11. THE CABIBBO-KOBAYASHI-MASKAWA QUARK-MIXING MATRIX

Revised January 2004 by F.J. Gilman (Carnegie-Mellon University), K. Kleinknecht and B. Renk (Johannes-Gutenberg Universität Mainz).

In the Standard Model with SU(2)×U(1) as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix relating these bases was defined for six quarks and given an explicit parametrization by Kobayashi and Maskawa [1] in 1973. This generalizes the four-quark case, where the matrix is described by a single parameter, the Cabibbo angle [2].

By convention, the mixing is often expressed in terms of a 3×3 unitary matrix V operating on the charge $-e/3$ quark mass eigenstates (d, s, and b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{11.1}$$

The values of individual matrix elements can in principle all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Using the eight tree-level constraints discussed below together with unitarity, and assuming only three generations, the 90% confidence limits on the magnitude of the elements of the complete matrix are

$$\begin{pmatrix} 0.9739 \text{ to } 0.9751 & 0.221 \text{ to } 0.227 & 0.0029 \text{ to } 0.0045 \\ 0.221 \text{ to } 0.227 & 0.9730 \text{ to } 0.9744 & 0.039 \text{ to } 0.044 \\ 0.0048 \text{ to } 0.014 & 0.037 \text{ to } 0.043 & 0.9990 \text{ to } 0.9992 \end{pmatrix} . \tag{11.2}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of others.

There are several parametrizations of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. We advocate a "standard" parametrization [3] of V that utilizes angles θ_{12}, θ_{23}, θ_{13}, and a phase, δ_{13}

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} , \tag{11.3}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for the "generation" labels $i, j = 1, 2, 3$. This has distinct advantages of interpretation, for the rotation angles are defined and labeled in a way which relate to the mixing of two specific generations and if one of these angles vanishes, so does the mixing between those two generations; in the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified as the Cabibbo angle [2]. This parametrization is exact to all orders, and has four parameters; the real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases.

The matrix elements in the first row and third column, which have been directly measured in decay processes, are all of a simple form, and, as c_{13} is known to deviate from unity only in the sixth decimal place, $V_{ud} = c_{12}$, $V_{us} = s_{12}$, $V_{ub} = s_{13}\ e^{-i\delta_{13}}$, $V_{cb} = s_{23}$, and $V_{tb} = c_{23}$ to an excellent approximation. The phase δ_{13} lies in the range $0 \leq \delta_{13} < 2\pi$, with non-zero values breaking CP invariance for the weak interactions. The generalization to the n generation case contains $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases. Using tree-level processes as constraints only, the matrix elements in Eq. (11.2) correspond to values of the sines of the angles of $s_{12} = 0.2243 \pm 0.0016$, $s_{23} = 0.0413 \pm 0.0015$, and $s_{13} = 0.0037 \pm 0.0005$.

If we use the loop-level processes discussed below as additional constraints, the central values of the sines of the angles do not change, and the CKM phase, sometimes referred to as the angle $\gamma = \phi_3$ of the unitarity triangle, is restricted to $\delta_{13} = (1.05 \pm 0.24)$ radians $= 60^o \pm 14^o$.

Kobayashi and Maskawa [1] originally chose a parametrization involving the four angles θ_1, θ_2, θ_3, and δ:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} , \tag{11.4}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1, 2, 3$. In the limit $\theta_2 = \theta_3 = 0$, this reduces to the usual Cabibbo mixing with θ_1 identified (up to a sign) with the Cabibbo angle [2]. Note that in this case V_{ub} and V_{td} are real and V_{cb} complex, illustrating a different placement of the phase than in the standard parametrization.

An approximation to the standard parametrization proposed by Wolfenstein [4] emphasizes the hierarchy in the size of the angles, $s_{12} \gg s_{23} \gg s_{13}$. Setting $\lambda \equiv s_{12}$, the sine of the Cabibbo angle, one expresses the other elements in terms of powers of λ:

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \ . \tag{11.5}$$

with A, ρ, and η real numbers that were intended to be of order unity. This approximate form is widely used, especially for B-physics, but care must be taken, especially for CP-violating effects in K-physics, since the phase enters V_{cd} and V_{cs} through terms that are higher order in λ. These higher order terms up to order (λ^5) are given in [5].

Another parametrization has been advocated [6] that arises naturally where one builds models of quark masses in which initially $m_u = m_d = 0$. With no phases in the third row or third column, the connection between measurements of CP-violating effects for B mesons and single CKM parameters is less direct than in the standard parametrization.

Different parametrizations shuffle the placement of phases between particular tree and loop (e.g., neutral meson mixing) amplitudes. No physics can depend on which of the above parametrizations (or any other) is used, as long as a single one is used consistently and care is taken to be sure that no other choice of phases is in conflict.

Our present knowledge of the matrix elements comes from the following sources:

(1)$|V_{ud}|$: Analyses have been performed comparing nuclear beta decays that proceed through a vector current to muon decay. Radiative corrections are essential to extracting the value of the matrix element. They already include effects [7] of order $Z\alpha^2$, and most of the theoretical argument centers on the nuclear mismatch and structure-dependent radiative corrections, [8], [9].

Taking the complete data set on superallowed $0^+ \rightarrow 0^+$ beta decays, [10], a value of $|V_{ud}| = 0.9740 \pm 0.0005$ has been obtained [11]. Calculations taking into account core polarization effects and charge symmetry breaking as well as charge independence breaking forces on the mean field potentials [12] get close results. This contradicts earlier results about changes in the charge-symmetry violation for quarks inside nucleons in nuclear matter. Therefore we do not apply further additional uncertainties.

The theoretical uncertainties in extracting a value of $|V_{ud}|$ from neutron decays are significantly smaller than for decays of mirror nuclei, but the value depends on both the value of g_A/g_V and the neutron lifetime. Experimental progress has been made on g_A/g_V using very highly polarized cold neutrons together with improved detectors. The recent experimental result [13], $g_A/g_V = -1.2739 \pm 0.0019$, by itself has a better precision than the former world average and results in $|V_{ud}| = 0.9713 \pm 0.0013$ if taken alone. Averaging over all recent experiments using polarizations of more than 90% [14] gives $g_A/g_V = -1.2720 \pm 0.0018$ and results in $|V_{ud}| = 0.9725 \pm 0.0013$ from neutron decay.

Since most of the contributions to the errors in these two determinations of $|V_{ud}|$ are independent, we average them to obtain

$$|V_{ud}| = 0.9738 \pm 0.0005 \ . \tag{11.6}$$

(2)$|V_{us}|$: The original analysis of K_{e3} decays yielded [15]

$$|V_{us}| = 0.2196 \pm 0.0023 \ . \tag{11.7}$$

With isospin violation taken into account in K^+ and K^0 decays, the extracted values of $|V_{us}|$ are in agreement at the 1% level. Radiative corrections have been recently calculated in chiral perturbation theory [16]. The combined effects of long-distance radiative corrections and nonlinear terms in the form factor can decrease the value of $|V_{us}|$ by up to 1% [17], and we take this into account by applying an additional correction of $(-0.5 \pm 0.5)\%$ which compensates the effect of radiative corrections in Ref. [16]. A new measurement of the K^+ semileptonic branching ratio [18] indicates a higher value of this quantity, in disagreement with the early measurements. It also would imply a contradiction to the value of $|V_{us}|$ derived from K^0 semileptonic decays. We average the new result with the older ones, leading mainly to an increase of the non-dominant experimental part of the uncertainty of $|V_{us}|$, and a slight increase of the derived value

$$|V_{us}| = 0.2200 \pm 0.0026 \ , \tag{11.8}$$

in very good agreement with the former analysis. New results on this will come from KLOE and NA48/2. The analysis [19] of hyperon decay data has larger theoretical uncertainties because of first order SU(3) symmetry breaking effects in the axial-vector couplings. This has been redone incorporating second order SU(3) symmetry breaking corrections in models [20] applied to the WA2 data [21] to give a value of $|V_{us}| = 0.2176 \pm 0.0026$, which is consistent with Eq. (11.8) using the "best-fit" model. A new analysis of the same hyperon decay data [22] yields $|V_{us}| = 0.2250 \pm 0.0027$, at variance with the earlier hyperon analysis. Since the values obtained in these models differ outside the errors and generally do not give good fits, we retain the value in Eq. (11.8) for $|V_{us}|$.

(3)$|V_{cd}|$: The magnitude of V_{cd} may be deduced from neutrino and antineutrino production of charm off valence d quarks. The dimuon production cross sections of the CDHS group [23] yield $\overline{B}_c\, |V_{cd}|^2 = (0.41 \pm 0.07) \times 10^{-2}$, where $\overline{B}_c$ is the semileptonic branching fraction of the charmed hadrons produced. The corresponding value from the more recent CCFR Tevatron experiment [24], where a next-to-leading-order QCD analysis has been carried out, is $0.534 \pm 0.046^{+0.025}_{-0.051} \times 10^{-2}$, where the last error is from the scale uncertainty. Assuming a similar scale error for the CDHS measurement and averaging these two values with the result from the Charm II experiment [25] $\overline{B}_c\, |V_{cd}|^2 = (0.442 \pm 0.049) \times 10^{-2}$, we obtain as an average $(0.463 \pm 0.034) \times 10^{-2}$. Supplementing this with data [26,27,28] on the mix of charmed particle species produced by neutrinos and values for their semileptonic branching fractions (to give $\overline{B}_c = 0.0923 \pm 0.0073$), this yields

$$|V_{cd}| = 0.224 \pm 0.012 \ . \tag{11.9}$$

(4)$|V_{cs}|$: Values for $|V_{cs}|$ obtained from neutrino production of charm and from semileptonic D decays have errors due to theoretical uncertainties that exceed 10%, as discussed in previous editions of this review. They have been superseded by direct measurements [29] of $|V_{cs}|$ in charm-tagged W decays that give $|V_{cs}| = 0.97 \pm 0.09$ (stat.) ± 0.07 (syst.). A tighter determination follows from the ratio of hadronic W decays to leptonic decays, which has been measured at LEP with the result [30] that $\sum_{i,j} |V_{ij}|^2 = 2.039 \pm 0.025 \pm 0.001$, where the sum extends over $i = u, c$ and $j = d, s, b$ and the last error is from knowledge of α_s. With a three-generation CKM matrix, unitarity requires that this sum has the value 2. Since five of the six CKM matrix elements in the sum are well measured or contribute negligibly to the measured sum of the squares, it can be converted into a greatly improved result [30]:

$$|V_{cs}| = 0.996 \pm 0.013 \ . \tag{11.10}$$

(5)$|V_{cb}|$: The heavy quark effective theory [31] (HQET) provides a nearly model-independent treatment of B semileptonic decays to charmed mesons, assuming that both the b and c quarks are heavy enough for the theory to apply. Measurements of the exclusive decay $B \to \overline{D}^* \ell^+ \nu_\ell$ have been used primarily to extract a value of $|V_{cb}|$ using corrections based on HQET. Exclusive $B \to \overline{D} \ell^+ \nu_\ell$ decays give a consistent, but less precise result. Analysis of inclusive decays, where the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual

$V-A$ interaction, depends on going from the quark to the hadron level and involves an assumption on the validity of quark-hadron duality. Improvements have been obtained in theoretical studies of the moments of inclusive semi leptonic and radiative decays and experimental measurements of such moments. The results for $|V_{cb}|$ from exclusive and inclusive decays generally are in good agreement. A more detailed discussion and references are found in a mini-review in the *Review of Particle Physics* [32]. We add an uncertainty due to the assumption of quark-hadron duality [32], [33] of 1% to the results from inclusive decays and average over the exclusive result $|V_{cb}| = (42.0 \pm 1.1 \pm 1.9) \times 10^{-3}$ and inclusive result $|V_{cb}| = (41.0 \pm 0.5 \pm 0.5 \pm 0.8) \times 10^{-3}$ with theoretical uncertainties combined linearly to obtain

$$|V_{cb}| = (41.3 \pm 1.5) \times 10^{-3} \ . \tag{11.11}$$

(6)$|V_{ub}|$: The decay $b \to u\ell\overline{\nu}$ and its charge conjugate can be observed in the semileptonic decay of B mesons produced on the $\Upsilon(4S)$ $(b\overline{b})$ resonance by measuring the lepton energy spectrum above the endpoint of the $b \to c\ell\overline{\nu}_\ell$ spectrum. There the $b \to u\ell\overline{\nu}_\ell$ decay rate can be obtained by subtracting the background from nonresonant e^+e^- reactions. This continuum background is determined from auxiliary measurements off the $\Upsilon(4S)$. The interpretation of this inclusive result in terms of $|V_{ub}|$ depends fairly strongly on the theoretical model used to generate the lepton energy spectrum, especially that for $b \to u$ transitions. At LEP, the separation between u-like and c-like decays is based on up to twenty different event parameters, and while the extraction of $|V_{ub}|$ is less sensitive to theoretical assumptions, it requires a detailed understanding of the decay $b \to c\ell\overline{\nu}_\ell$. The CLEO Collaboration [34] has recently employed an important technique that uses moments of measured distributions in $b \to s\gamma$ and $B \to D^*\ell\nu_\ell$ to fix the parameters in the inclusive distribution and thereby reduce the errors.

The huge data samples at the B factories, optimized cut variables which minimize theoretical uncertainties, measurements of spectral moments and event samples with fully reconstructed B decays contribute to an improved accuracy of $|V_{ub}|$.

The value of $|V_{ub}|$ can also be extracted from exclusive decays, such as $B \to \pi\ell\nu_\ell$ and $B \to \rho\ell\nu_\ell$, but there is an associated theoretical model dependence in the values of the matrix elements of the weak current between exclusive states. Detailed discussion and references on both the inclusive and exclusive analyses is found in the mini-review on $|V_{ub}|$ in the *Review of Particle Physics* [35]. They average the inclusive result $|V_{ub}| = (4.68 \pm 0.85) \times 10^{-3}$, with the exclusive result of $|V_{ub}| = (3.326 \pm 0.59) \times 10^{-3}$ to obtain a result dominated by the theoretical uncertainties,

$$|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3} \ . \tag{11.12}$$

(7)V_{tb}: The discovery of the top quark by the CDF and D0 collaborations utilized in part the semileptonic decays of t to b. The CDF experiment has published a limit on the fraction of decays of the form $t \to b\ \ell^+\ \nu_\ell$, as opposed to semileptonic t decays that involve the light s or d quarks, of [36]

$$\frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.94^{+0.31}_{-0.24} \ . \tag{11.13}$$

For most of the CKM matrix elements the principal error is no longer experimental, but rather theoretical. This arises from explicit model dependence in interpreting inclusive data or in the direct use of specific hadronic matrix elements to relate decay rates for exclusive processes to weak transitions of quarks. This type of uncertainty often is even larger at present in extracting CKM matrix elements from loop diagrams, as discussed below. Such theoretical errors are not distributed in a Gaussian manner. We have judged what is a reasonable range in assigning the theoretical errors.

While we use the central values with the quoted errors in a consistent way [37] performing a random exploration of the full parameter space to make a best overall fit to the CKM matrix (interpreting a "1 σ" range in a theoretical error as corresponding to a 68% confidence level that the true value lies within a range of "± 1 σ" of the central value in making those fits), the result should be taken with appropriate care. The issue of how to use appropriate statistical methods to deal with these errors has been intensively discussed in the last few years by a number of authors [38]. The different fitting methods, if they use the same input parameters, give essentially the same result. Our limited knowledge of some of the theoretical uncertainties makes us cautious in extending this to results for multi-standard-deviation determinations of the allowed regions for CKM matrix elements.

We determine the best fit by searching for the minimum chi-squared by scanning the parameter spaces of the four angles. The results for three generations of quarks, from Eqs. (11.6), (11.8), (11.9), (11.10), (11.11), (11.12), and (11.13) plus unitarity, are summarized in the matrix in Eq. (11.2). The ranges given there are different from those given in Eqs. (11.6) – (11.13) because of the inclusion of unitarity, but are consistent with the one-standard-deviation errors on the input matrix elements. Note in particular that the unitarity constraint has pushed $|V_{ud}|$ about 1.4 standard deviations higher than given in Eq. (11.6). We observe a violation of unitarity in the first row of the CKM matrix by more than 2 standard deviations. While this bears watching and encourages another more accurate measurement of $|V_{us}|$ as well as more theoretical work, we do not see this as a major challenge to the validity of the three-generation Standard Model.

The data do not preclude there being more than three generations. Moreover, the entries deduced from unitarity might be altered when the CKM matrix is expanded to accommodate more generations. Conversely, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that any additional element in the first row have a magnitude $|V_{ub'}| < 0.08$. When there are more than three generations the allowed ranges (at 90% CL) of the matrix elements connecting the first three generations are

$$\begin{pmatrix} 0.9730 \text{ to } 0.9746 & 0.2174 \text{ to } 0.2241 & 0.0030 \text{ to } 0.0044 \ldots \\ 0.213 \text{ to } 0.226 & 0.968 \text{ to } 0.975 & 0.039 \text{ to } 0.044 \ldots \\ 0 \text{ to } 0.08 & 0 \text{ to } 0.11 & 0.07 \text{ to } 0.9993 \ldots \\ \vdots & \vdots & \vdots \end{pmatrix}, \qquad (11.14)$$

where we have used unitarity (for the expanded matrix) and the measurements of the magnitudes of the CKM matrix elements (including the constraint from hadronic W decays), resulting in the weak bound $|V_{tb}| > 0.07$.

Direct and indirect information on the smallest matrix elements of the CKM matrix is neatly summarized in terms of the "unitarity triangle," one of six such triangles that correspond to the unitarity condition applied to two different rows or columns of the CKM matrix. Unitarity applied to the first and third columns yields

$$V_{ud}\ V_{ub}{}^* + V_{cd}\ V_{cb}{}^* + V_{td}\ V_{tb}{}^* = 0\ . \qquad (11.15)$$

The unitarity triangle is just a geometrical presentation of this equation in the complex plane [39], as in Fig. 11.1(a). We can always choose to orient the triangle so that $V_{cd}\ V_{cb}{}^*$ lies along the horizontal; in the standard parametrization, V_{cb} is real and V_{cd} is real to a very good approximation in any case. Setting cosines of small angles to unity, Eq. (11.15) becomes

$$V_{ub}{}^* + V_{td} \approx s_{12}\ V_{cb}{}^*\ , \qquad (11.16)$$

which is shown as the unitarity triangle. The sides of this triangle are of order 1% of the diagonal elements of the CKM matrix, which highlights the precision we are aiming to achieve of knowing each of these sides in turn to a precision of a few percent.

The angles α, β and γ of the triangle are also referred to as ϕ_2, ϕ_1, and ϕ_3, respectively, with β and $\gamma = \delta_{13}$ being the phases of the CKM elements V_{td} and V_{ub} as per

$$V_{td} = |V_{td}|e^{-i\beta}, V_{ub} = |V_{ub}|e^{-i\gamma}\ . \qquad (11.17)$$

Rescaling the triangle so that the base is of unit length, the coordinates of the vertices A, B, and C become respectively:

$$\big(\mathrm{Re}(V_{ud}\ V_{ub}^*)/|V_{cd}\ V_{cb}^*|,\ \mathrm{Im}(V_{ud}\ V_{ub}^*)/|V_{cd}\ V_{cb}^*|\big),\ (1,0),\ \&\ (0,0)\ . \qquad (11.18)$$

The coordinates of the apex of the rescaled unitarity triangle take the simple form $(\overline{\rho}, \overline{\eta})$, with $\overline{\rho} = \rho(1-\lambda^2/2)$ and $\overline{\eta} = \eta(1-\lambda^2/2)$ in the Wolfenstein approximation, [4] parametrization [4], as shown in Fig. 11.1(b).

CP-violating processes involve the phase in the CKM matrix, assuming that the observed CP violation is solely related to a nonzero value of this phase. More specifically, a necessary and sufficient condition for CP violation with three generations can be formulated in a parametrization-independent manner in terms of the non-vanishing of J, the determinant of the commutator of the mass matrices for the charge $2e/3$ and charge $-e/3$ quarks [40]. CP-violating amplitudes or differences of rates are all proportional to the product of CKM factors in this quantity, namely $s_{12}s_{13}s_{23}c_{12}c_{13}^2c_{23}\sin\delta_{13}$. This is just twice the area of the unitarity triangle.

Further information, particularly on CKM matrix elements involving the top quark, can be obtained from flavor-changing processes that occur at the one-loop level. We have not used this information up to this point since the derivation of values for V_{td} and V_{ts}

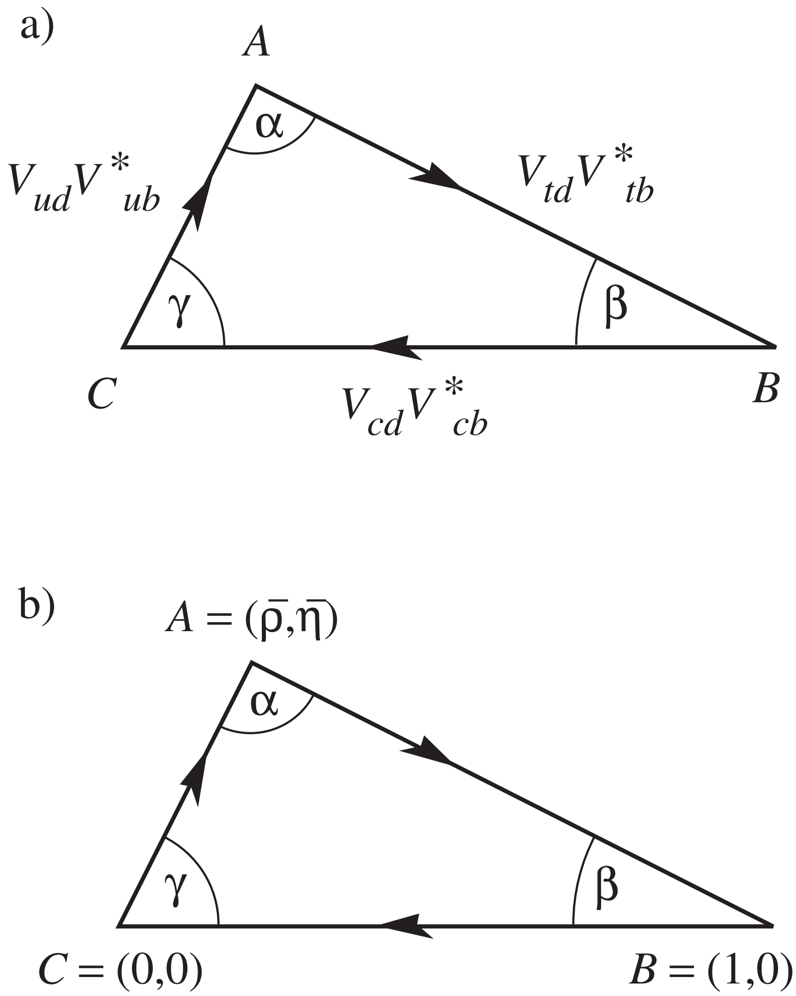


Figure 11.1: (a) Representation in the complex plane of the triangle formed by the CKM matrix elements $V_{ud}\ V_{ub}^*$, $V_{td}\ V_{tb}^*$, and $V_{cd}\ V_{cb}^*$. (b) Rescaled triangle with vertices A, B, and C at $(\overline{\rho}, \overline{\eta})$, $(1, 0)$, and $(0, 0)$, respectively.

in this manner from, for example, B mixing or $b \to s\gamma$, require an additional assumption that the top-quark loop, rather than new physics, gives the dominant contribution to the process in question. Conversely, when we find agreement between CKM matrix elements extracted from loop diagrams and the values above based on direct measurements plus the assumption of three generations, this can be used to place restrictions on new physics.

We first consider constraints from flavor-changing processes that are not CP-violating. The measured value [41] of $\Delta M_{B_d} = 0.502 \pm 0.007\ \mathrm{ps}^{-1}$ from $B_d{}^0 - \overline{B}_d{}^0$ mixing can be turned into information on $|V_{tb}{}^* V_{td}|$, assuming that the dominant contribution to the mass difference arises from the matrix element between a B_d and a $\overline{B}_d$ of an operator that corresponds to a box diagram with W bosons and top quarks as sides. Using the characteristic hadronic matrix element that then occurs, $\hat{B}_{B_d} \cdot f_{B_d}{}^2 = (1.26 \pm 0.10) \cdot (196 \pm 32\ \mathrm{MeV})^2$ from lattice QCD calculations [42], next-to-leading-order QCD corrections ($\eta_{\mathrm{QCD}} = 0.55$) [43], and the running top-quark mass, $\overline{m}_t(m_t) = (166 \pm 5)$ GeV as input, we obtain

$$|V_{tb}{}^{*} \cdot V_{td}| = 0.0083 \pm 0.0016 \ , \tag{11.19}$$

where the uncertainty comes primarily from that in the hadronic matrix elements, whose estimated errors are combined linearly.

In the ratio of B_s to B_d mass differences, many common factors (such as the QCD correction and dependence on the top-quark mass) cancel, and we have

$$\frac{\Delta M_{B_s}}{\Delta M_{B_d}} = \frac{M_{B_s}}{M_{B_d}} \frac{\widehat{B}_{B_s} f_{B_s}^2}{\widehat{B}_{B_d} f_{B_d}^2} \frac{|V_{tb}^{*} \cdot V_{ts}|^2}{|V_{tb}^{*} \cdot V_{td}|^2} \ . \tag{11.20}$$

With the experimentally measured masses, $\widehat{B}_{B_s} f_{B_s}^2 / (\widehat{B}_{B_d} f_{B_d}^2) = 1.56 \pm 0.26$ [42], and the experimental lower limit [41] at 95% CL of $\Delta M_{B_s} > 14.4$ ps^{-1} based on published data,

$$|V_{td}|/|V_{ts}| < 0.25 \ . \tag{11.21}$$

Since with three generations, $|V_{ts}| \approx |V_{cb}|$, this result converts to $|V_{td}| < 0.011$, which is a significant constraint by itself (see Figure 2).

The CLEO observation [44] of $b \to s\gamma$, confirmed by BELLE and BaBar [45], is in agreement with the Standard Model prediction. This observation can be restated, assuming the Standard Model, as a constraint [46]

$$V_{tb} V_{ts}{}^{*} = (-47 \pm 8) \times 10^{-3} \ . \tag{11.22}$$

This is consistent in both sign and magnitude with the value that follows from the measured magnitudes of CKM matrix elements and the assumption of three generations, but has a much larger uncertainty.

In $K^+ \to \pi^+ \nu \overline{\nu}$ there are significant contributions from loop diagrams involving both charm and top quarks. Experiment is just beginning to probe the level predicted in the Standard Model [47].

All these additional indirect constraints are consistent with the CKM elements obtained from the direct measurements plus unitarity, assuming three generations. Adding the results on B mixing together with theoretical improvements in lattice calculations reduces the range allowed for $|V_{td}|$.

Now we turn to CP-violating processes. Just the added constraint from CP violation in the neutral kaon system, taken together with the restrictions above on the magnitudes of the CKM matrix elements, is tight enough to restrict considerably the range of angles and the phase of the CKM matrix. For example, the constraint obtained from the CP-violating parameter ϵ in the neutral K system corresponds to the vertex A of the unitarity triangle lying on a hyperbola for fixed values of the (imprecisely known) hadronic matrix elements [48], [49].

In addition, following the initial evidence [50], it is now established that direct CP violation in the weak transition from a neutral K to two pions exists, i.e., that the parameter ϵ' is non-zero [51]. While theoretical uncertainties in hadronic matrix elements of canceling amplitudes presently preclude this measurement from giving a significant constraint on the unitarity triangle, it supports the assumption that the observed CP violation is related to a non-zero value of the CKM phase.

Ultimately in the neutral K system, the CP-violating process $K_L \to \pi^0 \nu\overline{\nu}$ offers the possibility of a theoretically clean, high precision measurement of the imaginary part of $V_{td} \cdot V_{ts}{}^*$ and the area of the unitarity triangle. Given $|V_{ts}|$, this will yield the altitude of the unitarity triangle. However, the experimental upper limit is presently many orders of magnitude away from the required sensitivity.

Turning to the B-meson system, for CP-violating asymmetries of neutral B mesons decaying to CP eigenstates, the interference between mixing and a single weak decay amplitude for certain final states directly relates the asymmetry in a given decay to $\sin 2\phi$, where $\phi = \alpha,\ \beta,\ \gamma$ is an appropriate angle of the unitarity triangle [39]. A new generation of experiments has established a non-vanishing asymmetry in the decays $B_d(\overline{B}_d) \to \psi K_S$ and in other B_d decay modes where the asymmetry is given by $\sin 2\beta$. The present experimental results from BaBar [52] and BELLE [53], when averaged yield

$$\sin 2\beta = 0.736 \pm 0.049 \ . \tag{11.23}$$

While the limits on the leptonic charge asymmetry for $B_d - \overline{B}_d$ mixing (measuring the analogue of 2Re ϵ in the neutral K system) have been reduced to the 1% level [41], this is still roughly an order of magnitude greater than the value expected without new physics. It provides no significant constraints on the CKM matrix for now [54].

The constraints on the apex of the unitarity triangle that follow from Eqs. (11.12), (11.19), (11.21), (11.23), and ϵ are shown in Fig. 11.2. Both the limit on ΔM_s and the value of ΔM_d indicate that the apex lies in the first rather than the second quadrant.

All constraints nicely overlap in one small area in the first quadrant with the sign of ϵ measured in the K system agreeing with the sign of $\sin 2\beta$ measured in the B system.

The situation with regard to the unitarity triangle has changed qualitatively in the past few years. Both the constraints from the lengths of the sides (from $|V_{ub}|$, $|V_{cb}|$, and $|V_{td}|$) and independently those from CP-violating processes (ϵ from the K system and $\sin 2\beta$ from the B system) indicate the same region for the apex of the triangle. The first major test of the full CKM picture and CP violation has been passed successfully.

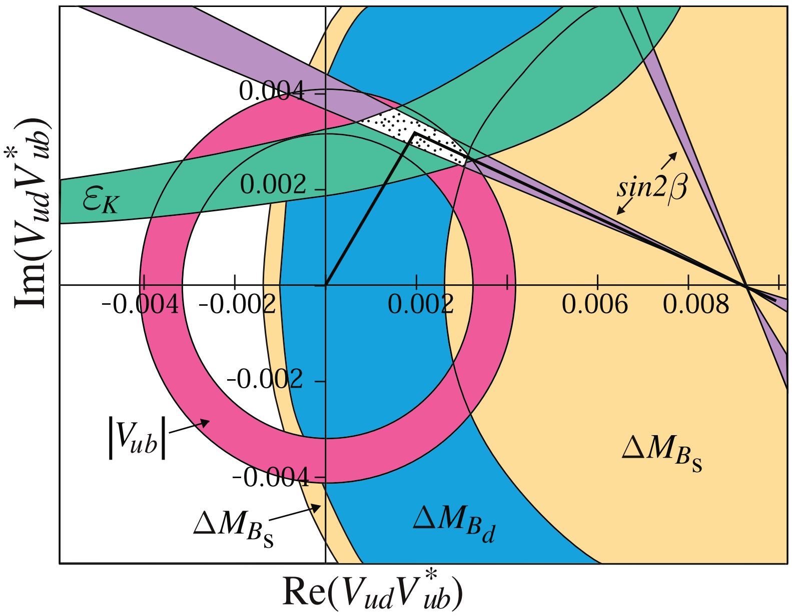
From a combined fit using the direct measurements, B mixing, ϵ, and $\sin 2\beta$, we obtain:

$$\text{Re}\ V_{td} = 0.0067 \pm 0.0008 \ , \tag{11.24}$$

$$\text{Im}\ V_{td} = -0.0031 \pm 0.0004 \ , \tag{11.25}$$

$$\overline{\rho} = 0.20 \pm 0.09 \ , \tag{11.26}$$

$$\overline{\eta} = 0.33 \pm 0.05 \ . \tag{11.27}$$



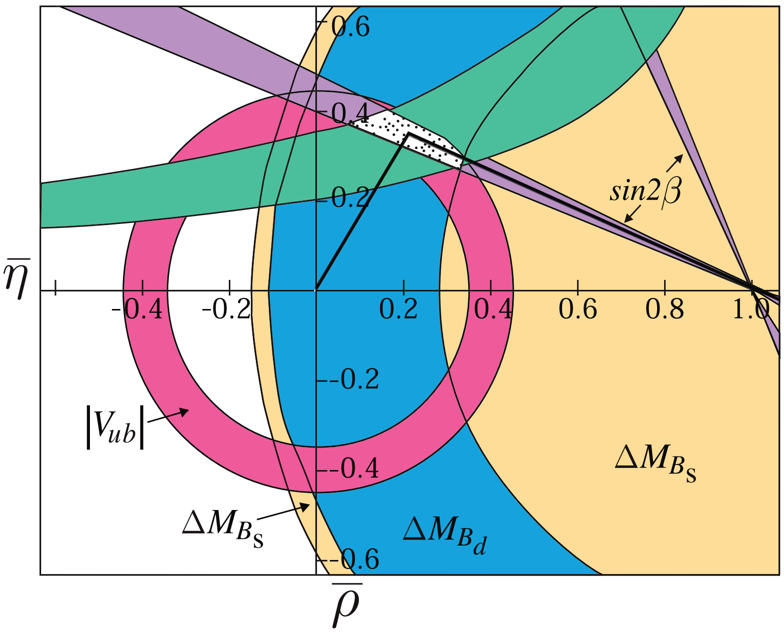


Figure 11.2: Constraints from the text on the position of the apex, A, of the unitarity triangle following from $|V_{ub}|$,B mixing, ϵ, and $\sin 2\beta$. A possible unitarity triangle is shown with A in the preferred region. See full-color version on color pages at end of book.

All processes can be quantitatively understood by one value of the CKM phase $\delta_{13} = \gamma = 60° \pm 14°$. The value of $\beta = 23.4° \pm 2°$ from the overall fit is consistent with the value from the CP-asymmetry measurements of $23.7° \pm 2.1°$. The invariant measure of CP violation is $J = (2.88 \pm 0.33) \times 10^{-5}$.

The limit in Eq. (11.21) is not far from the value we would expect from the other information on the unitarity triangle. This limit is more robust theoretically since it depends on ratios (rather than absolute values) of hadronic matrix elements and is independent of the top mass or QCD corrections (which cancel in the ratio). Thus, the significant increase in experimental sensitivity to B_s mixing that should become available in the CDF and D0 experiments in the next few years will lead either to an observation of mixing as predicted by our knowledge to date or to an indication of physics beyond the Standard Model.

Other experimental progress in the next few years includes: checking the unitarity of the first row of the CKM matrix by new precise measurements of $|V_{us}|$ in semileptonic decays of charged and neutral kaons; resolution of the apparent inconsistency between BELLE and BaBar in the measurement of the time-dependent particle-antiparticle asymmetry in the decay $B_d(\overline{B}_d) \to \phi K_S$; searches for direct CP violation in B decay modes; and measurement of the Dalitz plot asymmetry in $K^+(K^-) \to 3\pi$ at the 10^{-4} level by NA48/2.

Longer range, the frontiers are: extraction of the angle $\alpha = \phi_2$ from measurements of decays of B_d mesons; determination of the angle $\gamma = \phi_3$ from measurements of both B_d and B_s decays; and the pursuit of the CP-violating rare decay $K_L \to \pi^0 \nu\overline{\nu}$.

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References:

1. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
2. N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
3. L.-L. Chau and W.-Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984);
   H. Harari and M Leurer, Phys. Lett. **B181**, 123 (1986);
   H. Fritzsch and J. Plankl, Phys. Rev. **D35**, 1732 (1987);
   F.J. Botella and L.-L. Chao, Phys. Lett. **B168**, 97 (1986).
4. L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
5. A. Buras *et al.*, Phys. Rev. **D50**, 3433 (1994);
   see also M. Schmidtler and K. Schubert, Z. Phys. **C53**, 347, (1992).
6. C.D. Frogatt and H.B. Nielsen, Nucl. Phys. **B147**, 277 (1979);
   H. Fritzsch, Nucl. Phys. **B155**, 189 (1979);
   S. Dimopoulos, L.J. Hall, and S. Rabi, Phys. Rev. Lett. **68**, 1984 (1992);
   H. Fritzsch and Z.-Z. Xing, Phys. Lett. **B413**, 396 (1997).
7. W.J. Marciano and A. Sirlin, Phys. Rev. Lett. **56**, 22 (1986);

A. Sirlin and R. Zucchini, Phys. Rev. Lett. **57**, 1994 (1986);
W. Jaus and G. Rasche, Phys. Rev. **D35**, 3420 (1987);
A. Sirlin, Phys. Rev. **D35**, 3423 (1987).

8. B.A. Brown and W.E. Ormand, Phys. Rev. Lett. **62**, 866 (1989).
9. F.C. Barker *et al.*, Nucl. Phys. **A540**, 501 (1992);
F.C. Barker *et al.*, Nucl. Phys. **A579**, 62 (1994).
10. G. Savard *et al.*, Phys. Rev. Lett. **74**, 1521 (1995).
11. J.C. Hardy and I.S. Towner, talk at WEIN98, Santa Fe, June 14-21, 1998 and `nucl-th/9809087`.
12. H. Sagawa, Proc. of the Workshop "Quark-Mixing, CKM Unitarity", 2002, Heidelberg, 162.
13. H. Abele *et al.*, Phys. Rev. Lett. **88**, 211801 (2002) as a final result of J. Reich *et al.*, Nucl. Instrum. Methods **A440**, 535 (2000), and H. Abele *et al.*, Nucl. Phys. **A612**, 53 (1997).
14. Yu. A. Mostovoi *et al.*, Phys. Atomic Nucl. **64**, 1955 (2001);
P. Liaud, Nucl. Phys. **A612**, 53 (1997).
15. H. Leutwyler and M. Roos, Z. Phys. **C25**, 91 (1984);
See also the work of R.E. Shrock and L.-L. Wang, Phys. Rev. Lett. **41**, 1692 (1978).
16. V. Cirigliano *et al.*, Eur. Phys. J. **C23**, 121 (2002).
17. G. Calderon and G. Lopez Castro , Phys. Rev. **D65**, 073032 (2002).
18. J. Thompson, talk given at CKM03 workshop, Durham, UK, April 5th to 9th, (2003), `hep-ex/0307053` .
19. J.F. Donoghue, B.R. Holstein, and S.W. Klimt, Phys. Rev. **D35**, 934 (1987).
20. R. Flores-Mendieta, A. Garcia, and G. Sanchez-Col'on, Phys. Rev. **D54**, 6855 (1996).
21. M. Bourquin *et al.*, Z. Phys. **C21**, 27 (1983).
22. N. Cabibbo *et al.*, `hep-ph/0307214` ; `hep-ph/0307298` , to be published in *Ann. Rev. Nucl. Part. Sci.*, Vol. 53 (2003).
23. H. Abramowicz *et al.*, Z. Phys. **C15**, 19 (1982).
24. S.A. Rabinowitz *et al.*, Phys. Rev. Lett. **70**, 134 (1993);
A.O. Bazarko *et al.*, Z. Phys. **C65**, 189 (1995).
25. P. Vilain *et al.*, Eur. Phys. J. **C11**, 19 (1999).
26. N. Ushida *et al.*, Phys. Lett. **B206**, 375 (1988).
27. T. Bolton, `hep-ex/9708014` (1997).
28. A. Kayis-Topasku *et al.*, Phys. Lett. **B549**, 48 (2002).
29. P. Abreu *et al.*, Phys. Lett. **B439**, 209 (1998);
R. Barate *et al.*, Phys. Lett. **B465**, 349 (1999).
30. The LEP Collaborations, the LEP Electroweak Working Group and the SLD Heavy Flavour and Electroweak Groups, `hep-ex/0112021v2` (2002).

31. N. Isgur and M.B. Wise, Phys. Lett. **B232**, 113 (1989), and Phys. Lett. **B237**, 527 (1990) E;

    E. Eichten and B. Hill, Phys. Lett. **B234**, 511 (1990);

    M.E. Luke, Phys. Lett. **B252**, 447 (1990).
32. See the review on "Determination of $|V_{cb}|$" by M. Artuso and E. Barberio in this *Review*.
33. A. Falk, presentation at the Fifth KEK Topical Conference, Tsukuba, Japan, November 20–22, 2001 and `hep-ph/0201094` .
34. A. Bornheim *et al.* (CLEO Collaboration), `hep-ex/0202019` , 2002.
35. See the review on "Determination of $|V_{ub}|$" by M. Battaglia and L. Gibbons in this *Review*.
36. T. Affolder *et al.*, Phys. Rev. Lett. **86**, 3233 (2001).
37. K. Kleinknecht and B. Renk, Phys. Lett. **86**, 130B (1983);

    Z. Phys. **C34**, 209 (1987).
38. A. Hocker *et al.*, Eur. Phys. J. **C21**, 225 (2001);

    C. Ciuchini *et al.*, JHEP **0107**, 013 (2001).
39. L.-L. Chau and W.Y. Keung, Ref. 3;

    J.D. Bjorken, private communication and Phys. Rev. **D39**, 1396 (1989);

    C. Jarlskog and R. Stora, Phys. Lett. **B208**, 268 (1988);

    J.L. Rosner, A.I. Sanda, and M.P. Schmidt, in *Proceedings of the Workshop on High Sensitivity Beauty Physics at Fermilab*, Fermilab, November 11–14, 1987, edited by A.J. Slaughter, N. Lockyer, and M. Schmidt (Fermilab, Batavia, 1988), p. 165;

    C. Hamzaoui, J.L. Rosner, and A.I. Sanda, *ibid.*, p. 215.
40. C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985) and Z. Phys. **C29**, 491 (1985).
41. See the review on "B–$\overline{B}$ Mixing" by O. Schneider in this *Review*.
42. A. Kronfeld, `hep-lat/0310063v1`.
43. A.J. Buras *et al.*, Nucl. Phys. **B347**, 491 (1990).
44. M. S. Alam *et al.* (CLEO Collab.), Phys. Rev. Lett. **74**, 2885 (1995);

    S. Chen *et al.* (CLEO Collab.), Phys. Rev. Lett. **87**, 1807 (2001).
45. K. Abe *et al.* (BELLE Collab.), Phys. Lett. **B511**, 157 (2001);

    B. Aubert *et al.* (BaBar Collab.), `hep-ex/0207074` and `hep-ex/0207076` (2002).
46. A. Ali and M. Misiak, `hep-ph/0304132`, (2003).
47. S. Adler *et al.*, `hep-ex/0111091` (2001).
48. The relevant QCD corrections in leading order in F.J. Gilman and M.B. Wise Phys. Lett. **B93**, 129 (1980), and Phys. Rev. **D27**, 1128 (1983), have been extended to next-to-leading-order by A. Buras *et al.*, Ref. 43;

    S. Herrlich and U. Nierste Nucl. Phys. **B419**, 292 (1992) and Nucl. Phys. **B476**, 27 (1996).

49. The limiting curves in Fig. 11.2 arising from the value of $|\epsilon|$ correspond to values of the hadronic matrix element expressed in terms of the renormalization group invariant parameter $\widehat{B}_K$ from 0.68 to 1.06 . See, for example, D. Becirevic, plenary talk at Lattice 2003, Tsukuba, Japan, July 15 - 19, 2003.
50. H. Burkhardt *et al.*, Phys. Lett. **B206**, 169 (1988).
51. G.D. Barr *et al.*, Phys. Lett. **B317**, 233 (1993);
    L.K. Gibbons *et al.*, Phys. Rev. Lett. **70**, 1203 (1993);
    V. Fanti *et al.*, Phys. Lett. **B465**, 335 (1999);
    A. Alavi-Harati *et al.*, Phys. Rev. Lett. **83**, 22 (1999);
    A. Lai *et al.*, Eur. Phys. J. **C22**, 231 (2001);
    J.R. Batley *et al.*, Phys. Lett. **B544**, 97 (2002).
52. B. Aubert *et al.*, Phys. Rev. Lett. **89**, 201802 (2002).
53. K. Abe *et al.* Belle-CONF-0353, LP'03 (2003).
54. S. Laplace *et al.*, Phys. Rev. **D65**, 094040 (2002).