39. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

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Setting aside leptoproduction (for which, see Sec. 16), the cross sections of primary interest are those with light incident particles, e^+e^-, $\gamma\gamma$, $q\overline{q}$, gq , gg, *etc.*, where g and q represent gluons and light quarks. The produced particles include both light particles and heavy ones - t, W, Z, and the Higgs boson H. We provide the production cross sections calculated within the Standard Model for several such processes.

39.1. Resonance Formation

Resonant cross sections are generally described by the Breit-Wigner formula (Sec. 16 of this *Review*).

$$\sigma(E) = \frac{2J+1}{(2S_1+1)(2S_2+1)}\frac{4\pi}{k^2}\left[\frac{\Gamma^2/4}{(E-E_0)^2+\Gamma^2/4}\right] B_{in}B_{out}, \tag{39.1}$$

where E is the c.m. energy, J is the spin of the resonance, and the number of polarization states of the two incident particles are $2S_1+1$ and $2S_2+2$. The c.m. momentum in the initial state is k, E_0 is the c.m. energy at the resonance, and Γ is the full width at half maximum height of the resonance. The branching fraction for the resonance into the initial-state channel is B_{in} and into the final-state channel is B_{out}. For a narrow resonance, the factor in square brackets may be replaced by $\pi\Gamma\delta(E-E_0)/2$.

39.2. Production of light particles

The production of point-like, spin-1/2 fermions in e^+e^- annihilation through a virtual photon, $e^+e^- \to \gamma^* \to f\overline{f}$, at c.m. energy squared s is given by

$$\frac{d\sigma}{d\Omega} = N_c\frac{\alpha^2}{4s}\beta\left[1+\cos^2\theta+(1-\beta^2)\sin^2\theta\right]Q_f^2 \ , \tag{39.2}$$

where β is v/c for the produced fermions in the c.m., θ is the c.m. scattering angle, and Q_f is the charge of the fermion. The factor N_c is 1 for charged leptons and 3 for quarks. In the ultrarelativistic limit, $\beta \to 1$,

$$\sigma = N_c Q_f^2\frac{4\pi\alpha^2}{3s} = N_c Q_f^2\frac{86.8\ nb}{s(\mathrm{GeV}^2)^2} \ . \tag{39.3}$$

The cross section for the annihilation of a $q\overline{q}$ pair into a distinct pair $q'\overline{q}'$ through a gluon is completely analogous up to color factors, with the replacement $\alpha \to \alpha_s$. Treating all quarks as massless, averaging over the colors of the initial quarks and defining $t=-s\sin^2(\theta/2)$, $u=-s\cos^2(\theta/2)$, one finds [1]

$$\frac{d\sigma}{d\Omega}(q\overline{q}\to q'\overline{q}') = \frac{\alpha^2}{9s}\frac{t^2+u^2}{s^2} \ . \tag{39.4}$$

Crossing symmetry gives

$$\frac{d\sigma}{d\Omega}(qq' \to qq') = \frac{\alpha^2}{9s}\frac{s^2+u^2}{t^2} \ . \tag{39.5}$$

If the quarks q and q' are identical, we have

$$\frac{d\sigma}{d\Omega}(q\overline{q} \to q\overline{q}) = \frac{\alpha_s^2}{9s}\left[\frac{t^2+u^2}{s^2} + \frac{s^2+u^2}{t^2} - \frac{2u^2}{3st}\right] \ , \tag{39.6}$$

and by crossing

$$\frac{d\sigma}{d\Omega}(qq \to qq) = \frac{\alpha_s^2}{9s}\left[\frac{t^2+s^2}{u^2} + \frac{s^2+u^2}{t^2} - \frac{2s^2}{3ut}\right] . \tag{39.7}$$

Annihilation of e^+e^- into $\gamma\gamma$ has the cross section

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\frac{u^2+t^2}{tu} \ . \tag{39.8}$$

The related QCD process also has a triple-gluon coupling. The cross section is

$$\frac{d\sigma}{d\Omega}(q\overline{q} \to gg) = \frac{8\alpha_s^2}{27s}(t^2+u^2)\left(\frac{1}{tu} - \frac{9}{4s^2}\right) \ . \tag{39.9}$$

The crossed reactions are

$$\frac{d\sigma}{d\Omega}(qg \to qg) = \frac{\alpha_s^2}{9s}(s^2+u^2)(-\frac{1}{su} + \frac{9}{4t^2}) \tag{39.10}$$

and

$$\frac{d\sigma}{d\Omega}(gg \to q\overline{q}) = \frac{\alpha_s^2}{24s}(t^2+u^2)(\frac{1}{tu} - \frac{9}{4s^2}) \ . \tag{39.11}$$

Finally,

$$\frac{d\sigma}{d\Omega}(gg \to gg) = \frac{9\alpha_s^2}{8s}(3 - \frac{ut}{s^2} - \frac{su}{t^2} - \frac{st}{u^2}) \ . \tag{39.12}$$

39.3. Production of Weak Gauge Bosons

39.3.1. *W and Z resonant production* :

Resonant production of a single W or Z is governed by the partial widths

$$\Gamma(W \to \ell_i \overline{\nu}_i) = \frac{\sqrt{2} G_F m_W^3}{12\pi} \tag{39.13}$$

$$\Gamma(W \to q_i \overline{q}_j) = 3 \frac{\sqrt{2} G_F |V_{ij}|^2 m_W^3}{12\pi} \tag{39.14}$$

$$\begin{aligned}\Gamma(Z \to f\overline{f}) = N_c &\frac{\sqrt{2} G_F m_Z^3}{6\pi} \\ &\times \left[(T_3 - Q_f \sin^2\theta_W)^2 + (Q_f \sin\theta_W)^2 \right] .\end{aligned} \tag{39.15}$$

The weak mixing angle is θ_W. The CKM matrix elements are indicated by V_{ij} and N_c is 3 for $q\overline{q}$ final states and 1 for leptonic final states.

The full differential cross section for $f_i \overline{f}_j \to (W, Z) \to f_{i'} \overline{f}_{j'}$ is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega} = &\frac{N_c^f}{N_c^i} \cdot \frac{1}{256\pi^2 s} \cdot \frac{s^2}{(s - M^2)^2 + s\Gamma^2} \\ &\times \Big[(L^2 + R^2)(L'^2 + R'^2)(1 + \cos^2\theta) \\ &+ (L^2 - R^2)(L'^2 - R'^2) 2\, cos\theta \Big]\end{aligned} \tag{39.16}$$

where M is the mass of the W or Z. The couplings for the W are $L = (8G_F m_W^2/\sqrt{2})^{1/2} V_{ij}/\sqrt{2}; R = 0$ where V_{ij} is the corresponding CKM matrix element, with an analogous expression for L' and R'. For Z, the couplings are $L = (8G_F m_Z^2/\sqrt{2})^{1/2}(T_3 - \sin^2\theta_W Q); R = -(8G_F m_Z^2/\sqrt{2})^{1/2} \sin^2\theta_W Q$, where T_3 is the weak isospin of the initial left-handed fermion and Q is the initial fermion's electric charge. The expressions for L' and R' are analogous. The color factors $N_c^{i,f}$ are 3 for initial or final quarks and 1 for initial or final leptons.

39.3.2. *Production of pairs of weak gauge bosons* :

The cross section for $f\overline{f} \to W^+W^-$ is given in term of the couplings of the left-handed and right-handed fermion f, $\ell = 2(T_3 - Qx_W)$, $r = -2Qx_W$, where T_3 is the third component of weak isospin for the left-handed f, Q is its electric charge (in units of the proton charge), and $x_W = \sin^2\theta_W$:

$$\begin{aligned}\frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{N_c s^2}\Bigg\{&\left[\left(Q + \frac{\ell + r}{4x_W}\frac{s}{s - m_Z^2}\right)^2 + \left(\frac{\ell + r}{4x_W}\frac{s}{s - m_Z^2}\right)^2\right] A(s,t,u) \\ &+ \frac{1}{2x_W}\left(Q + \frac{\ell}{2x_W}\frac{s}{s - m_Z^2}\right)\left(\Theta(-Q)I(s,t,u) - \Theta(Q)I(s,u,t)\right) \\ &+ \frac{1}{8x_W^2}\left(\Theta(-Q)E(s,t,u) + \Theta(Q)E(s,u,t)\right)\Bigg\}\,,\end{aligned} \tag{39.17}$$

where $\Theta(x)$ is 1 for $x > 0$ and 0 for $x < 0$, and where

$$\begin{aligned}A(s,t,u) &= \left(\frac{tu}{m_W^4} - 1\right)\left(\frac{1}{4} - \frac{m_W^2}{s} + 3\frac{m_W^4}{s^2}\right) + \frac{s}{m_W^2} - 4, \\ I(s,t,u) &= \left(\frac{tu}{m_W^4} - 1\right)\left(\frac{1}{4} - \frac{m_W^2}{2s} - \frac{m_W^4}{st}\right) + \frac{s}{m_W^2} - 2 + 2\frac{m_W^2}{t}, \\ E(s,t,u) &= \left(\frac{tu}{m_W^4} - 1\right)\left(\frac{1}{4} + \frac{m_W^2}{t}\right) + \frac{s}{m_W^2},\end{aligned} \tag{39.18}$$

and s, t, u are the usual Mandelstam variables with $s = (p_f + p_{\overline{f}})^2, t = (p_f - p_{W^-})^2, u = (p_f - p_{W^+})^2$. The factor N_c is 3 for quarks and 1 for leptons.

The analogous cross-section for $q_i\overline{q}_j \to W^\pm Z^0$ is

$$\begin{aligned}\frac{d\sigma}{dt} = \frac{\pi\alpha^2|V_{ij}|^2}{6s^2x_W^2}\Bigg\{&\left(\frac{1}{s - m_W^2}\right)^2\Bigg[\left(\frac{9 - 8x_W}{4}\right)\left(ut - m_W^2 m_Z^2\right) \\ &+ (8x_W - 6)\, s\left(m_W^2 + m_Z^2\right)\Bigg] \\ &+ \left[\frac{ut - m_W^2 m_Z^2 - s(m_W^2 + m_Z^2)}{s - m_W^2}\right]\left[\frac{\ell_j}{t} - \frac{\ell_i}{u}\right] \\ &+ \frac{ut - m_W^2 m_Z^2}{4(1 - x_W)}\left[\frac{\ell_j^2}{t^2} + \frac{\ell_i^2}{u^2}\right] + \frac{s(m_W^2 + m_Z^2)}{2(1 - x_W)}\frac{\ell_i \ell_j}{tu}\Bigg\},\end{aligned} \tag{39.19}$$

where ℓ_i and ℓ_j are the couplings of the left-handed q_i and q_j as defined above. The CKM matrix element between q_i and q_j is V_{ij}.

The cross section for $q_i\overline{q}_i \to Z^0Z^0$ is

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2}{96} \frac{\ell_i^4 + r_i^4}{x_W^2(1-x_W^2)s^2} \left[\frac{t}{u} + \frac{u}{t} + \frac{4m_Z^2 s}{tu} - m_Z^4 \left(\frac{1}{t^2} + \frac{1}{u^2} \right) \right]. \tag{39.20}$$

39.4. Production of Higgs Bosons

39.4.1. *Resonant Production* :

The Higgs boson of the Standard Model can be produced resonantly in the collisions of quarks, leptons, W or Z bosons, gluons, or photons. The production cross section is thus controlled by the partial width of the Higgs boson into the entrance channel and its total width. The branching fractions for the Standard Model Higgs boson are shown in Fig. 1 of the "Searches for Higgs bosons" review in the Particle Listings section, as a function of the Higgs boson mass. The partial widths are given by the relations

$$\Gamma(H \to f\overline{f}) = \frac{G_F m_f^2 m_H N_c}{4\pi\sqrt{2}} \left(1 - 4m_f^2/m_H^2\right)^{3/2}, \tag{39.21}$$

$$\Gamma(H \to W^+W^-) = \frac{G_F m_H^3 \beta_W}{32\pi\sqrt{2}} \left(4 - 4a_W + 3a_W^2\right), \tag{39.22}$$

$$\Gamma(H \to ZZ) = \frac{G_F m_H^3 \beta_Z}{64\pi\sqrt{2}} \left(4 - 4a_Z + 3a_Z^2\right), \tag{39.23}$$

where N_c is 3 for quarks and 1 for leptons and where $a_W = 1 - \beta_W^2 = 4m_W^2/m_H^2$ and $a_Z = 1 - \beta_Z^2 = 4m_Z^2/m_H^2$. The decay to two gluons proceeds through quark loops, with the t quark dominating [2]. Explicitly,

$$\Gamma(H \to gg) = \frac{\alpha_s^2 G_F m_H^3}{36\pi^3\sqrt{2}} \left| \sum_q I(m_q^2/m_H^2) \right|^2, \tag{39.24}$$

where $I(z)$ is complex for $z < 1/4$. For $z < 2 \times 10^{-3}$, $|I(z)|$ is small so the light quarks contribute negligibly. For $m_H < 2m_t$, $z > 1/4$ and

$$I(z) = 3 \left[2z + 2z(1-4z) \left(\sin^{-1} \frac{1}{2\sqrt{z}} \right)^2 \right], \tag{39.25}$$

which has the limit $I(z) \to 1$ as $z \to \infty$.

39.4.2. *Higgs Boson Production in W^* and Z^* decay* :

The Standard Model Higgs boson can be produced in the decay of a virtual W or Z ("Higgstrahlung") [3,4]: In particular, if k is the c.m. momentum of the Higgs boson,

$$\sigma(q_i\overline{q}_j \to WH) = \frac{\pi\alpha^2|V_{ij}|^2}{36\sin^4\theta_W}\frac{2k}{\sqrt{s}}\frac{k^2+3m_W^2}{(s-m_W^2)^2} \tag{39.26}$$

$$\sigma(f\overline{f} \to ZH) = \frac{2\pi\alpha^2(\ell_f^2+r_f^2)}{48N_c\sin^4\theta_W\cos^4\theta_W}\frac{2k}{\sqrt{s}}\frac{k^2+3m_Z^2}{(s-m_Z^2)^2}, \tag{39.27}$$

where ℓ and r are defined as above.

39.4.3. *W and Z Fusion* :

Just as high-energy electrons can be regarded as sources of virtual photon beams, at very high energies they are sources of virtual W and Z beams. For Higgs boson production, it is the longitudinal components of the Ws and Zs that are important [5]. The distribution of longitudinal Ws carrying a fraction y of the electron's energy is [6]

$$f(y) = \frac{g^2}{16\pi^2}\frac{1-y}{y}, \tag{39.28}$$

where $g = e/\sin\theta_W$. In the limit $s \gg m_H \gg m_W$, the partial decay rate is $\Gamma(H \to W_L W_L) = (g^2/64\pi)(m_H^3/m_W^2)$ and in the equivalent W approximation [7]

$$\sigma(e^+e^- \to \overline{\nu}_e\nu_e H) = \frac{1}{16m_W^2}\left(\frac{\alpha}{\sin^2\theta_W}\right)^3 \times\left[\left(1+\frac{m_H^2}{s}\right)\log\frac{s}{m_H^2} - 2 + 2\frac{m_H^2}{s}\right]. \tag{39.29}$$

There are significant corrections to this relation when m_H is not large compared to m_W [8]. For $m_H = 150$ GeV, the estimate is too high by 51% for $\sqrt{s} = 1000$ GeV, 32% too high at $\sqrt{s} = 2000$ GeV, and 22% too high at $\sqrt{s} = 4000$ GeV. Fusion of ZZ to make a Higgs boson can be treated similarly. Identical formulae apply for Higgs production in the collisions of quarks whose charges permit the emission of a W^+ and a W^-, except that QCD corrections and CKM matrix elements are required. Even in the absence of QCD corrections, the fine-structure constant ought to be evaluated at the scale of the collision, say m_W. All quarks contribute to the ZZ fusion process.

39.5. Inclusive hadronic reactions

One-particle inclusive cross sections $Ed^3\sigma/d^3p$ for the production of a particle of momentum p are conveniently expressed in terms of rapidity y (see above) and the momentum p_T transverse to the beam direction (in the c.m.):

$$E\frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi\, dy\, p_T\, dp_T^2} \ . \tag{39.30}$$

In appropriate circumstances, the cross section may be decomposed as a partonic cross section multiplied by the probabilities of finding partons of the prescribed momenta:

$$\sigma_{\text{hadronic}} = \sum_{ij} \int dx_1\ dx_2\ f_i(x_1)\ f_j(x_2)\ d\widehat{\sigma}_{\text{partonic}} \ , \tag{39.31}$$

The probability that a parton of type i carries a fraction of the incident particle's that lies between x_1 and $x_1 + dx_1$ is $f_i(x_1)dx_1$ and similarly for partons in the other incident particle. The partonic collision is specified by its c.m. energy squared $\hat{s} = x_1x_2s$ and the momentum transfer squared $\hat{t}$. The final hadronic state is more conveniently specified by the rapidities y_1, y_2 of the two jets resulting from the collision and the transverse momentum p_T. The connection between the differentials is

$$dx_1dx_2d\hat{t} = dy_1dy_2\frac{\hat{s}}{s}dp_T^2, \tag{39.32}$$

so that

$$\frac{d^3\sigma}{dy_1dy_2dp_T^2} = \frac{\hat{s}}{s}\left[f_i(x_1)f_j(x_2)\frac{d\hat{\sigma}}{d\hat{t}}(\hat{s},\hat{t},\hat{u}) + f_i(x_2)f_j(x_1)\frac{d\hat{\sigma}}{d\hat{t}}(\hat{s},\hat{u},\hat{t})\right], \tag{39.33}$$

where we have taken into account the possibility that the incident parton types might arise from either incident particle. The second term should be dropped if the types are identical: $i = j$.

39.6. Two-photon processes

In the Weizsäcker-Williams picture, a high-energy electron beam is accompanied by a spectrum of virtual photons of energies ω and invariant-mass squared $q^2 = -Q^2$, for which the photon number density is

$$dn = \frac{\alpha}{\pi}\left[1 - \frac{\omega}{E} + \frac{\omega^2}{E^2} - \frac{m_e^2\ \omega^2}{Q^2E^2}\right]\frac{d\omega}{\omega}\frac{dQ^2}{Q^2}, \tag{39.34}$$

where E is the energy of the electron beam. The cross section for $e^+e^- \to e^+e^- X$ is then [9]

$$d\sigma_{e^+e^- \to e^+e^- X}(s) = dn_1 dn_2 d\sigma_{\gamma\gamma\to X}(W^2), \tag{39.35}$$

where $W^2 = m_X^2$. Integrating from the lower limit $Q^2 = m_e^2 \dfrac{\omega_i^2}{E_i(E_i - \omega_i)}$ to a maximum Q^2 gives

$$\begin{aligned}\sigma_{e^+e^- \to e^+e^- X}(s) &= \frac{\alpha^2}{\pi^2}\int_{z_{th}}^{1}\frac{dz}{z} \\ &\times \left[\left(\ln\frac{Q^2_{max}}{zm_e^2} - 1\right)^2 f(z) + \frac{1}{3}(\ln z)^3\right]\sigma_{\gamma\gamma\to X}(zs),\end{aligned} \tag{39.36}$$

where

$$f(z) = \left(1 + \tfrac{1}{2}z\right)^2 \ln(1/z) - \tfrac{1}{2}(1-z)(3+z). \tag{39.37}$$

The appropriate value of Q^2_{max} depends on the properties of the produced system X. For production of hadronic systems, $Q^2_{max} \approx m_\rho^2$, while for lepton-pair production, $Q^2 \approx W^2$. For production of a resonance with spin $J \neq 1$, we have

$$\begin{aligned}\sigma_{e^+e^- \to e^+e^- R}(s) &= (2J+1)\frac{8\alpha^2\Gamma_{R\to\gamma\gamma}}{m_R^3} \\ &\times \left[f(m_R^2/s)\left(\ln\frac{m_V^2 s}{m_e^2 m_R^2} - 1\right)^2 - \frac{1}{3}\left(\ln\frac{s}{M_R^2}\right)^3\right]\end{aligned} \tag{39.38}$$

where m_V is the mass that enters into the form factor for the $\gamma\gamma \to R$ transition, typically m_ρ.

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