CPT INVARIANCE TESTS IN NEUTRAL KAON DECAY

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The time evolution of a neutral kaon state state is described by

$$\frac{d}{dt}\Psi = -i\Lambda\Psi \ , \qquad \Lambda \equiv M - \frac{i}{2}\Gamma \tag{1}$$

where M and Γ are Hermitian 2×2 matrices known as the mass and decay matrices. The corresponding eigenvalues are $\lambda_{L,S} = m_{L,S} - \frac{i}{2}\gamma_{L,S}$. *CPT* invariance requires the diagonal elements of Λ to be equal. The *CPT*-violation complex parameter δ is defined as

$$\begin{aligned} \delta &= \frac{\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0}}{2(\lambda_L - \lambda_S)} \\ &= \delta_{\parallel}\exp(i\phi_{SW}) + \delta_{\perp}\exp(i(\phi_{SW} + \frac{\pi}{2})) \end{aligned} \tag{2}$$

where we have introduced the projections $\delta_{\parallel}$ and $\delta_{\perp}$ respectively parallel and perpendicular to the superweak direction $\phi_{SW} = \tan^{-1}(2\Delta m/\Delta\gamma)$, where $\Delta m = m_L - m_S$ and $\Delta\gamma = \gamma_S - \gamma_L$, the positive mass and width differences between K_L and K_S. These projections are linked to the mass and width difference between K^0 and $\overline{K}^0$:

$$\delta_{\parallel} = \frac{1}{4}\frac{\gamma_{K^0} - \gamma_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ , \ \delta_{\perp} = \frac{1}{2}\frac{m_{K^0} - m_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ . \tag{3}$$

Re(δ) can be directly measured by studying the time evolution of the strangeness content of initially pure K^0 and $\overline{K}^0$ states, for example through the asymmetry

$$A_{CPT} = \frac{P[\overline{K}^0 \to \overline{K}^0(t)] - P[K^0 \to K^0(t)]}{P[\overline{K}^0 \to \overline{K}^0(t)] + P[K^0 \to K^0(t)]} = 4\mathrm{Re}(\delta) \tag{4}$$

where $P[a \to b(t)]$ is the probability that the pure initial state a is seen as state b at proper time t. This method has been used by tagging the initial strangeness with strong interactions and the final strangeness with the semileptonic decay (a more appropriate combination of semileptonic rates allows to be independent of any direct *CPT* violation in the decay itself) and yields today's best value of Re(δ), compatible with zero with an error of $\sim 3 \times 10^{-4}$.

As an alternative it has been proposed to compare the semileptonic charge asymmetries for K_L and K_S

$$A_{L,S} = \frac{R(K_{L,S} \to \pi^- \ell^+ \nu) - R(K_{L,S} \to \pi^+ \ell^- \overline{\nu})}{R(K_{L,S} \to \pi^- \ell^+ \nu) + R(K_{L,S} \to \pi^+ \ell^- \overline{\nu})} ,$$
$$A_S - A_L = 4\mathrm{Re}(\delta) \ . \tag{5}$$

A_L has been accurately measured. A_S has been recently measured with tagged K_S at ϕ factories, however not yet with the required accuracy. Note however that Eq. (5) assumes CPT invariance in the $\Delta S = -\Delta Q$ semileptonic decay amplitude.

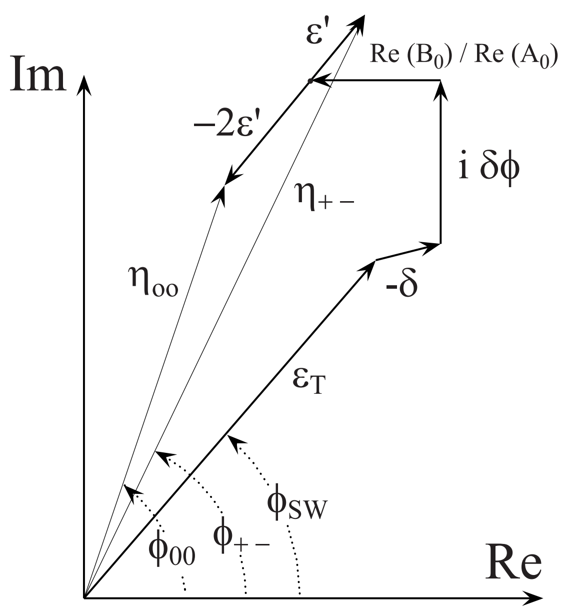

Figure 1: CP- and CPT-violation parameters in 2π decay.

$\delta_\perp$ can be obtained from the measurement of the $\pi\pi$ decays CP-violation parameters η_{+-} and η_{00}. Figure 1 shows the various contributions to $\eta_{\pi\pi}$ [1]. The T-violation parameter ϵ_T

$$\epsilon_T = i \frac{|\Lambda_{K^0 \overline{K}^0}|^2 - |\Lambda_{\overline{K}^0 K^0}|^2}{\Delta\gamma(\lambda_L - \lambda_S)} \tag{6}$$

has been defined in such a way that it is exactly aligned along the superweak direction [‡]. A_I (resp. B_I) is the CPT-conserving (resp. violating) decay amplitude for the $\pi\pi$ Isospin I state, ε' is the direct CP/CPT-violation parameter $[\varepsilon' = 1/3(\eta_{+-} - \eta_{00})]$ and $\delta\phi = \frac{1}{2}\left[\varphi_\Gamma - \arg(A_0^*\overline{A}_0)\right]$ is the phase difference between the $I = 0$ component of the decay amplitude and the matrix element $\Gamma_{K^0\overline{K}^0}$. From Fig. 1 one obtains

$$\begin{aligned}\delta_\perp =& |\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00}) \\ & - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)}\sin(\phi_{SW}) + \delta\phi \;\; \cos(\phi_{SW}) \; . \end{aligned} \quad (7)$$

The present accuracy on the term $|\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00})$ is 2.6×10^{-5}. $\delta\phi$ gets contributions from CP violation in semileptonic and 3π decays [2,3] and can only be neglected at the present time if one assumes that η_{000} is not significantly larger than η_{+-0}. Furthermore, B_0 is not directly measured, so additional assumptions (for example, CPT conservation in the decay which implies $B_0 = 0$) or a combination with other measurements are necessary to obtain $\delta_\perp$.

If one assumes unitarity, one can measure $\mathrm{Im}(\delta)$ using the Bell-Steinberger relation which relates K_S and K_L decay amplitudes into all final states f:

$$\mathrm{Re}(\epsilon_T) - i\mathrm{Im}(\delta) = \frac{1}{2(i\Delta m + \frac{1}{2}(\gamma_L + \gamma_S))} \times \sum A_{f_L} A^*_{f_S} \; . \quad (8)$$

Since the $\pi\pi$ amplitudes dominate, the result relies also strongly on the $\phi_{\pi\pi}$ phase measurements. The advantage is that B_0 does not enter. Using all available data, one obtains a value of $\mathrm{Im}(\delta)$ compatible with zero with a precision of 2×10^{-5}. The precision here is limited by the measurement of η_{+-}.

The results on $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ can be combined to obtain $\delta_\parallel$ and $\delta_\perp$ and therefore the K^0–$\overline{K}^0$ mass and width difference shown in Fig. 2. The current accuracy is a few 10^{-18} GeV for both.

If one assumes that CPT is conserved in the decays ($\gamma_{K^0} = \gamma_{\overline{K}^0}$, $\delta_\parallel = 0$, $B_I = 0$), the phase of δ is known, and the $\delta_\perp$ and Bell-Steinberger methods are identical. One in this case obtains a limit for $|m_{K^0} - m_{\overline{K}^0}|$ of 4.7×10^{-19} GeV (90%CL).

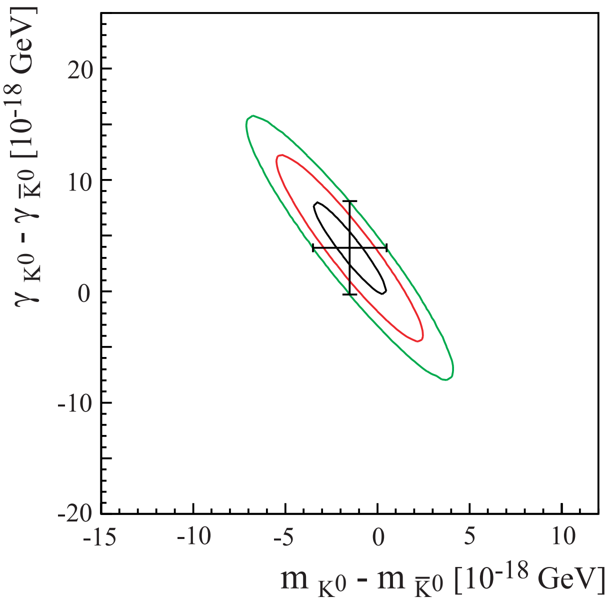

Figure 2: K^0–$\overline{K}^0$ mass vs width difference.

Footnotes and References

[‡] Many authors have a different definition of the T-violation parameter, $\epsilon = (\Lambda_{\overline{K}^0 K^0} - \Lambda_{K^0\overline{K}^0})/(2(\lambda_L - \lambda_S))$. ϵ is not exactly aligned with the superweak direction. The two definitions can be related through $\epsilon = \epsilon_T + i\delta\phi$.

1. See for instance, C.D. Buchanan *et al.*, Phys. Rev. **D45**, 4088 (1992). See also the Second Daphne Handbook, Ed. L.Maiani *et al.*, INFN Frascati (1995).
2. V.V. Barmin *et al.*, Nucl. Phys. **B247**, 293 (1984).
3. L. Lavoura, Mod. Phys. Lett. **A7**, 1367 (1992).