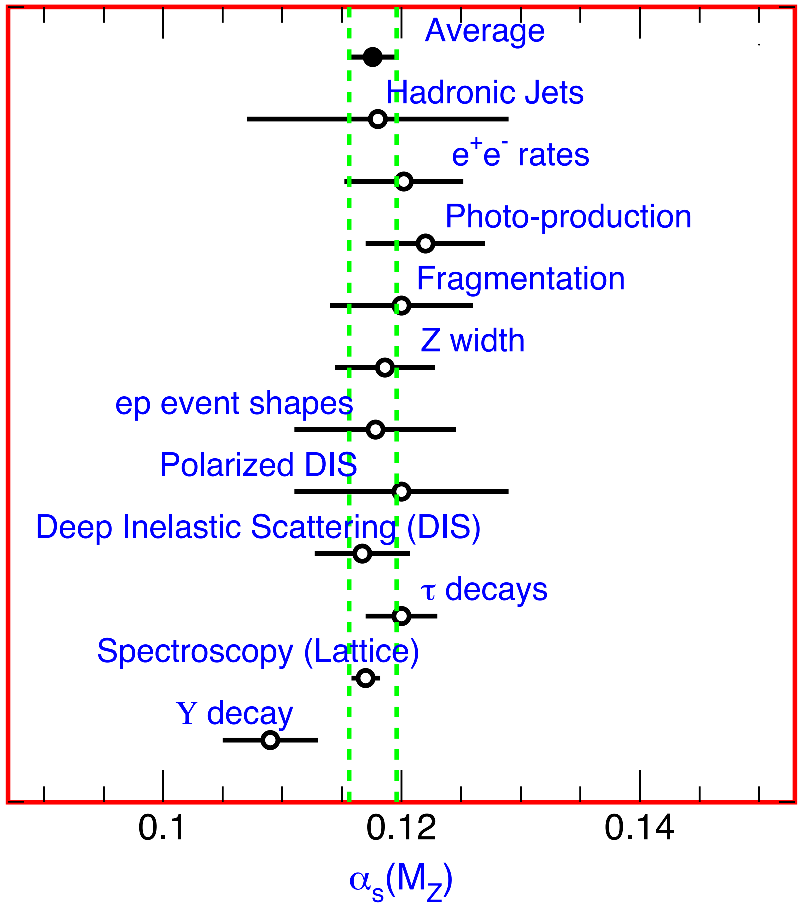

Figure 9.1: Summary of the value of $\alpha_s(M_Z)$ from various processes. The values shown indicate the process and the measured value of α_s extrapolated to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties. The average quoted in this report which comes from these measurements is also shown. See text for discussion of errors.

consistent with the theoretical estimates. If the nonperturbative terms are omitted from the fit, the extracted value of $\alpha_s(m_\tau)$ decreases by ~ 0.02.

For $\alpha_s(m_\tau) = 0.35$ the perturbative series for R_τ is $R_\tau \sim 3.058(1+0.112+0.064+0.036)$. The size (estimated error) of the nonperturbative term is 20% (7%) of the size of the order α_s^3 term. The perturbation series is not very well convergent; if the order α_s^3 term is omitted, the extracted value of $\alpha_s(m_\tau)$ increases by 0.05. The order α_s^4 term has been estimated [47] and attempts made to resum the entire series [48,49]. These estimates can be used to obtain an estimate of the errors due to these unknown terms [50,51]. Another approach to estimating this α_s^4 term gives a contribution that is slightly larger than the α_s^3 term [52].

R_τ can be extracted from the semi-leptonic branching ratio from the relation $R_\tau = 1/\mathrm{B}(\tau \to e\nu\overline{\nu}) - 1.97256$; where $\mathrm{B}(\tau \to e\nu\overline{\nu})$ is measured directly or extracted from the lifetime, the muon mass, and the muon lifetime assuming universality of lepton couplings. Using the average lifetime of 290.6 ± 1.1 fs and a τ mass of 1776.99 ± 0.29

MeV from the PDG fit gives $R_\tau = 3.645 \pm 0.020$. The direct measurement of $\mathrm{B}(\tau \to e\nu\overline{\nu})$ can be combined with $\mathrm{B}(\tau \to \mu\nu\overline{\nu})$ to give $\mathrm{B}(\tau \to e\nu\overline{\nu}) = 0.1785 \pm 0.0005$ which gives $R_\tau = 3.629 \pm 0.015$. Averaging these yields $\alpha_s(m_\tau) = 0.338 \pm 0.004$ using the experimental error alone. We assign a theoretical error equal to 40% of the contribution from the order α^3 term and all of the nonperturbative contributions. This then gives $\alpha_s(m_\tau) = 0.34 \pm 0.03$ for the final result. This corresponds to $\alpha_s(M_Z) = 0.120 \pm 0.003$. This result is consistent with that obtained by using the moments [53] of the integrand and is used in the average below.

9.5. QCD in high-energy hadron collisions

There are many ways in which perturbative QCD can be tested in high-energy hadron colliders. The quantitative tests are only useful if the process in question has been calculated beyond leading order in QCD perturbation theory. The production of hadronic jets with large transverse momentum in hadron-hadron collisions provides a direct probe of the scattering of quarks and gluons: $qq \to qq$, $qg \to qg$, $gg \to gg$, *etc.* Higher–order QCD calculations of the jet rates [54] and shapes are in impressive agreement with data [55]. This agreement has led to the proposal that these data could be used to provide a determination of α_s [56]. A set of structure functions is assumed and jet data are fitted over a very large range of transverse momenta to the QCD prediction for the underlying scattering process that depends on α_s. The evolution of the coupling over this energy range (40 to 250 GeV) is therefore tested in the analysis. CDF obtains $\alpha_s(M_Z) = 0.1178 \pm 0.0001$ (stat.) ± 0.0085 (syst.) [57]. Estimation of the theoretical errors is not straightforward. The structure functions used depend implicitly on α_s and an iteration procedure must be used to obtain a consistent result; different sets of structure functions yield different correlations between the two values of α_s. CDF includes a scale error of 4% and a structure function error of 5% in the determination of α_s. Ref. 56 estimates the error from unknown higher order QCD corrections to be ± 0.005. Combining these then gives $\alpha_s(M_Z) = 0.118 \pm 0.011$ which is used in the final average. For additional comments on comparisons between these data and theory see Ref. 4. Data are also available on the angular distribution of jets; these are also in agreement with QCD expectations [58,59].

QCD corrections to Drell-Yan type cross sections (*i.e.*, the production in hadron collisions by quark-antiquark annihilation of lepton pairs of invariant mass Q from virtual photons, or of real W or Z bosons), are known [60]. These $\mathcal{O}(\alpha_s)$ QCD corrections are sizable at small values of Q. The correction to W and Z production, as measured in $p\overline{p}$ collisions at $\sqrt{s} = 0.63$ TeV and $\sqrt{s} = 1.8,\ 1.96$ TeV, is of order 30%. The NNLO corrections to this process are known [61].

The production of W and Z bosons and photons at large transverse momentum can also be used to test QCD. The leading-order QCD subprocesses are $q\overline{q} \to Vg$ and $qg \to Vq$ ($V = W, Z, \gamma$). If the parton distributions are taken from other processes and a value of α_s assumed, then an absolute prediction is obtained. Conversely, the data can be used to extract information on quark and gluon distributions and on the value of α_s. The next-to-leading-order QCD corrections are known for photons [62,63], and for W/Z production [64], and so a precision test is possible. Data exist on photon production from the CDF and DØ collaborations [65,66] and from fixed target experiments [67]. Detailed comparisons with QCD predictions [68] may indicate an excess of the data over the theoretical prediction at low value of transverse momenta, although other authors [69] find smaller excesses.

The UA2 collaboration [70] extracted a value of $\alpha_s(M_W) = 0.123 \pm 0.018$ (stat.) $\pm$ 0.017 (syst.) from the measured ratio $R_W = \frac{\sigma(W + 1\text{jet})}{\sigma(W + 0\text{jet})}$. The result depends on the algorithm used to define a jet, and the dominant systematic errors due to fragmentation and corrections for underlying events (the former causes jet energy to be lost, the latter causes it to be increased) are connected to the algorithm. There is also dependence on the parton distribution functions, and hence, α_s appears explicitly in the formula for R_W, and implicitly in the distribution functions. The UA2 result is not used in the final average. Data from CDF and DØ on the W p_t distribution [71] are in agreement with QCD but are not able to determine α_s with sufficient precision to have any weight in a global average.

In the region of low p_t, the fixed order perturbation theory is not applicable; one must sum terms of order $\alpha_s^n \ln^n(p_t/M_W)$ [72]. Data from DØ [73] on the p_t distribution of Z bosons agree well with these predictions.

The production rates of b quarks in $p\overline{p}$ have been used to determine α_s [74]. The next-to-leading-order QCD production processes [75] have been used. By selecting events where the b quarks are back-to-back in azimuth, the next-to-leading-order calculation can be used to compare rates to the measured value and a value of α_s extracted. The errors are dominated by the measurement errors, the choice of μ and the scale at which the structure functions are evaluated, and uncertainties in the choice of structure functions. The last were estimated by varying the structure functions used. The result is $\alpha_s(M_Z) = 0.113^{+0.009}_{-0.013}$, which is not included in the final average, as the measured $b\overline{b}$ cross section is not in very good agreement with perturbative QCD [76] and it is therefore difficult to interpret this result. Recent improvments in the theoretical undestanding [77] and measurements from CDF [78] now show good agreement between the measured cross-sections and the QCD predictions but there is no extraction of α_s using these.

9.6. QCD in heavy-quarkonium decay

Under the assumption that the hadronic and leptonic decay widths of heavy $Q\overline{Q}$ resonances can be factorized into a nonperturbative part—dependent on the confining potential—and a calculable perturbative part, the ratios of partial decay widths allow measurements of α_s at the heavy-quark mass scale. The most precise data come from the decay widths of the 1^{--} $J/\psi(1S)$ and Υ resonances. The total decay width of the Υ is predicted by perturbative QCD [79,80]

$$\begin{aligned} R_\mu(\Upsilon) &= \frac{\Gamma(\Upsilon \to \text{hadrons})}{\Gamma(\Upsilon \to \mu^+\mu^-)} \\ &= \frac{10(\pi^2 - 9)\alpha_s^3(M_b)}{9\pi\alpha_{\text{em}}^2} \end{aligned}$$

$$\times \left[1 + \frac{\alpha_s}{\pi} \left(-14.05 + \frac{3\beta_0}{2} \left(1.161 + \ln\Bigl(\frac{2M_b}{M_\Upsilon} \Bigr) \right) \right) \right] . \qquad (9.11)$$

Data are available for the Υ, Υ', Υ'', and J/ψ. The result is very sensitive to α_s and the data are sufficiently precise ($R_\mu(\Upsilon) = 37.28 \pm 0.75$) [81] that the theoretical errors will dominate. There are theoretical corrections to this formula due to the relativistic nature of the $Q\overline{Q}$ system which have been calculated [80] to order v^2/c^2. These corrections are more severe for the J/ψ. There are also nonperturbative corrections arising from annihilation from higher Fock states ("color octet" contribution) which can only be estimated [82]; again these are more severe for the J/ψ. The Υ gives $\alpha_s(M_b) = 0.183 \pm 0.01$, where the error includes that from the "color octet" term and the choice of scale which together dominate. The ratio of widths $\dfrac{\Upsilon \to \gamma gg}{\Upsilon \to ggg}$ has been measured by the CLEO collaboration and can be used to determine $\alpha_s(M_b) = 0.189 \pm 0.01 \pm 0.01$. The error is dominated by theoretical uncertainties associated with the scale choice; the uncertainty due to the "color octet" piece is not present in this case [83]. The theoretical uncertainties due to the production of photons in fragmentation [84] are small [85]. Higher order QCD calculations of the photon energy distribution are available [86]; this distribution could now be used to further test the theory. The width $\Gamma(\Upsilon \to e^+e^-)$ can also be used to determine α_s by using moments of the quantity $R_b(s) = \dfrac{\sigma(e^+e^- \to b\overline{b})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ defined by $M_n = \int_0^\infty \dfrac{R_b(s)}{s^{n+1}}$ [87]. At large values of n, M_n is dominated by $\Gamma(\Upsilon \to e^+e^-)$. Higher order corrections are available and the method gives $\alpha_s(M_b) = 0.220 \pm 0.027$ [88]. The dominant error is theoretical and is dominated by the choice of scale and by uncertainties in Coulomb corrections that have been resummed in Ref. 89. These various Υ decay measurements can be combined and give $\alpha_s(M_b) = 0.185 \pm 0.01$ corresponding to $\alpha_s(M_Z) = 0.109 \pm 0.004$ which is used in the final average [83]. The mass of charmonium can also be used for determination of α_s after taking into account effects of analytic continuation (π^2-terms summation), and Coulomb summation [90].

9.7. Perturbative QCD in e^+e^- collisions

The total cross section for $e^+e^- \to$ hadrons is obtained (at low values of $\sqrt{s}$) by multiplying the muon-pair cross section by the factor $R = 3\Sigma_q e_q^2$. The higher-order QCD corrections to this quantity have been calculated, and the results can be expressed in terms of the factor:

$$R = R^{(0)} \left[1 + \frac{\alpha_s}{\pi} + C_2 \left(\frac{\alpha_s}{\pi} \right)^2 + C_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \cdots \right] , \qquad (9.12)$$

where $C_2 = 1.411$ and $C_3 = -12.8$ [91].

$R^{(0)}$ can be obtained from the formula for $d\sigma/d\Omega$ for $e^+e^- \to f\overline{f}$ by integrating over Ω. The formula is given in Sec. 39.2 of this *Review*. This result is only correct in the zero-quark-mass limit. The $\mathcal{O}(\alpha_s)$ corrections are also known for massive quarks [92].

The principal advantage of determining α_s from R in e^+e^- annihilation is that there is no dependence on fragmentation models, jet algorithms, *etc.*

A measurement by CLEO [93] at $\sqrt{s} = 10.52$ GeV yields $\alpha_s(10.52 \text{ GeV}) = 0.20 \pm 0.01 \pm 0.06$, which corresponds to $\alpha_s(M_Z) = 0.130 \pm 0.005 \pm 0.03$. A comparison of the theoretical prediction of Eq. (9.12) (corrected for the b-quark mass), with all the available data at values of $\sqrt{s}$ between 20 and 65 GeV, gives [94] $\alpha_s(35 \text{ GeV}) = 0.146 \pm 0.030$. The size of the order α_s^3 term is of order 40% of that of the order α_s^2 and 3% of the order α_s. If the order α_s^3 term is not included, a fit to the data yields α_s (35 GeV) $= 0.142 \pm 0.030$, indicating that the theoretical uncertainty is smaller than the experimental error.

Measurements of the ratio of hadronic to leptonic width of the Z at LEP and SLC, Γ_h/Γ_μ probe the same quantity as R. Using the average of $\Gamma_h/\Gamma_\mu = 20.767 \pm 0.025$ gives $\alpha_s(M_Z) = 0.1226 \pm 0.0038$ [95]. In performing this extraction it is necessary to include the electroweak corrections to the Z width. As these must be calculated beyond leading order, they depend on the top top-quark and Higgs masses. The latter is not yet meausured an is inferred from global fits to the electroweak data. There are additional theoretical errors arising from the choice of QCD scale. While this method has small theoretical uncertainties from QCD itself, it relies sensitively on the electroweak couplings of the Z to quarks [96]. The presence of new physics which changes these couplings via electroweak radiative corrections would invalidate the value of $\alpha_s(M_Z)$. An illustration of the sensitivity can be obtained by comparing this value with the one obtained from the global fits [95] of the various precision measurements at LEP/SLC and the W and top masses: $\alpha_s(M_Z) = 0.1186 \pm 0.0027$. The difference between these two values may be accounted for by systematic uncertainties as large as ± 0.003 [95], therefore $\alpha_s(M_Z) = 0.1186 \pm 0.0042$ will be used in the final average.

An alternative method of determining α_s in e^+e^- annihilation is from measuring quantities that are sensitive to the relative rates of two-, three-, and four-jet events. A review should be consulted for more details [97] of the issues mentioned briefly here. In addition to simply counting jets, there are many possible choices of such "shape variables": thrust [98], energy-energy correlations [99], average jet mass, *etc.* All of these are infrared safe, which means they can be reliably calculated in perturbation theory. The starting point for all these quantities is the multijet cross section. For example, at order α_s, for the process $e^+e^- \to q\overline{q}g$: [100]

$$\frac{1}{\sigma}\,\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\ , \tag{9.13}$$

$$x_i = \frac{2E_i}{\sqrt{s}} \tag{9.14}$$

where x_i are the center-of-mass energy fractions of the final-state (massless) quarks. A distribution in a "three-jet" variable, such as those listed above, is obtained by integrating this differential cross section over an appropriate phase space region for a fixed value of the variable. The order α_s^2 corrections to this process have been computed, as well as the 4-jet final states such as $e^+e^- \to qqgg$ [101].

There are many methods used by the e^+e^- experimental groups to determine α_s from the event topology. The jet-counting algorithm, originally introduced by the JADE collaboration [102], has been used by many other groups. Here, particles of momenta p_i and p_j are combined into a pseudo-particle of momentum $p_i + p_j$ if the invariant mass of the pair is less than $y_0\sqrt{s}$. The process is iterated until all pairs of particles or pseudoparticles have a mass-measure that exceeds $y_0\sqrt{s}$; the remaining number is then defined to be the jet multiplicity. The remaining number is then defined to be the number of jets in the event, and can be compared to the QCD prediction. The Durham algorithm is slightly different: in combining a pair of partons, it uses $M^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$ for partons of energies E_i and E_j separated by angle θ_{ij} [103].

There are theoretical ambiguities in the way this process is carried out. Quarks and gluons are massless, whereas the observed hadrons are not, so that the massive jets that result from this scheme cannot be compared directly to the jets of perturbative QCD. Different recombination schemes have been tried, for example combining 3-momenta and then rescaling the energy of the cluster so that it remains massless. These schemes result in the same data giving slightly different values [104,105] of α_s. These differences can be used to determine a systematic error. In addition, since what is observed are hadrons rather than quarks and gluons, a model is needed to describe the evolution of a partonic final state into one involving hadrons, so that detector corrections can be applied. The QCD matrix elements are combined with a parton-fragmentation model. This model can then be used to correct the data for a direct comparison with the parton calculation. The different hadronization models that are used [106–109] model the dynamics that are controlled by nonperturbative QCD effects which we cannot yet calculate. The fragmentation parameters of these Monte Carlos are tuned to get agreement with the observed data. The differences between these models contribute to the systematic errors. The systematic errors from recombination schemes and fragmentation effects dominate over the statistical and other errors of the LEP/SLD experiments.

The scale M at which $\alpha_s(M)$ is to be evaluated is not clear. The invariant mass of a typical jet (or $\sqrt{sy_0}$) is probably a more appropriate choice than the e^+e^- center-of-mass energy. While there is no justification for doing so, if the value is allowed to float in the fit to the data, the fit improves and the data tend to prefer values of order $\sqrt{s}/10$ GeV for some variables [105,110]; the exact value depends on the variable that is fitted.

The perturbative QCD formulae can break down in special kinematical configurations. For example, the thrust (T) distribution contains terms of the type $\alpha_s \ln^2(1-T)$. The higher orders in the perturbation expansion contain terms of order $\alpha_s^n \ln^m(1-T)$. For $T \sim 1$ (the region populated by 2-jet events), the perturbation expansion is unreliable. The terms with $n \leq m$ can be summed to all orders in α_s [111]. If the jet recombination methods are used higher-order terms involve $\alpha_s^n \ln^m(y_0)$, these too can be resummed [112] The resummed results give better agreement with the data at large values of T. Some caution should be exercised in using these resummed results because of the possibility of overcounting; the showering Monte Carlos that are used for the fragmentation corrections also generate some of these leading-log corrections. Different schemes for combining the order α_s^2 and the resummations are available [113]. These different schemes result in shifts in α_s of order ± 0.002. The use of the resummed results improves the agreement between the data and the theory; for more details see Ref. 114. An average of results at the Z resonance from SLD [105], OPAL [115], L3 [116], ALEPH [117], and DELPHI [118], using the combined α_s^2 and resummation fitting to a large set of shape variables, gives $\alpha_s(M_Z) = 0.122 \pm 0.007$. The errors in the values of $\alpha_s(M_Z)$ from these shape variables are totally dominated by the theoretical uncertainties associated with the choice of scale, and the effects of hadronization Monte Carlos on the different quantities fitted.

Estimates are available for the nonperturbative corrections to the mean value of $1 - T$ [119]. These are of order $1/E$ and involve a single parameter to be determined from experiment. These corrections can then be used as an alternative to those modeled by the fragmentation Monte Carlos. The DELPHI collaboration has fitted its data using an additional parameter to take into account these $1/E$ effects [120] and quotes for the $\overline{\rm MS}$ scheme $\alpha_s = 0.1217 \pm 0.0046$ and a significant $1/E$ term. This term vanishes in the RGI/ECH scheme and the data are well described by pure perturbation theory with consistent $\alpha_s = 0.1201 \pm 0.0020$.

Studies have been carried out at energies between $\sim$130 GeV [121] and $\sim$200 GeV [122]. These can be combined to give $\alpha_s(130\ \text{GeV}) = 0.114 \pm 0.008$ and $\alpha_s(189\ \text{GeV}) = 0.1104 \pm 0.005$. The dominant errors are theoretical and systematic and, most of these are in common at the two energies. These data and those at the Z resonance and below provide clear confirmation of the expected decrease in α_s as the energy is increased.

The LEP QCD working group [123] uses all LEP data Z mass and higher energies to perform a global fit using a large number of shape variables. It determines $\alpha_s(M_Z) = 0.1202 \pm 0.0003\ (\text{stat}) \pm 0.0049\ (\text{syst})$, (result quoted in Ref. 6) the error being dominated by theoretical uncertainties which are the most difficult to quantify.

Similar studies on event shapes have been undertaken at lower energies at TRISTAN, PEP/PETRA, and CLEO. A combined result from various shape parameters by the TOPAZ collaboration gives $\alpha_s(58\ \text{GeV}) = 0.125 \pm 0.009$, using the fixed order QCD result, and $\alpha_s(58\ \text{GeV}) = 0.132 \pm 0.008$ (corresponding to $\alpha_s(M_Z) = 0.123 \pm 0.007$), using the same method as in the SLD and LEP average [124]. The measurements of event shapes at PEP/PETRA are summarized in earlier editions of this note. A recent reevaluation of the JADE data [125] obtained using resummed QCD results with modern models of jet fragmentation and by averaging over several shape variables gives $\alpha_s(22\ \text{GeV}) = 0.151 \pm 0.004\ (\text{expt})\ {}^{+0.014}_{-0.012}\ (\text{theory})$ which is used in the final average. These results also attempt to constrain the non-perurbative parameters and show a remarkable agreement with QCD even at low energies [126]. An analysis by the TPC group [127] gives $\alpha_s(29\ \text{GeV}) = 0.160 \pm 0.012$, using the same method as TOPAZ.

The CLEO collaboration fits to the order α_s^2 results for the two jet fraction at $\sqrt{s} = 10.53$ GeV, and obtains $\alpha_s(10.53\ \text{GeV}) = 0.164 \pm 0.004\ (\text{expt.}) \pm 0.014\ (\text{theory})$ [128]. The dominant systematic error arises from the choice of scale (μ), and is determined from the range of α_s that results from fit with $\mu = 10.53$ GeV, and a fit where μ is allowed to vary to get the lowest χ^2. The latter results in $\mu = 1.2$ GeV. Since the quoted result corresponds to $\alpha_s(1.2\ \text{GeV}) = 0.35$, it is by no means clear that the perturbative QCD expression is reliable and the resulting error should, therefore, be treated with caution. A fit to many different variables as is done in the LEP/SLC analyses would give added confidence to the quoted error.

All these measurements are consistent with the LEP average quoted above which has the smallest statistical error; the systematic errors being mostly theoretical are likely to be strongly correlated between the measurements. The value of $\alpha_s(M_Z) = 0.1202 \pm 0.005$ is used in the final average.

The four jet final states can be used to measure the color factors of QCD, related to the relate strength of the couplings of quarks and gluons to each other. While these factors are not free parameters, the agreement between the measurements and expectations provides more evidence for the validity of QCD. The results are summarized in Ref. 129.

9.8. Scaling violations in fragmentation functions

Measurements of the fragmentation function $d_i(z, E)$, (the probability that a hadron of type i be produced with energy zE in e^+e^- collisions at $\sqrt{s} = 2E$) can be used to determine α_s. (Detailed definitions and a discussion of the properties of fragmentation functions can be found in Sec. 17 of this *Review*). As in the case of scaling violations in structure functions, perturbative QCD predicts only the E dependence. Hence, measurements at different energies are needed to extract a value of α_s. Because the QCD evolution mixes the fragmentation functions for each quark flavor with the gluon fragmentation function, it is necessary to determine each of these before α_s can be extracted.

The ALEPH collaboration has used data from energies ranging from $\sqrt{s} = 22$ GeV to $\sqrt{s} = 91$ GeV. A flavor tag is used to discriminate between different quark species, and the longitudinal and transverse cross sections are used to extract the gluon fragmentation function [130]. The result obtained is $\alpha_s(M_Z) = 0.126 \pm 0.007\ (\text{expt.}) \pm 0.006\ (\text{theory})$ [131]. The theory error is due mainly to the choice of scale. The OPAL collaboration [132] has also extracted the separate fragmentation functions. DELPHI [133] has also performed a similar analysis using data from other experiments at lower energy with the result $\alpha_s(M_Z) = 0.124 \pm 0.007\ (\text{expt.}) \pm 0.009\ (\text{theory})$. The larger theoretical error is due to the larger range of scales that were used in the fit. These results can be combined to give $\alpha_s(M_Z) = 0.125 \pm 0.005\ (\text{expt.}) \pm 0.008\ (\text{theory})$.

A global analysis [134] uses data on the production of π, K, p, and $\overline{p}$ from SLC [135], DELPHI [136], OPAL [137], ALEPH [138], and lower-energy data from the TPC collaboration [139]. A flavor tag and a three-jet analysis is used to disentangle the quark and gluon fragmentation functions. The value $\alpha_s(M_Z) = 0.1172^{+0.0055+0.0017}_{-0.0069-0.0025}$ is obtained. The second error is a theoretical one arising from the choice of scale. The fragmentation functions resulting from this fit are consistent with a recent fit of [140].

It is unclear how to combine the measurements discussed in the two previous paragraphs as much of the data used are common to both. If the theoretical errors dominate then a simple average is appropriate as the methods are different. For want of a better solution, the naive average of $\alpha_s(M_Z) = 0.1201 \pm 0.006$ is used in the average value quoted below.

9.9. Photon structure functions

e^+e^- can also be used to study photon-photon interactions, which can be used to measure the structure function of a photon [141], by selecting events of the type $e^+e^- \rightarrow e^+e^- + hadrons$ which proceeds via two photon scattering. If events are selected where one of the photons is almost on mass shell and the other has a large invariant mass Q, then the latter probes the photon structure function at scale Q; the process is analogous to deep inelastic scattering where a highly virtual photon is used to probe the proton structure. This process was included in earlier versions of this *Review* which can be consulted for details on older measurements [142–145]. A review of the data can be found in [146]. Data are available from LEP [147–151] and from TRISTAN [152,153] which extend the range of Q^2 to of order 300 GeV^2 and x as low as $2 \times 10^{-3}$and show Q^2 dependence of the structure function that is consistent with QCD expectations. There is evidence for a hadronic (non-perturbative) component to the photon structure function that complicates attempts to extract a value of α_s from the data.

Ref. 154 uses data from PETRA, TRISTAN, and LEP to perform a combined fit. The higher data from LEP extend to higher Q^2 ($<$ 780 GeV^2) and enable a measurement: $\alpha_s(m_Z) = 0.1198 \pm 0.0054$ which now is competitive with other results.

Experiments at HERA can also probe the photon structure function by looking at jet production in γp collisions; this is analogous to the jet production in hadron-hadron collisions which is sensitive to hadron structure functions. The data [155] are consistent with theoretical models [156].

9.10. Jet rates in *ep* collisions

At lowest order in α_s, the ep scattering process produces a final state of (1+1) jets, one from the proton fragment and the other from the quark knocked out by the process $e + quark \rightarrow e + quark$. At next order in α_s, a gluon can be radiated, and hence a (2+1) jet final state produced. By comparing the rates for these (1+1) and (2+1) or (2+1) and (3+1) jet processes, a value of α_s can be obtained. A NLO QCD calculation is available [157]. The basic methodology is similar to that used in the jet counting experiments in e^+e^- annihilation discussed above. Unlike those measurements, the ones in ep scattering are not at a fixed value of Q^2. In addition to the systematic errors associated with the jet definitions, there are additional ones since the structure functions enter into the rate calculations. A summary of the measurements from HERA can be found in Ref. 158, which clearly demonstrates the evidence for the evolution of $\alpha_s(Q^2)$ with Q^2. Results from H1 [159] $\alpha_s(M_Z) = 0.1175 \pm 0.0057$ (expt.) $\pm\, 0.0053$ (theor.) and ZEUS [160] $\alpha_s(M_Z) = 0.1179 \pm 0.0040$ (expt.) $\pm\, 0.005$ (theor.) can be combined to give $\alpha_s(M_Z) = 0.1178 \pm 0.0033$ (expt.) $\pm\, 0.006$ (theor.) which is used in the final average. The theoretical errors arise from scale choice, structure functions, and hadronization correction.

Photoproduction of two or more jets via processes such as $\gamma + g \to q\overline{q}$ can also be observed at HERA. The process is similar to jet production in hadron-hadron collisions. Agreement with perturbative QCD is excellent and ZEUS [161] quotes $\alpha_s(M_Z) = 0.1224 \pm 0.0020$ (expt) ± 0.0050 (theory) which is used in the average below.

9.11. QCD in diffractive events

In approximately 10% of the deep-inelastic scattering events at HERA a rapidity gap is observed [162]; that is events are seen where there are almost no hadrons produced in the direction of the incident proton. This was unexpected; QCD based models of the final state predicted that the rapidity interval between the quark that is hit by the electron and the proton remnant should be populated approximately evenly by the hadrons. Similar phenomena have been observed at the Tevatron in W and jet production. For a review see Ref. 163.

9.12. Lattice QCD

Lattice gauge theory can be used to calculate, using non-perturbative methods, a physical quantity that can be measured experimentally. The value of this quantity can then be used to determine the QCD coupling that enters in the calculation. The main theoretical differance between this approach and those discussed above is that, in the previous cases, precise calculations are restricted to high energy phenomena where perturbation theory can be applied due to the smallness of α_s in the appropriate energy regime. Lattice calculations enable reliable calculations to be done without this restriction. It is important to emphasize that this is exactly the same methodology used in the cases discussed above. The main quantitative difference is that the experimental measurements involved, such as the masses of Υ states, are so precise that their uncertainties have almost no impact on the final comparisons. A discussion of the uncertainties that enter into the QCD tests and determination of α_s is therefore almost exclusively a discussion of the techniques used in the calculations. In addition to α_s, other physical quantities such as the masses of the light quarks can be obtained. For a review of the methodology, see Ref. 164 [165]. For example, the energy levels of a $Q\overline{Q}$ system can be determined and then used to extract α_s. The masses of the $Q\overline{Q}$ states depend only on the quark mass and on α_s. Until a few years ago, calculations have not been performed for three light quark flavors. Results for zero ($n_f = 0$, quenched approximation) and two light flavors were extrapolated to $n_f = 3$. This major limitation has now been removed and a qualitative improvement in the calculations has occurred. Using the mass differences of Υ and Υ' and Υ'' and χ_b, Mason *et al.* [167] extract a value of $\alpha_s(M_Z) = 0.1170 \pm 0.0012$. Many other quantities such as the pion decay constant, and the masses of the B_s meson and Ω baryon are used and the overall consistency is excellent.

There have also been investigations of the running of α_s [173]. These show remarkable agreement with the two loop perturbative result of Eq. (9.5).

There are several sources of error in these estimates of $\alpha_s(M_Z)$. The experimental error associated with the measurements of the particle masses is negligible. The limited statistics of the Monte-Carlo calculation which can be improved only with more computational resources is one dominant error. The conversion from the lattice coupling constant to the $\overline{\text{MS}}$ constant is obtained using a perturbative expansion where one coupling expanded as a power series in the other. The series is known to third order and this leads to the second largest uncertainty [166]. Extra degrees of freedom introduced by calculating (using staggered fermions) on a lattice have to be removed: see Ref. 174 for a discussion of this point and the possible uncertainties related to it. The use of Wilson fermions involves a differant systematic. Results from Ref. 175 using this method with two light quark flavors are $\alpha_s(M_Z) = 0.112 \pm 0.003$, which illustrates the tendency for results using Wilson fermions to be systematically lower than those from staggered fermions.

In this review, we will use only the new result [166] of $\alpha_s(M_Z) = 0.1170 \pm 0.0012$, which is consistent with the value $\alpha_s(M_Z) = 0.121 \pm 0.003$ used in the last version of this review [167].

In addition to the strong coupling constant other quantities can be determined including the light quark masses [168]. Of particular interest are the decay constants of charmed and bottom mesons. These are required, for example, to facilitate the extraction of CKM elements from measurements of charm and bottom decay rates [169,170]. Some of these quantities such as the D-meson decay constant have been found to be in excellent agreement with experiment [171].

9.13. Conclusions

The need for brevity has meant that many other important topics in QCD phenomenology have had to be omitted from this review. One should mention in particular the study of exclusive processes (form factors, elastic scattering, ...), the behavior of quarks and gluons in nuclei, the spin properties of the theory, and QCD effects in hadron spectroscopy.

We have focused on those high-energy processes which currently offer the most quantitative tests of perturbative QCD. Figure 9.1 shows the values of $\alpha_s(M_Z)$ deduced from the various experiments. Figure 9.2 shows the values and the values of Q where they are measured. This figure clearly shows the experimental evidence for the variation of $\alpha_s(Q)$ with Q.

An average of the values in Fig. 9.1 gives $\alpha_s(M_Z) = 0.1176$, with a total χ^2 of 9 for eleven fitted points, showing good consistency among the data. The error on this average, assuming that all of the errors in the contributing results are uncorrelated, is ± 0.0009, and may be an underestimate. Almost all of the values used in the average are dominated by systematic, usually theoretical, errors. Only some of these, notably from the choice of scale, are correlated. The error on the lattice gauge theory result is the smallest and then there are several results with comparable small errors: these are the ones from τ decay, deep inelastic scattering, Υ decay and the Z^0 width. Omitting the lattice-QCD result from the average changes it to $\alpha_s(M_Z) = 0.1185$ or 1σ. All of the results that dominate the average are from NNLO. The NLO results have little weight, there are no LO results used. Almost all of the results have errors that are dominated by

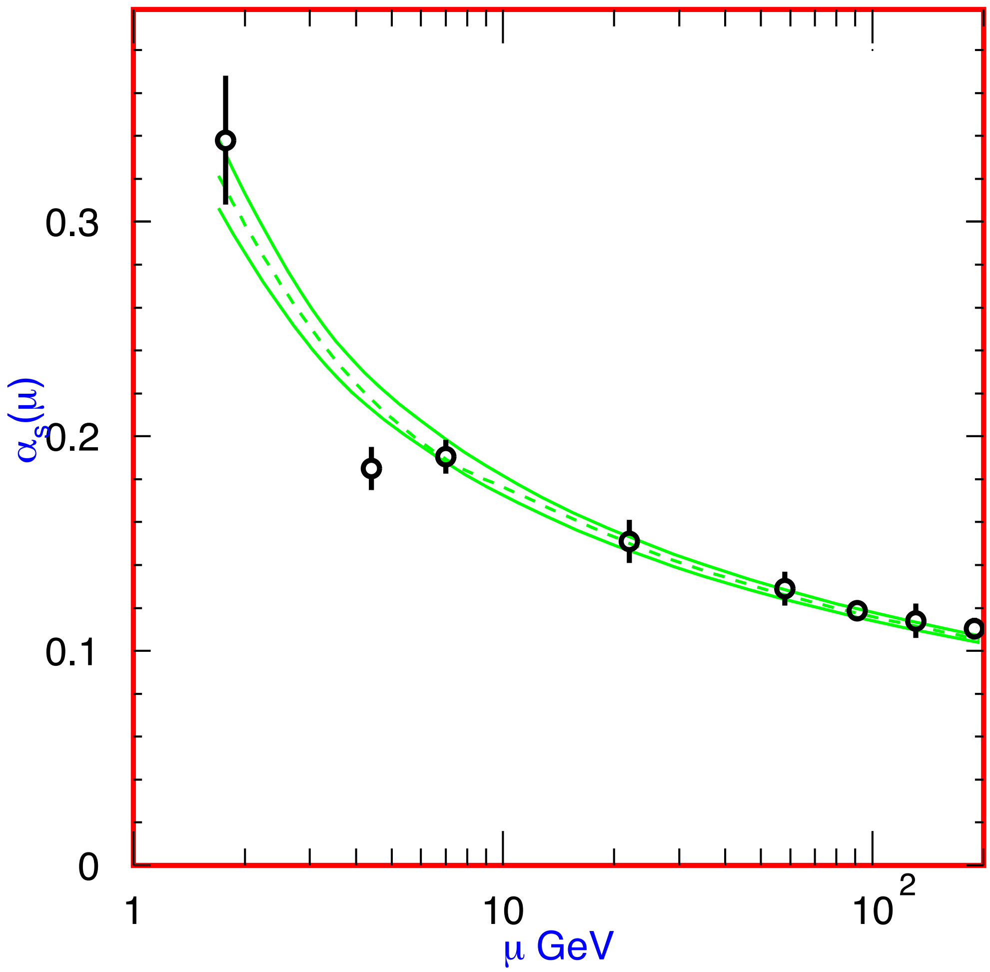

Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ. The data are, in increasing order of μ, τ width, Υ decays, deep inelastic scattering, e^+e^- event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV, Z width, and e^+e^- event shapes at 135 and 189 GeV.

theoretical issues, either from unknown higher order perturbative corrections or estimates of non-perturbative contributions. It is therefore prudent be conservative and quote our average value as $\alpha_s(M_Z) = 0.1176 \pm 0.002$. Note that the average has moved by less than 1σ from the last version of this review. Future experiments can be expected to improve the measurements of α_s somewhat.

The value of α_s at any scale corresponding to our average can be obtained from `http://www-theory.lbl.gov/~ianh/alpha/alpha.html` which uses Eq. (9.5) to interpolate.

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