13. NEUTRINO MASS, MIXING, AND FLAVOR CHANGE

Revised September 2005 by B. Kayser (Fermilab).

There is now convincing evidence that atmospheric, solar, and reactor neutrinos change from one flavor to another. There is also very strong evidence that accelerator neutrinos do this as well. Barring exotic possibilities, neutrino flavor change implies that neutrinos have masses and that leptons mix. In this review, we discuss the physics of flavor change and the evidence for it, summarize what has been learned so far about neutrino masses and leptonic mixing, consider the relation between neutrinos and their antiparticles, and discuss the open questions about neutrinos to be answered by future experiments.

I. The physics of flavor change: If neutrinos have masses, then there is a spectrum of three or more neutrino mass eigenstates, ν_1, ν_2, ν_3, ..., that are the analogues of the charged-lepton mass eigenstates, e, μ, and τ. If leptons mix, the weak interaction coupling the W boson to a charged lepton and a neutrino can couple any charged-lepton mass eigenstate ℓ_α to any neutrino mass eigenstate ν_i. Here, $\alpha = e, \mu$, or τ, and ℓ_e is the electron, *etc.*. Leptonic W^+ decay can yield a particular ℓ_α^+ in association with any ν_i. The amplitude for this decay to produce the specific combination $\ell_\alpha^+ + \nu_i$ is $U_{\alpha i}^*$, where U is the unitary leptonic mixing matrix [1]. Thus, the neutrino state created in the decay $W^+ \to \ell_\alpha^+ + \nu$ is the state

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle \ . \tag{13.1}$$

This superposition of neutrino mass eigenstates, produced in association with the charged lepton of "flavor" α, is the state we refer to as the neutrino of flavor α.

While there are only three (known) charged lepton mass eigenstates, the experimental results suggest that perhaps there are more than three neutrino mass eigenstates. If, for example, there are four ν_i, then one linear combination of them,

$$|\nu_s\rangle = \sum_i U_{si}^* |\nu_i\rangle \ , \tag{13.2}$$

does not have a charged-lepton partner, and consequently does not couple to the Standard Model W boson. Indeed, since the decays $Z \to \nu_\alpha \overline{\nu}_\alpha$ of the Standard Model Z boson have been found to yield only three distinct neutrinos ν_α of definite flavor [2], ν_s does not couple to the Z boson either. Such a neutrino, which does not have any Standard Model weak couplings, is referred to as a "sterile" neutrino.

To understand neutrino flavor change, or "oscillation," in vacuum, let us consider how a neutrino born as the ν_α of Eq. (13.1) evolves in time. First, we apply Schrödinger's equation to the ν_i component of ν_α in the rest frame of that component. This tells us that

$$|\nu_i(\tau_i)\rangle = e^{-im_i\tau_i}|\nu_i(0)\rangle \ , \tag{13.3}$$

where m_i is the mass of ν_i, and τ_i is time in the ν_i frame.

In terms of the time t and position L in the laboratory frame, the Lorentz-invariant phase factor in Eq. (13.3) may be written

$$e^{-im_i\tau_i} = e^{-i(E_i t - p_i L)} \ . \tag{13.4}$$

Here, E_i and p_i are respectively the energy and momentum of ν_i in the laboratory frame. In practice, our neutrino will be extremely relativistic, so we will be interested in evaluating the phase factor of Eq. (13.4) with $t \approx L$, where it becomes $\exp[-i(E_i - p_i)L]$.

Imagine now that our ν_α has been produced with a definite momentum p, so that all of its mass-eigenstate components have this common momentum. Then the ν_i component has $E_i = \sqrt{p^2 + m_i^2} \approx p + m_i^2/2p$, assuming that all neutrino masses m_i are small compared to the neutrino momentum. The phase factor of Eq. (13.4) is then approximately

$$e^{-i(m_i^2/2p)L} \ . \tag{13.5}$$

From this expression and Eq. (13.1), it follows that after a neutrino born as a ν_α has propagated a distance L, its state vector has become

$$|\nu_\alpha(L)\rangle \approx \sum_i U^*_{\alpha i} e^{-i(m_i^2/2E)L} |\nu_i\rangle \ . \tag{13.6}$$

Here, $E \simeq p$ is the average energy of the various mass eigenstate components of the neutrino. Using the unitarity of U to invert Eq. (13.1), and inserting the result in Eq. (13.6), we find that

$$|\nu_\alpha(L)\rangle \approx \sum_\beta \left[\sum_i U^*_{\alpha i} e^{-i(m_i^2/2E)L} U_{\beta i} \right] |\nu_\beta\rangle \ . \tag{13.7}$$

We see that our ν_α, in traveling the distance L, has turned into a superposition of all the flavors. The probability that it has flavor β, $P(\nu_\alpha \to \nu_\beta)$, is obviously $|\langle \nu_\beta | \nu_\alpha(L)\rangle|^2$. From Eq. (13.7) and the unitarity of U, we easily find that

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} & \\ -4 \sum_{i>j} & \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2[1.27\, \Delta m^2_{ij}(L/E)] \\ +2 \sum_{i>j} & \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin[2.54\, \Delta m^2_{ij}(L/E)] \ . \end{aligned} \tag{13.8}$$

Here, $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$ is in eV^2, L is in km, and E is in GeV. We have used the fact that when the previously omitted factors of $\hbar$ and c are included,

$$\Delta m^2_{ij}(L/4E) \simeq 1.27\, \Delta m^2_{ij}(\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})} \ . \tag{13.9}$$

The quantum mechanics of neutrino oscillation leading to the result Eq. (13.8) is somewhat subtle. To do justice to the physics requires a more refined treatment [3] than the one we have given. Sophisticated treatments continue to yield new insights [4].

Assuming that CPT invariance holds,

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\beta \to \nu_\alpha) \ . \tag{13.10}$$

But, from Eq. (13.8) we see that

$$P(\nu_\beta \to \nu_\alpha; U) = P(\nu_\alpha \to \nu_\beta; U^*) \ . \tag{13.11}$$

Thus, when CPT holds,

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta; U) = P(\nu_\alpha \to \nu_\beta; U^*) \ . \tag{13.12}$$

That is, the probability for oscillation of an anti-neutrino is the same as that for a neutrino, except that the mixing matrix U is replaced by its complex conjugate. Thus, if U is not real, the neutrino and anti-neutrino oscillation probabilities can differ by having opposite values of the last term in Eq. (13.8). When CPT holds, any difference between these probabilities indicates a violation of CP invariance.

As we shall see, the squared-mass splittings Δm^2_{ij} called for by the various reported signals of oscillation are quite different from one another. It may be that one splitting, ΔM^2, is much bigger than all the others. If that is the case, then for an oscillation experiment with L/E such that $\Delta M^2 L/E = \mathcal{O}(1)$, Eq. (13.8) simplifies considerably, becoming

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \simeq S_{\alpha\beta}\, \sin^2[1.27\, \Delta M^2 (L/E)] \tag{13.13}$$

for $\beta \neq \alpha$, and

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \simeq 1 - 4\, T_\alpha (1 - T_\alpha) \sin^2[1.27\, \Delta M^2 (L/E)] \ . \tag{13.14}$$

Here,

$$S_{\alpha\beta} \equiv 4 \left| \sum_{i \ \mathrm{Up}} U^*_{\alpha i} U_{\beta i} \right|^2 \tag{13.15}$$

and

$$T_\alpha \equiv \sum_{i \ \mathrm{Up}} |U_{\alpha i}|^2 \ , \tag{13.16}$$

where "i Up" denotes a sum over only those neutrino mass eigenstates that lie *above* ΔM^2 or, alternatively, only those that lie *below* it. The unitarity of U guarantees that summing over either of these two clusters will yield the same results for $S_{\alpha\beta}$ and for $T_\alpha(1 - T_\alpha)$.

The situation described by Eqs. (13.13)–(13.16) may be called "quasi-two-neutrino oscillation." It has also been called "one mass scale dominance" [5]. It corresponds to an experiment whose L/E is such that the experiment can "see" only the big splitting ΔM^2. To this experiment, all the neutrinos above ΔM^2 appear to be a single neutrino, as do all those below ΔM^2.

The relations of Eqs. (13.13)–(13.16) also apply to the special case where, to a good approximation, only two mass eigenstates, and two corresponding flavor eigenstates (or two linear combinations of flavor eigenstates), are relevant. One encounters this case when, for example, only two mass eigenstates couple significantly to the charged lepton with which the neutrino being studied is produced. When only two mass eigenstates count, there is only a single splitting, Δm^2, and, omitting irrelevant phase factors, the unitary mixing matrix U takes the form

$$U = \begin{array}{c c} & \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} & \left[\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right] \end{array} \quad . \tag{13.17}$$

Here, the symbols above and to the left of the matrix label the columns and rows, and θ is referred to as the mixing angle. From Eqs. (13.15) and (13.16), we now have $S_{\alpha\beta} = \sin^2 2\theta$ and $4\,T_\alpha(1 - T_\alpha) = \sin^2 2\theta$, so that Eqs. (13.13) and (13.14) become, respectively,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \, \sin^2[1.27\,\Delta m^2(L/E)] \tag{13.18}$$

with $\beta \neq \alpha$, and

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \, \sin^2[1.27\,\Delta m^2(L/E)] \quad . \tag{13.19}$$

Many experiments have been analyzed using these two expressions. Some of these experiments actually have been concerned with quasi-two-neutrino oscillation, rather than a genuinely two-neutrino situation. For these experiments, "$\sin^2 2\theta$" and "Δm^2" have the significance that follows from Eqs. (13.13)–(13.16).

When neutrinos travel through matter (*e.g.*in the Sun, Earth, or a supernova), their coherent forward scattering from particles they encounter along the way can significantly modify their propagation [6]. As a result, the probability for changing flavor can be rather different than it is in vacuum [7]. Flavor change that occurs in matter, and that grows out of the interplay between flavor-nonchanging neutrino-matter interactions and neutrino mass and mixing, is known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

To a good approximation, one can describe neutrino propagation through matter via a Schrödinger-like equation. This equation governs the evolution of a neutrino state vector with several components, one for each flavor. The effective Hamiltonian in the equation, a matrix $\mathcal{H}$ in neutrino flavor space, differs from its vacuum counterpart by the addition of interaction energies arising from the coherent forward neutrino scattering. For example, the ν_e–ν_e element of $\mathcal{H}$ includes the interaction energy

$$V = \sqrt{2}\, G_F N_e \quad , \tag{13.20}$$

arising from W-exchange-induced ν_e forward scattering from ambient electrons. Here, G_F is the Fermi constant, and N_e is the number of electrons per unit volume. In addition, the

ν_e–ν_e, ν_μ–ν_μ, and ν_τ–ν_τ elements of $\mathcal{H}$ all contain a common interaction energy growing out of Z-exchange-induced forward scattering. However, when one is not considering the possibility of transitions to sterile neutrino flavors, this common interaction energy merely adds to $\mathcal{H}$ a multiple of the identity matrix, and such an addition has no effect on flavor transitions.

The effect of matter is illustrated by the propagation of solar neutrinos through solar matter. When combined with information on atmospheric neutrino oscillation, the experimental bounds on short-distance ($L \lesssim 1$ km) oscillation of reactor $\overline{\nu}_e$ [8] tell us that, if there are no sterile neutrinos, then only two neutrino mass eigenstates, ν_1 and ν_2, are significantly involved in the evolution of the solar neutrinos. Correspondingly, only two flavors are involved: the ν_e flavor with which every solar neutrino is born, and the effective flavor ν_x — some linear combination of ν_μ and ν_τ — which it may become. The Hamiltonian $\mathcal{H}$ is then a 2×2 matrix in ν_e–ν_x space. Apart from an irrelevant multiple of the identity, for a distance r from the center of the Sun, $\mathcal{H}$ is given by

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_V + \mathcal{H}_M(r) \\ &= \frac{\Delta m^2_\odot}{4E}\begin{bmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{bmatrix} + \begin{bmatrix} V(r) & 0 \\ 0 & 0 \end{bmatrix} .\end{aligned} \tag{13.21}$$

Here, the first matrix $\mathcal{H}_V$ is the Hamiltonian in vacuum, and the second matrix $\mathcal{H}_M(r)$ is the modification due to matter. In $\mathcal{H}_V$, $\theta_\odot$ is the solar mixing angle defined by the two-neutrino mixing matrix of Eq. (13.17) with $\theta = \theta_\odot, \nu_\alpha = \nu_e$, and $\nu_\beta = \nu_x$. The splitting $\Delta m^2_\odot$ is $m_2^2 - m_1^2$, and for the present purpose we *define* ν_2 to be the heavier of the two mass eigenstates, so that $\Delta m^2_\odot$ is positive. In $\mathcal{H}_M(r)$, $V(r)$ is the interaction energy of Eq. (13.20) with the electron density $N_e(r)$ evaluated at distance r from the Sun's center.

From Eqs. (13.18–13.19) (with $\theta = \theta_\odot$), we see that two-neutrino oscillation in vacuum cannot distinguish between a mixing angle $\theta_\odot$ and an angle $\theta'_\odot = \pi/2 - \theta_\odot$. But these two mixing angles represent physically different situations. Suppose, for example, that $\theta_\odot < \pi/4$. Then, from Eq. (13.17) we see that if the mixing angle is $\theta_\odot$, the lighter mass eigenstate (defined to be ν_1) is more ν_e than ν_x, while if it is $\theta'_\odot$, then this mass eigenstate is more ν_x than ν_e. While oscillation in vacuum cannot discriminate between these two possibilities, neutrino propagation through solar matter can do so. The neutrino interaction energy V of Eq. (13.20) is of definite, positive sign [9]. Thus, the ν_e–ν_e element of the solar $\mathcal{H}$, $-(\Delta m^2_\odot/4E)\cos 2\theta_\odot + V(r)$, has a different size when the mixing angle is $\theta'_\odot = \pi/2 - \theta_\odot$ than it does when this angle is $\theta_\odot$. As a result, the flavor content of the neutrinos coming from the Sun can be different in the two cases [10].

Solar and long-baseline reactor neutrino data establish that the behavior of solar neutrinos is governed by a Large-Mixing-Angle (LMA) MSW effect (see Sec. II). Let us estimate the probability $P(\nu_e \to \nu_e)$ that a solar neutrino which undergoes the LMA-MSW effect in the Sun still has its original ν_e flavor when it arrives at the Earth. We focus on the neutrinos produced by ^{8}B decay, which are at the high-energy end of the solar neutrino spectrum. At $r \simeq 0$, where the solar neutrinos are created, the electron density $N_e \simeq 6 \times 10^{25}/\text{cm}^3$ [11] yields for the interaction energy V of Eq. (13.20) the value 0.75×10^{-5} eV^2/MeV. Thus, for $\Delta m^2_\odot$ in the favored region, around 8×10^{-5} eV^2, and E a typical ^{8}B neutrino energy ($\sim$6-7 MeV), $\mathcal{H}_M$ dominates over $\mathcal{H}_V$. This means that, in first approximation, $\mathcal{H}(r \simeq 0)$ is diagonal. Thus, a ^{8}B neutrino is born not only in a ν_e flavor eigenstate, but also, again in first approximation, in an eigenstate of the Hamiltonian $\mathcal{H}(r \simeq 0)$. Since $V > 0$, the neutrino will be in the heavier of the two eigenstates. Now, under the conditions where the LMA-MSW effect occurs, the propagation of a neutrino from $r \simeq 0$ to the outer edge of the Sun is adiabatic. That is, $N_e(r)$ changes sufficiently slowly that we may solve Schrödinger's equation for one r at a time, and then patch together the solutions. This means that our neutrino propagates outward through the Sun as one of the r-dependent eigenstates of the r-dependent $\mathcal{H}(r)$. Since the eigenvalues of $\mathcal{H}(r)$ do not cross at any r, and our neutrino is born in the heavier of the two $r = 0$ eigenstates, it emerges from the Sun in the heavier of the two $\mathcal{H}_V$ eigenstates. The latter is the mass eigenstate we have called ν_2, given according to Eq. (13.17) by

$$\nu_2 = \nu_e \sin\theta_\odot + \nu_x \cos\theta_\odot \ . \tag{13.22}$$

Since this is an eigenstate of the vacuum Hamiltonian, the neutrino remains in it all the way to the surface of the Earth. The probability of observing the neutrino as a ν_e on Earth is then just the probability that ν_2 is a ν_e. That is [cf. Eq. (13.22)] [12],

$$P(\nu_e \to \nu_e) = \sin^2\theta_\odot \ . \tag{13.23}$$

We note that for $\theta_\odot < \pi/4$, this ν_e survival probability is less than 1/2. In contrast, when matter effects are negligible, the energy-averaged survival probability in two-neutrino oscillation cannot be less than 1/2 for any mixing angle [see Eq. (13.19)] [13].

II. The evidence for flavor metamorphosis: The persuasiveness of the evidence that neutrinos actually do change flavor in nature is summarized in Table 13.1. We discuss the different pieces of evidence.

Table 13.1: The persuasiveness of the evidence for neutrino flavor change. The symbol L denotes the distance travelled by the neutrinos. LSND is the Liquid Scintillator Neutrino Detector experiment.

Neutrinos	Evidence for Flavor Change
Atmospheric	Compelling
Accelerator ($L = 250$ km)	Very Strong
Solar	Compelling
Reactor ($L \sim 180$ km)	Compelling
From Stopped μ^+ Decay (LSND)	Unconfirmed

The atmospheric neutrinos are produced in the Earth's atmosphere by cosmic rays, and then detected in an underground detector. The flux of cosmic rays that lead to neutrinos with energies above a few GeV is isotropic, so that these neutrinos are produced at the same rate all around the Earth. This can easily be shown to imply that at any underground site, the downward- and upward-going fluxes of multi-GeV neutrinos of a given flavor must be equal. That is, unless some mechanism changes the flux of neutrinos of the given flavor as they propagate, the flux coming down from zenith angle θ_Z must equal that coming up from angle $\pi - \theta_Z$ [14].

The underground Super-Kamiokande (SK) detector finds that for multi-GeV atmospheric muon neutrinos, the θ_Z event distribution looks nothing like the expected $\theta_Z \Leftrightarrow \pi - \theta_Z$ symmetric distribution. For $\cos\theta_Z \gtrsim 0.3$, the observed ν_μ flux coming up from zenith angle $\pi - \theta_Z$ is only about half that coming down from angle θ_Z [15]. Thus, some mechanism does change the ν_μ flux as the neutrinos travel to the detector. The most attractive candidate for this mechanism is the oscillation $\nu_\mu \rightarrow \nu_X$ of the muon neutrinos into neutrinos ν_X of another flavor. Since the upward-going muon neutrinos come from the atmosphere on the opposite side of the Earth from the detector, they travel much farther than the downward-going ones to reach the detector. Thus, they have more time to oscillate away into the other flavor, which explains why Flux Up < Flux Down. The null results of short-baseline reactor neutrino experiments [8] imply limits on $P(\overline{\nu}_e \rightarrow \overline{\nu}_\mu)$, which, assuming CPT invariance, are also limits on $P(\nu_\mu \rightarrow \nu_e)$. From the latter, we know that ν_X is not a ν_e, except possibly a small fraction of the time. Thus, ν_X is a ν_τ, a sterile neutrino ν_s, or sometimes one and sometimes the other. All of the voluminous, detailed SK atmospheric neutrino data are very well described by the hypothesis that the oscillation is purely $\nu_\mu \rightarrow \nu_\tau$, and that it is a quasi-two-neutrino oscillation with a splitting $\Delta m^2_{\rm atm}$ and a mixing angle $\theta_{\rm atm}$ that, at 90% CL, are in the ranges [16]

$$1.9 \times 10^{-3}\,{\rm eV}^2 < \Delta m^2_{\rm atm} < 3.0 \times 10^{-3}\,{\rm eV}^2 \quad , \tag{13.24}$$

and

$$\sin^2 2\theta_{\rm atm} > 0.90 \quad . \tag{13.25}$$

Other experiments yield results consistent with these [17,18]. We note that the constraint (13.24) implies that at least one mass eigenstate ν_i has a mass exceeding 40 meV. From several pieces of evidence, the 90% CL upper limit on the fraction of ν_X that is sterile is 19% [19].

From Eq. (13.14), we see that the quasi-two-neutrino atmospheric $\nu_\mu \rightarrow \nu_\tau$ oscillation with dominant splitting $\Delta M^2 = \Delta m^2_{\rm atm}$ should lead to a minimum in the survival probability $P(\nu_\mu \rightarrow \nu_\mu)$ when $1.27\Delta m^2_{\rm atm}(L/E) = \pi/2$. For $\Delta m^2_{\rm atm}$ in the range of Eq. (13.24), this minimum is predicted to occur when $L/E \simeq 500\,{\rm km/GeV}$. The Super-Kamiokande experiment has confirmed that a dip in the ν_μ survival probability does occur at this L/E [16,20].

The oscillation interpretation of the atmospheric neutrino data has received support from the KEK to Kamioka (K2K) long-baseline experiment. This experiment produced a ν_μ beam using an accelerator, measured the beam intensity with a complex of near detectors, and then measured the ν_μ flux still in the beam 250 km away using the SK detector. The L/E of this experiment was such that one expected to see an oscillation dominated by the atmospheric squared-mass splitting $\Delta m^2_{\rm atm}$. From its near detector measurements, K2K would have seen $155.9 \pm 0.3\ ^{+13.6}_{-15.6}$ events in the distant SK detector if there had been no oscillation. However, it actually saw only 112 events in SK [20]. In addition, the spectrum of ν_μ events observed in SK was distorted relative to the no-oscillation spectrum. The anomalously small number of events and spectral distortion seen by SK are consistent with a neutrino oscillation interpretation, with parameters $\Delta m^2_{\rm atm}$ and $\sin^2 2\theta_{\rm atm}$ compatible with those that fit the atmospheric neutrino data [20].

The neutrinos created in the Sun have been detected on Earth by several experiments, as discussed by K. Nakamura in this *Review*. The nuclear processes that power the Sun make only ν_e, not ν_μ or ν_τ. For years, solar neutrino experiments had been finding that the solar ν_e flux arriving at the Earth is below the one expected from neutrino production calculations. Now, thanks especially to the Sudbury Neutrino Observatory (SNO), we have compelling evidence that the missing ν_e have simply changed into neutrinos of other flavors.

SNO has studied the flux of high-energy solar neutrinos from ^{8}B decay. This experiment detects these neutrinos via the reactions

$$\nu + d \to e^- + p + p \ , \tag{13.26}$$

$$\nu + d \to \nu + p + n \ , \tag{13.27}$$

and

$$\nu + e^- \to \nu + e^- \ . \tag{13.28}$$

The first of these reactions, charged-current deuteron breakup, can be initiated only by a ν_e. Thus, it measures the flux $\phi(\nu_e)$ of ν_e from ^{8}B decay in the Sun. The second reaction, neutral-current deuteron breakup, can be initiated with equal cross sections by neutrinos of all active flavors. Thus, it measures $\phi(\nu_e) + \phi(\nu_{\mu,\tau})$, where $\phi(\nu_{\mu,\tau})$ is the flux of ν_μ and/or ν_τ from the Sun. Finally, the third reaction, neutrino electron elastic scattering, can be triggered by a neutrino of any active flavor, but $\sigma(\nu_{\mu,\tau}\, e \to \nu_{\mu,\tau}\, e) \simeq \sigma(\nu_e\, e \to \nu_e\, e)/6.5$. Thus, this reaction measures $\phi(\nu_e) + \phi(\nu_{\mu,\tau})/6.5$.

Recently, SNO has reported the results of measurements made with increased sensitivity to the neutral-current deuteron breakup, and with an enhanced ability to statistically separate the neutral-current breakups from the charged-current ones [21]. These improvements were achieved by adding salt to the heavy-water-filled SNO detector. From its observed rates for the two deuteron breakup reactions, SNO finds that [21]

$$\frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})} = 0.340 \pm 0.023\,({\rm stat})\ ^{+0.029}_{-0.031}\,({\rm syst}) \ . \tag{13.29}$$

Clearly, $\phi(\nu_{\mu,\tau})$ is not zero. This non-vanishing $\nu_{\mu,\tau}$ flux from the Sun is "smoking-gun" evidence that some of the ν_e produced in the solar core do indeed change flavor.

Corroborating information comes from the detection reaction $\nu e^- \to \nu e^-$, studied by both SNO and SK [22].

Change of neutrino flavor, whether in matter or vacuum, does not change the total neutrino flux. Thus, unless some of the solar ν_e are changing into sterile neutrinos, the total active high-energy flux measured by the neutral-current reaction

(13.27) should agree with the predicted total ^{8}B solar neutrino flux based on calculations of neutrino production in the Sun. This predicted total is $(5.69 \pm 0.91) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ [23]. By comparison, the total active flux measured by reaction (13.27) is $[4.94 \pm 0.21\,(\mathrm{stat})\,^{+0.38}_{-0.34}\,(\mathrm{syst})] \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$, in good agreement. This agreement provides evidence that neutrino production in the Sun is correctly understood, further strengthens the evidence that neutrinos really do change flavor, and strengthens the evidence that the previously-reported deficits of solar ν_e flux are due to this change of flavor.

The strongly favored explanation of solar neutrino flavor change is the LMA-MSW effect. As pointed out after Eq. (13.23), a ν_e survival probability below 1/2, which is indicated by Eq. (13.29), requires that solar matter effects play a significant role [24]. The LMA-MSW interpretation of solar neutrino behavior implies that a substantial fraction of reactor $\overline{\nu}_e$ that travel more than a hundred kilometers should disappear into anti-neutrinos of other flavors. The KamLAND experiment, which studies reactor $\overline{\nu}_e$ that typically travel ~ 180 km to reach the detector, finds that, indeed, the $\overline{\nu}_e$ flux at the detector is only $0.658 \pm 0.044\,(\mathrm{stat}) \pm 0.047\,(\mathrm{syst})$ of what it would be if no $\overline{\nu}_e$ were vanishing [25]. In addition, KamLAND observes the energy spectrum of the $\overline{\nu}_e$ that do reach the detector to be distorted, relative to the spectrum expected in the absence of oscillation. Plotting the ratio of the observed spectrum to the no-oscillation spectrum versus $1/E$, the KamLAND Collaboration finds evidence for the oscillatory L/E dependence that, as illustrated by Eq. (13.19), is a signature feature of neutrino oscillation [25].

The KamLAND data establish that the "solar" mixing angle $\theta_\odot$ is indeed large. In addition, KamLAND helps to confirm the LMA-MSW explanation of solar neutrino behavior since both the KamLAND result and all the solar neutrino data [26] can be described by the same neutrino parameters, in the LMA-MSW region. A global fit to both the solar and KamLAND data constrains these parameters, the solar $\Delta m^2_\odot$ and $\theta_\odot$ defined after Eq. (13.21), to lie in the region shown in Fig. 13.1 [21]. That θ_{atm}, Eq. (13.25), and $\theta_\odot$, Fig. 13.1, are both large, in striking contrast to all quark mixing angles, is very interesting.

While the total active solar neutrino flux measured by SNO via neutral-current deuteron breakup is compatible with the theoretically predicted total ^{8}B neutrino production by the Sun, we have seen that the uncertainties in these quantities are not negligible. It remains possible that some of the solar ν_e that change their flavor become sterile. Taking into account both the solar and KamLAND data, but not assuming the total ^{8}B solar neutrino flux to be known from theory, it has been found that, at $3\,\sigma$, the sterile fraction of the non-ν_e solar neutrino flux at Earth is less than 35% [27].

The neutrinos studied by the LSND experiment [28] come from the decay $\mu^+ \to e^+\nu_e\overline{\nu}_\mu$ of muons at rest. While this decay does not produce $\overline{\nu}_e$, an excess of $\overline{\nu}_e$ over expected background is reported by the experiment. This excess is interpreted as due to oscillation of some of the $\overline{\nu}_\mu$ produced by μ^+ decay into $\overline{\nu}_e$. The related KArlsruhe Rutherford Medium Energy Neutrino (KARMEN) experiment [29] sees no indication for such an oscillation. However, the LSND and KARMEN experiments are not identical; at LSND the neutrino travels a distance $L \approx 30\,\mathrm{m}$ before detection, while at KARMEN it travels

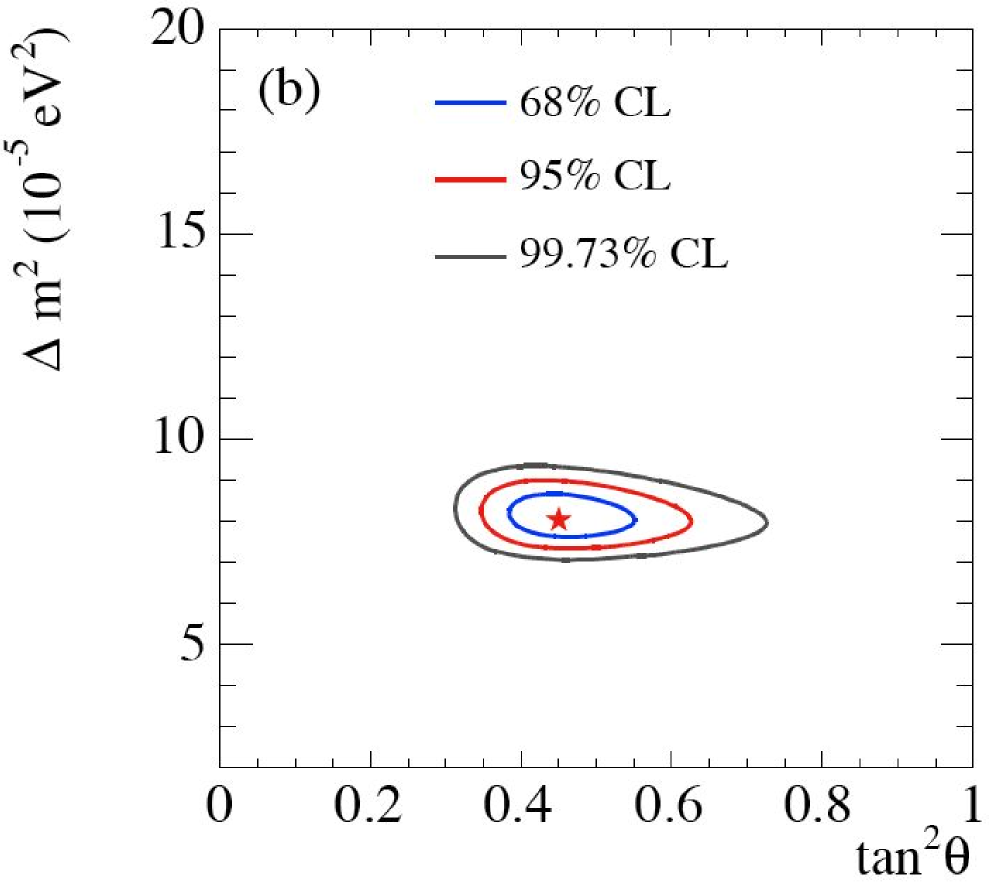

Figure 13.1: The region allowed for the neutrino parameters $\Delta m^2_\odot$ and $\theta_\odot$ by the solar and KamLAND data. The best-fit point, indicated by the star, is $\Delta m^2_\odot = (8.0^{+0.6}_{-0.4}) \times 10^{-5}\,\mathrm{eV}^2$ and $\theta_\odot = (33.9^{+2.4}_{-2.2})^\circ$. [21] See full-color version on color pages at end of book.

$L \approx 18\,\mathrm{m}$. The KARMEN results exclude a portion of the neutrino parameter region favored by LSND, but not all of it. A joint analysis [30] of the results of both experiments finds that a splitting $0.2 \lesssim \Delta m^2_{\mathrm{LSND}} \lesssim 1\,\mathrm{eV}^2$ and mixing $0.003 \lesssim \sin^2 2\theta_{\mathrm{LSND}} \lesssim 0.03$, or a splitting $\Delta m^2_{\mathrm{LSND}} \simeq 7\,\mathrm{eV}^2$ and mixing $\sin^2 2\theta_{\mathrm{LSND}} \simeq 0.004$, might explain both experiments.

The regions of neutrino parameter space favored or excluded by various neutrino oscillation experiments are shown in Fig. 13.2.

III. Neutrino spectra and mixings: If there are only three neutrino mass eigenstates, ν_1, ν_2 and ν_3, then there are only three mass splittings Δm^2_{ij}, and they obviously satisfy

$$\Delta m^2_{32} + \Delta m^2_{21} + \Delta m^2_{13} = 0 \quad . \tag{13.30}$$

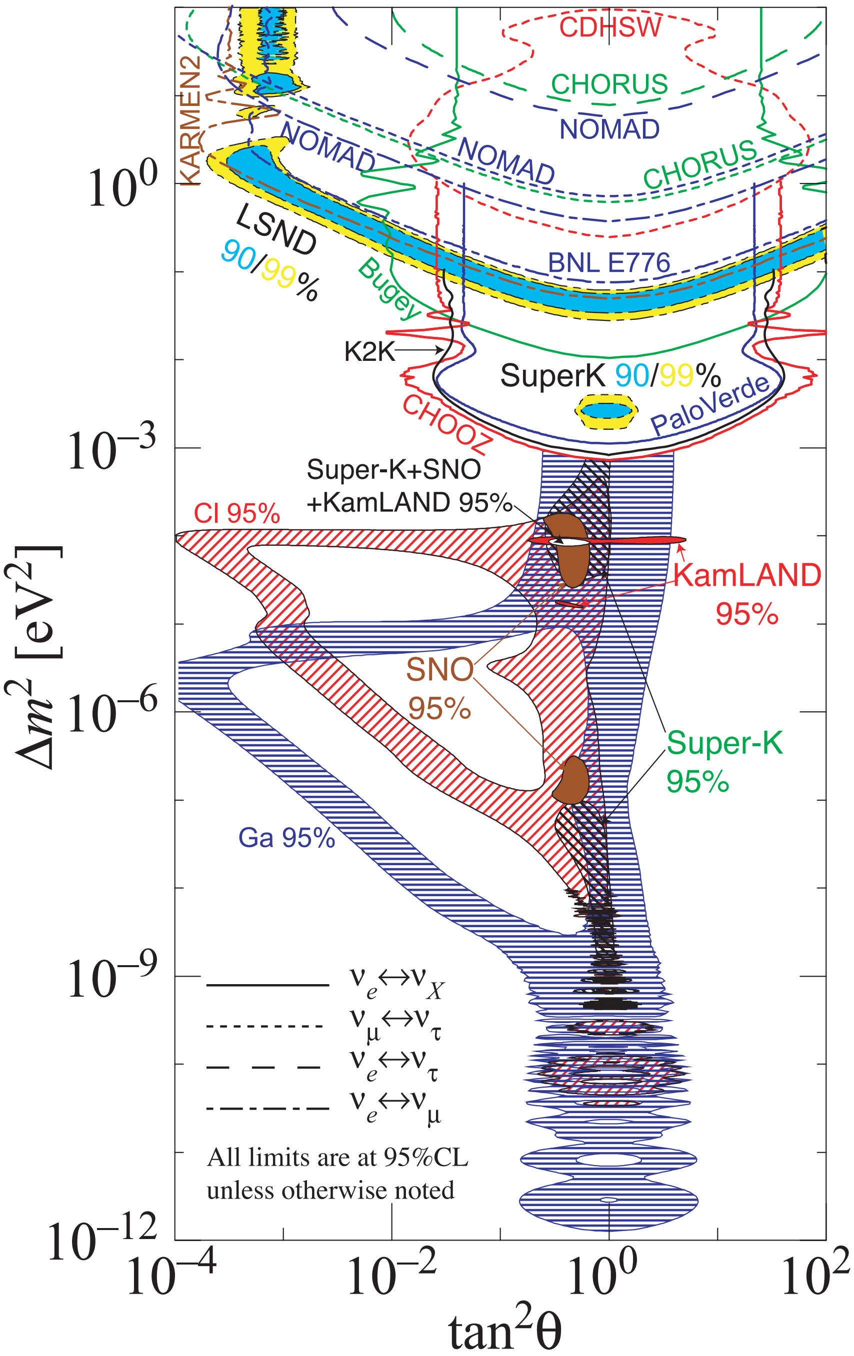

Figure 13.2: The regions of squared-mass splitting and mixing angle favored or excluded by various experiments. This figure was contributed by H. Murayama (University of California, Berkeley). References to the data used in the figure can be found at http://hitoshi.berkeley.edu/neutrino/ref2006.html. See full-color version on color pages at end of book.

However, as we have seen, the Δm^2 values required to explain the flavor changes of the atmospheric, solar, and LSND neutrinos are of three different orders of magnitude. Thus, they cannot possibly obey the constraint of Eq. (13.30). If all of the reported changes of flavor are genuine, then nature must contain at least four neutrino mass eigenstates [31]. As explained in Sec. I, one linear combination of these mass eigenstates would have to be sterile.

If the LSND oscillation is not confirmed, then nature may well contain only three neutrino mass eigenstates. The neutrino spectrum then contains two mass eigenstates separated by the splitting $\Delta m^2_{\odot}$ needed to explain the solar and KamLAND data, and a third eigenstate separated from the first two by the larger splitting $\Delta m^2_{\mathrm{atm}}$ called for by the atmospheric and K2K data. Current experiments do not tell us whether the solar pair — the two eigenstates separated by $\Delta m^2_{\odot}$ — is at the bottom or the top of the spectrum. These two possibilities are usually referred to, respectively, as a normal and an inverted spectrum. The study of flavor changes of accelerator-generated neutrinos and anti-neutrinos that pass through matter can discriminate between these two spectra (see Sec. V). If the solar pair is at the bottom, then the spectrum is of the form shown in Fig. 13.3. There we include the approximate flavor content of each mass eigenstate, the flavor-α fraction of eigenstate ν_i being simply $|\langle \nu_\alpha | \nu_i \rangle|^2 = |U_{\alpha i}|^2$. The flavor content shown assumes that the atmospheric mixing angle of Eq. (13.25) is maximal (which gives the best fit to the atmospheric data [20]), and takes into account the now-established LMA-MSW explanation of solar neutrino behavior. For simplicity, it neglects the small, as-yet-unknown ν_e fraction of ν_3 (see below).

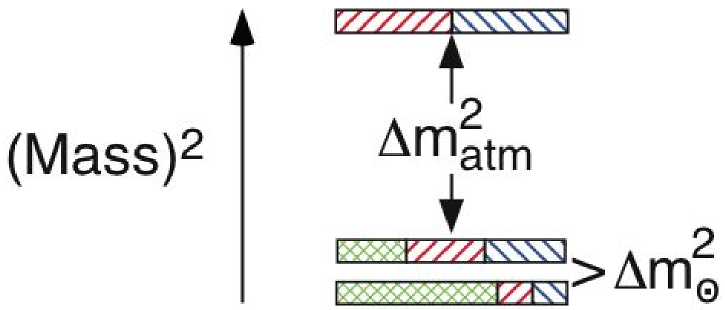

Figure 13.3: A three-neutrino squared-mass spectrum that accounts for the observed flavor changes of solar, reactor, atmospheric, and long-baseline accelerator neutrinos. The ν_e fraction of each mass eigenstate is crosshatched, the ν_μ fraction is indicated by right-leaning hatching, and the ν_τ fraction by left-leaning hatching.

When there are only three neutrino mass eigenstates, and the corresponding three familiar neutrinos of definite flavor, the leptonic mixing matrix U can be written as

$$\nu_1 \qquad\qquad\qquad \nu_2 \qquad\qquad \nu_3$$

$$
U = \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array}
\begin{bmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{bmatrix}
\times \mathrm{diag}(e^{i\alpha_1/2},\ e^{i\,\alpha_2/2},\ 1)\ . \tag{13.31}
$$

Here, ν_1 and ν_2 are the members of the solar pair, with $m_2 > m_1$, and ν_3 is the isolated neutrino, which may be heavier or lighter than the solar pair. Inside the matrix, $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, where the three θ_{ij}'s are mixing angles. The quantities δ, α_1, and α_2 are CP-violating phases. The phases α_1 and α_2, known as Majorana phases, have physical consequences only if neutrinos are Majorana particles, identical to their antiparticles. Then these phases influence neutrinoless double beta decay [see Sec. IV] and other processes [32]. However, as we see from Eq. (13.8), α_1 and α_2 do not affect neutrino oscillation, regardless of whether neutrinos are Majorana particles. Apart from the phases α_1, α_2, which have no quark analogues, the parametrization of the leptonic mixing matrix in Eq. (13.31) is identical to that [33] advocated for the quark mixing matrix by Gilman, Kleinknecht, and Renk in their article in this *Review*.

From bounds on the short-distance oscillation of reactor $\overline{\nu}_e$ [8] and other data, at $2\,\sigma$, $|U_{e3}|^2 \lesssim 0.032$ [34]. (Thus, the ν_e fraction of ν_3 would have been too small to see in Fig. 13.3; this is the reason it was neglected.) From Eq. (13.31), we see that the bound on $|U_{e3}|^2$ implies that $s_{13}^2 \lesssim 0.032$. From Eq. (13.31), we also see that the CP-violating phase δ, which is the sole phase in the U matrix that can produce CP violation in neutrino oscillation, enters U only in combination with s_{13}. Thus, the size of CP violation in oscillation will depend on s_{13}.

Given that s_{13} is small, Eqs. (13.31), (13.14), and (13.16) imply that the atmospheric mixing angle θ_{atm} extracted from ν_μ disappearance measurements is approximately θ_{23}, while Eqs. (13.31) and (13.17) (with $\nu_\alpha = \nu_e$ and $\theta = \theta_\odot$) imply that $\theta_\odot \simeq \theta_{12}$.

If the LSND oscillation is confirmed, then, as already noted, there must be at least four mass eigenstates. It is found that if there are *exactly* four, a statistically satisfactory fit to all the neutrino data is not possible. However, if there are *at least* four neutrino mass eigenstates, there is no strong reason to believe that there are *exactly* four. The presence of more states may improve the quality of the fit. For example, it has been found that a "3+2" spectrum fits all the short-baseline data significantly better than a 3+1 spectrum [35].

IV. The neutrino-anti-neutrino relation: Unlike quarks and charged leptons, neutrinos may be their own antiparticles. Whether they are depends on the nature of the physics that gives them mass.

In the Standard Model (SM), neutrinos are assumed to be massless. Now that we know they do have masses, it is straightforward to extend the SM to accommodate these masses in the same way that this model accommodates quark and charged lepton masses. When a neutrino ν is assumed to be massless, the SM does not contain the chirally right-handed neutrino field ν_R, but only the left-handed field ν_L that couples to the W and Z bosons. To accommodate the ν mass in the same manner as quark masses are accommodated, we add ν_R to the Model. Then we may construct the "Dirac mass term"

$$\mathcal{L}_D = -m_D \, \overline{\nu}_L \, \nu_R + h.c. \quad , \tag{13.32}$$

in which m_D is a constant. This term, which mimics the mass terms of quarks and charged leptons, conserves the lepton number L that distinguishes neutrinos and negatively-charged leptons on the one hand from anti-neutrinos and positively-charged leptons on the other. Since everything else in the SM also conserves L, we then have an L-conserving world. In such a world, each neutrino mass eigenstate ν_i differs from its antiparticle $\overline{\nu}_i$, the difference being that $L(\overline{\nu}_i) = -L(\nu_i)$. When $\overline{\nu}_i \neq \nu_i$, we refer to the $\nu_i - \overline{\nu}_i$ complex as a "Dirac neutrino."

Once ν_R has been added to our description of neutrinos, a "Majorana mass term,"

$$\mathcal{L}_M = -m_R \, \overline{\nu_R^c} \, \nu_R + h.c. \quad , \tag{13.33}$$

can be constructed out of ν_R and its charge conjugate, ν_R^c. In this term, m_R is another constant. Since both ν_R and $\overline{\nu_R^c}$ absorb ν and create $\overline{\nu}$, $\mathcal{L}_M$ mixes ν and $\overline{\nu}$. Thus, a Majorana mass term does not conserve L. There is then no conserved lepton number to distinguish a neutrino mass eigenstate ν_i from its antiparticle. Hence, when Majorana mass terms are present, $\overline{\nu}_i = \nu_i$. That is, for a given helicity h, $\overline{\nu}_i(h) = \nu_i(h)$. We then refer to ν_i as a "Majorana neutrino."

Suppose the right-handed neutrinos required by Dirac mass terms have been added to the SM. If we insist that this extended SM conserve L, then, of course, Majorana mass terms are forbidden. However, if we do not impose L conservation, but require only the general principles of gauge invariance and renormalizability, then Majorana mass terms like that of Eq. (13.33) are expected to be present. As a result, L is violated, and neutrinos are Majorana particles [36].

In the see-saw mechanism [37], which is the most popular explanation of why neutrinos — although massive — are nevertheless so light, both Dirac and Majorana mass terms are present. Hence, the neutrinos are Majorana particles. However, while half of them are the familiar light neutrinos, the other half are extremely heavy Majorana particles referred to as the N_i, with masses possibly as large as the GUT scale. The N_i may have played a crucial role in baryogenesis in the early universe, as we shall discuss in Sec. V.

How can the theoretical expectation that L is violated and neutrinos are Majorana particles be confirmed experimentally? The interactions of neutrinos are well described by the SM, and the SM interactions conserve L. If we may neglect any non-SM L-violating interactions, then the only sources of L violation are the neutrino Majorana mass terms. This means that all L-violating effects disappear in the limit of vanishing neutrino masses. Thus, any experimental approach to confirming the violation of L, and the consequent Majorana character of neutrinos, must be able to see an L violation that is going to be very small because of the smallness of the neutrino masses that drive it. One approach that shows great promise is the search for neutrinoless double beta decay ($0\nu\beta\beta$). This is the process $(A, Z) \rightarrow (A, Z+2) + 2e^-$, in which a nucleus containing A nucleons, Z of which are protons, decays to a nucleus containing $Z+2$ protons by emitting two electrons.

This process manifestly violates L conservation, so we expect it to be suppressed. However, if (A, Z) is a nucleus that is stable against single β (and α and γ) decay, then it can decay only via the process we are seeking, and the L-conserving two-neutrino process $(A, Z) \to (A, Z+2) + 2e^- + 2\overline{\nu}_e$. The latter decay mode is suppressed by the small phase space associated with the four light particles in the final state, so we have a chance to observe the neutrinoless mode, $(A, Z) \to (A, Z+2) + 2e^-$.

While $0\nu\beta\beta$ can in principle receive contributions from a variety of mechanisms (R-parity-violating supersymmetric couplings, for example), it is easy to show explicitly that the observation of $0\nu\beta\beta$ at any non-vanishing rate would imply that nature contains at least one Majorana neutrino mass term [38]. Now, quarks and charged leptons cannot have Majorana mass terms, because such terms mix fermion and antifermion, and $q \leftrightarrow \overline{q}$ or $\ell \leftrightarrow \overline{\ell}$ would not conserve electric charge. Thus, the discovery of $0\nu\beta\beta$ would demonstrate that the physics of neutrino masses is unlike that of the masses of all other fermions.

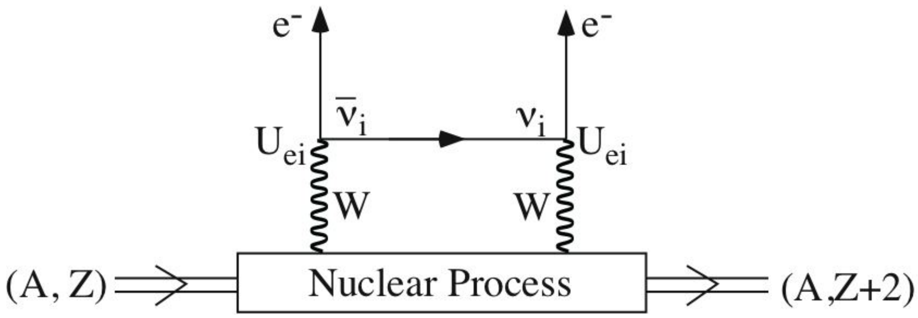

Figure 13.4: The dominant mechanism for $0\nu\beta\beta$. The diagram does not exist unless $\overline{\nu}_i = \nu_i$.

The dominant mechanism for $0\nu\beta\beta$ is expected to be the one depicted in Fig. 13.4. There, a pair of virtual W bosons are emitted by the parent nucleus, and then these W bosons exchange one or another of the light neutrino mass eigenstates ν_i to produce the outgoing electrons. The $0\nu\beta\beta$ amplitude is then a sum over the contributions of the different ν_i. It is assumed that the interactions at the two leptonic W vertices are those of the SM.

Since the exchanged ν_i is created together with an e^-, the left-handed SM current that creates it gives it the helicity we associate, in common parlance, with an "anti-neutrino." That is, the ν_i is almost totally right-handed, but has a small left-handed-helicity component, whose amplitude is of order m_i/E, where E is the ν_i energy. At the vertex where this ν_i is absorbed, the absorbing left-handed SM current can absorb only its small left-handed-helicity component without further suppression. Consequently, the ν_i-exchange contribution to the $0\nu\beta\beta$ amplitude is proportional to m_i. From Fig. 13.4, we see that this contribution is also proportional to U_{ei}^2. Thus, summing over the contributions of all the ν_i, we conclude that the amplitude for $0\nu\beta\beta$ is proportional to the quantity

$$\left|\sum_i m_i U_{ei}^2\right| \equiv |< m_{\beta\beta} >| \quad , \tag{13.34}$$

commonly referred to as the "effective Majorana mass for neutrinoless double beta decay" [39].

That the $0\nu\beta\beta$ amplitude arising from the diagram in Fig. 13.4 is proportional to neutrino mass is no surprise, and illustrates our earlier general discussion. The diagram in Fig. 13.4 is manifestly L-nonconserving. But we are assuming that the interactions in this diagram are L-conserving. Thus, the L-nonconservation in the diagram as a whole must be coming from underlying Majorana neutrino mass terms. Hence, if all the neutrino masses vanish, the L-nonconservation will vanish as well.

To how small an $|< m_{\beta\beta} >|$ should a $0\nu\beta\beta$ search be sensitive? In answering this question, it makes sense to assume there are only three neutrino mass eigenstates — if there are more, $|< m_{\beta\beta} >|$ might be larger. Suppose that there are just three mass eigenstates, and that the solar pair, ν_1 and ν_2, is at the top of the spectrum, so that we have an inverted spectrum. If the various ν_i are not much heavier than demanded by the observed splittings $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\odot}$, then in $|< m_{\beta\beta} >|$, Eq. (13.34), the contribution of ν_3 may be neglected, because both m_3 and $|U_{e3}^2| = s_{13}^2$ are small. From Eqs. (13.34) and (13.31), approximating c_{13} by unity, we then have that

$$|< m_{\beta\beta} >| \simeq m_0 \sqrt{1 - \sin^2 2\theta_{\odot} \sin^2\left(\frac{\Delta\alpha}{2}\right)} \quad . \tag{13.35}$$

Here, m_0 is the average mass of the members of the solar pair, whose splitting will be invisible in a practical $0\nu\beta\beta$ experiment, and $\Delta\alpha \equiv \alpha_2 - \alpha_1$ is a CP-violating phase. Although $\Delta\alpha$ is completely unknown, we see from Eq. (13.35) that

$$|< m_{\beta\beta} >| \geq m_0 \cos 2\theta_{\odot} \quad . \tag{13.36}$$

Now, in an inverted spectrum, $m_0 \geq \sqrt{\Delta m^2_{\rm atm}}$. At 90% CL, $\sqrt{\Delta m^2_{\rm atm}} > 40$ meV [16], while at 95% CL, $\cos 2\theta_{\odot} > 0.28$ [25]. Thus, if neutrinos are Majorana particles, and the spectrum is as we have assumed, a $0\nu\beta\beta$ experiment sensitive to $|< m_{\beta\beta} >| \gtrsim 10$ meV would have an excellent chance of observing a signal. If the spectrum is inverted, but the ν_i masses are larger than the $\Delta m^2_{\rm atm}$- and $\Delta m^2_{\odot}$-demanded minimum values we have assumed above, then once again $|< m_{\beta\beta} >|$ is larger than 10 meV [40], and an experiment sensitive to 10 meV still has an excellent chance of seeing a signal.

If the solar pair is at the bottom of the spectrum, rather than at the top, then $|< m_{\beta\beta} >|$ is not as tightly constrained, and can be anywhere from the present bound of 0.3–1.0 eV down to invisibly small [40,41]. For a discussion of the present bounds, see the article by Vogel and Piepke in this *Review* [42].

V. Questions to be answered: The strong evidence for neutrino flavor metamorphosis — hence neutrino mass — opens many questions about the neutrinos. These questions, which hopefully will be answered by future experiments, include the following:

i) How many neutrino species are there? Do sterile neutrinos exist?

This question is being addressed by the MiniBooNE experiment [43],whose purpose is to confirm or refute LSND. MiniBooNE expects to have a result at the end of 2005. If this result is positive, the implications will be far-reaching. We will have learned that either there are more than three neutrino species and at least one of these species is sterile, or else there is an even more amazing departure from what has been our picture of the neutrino world.

ii) What are the masses of the mass eigenstates ν_i?

The sizes of the squared-mass splittings $\Delta m^2_\odot$, $\Delta m^2_{\rm atm}$, and, if present, one or more large splittings $\Delta m^2_{\rm LSND}$, can be determined more precisely than they are currently known through future neutrino oscillation measurements. Importantly, if there are only three ν_i, then one can find out whether the solar pair, $\nu_{1,2}$, is at the bottom of the spectrum or at its top by exploiting matter effects in long-baseline neutrino and anti-neutrino oscillations. These matter effects will determine the sign one wishes to learn — that of $\{m_3^2 - [(m_2^2 + m_1^2)/2]\}$ — relative to a sign that is already known — that of the interaction energy of Eq. (13.20). Grand unified theories favor a spectrum with the closely-spaced solar pair at the bottom [44]. The neutrino spectrum would then resemble the spectra of the quarks, to which grand unified theories relate the neutrinos.

While flavor-change experiments can determine a spectral pattern such as the one in Fig. 13.3, they cannot tell us the distance of the entire pattern from the zero of squared-mass. One might discover that distance via study of the β energy spectrum in tritium β decay, if the mass of some ν_i with appreciable coupling to an electron is large enough to be within reach of a feasible experiment. One might also gain some information on the distance from zero by measuring $|< m_{\beta\beta} >|$, the effective Majorana mass for neutrinoless double beta decay [40–42] (see Vogel and Piepke in this *Review*). Finally, one might obtain information on this distance from cosmology or astrophysics. Indeed, from current cosmological data and some cosmological assumptions, it is already concluded that [45]

$$\sum_i m_i < (0.4 - 1.0)\ \text{eV} \ . \tag{13.37}$$

Here, the sum runs over the masses of all the light neutrino mass eigenstates ν_i that may exist and that were in thermal equilibrium in the early universe. The range quoted in Eq. (13.37) reflects the dependence of this upper bound on the underlying cosmological assumptions and on which data are used [45].

If there are just three ν_i, and their spectrum is either the one shown in Fig. 13.3 or its inverted version, then Eq. (13.37) implies that the mass of the heaviest ν_i, Mass [Heaviest ν_i], cannot exceed (0.2 – 0.4) eV. Moreover, Mass [Heaviest ν_i] obviously cannot be less than $\sqrt{\Delta m^2_{\rm atm}}$, which in turn is not less than 0.04 eV, as previously noted. Thus, if the cosmological assumptions behind Eq. (13.37) are correct, then

$$0.04\,\mathrm{eV} < \mathrm{Mass}\,[\mathrm{Heaviest}\ \nu_i] < (0.2 - 0.4)\,\mathrm{eV} \ . \tag{13.38}$$

iii) Are the neutrino mass eigenstates Majorana particles?

The confirmed observation of neutrinoless double beta decay would establish that the answer is "yes." If there are only three ν_i, knowledge that the spectrum is inverted and a definitive upper bound on $| < m_{\beta\beta} > |$ that is well below 0.01 eV would establish that it is "no" [see discussion after Eq. (13.36)] [40,41].

iv) What are the mixing angles in the leptonic mixing matrix U?

The solar mixing angle $\theta_\odot$ is already rather well determined.

The atmospheric mixing angle θ_{atm} is constrained by the most stringent analysis to lie, at 90% CL, in the region where $\sin^2 2\theta_{\mathrm{atm}} > 0.92$ [20]. This region is still fairly large: 37° to 53°. A more precise value of $\sin^2 2\theta_{\mathrm{atm}}$, and, in particular, its deviation from unity, can be sought in precision long-baseline ν_μ disappearance experiments. If $\sin^2 2\theta_{\mathrm{atm}} \neq 1$, so that $\theta_{\mathrm{atm}} \neq 45°$, one can determine whether it lies below or above 45° by a reactor $\overline{\nu}_e$ experiment [46,47]. Once we know whether the neutrino spectrum is normal or inverted, this determination will tell us whether the heaviest mass eigenstate is more ν_τ than ν_μ, as naively expected, or more ν_μ than ν_τ [cf. Eq. (13.31)].

A knowledge of the small mixing angle θ_{13} is important not only to help complete our picture of leptonic mixing, but also because, as Eq. (13.31) made clear, all CP-violating effects of the phase δ are proportional to $\sin\theta_{13}$. Thus, a knowledge of the order of magnitude of θ_{13} would help guide the design of experiments to probe CP violation. From Eq. (13.31), we recall that $\sin^2\theta_{13}$ is the ν_e fraction of ν_3. The ν_3 is the isolated neutrino that lies at one end of the atmospheric squared-mass gap $\Delta m^2_{\mathrm{atm}}$, so an experiment seeking to measure θ_{13} should have an L/E that makes it sensitive to $\Delta m^2_{\mathrm{atm}}$, and should involve ν_e. Possibilities include a sensitive search for the disappearance of reactor $\overline{\nu}_e$ while they travel a distance $L \sim 1$ km, and an accelerator neutrino search for $\nu_\mu \to \nu_e$ or $\nu_e \to \nu_\mu$ with a beamline $L >$ several hundred km.

If LSND is confirmed, then (barring the still more revolutionary) the matrix U is at least 4×4, and contains many more than three angles. A rich program, including short baseline experiments with multiple detectors, will be needed to learn about both the squared-mass spectrum and the mixing matrix.

Given the large sizes of θ_{atm} and $\theta_\odot$, we already know that leptonic mixing is very different from its quark counterpart, where all the mixing angles are small. This difference, and the striking contrast between the tiny neutrino masses and the very much larger quark masses, suggest that the physics underlying neutrino masses and mixing may be very different from the physics behind quark masses and mixing.

v) Does the behavior of neutrinos violate CP?

From Eqs. (13.8), (13.12), and (13.31), we see that if the CP-violating phase δ and the small mixing angle θ_{13} are both non-vanishing, there will be CP-violating differences between neutrino and anti-neutrino oscillation probabilities. Observation of these differences would establish that CP violation is not a peculiarity of quarks.

The CP-violating difference $P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$ between "neutrino" and " anti-neutrino" oscillation probabilities is independent of whether the mass eigenstates ν_i are Majorana or Dirac particles. To study $\nu_\mu \to \nu_e$ with a super-intense but conventionally-generated neutrino beam, for example, one would create the beam via the process $\pi^+ \to \mu^+ \nu_i$, and detect it via $\nu_i + \text{target} \to e^- + \ldots$. To study $\overline{\nu}_\mu \to \overline{\nu}_e$, one would create the beam via $\pi^- \to \mu^- \overline{\nu}_i$, and detect it via $\overline{\nu}_i + \text{target} \to e^+ + \ldots$. Whether $\overline{\nu}_i = \nu_i$ or not, the amplitudes for the latter two processes are proportional to $U_{\mu i}$ and U^*_{ei}, respectively. In contrast, the amplitudes for their $\nu_\mu \to \nu_e$ counterparts are proportional to $U^*_{\mu i}$ and U_{ei}. As this illustrates, Eq. (13.12) relates "neutrino" and "anti-neutrino" oscillation probabilities even when the neutrino mass eigenstates are their own antiparticles.

The baryon asymmetry of the universe could not have developed without some violation of CP during the universe's early history. The one known source of CP violation — the complex phase in the quark mixing matrix — could not have produced sufficiently large effects. Thus, perhaps *leptonic* CP violation is responsible for the baryon asymmetry. The see-saw mechanism predicts very heavy Majorana neutral leptons N_i (see Sec. IV), which would have been produced in the Big Bang. Perhaps CP violation in the leptonic decays of an N_i led to the inequality

$$\Gamma(N_i \to \ell^+ + \ldots) \neq \Gamma(N_i \to \ell^- + \ldots) \ , \tag{13.39}$$

which would have resulted in unequal numbers of ℓ^+ and ℓ^- in the early universe [48]. This leptogenesis could have been followed by nonperturbative SM processes that would have converted the lepton asymmetry, in part, into the observed baryon asymmetry [49].

While the connection between the CP violation that would have led to leptogenesis, and that which we hope to observe in neutrino oscillation, is model-dependent, it is not likely that we have either of these without the other [50]. This makes the search for CP violation in neutrino oscillation very interesting indeed. Depending on the rough size of θ_{13}, this CP violation may be observable with a very intense conventional neutrino beam, or may require a "neutrino factory," whose neutrinos come from the decay of stored muons or radioactive nuclei. The detailed study of CP violation may require a neutrino factory in any case.

vi) Will we encounter the completely unexpected?

The study of neutrinos has been characterized by surprises. It would be surprising if further surprises were not in store. The possibilities include new, non-Standard-Model interactions, unexpectedly large magnetic and electric dipole moments, unexpectedly short lifetimes, and violations of CPT invariance, Lorentz invariance, or the equivalence principle.

The questions we have discussed, and other questions about the world of neutrinos, will be the focus of a major experimental program in the years to come.

Acknowledgements

I am grateful to Susan Kayser for her crucial role in the production of this manuscript.

References:

1. This matrix is sometimes referred to as the Maki-Nakagawa-Sakata matrix, or as the Pontecorvo-Maki-Nakagawa-Sakata matrix, in recognition of the pioneering contributions of these scientists to the physics of mixing and oscillation. See Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962);

   B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)].
2. D. Karlen in this *Review*.
3. B. Kayser, Phys. Rev. **D24**, 110 (1981);

   F. Boehm and P. Vogel, *Physics of Massive Neutrinos* (Cambridge University Press, Cambridge, 1987) p. 87;

   C. Giunti, C. Kim, and U. Lee, Phys. Rev. **D44**, 3635 (1991);

   J. Rich, Phys. Rev. **D48**, 4318 (1993);

   H. Lipkin, Phys. Lett. **B348**, 604 (1995);

   W. Grimus and P. Stockinger, Phys. Rev. **D54**, 3414 (1996);

   T. Goldman, `hep-ph/9604357`;

   Y. Grossman and H. Lipkin, Phys. Rev. **D55**, 2760 (1997);

   W. Grimus, S. Mohanty, and P. Stockinger, in *Proc. of the 17th Int. Workshop on Weak Interactions and Neutrinos*, eds. C. Dominguez and R. Viollier (World Scientific, Singapore, 2000) p. 355.
4. L. Stodolsky, Phys. Rev. **D58**, 036006 (1998);

   C. Giunti, Phys. Scripta **67**, 29 (2003);

   M. Beuthe, Phys. Rept. **375**, 105 (2003) and Phys. Rev. **D66**, 013003 (2002), and references therein;

   H. Lipkin, Phys. Lett. **B579**, 355 (2004).
5. G. Fogli, E. Lisi, and G. Scioscia, Phys. Rev. **D52**, 5334 (1995).
6. L. Wolfenstein, Phys. Rev. **D17**, 2369 (1978).
7. S. Mikheyev and A. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1986)]; Zh. Eksp. Teor. Fiz. **91**, 7, (1986) [Sov. Phys. JETP **64**, 4 (1986)]; Nuovo Cimento **9C**, 17 (1986).
8. The Bugey Collaboration (B. Achkar *et al.*), Nucl. Phys. **B434**, 503 (1995);

   The Palo Verde Collaboration (F. Boehm *et al.*), Phys. Rev. **D64**, 112001 (2001);

   The CHOOZ Collaboration (M. Apollonio *et al.*), Eur. Phys. J. **C27**, 331 (2003).
9. P. Langacker, J. Leveille, and J. Sheiman, Phys. Rev. **D27**, 1228 (1983);

   The corresponding energy for anti-neutrinos is negative.
10. G. L. Fogli, E. Lisi, and D. Montanino, Phys. Rev. **D54**, 2048 (1996);

    A. de Gouvêa, A. Friedland, and H. Murayama, Phys. Lett. **B490**, 125 (2000).

11. J. Bahcall, *Neutrino Astrophysics*, (Cambridge Univ. Press, Cambridge, UK 1989).
12. S. Parke, Phys. Rev. Lett. **57**, 1275 (1986).
13. We thank J. Beacom and A. Smirnov for invaluable conversations on how LMA-MSW works. For an early description, see S. Mikheyev and A. Smirnov, Ref. 7 (first paper).
14. D. Ayres *et al.*, in *Proc. of the 1982 DPF Summer Study on Elementary Particle Physics and Future Facilities*, p. 590;

    G. Dass and K. Sarma, Phys. Rev. **D30**, 80 (1984);

    J. Flanagan, J. Learned, and S. Pakvasa, Phys. Rev. **D57**, 2649 (1998);

    B. Kayser, in *Proc. of the 17th Int. Workshop on Weak Interactions and Neutrinos*, eds. C. Dominguez and R. Viollier (World Scientific, Singapore, 2000) p. 339.
15. The Super-Kamiokande Collaboration (Y. Ashie *et al.*), Phys. Rev. **D71**, 112005 (2005).
16. From the L/E analysis reported by the Super-Kamiokande Collaboration (Y. Ashie *et al.*), Phys. Rev. Lett. **93**, 101801 (2004).
17. MACRO Collaboration (M. Ambrosio *et al.*), Phys. Lett. **B566**, 35 (2003);

    MACRO Collaboration, Eur. Phys. J. C, **36**, 323 (2004).
18. M. Sanchez *et al.*, Phys. Rev. **D68**, 113004 (2003);

    W. W. M. Allison *et al.*, Phys. Rev. **D72**, 052005 (2005).
19. M. Shiozawa, presented at the XXth Int. Conf. on Neutrino Physics and Astrophysics, Munich, May, 2002.
20. Y. Suzuki, presented at the XXII Int. Symp. on Lepton and Photon Interactions at High Energies (Lepton Photon 2005), Uppsala, Sweden, July, 2005.
21. The SNO Collaboration (B. Aharmim *et al.*), `nucl-ex/0502021`.
22. Y. Koshio, in *2003 Electroweak Interactions and Unified Theories* (Proceedings of the 38th Rencontres de Moriond), ed. J. Trân Thanh Vân (The Gioi, Vietnam, 2003) p. 3;

    The Super-Kamiokande Collaboration (J. Hosaka *et al.*), `hep-ex/0508053`.
23. J. Bahcall, S. Basu, and A. Serenelli, Astrophys. J. **621**, L85 (2005).
24. G. Fogli, E. Lisi, A. Marrone, and A. Palazzo, Phys. Lett. **B583**, 149 (2004).
25. The KamLAND Collaboration (T. Araki *et al.*), Phys. Rev. Lett. **94**, 081801 (2005).
26. The latter include the data from the chlorine experiment: B. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998);

    and from the gallium experiments: GNO Collaboration (M. Altmann *et al.*), Phys. Lett. **B616**, 174 (2005);

    SAGE Collaboration (J. N. Abdurashitov *et al.*), JETP **95**, 181 (2002).
27. J. Bahcall, M.C. Gonzalez-Garcia, and C. Peña-Garay, JHEP **0408**, 016 (2004);

    See also P. Holanda and A. Smirnov, `hep-ph/0211264` and Phys. Rev. **D69**, 113002 (2004).

28. The LSND Collaboration (A. Aguilar *et al.*), Phys. Rev. **D64**, 112007 (2001).
29. The KARMEN Collaboration (B. Armbruster *et al.*), Phys. Rev. **D65**, 112001 (2002).
30. E. Church *et al.*, Phys. Rev. **D66**, 013001 (2003).
31. For an alternative possibility entailing CPT violation, see H. Murayama and T. Yanagida, Phys. Lett. **B520**, 263 (2001);

    G. Barenboim *et al.*, JHEP **0210**, 001 (2002);

    However, after KamLAND, this alternative is disfavored. M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, Phys. Rev. **D68**, 053007 (2003);

    G. Barenboim, L. Borissov, and J. Lykken, `hep-ph/0212116 v2`.
32. J. Schechter and J. Valle, Phys. Rev. **D23**, 1666 (1981);

    J. Nieves and P. Pal, Phys. Rev. **D64**, 076005 (2001);

    A. de Gouvêa, B. Kayser, and R. Mohapatra, Phys. Rev. **D67**, 053004 (2003).
33. L.-L. Chau and W.-Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984);

    H. Harari and M. Leurer, Phys. Lett. **B181**, 123 (1986);

    F.J. Botella and L.-L. Chau, Phys. Lett. **B168**, 97 (1986);

    H. Fritzsch and J. Plankl, Phys. Rev. **D35**, 1732 (1987).
34. G. Fogli, E. Lisi, A. Marrone, and A. Palazzo, `hep-ph/0506083`, to appear in Prog. Part. Nucl. Phys..
35. M. Sorel, J. Conrad, and M. Shaevitz, Phys. Rev. **D70**, 073004 (2004).
36. We thank Belen Gavela for introducing us to this argument.
37. M. Gell-Mann, P. Ramond, and R. Slansky, in: *Supergravity*, eds. D. Freedman and P. van Nieuwenhuizen (North Holland, Amsterdam, 1979) p. 315;

    T. Yanagida, in: *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe,* eds. O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979);

    R. Mohapatra and G. Senjanovic: Phys. Rev. Lett. **44**, 912 (1980) and Phys. Rev. **D23**, 165 (1981);

    P. Minkowski, Phys. Lett. **B67**, 421 (1977).
38. J. Schechter and J. Valle, Phys. Rev. **D25**, 2951 (1982).
39. The physics of Majorana neutrinos and $0\nu\beta\beta$ are discussed in S. Bilenky and S. Petcov, Rev. Mod. Phys. **59**, 671 (1987) [Erratum–*ibid.* **61**, 169 (1987)];

    B. Kayser, F. Gibrat-Debu, and F. Perrier, *The Physics of Massive Neutrinos* (World Scientific, Singapore, 1989);

    B. Kayser, `hep-ph/0504052,` to appear in *Neutrino Physics, Proceedings of the Nobel Symposium 2004*, eds. L. Bergström, O Botner, P. Carlson, P.O. Hulth, and T. Ohlsson.
40. S. Pascoli and S.T. Petcov, Phys. Lett. **B580**, 280 (2003).
41. Analyses of the possible values of $| < m_{\beta\beta} > |$ have been given by H. Murayama and C. Peña-Garay, Phys. Rev. **D69**, 031301 (2004);

S. Pascoli and S. Petcov, Phys. Lett. **B544**, 239 (2002);

S. Bilenky, S. Pascoli, and S. Petcov, Phys. Rev. **D64**, 053010 (2001), and Phys. Rev. **D64**, 113003 (2001);

H. Klapdor-Kleingrothaus, H. Päs, and A. Smirnov, Phys. Rev. **D63**, 073005 (2001);

S. Bilenky *et al.*, Phys. Lett. **B465**, 193 (1999);

References in these papers.

42. See also S. Elliott and P Vogel, Ann. Rev. Nucl. Part. Sci. **52**, 115 (2002), and references therein.

43. The MiniBooNE Collaboration (E. Church *et al.*) FERMILAB-P-0898 (1997), available at http://library.fnal.gov/archive/ test-proposal/0000/fermilab-proposal-0898.shtml.

44. We thank Carl Albright for discussions of this point.

45. S. Pastor, in the *Proceedings of the Eleventh International Workshop on Neutrino Telescopes,* ed. M. Baldo Ceolin (Italy, 2005) p. 371;

U. Seljak *et al.*, Phys. Rev. **D71**, 103515 (2005).

46. K. McConnel and M. Shaevitz, `hep-ex/0409028`.

47. For an alternative approach to determining $\theta_{\rm atm}$, see M. Gonzalez-Garcia, M. Maltoni and A. Smirnov, Phys. Rev. D **70**, 093005 (2004), and references therein..

48. M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).

49. G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976);

V. Kuzmin, V. Rubakov, and M. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).

50. S. Pascoli, S. Petcov, and W. Rodejohann, Phys. Rev. **D68**, 093007 (2003);

S. Davidson, S. Pascoli, and S. Petcov, private communications.