## REVIEW OF CHARM DALITZ-PLOT ANALYSES

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Weak nonleptonic decays of charm mesons are expected to proceed dominantly through resonant two-body decays in several theoretical models [1]. The Dalitz-plot analysis technique [2,3] has been applied to the decays $D \to rc$, $r \to ab$ where the decay products $a$, $b$, and $c$ are $K$ or $\pi$ and the intermediate state $r$ is a scalar, vector, or tensor meson. Table 1 lists published analyses of $D \to \overline{K}\pi\pi$, $\to \pi\pi\pi$, $\to \overline{K}K\pi$, and $\to \overline{K}K\overline{K}$ decays. The analyses include studies of doubly Cabibbo-suppressed decays [4,5], searches for $CP$ violation [5–8], properties of established light mesons [9–11], and properties of $\pi\pi$ [4,11,12] and $K\pi$ [13] S-wave states. Future studies could improve sensitivity to $D^0$–$\overline{D}^0$ mixing [14].

The amplitude of the process, $D \to rc, r \to ab$, is given by the product of three factors: the angular distributions [15,16] of final-state particles, the barrier form factors [17,18] for the production of $rc$ and $ab$, respectively, and the dynamical function describing the resonance $r$. Usually $r$ is modeled with a Breit-Wigner, and the nonresonant contribution is parameterized as an S-wave with no variation in magnitude or phase across the Dalitz plot. Some more recent analyses have used the $K$-matrix formalism [19] with the $P$-vector approximation [20] to describe the $\pi\pi$ S-wave.

In the following, we discuss a number of subjects of current interest.

**$D^0 \to K^0_S\pi^+\pi^-$** — Several experiments have analyzed the decay $D^0 \to K^0_S\pi^+\pi^-$. The earliest analyses, by Mark II [21], Mark III [22], and E687 [23], assumed only two intermediate resonances, $K^0_S\rho^0$, $K^*(892)^-\pi^+$, and a significant nonresonant component. Additional resonances were considered by E691 [24] but were not found to be statistically significant. ARGUS [25] and E687 [26], with more events, fit the Dalitz plot with six intermediate resonances: $K^*(892)^-\pi^+$, $K^*_0(1430)^-\pi^+$, $K^0_S\rho^0$, $K^0_Sf_0(975)$, $K^0_Sf_2(1270)$, and $K^0_Sf_0(1400)$. The nonresonant contribution was negligible. The early and later E687 results [23, 26] were consistent under similar assumptions. The most

**Table 1:** Reported Dalitz Plot Analyses.

| Decay | Experiment(s) |
|---|---|
| $D^0 \to K^0_S \pi^+\pi^-$ | Mark II [21], Mark III [22], E691 [24], E687 [23,26], ARGUS [25], CLEO [4] |
| $D^0 \to K^-\pi^+\pi^0$ | Mark III [22], E687 [26], E691 [24], CLEO [6] |
| $D^0 \to \overline{K}^0 K^+\pi^-$ | BABAR [27] |
| $D^0 \to K^0 K^-\pi^+$ | BABAR [27] |
| $D^0 \to \pi^+\pi^-\pi^0$ | CLEO [8] |
| $D^0 \to K^0_S K^+K^-$ | BABAR [27] |
| $D^+ \to K^-\pi^+\pi^+$ | Mark III [22], E687 [26], E691 [24], E791 [13] |
| $D^+ \to \overline{K}^0\pi^+\pi^0$ | Mark III [22] |
| $D^+ \to \pi^+\pi^+\pi^-$ | E687 [9], E791 [12], FOCUS [11] |
| $D^+ \to K^+K^-\pi^+$ | FOCUS [29], E687 [30] |
| $D_s^+ \to K^+K^-\pi^+$ | E687 [30], FOCUS [29] |
| $D_s^+ \to \pi^+\pi^+\pi^-$ | E687 [9], E791 [10], FOCUS [11] |

precise results are from CLEO [4], which includes three additional resonances: $K^0_S\omega, K^*(1680)^-\pi^+$ and the doubly Cabibbo-suppressed $K^*(892)^+\pi^-$. They find a much smaller nonresonant contribution than did the earliest experiments.

It is not straightforward to compare or combine results using different descriptions of the angular distributions, barrier factors, resonant parametrizations, and different sets of resonances. Some of the earlier results [22–24], did not include barrier factors [17, 18]. Most of the earlier results [22–24, 26] used the Zemach formalism [15] to describe the angular shape of the decay pattern, while the more recent results [25, 4] use the helicity formalism [16].

The significance of the nonresonant component in the smaller data samples has been attributed to the presence of the broad scalar resonances $K^*_0(1430)^-$ and $f_0(1370)$ that were later observed in the larger data samples. The observation of a small but significant nonresonant component in the largest data samples suggests the presence of additional broad scalar resonances, the $\kappa(800)$ and $\sigma(500)$. The CLEO analysis could accomodate the $\sigma(500)$ in lieu of the nonresonant component, but found no evidence for the $\kappa(800)$.

**$D^0 \to \pi^+\pi^-\pi^0$ and $D^0 \to \overline{K}^0 K^+K^-$** — The only significant contribution to the resonant substructure of $D^0 \to \pi^+\pi^-\pi^0$ is in the $\rho\pi$ channels. A small nonresonant component is observed but all other $\pi\pi$ resonances, including the $\sigma(500)$, yielded fit fractions consistent with zero. The CLEO [8] results for $D^0 \to \pi^+\pi^-\pi^0$ are given in Table 2.

**Table 2:** Dalitz fit results of $D^0 \to \pi^+\pi^-\pi^0$.

| Resonance | Amplitude | Phase(°) | Fit fraction(%) |
|---|---|---|---|
| $\rho^+$ | 1. (fixed) | 0. (fixed) | $76.5 \pm 1.8 \pm 4.8$ |
| $\rho^0$ | $0.56 \pm 0.02 \pm 0.07$ | $10 \pm 3 \pm 3$ | $23.9 \pm 1.8 \pm 4.6$ |
| $\rho^-$ | $0.65 \pm 0.03 \pm 0.04$ | $-4 \pm 3 \pm 4$ | $32.3 \pm 2.1 \pm 2.2$ |
| nonresonant | $1.03 \pm 0.17 \pm 0.31$ | $77 \pm 8 \pm 11$ | $2.7 \pm 0.9 \pm 1.7$ |

The BABAR [27] results for $D^0 \to \overline{K}^0 K^+K^-$ are given in Table 3. The non-$\phi$ resonant substructure in $K^+K^-$ is significant. Resonant contributions from $a_0(980)^0$, $a_0(980)^+$, and $f_0(980)$ are observed. The nonresonant and the doubly Cabibbo-suppressed contributions are consistent with zero.

**Table 3:** Dalitz fit results of $D^0 \to \overline{K}^0 K^+K^-$.

| Resonance | Phase(°) | Fit fraction(%) |
|---|---|---|
| $\overline{K}^0\phi$ | 0. (fixed) | $45.4 \pm 1.6 \pm 1.0$ |
| $\overline{K}^0 a_0(980)$ | $109 \pm 5$ | $60.9 \pm 7.5 \pm 13.3$ |
| $\overline{K}^0 f_0(980)$ | $-161 \pm 14$ | $12.2 \pm 3.1 \pm 8.6$ |
| $a_0(980)^+K^-$ | $-53 \pm 4$ | $34.3 \pm 3.2 \pm 6.8$ |
| $a_0(980)^-K^+$ | $-13 \pm 15$ | $3.2 \pm 1.9 \pm 0.5$ |
| nonresonant | $40 \pm 44$ | $0.4 \pm 0.3 \pm 0.8$ |

Charm Dalitz-plot analyses might be useful for calibrating tools used in $B$ decays: specifically, to extract $\alpha$ from $B^0 \to \pi^+\pi^-\pi^0$, $\beta$ from $B^0 \to \overline{K}^0 K^+K^-$, and $\gamma$ from $B^\pm \to DK^\pm$ followed by $D \to \overline{K}^0 K^+K^-$ or $D \to \overline{K}^0\pi^+\pi^-$ [28].

***$D^+ \to \pi^+\pi^+\pi^-$: a $\sigma(500)$ or $f_0(600)$*** — The decay $D^+ \to \pi^+\pi^+\pi^-$ has been studied by the E687 [9], E791 [12] and FOCUS [11] experiments. The E687 experiment considered the $\rho(770)^0\pi^+$, $f_0(980)\pi^+$, $f_2(1270)\pi^+$, and a nonresonant component. The E791 experiment included in addition $f_0(1370)\pi^+$ and $\rho(1450)^0\pi^+$. Both analyses found a very large fraction ($\sim$ 50%) for the nonresonant contribution, perhaps indicating a broad scalar contribution. E791 found the nonresonant amplitude to be consistent with zero if a broad scalar resonance was included in the fit. FOCUS analyzed its data sample using both the Breit-Wigner formalism and the $K$-matrix formalism. The Breit-Wigner analysis included $\rho(770), f_0(980), f_2(1270), f_0(1500), \sigma(500)$, and a nonresonant contribution. Applying the $K$-matrix formalism to the S wave and parameterizing the $\rho(770)$ and $f_2(1270)$ with the Breit-Wigner functions also described the FOCUS data well.

None of these analyses has modeled the dynamics of the $\pi^+\pi^+$ interaction. Consideration of the $I = 2$ S-wave and D-wave phase shifts, also measured in $\pi^+ p \to \pi^+\pi^+ n$ [31], could affect the $\pi^+\pi^-$ S-wave result.

E791 finds additional evidence that the low mass $\pi\pi$ feature is resonant by examining the phase of the $\pi\pi$ amplitude in the vicinity of the reported $\sigma(500)$ mass. A phase variation with invariant $\pi\pi$ mass is consistent with a resonant contribution [32].

Table 4 gives the parameters of the $\sigma(500)$ determined in charm Dalitz plot analyses. A consistent relative phase between the $\sigma(500)$ and $\rho(770)$ resonances is observed.

**Table 4:** Parameters of the $\sigma(500)$ resonance. The amplitude and phase are relative to the $\rho(770)$.

| Experiment | E791 [12] | CLEO [4] | FOCUS [11] |
|---|---|---|---|
| Decay Mode | $D^+ \to \pi^+\pi^+\pi^-$ | $D^0 \to K^0_S\pi^+\pi^-$ | $D^+ \to \pi^+\pi^+\pi^-$ |
| Amplitude | $1.17 \pm 0.13 \pm 0.06$ | $0.57 \pm 0.13$ | — |
| Phase(°) | $205.7 \pm 8.0 \pm 5.2$ | $214 \pm 11$ | $200 \pm 31$ |
| $m(\mathrm{MeV}/c^2)$ | $478^{+24}_{-23} \pm 17$ | $513 \pm 32$ | $443 \pm 27$ |
| $\Gamma(\mathrm{MeV}/c^2)$ | $324^{+42}_{-40} \pm 21$ | $335 \pm 67$ | $443 \pm 80$ |

***$D^+ \to K^-\pi^+\pi^+$: a $\kappa(800)$?*** — Indication of a broad $K\pi$ scalar intermediate resonance has been reported by E791 in the decay $D^+ \to K^-\pi^+\pi^+$ [13]. Fitting the Dalitz plot with $\overline{K}^*(892)^0\pi^+$, $\overline{K}_0^*(1430)^0\pi^+$, $\overline{K}_2^*(1430)^0\pi^+$, and $\overline{K}^*(1680)^0\pi^+$, plus a constant nonresonant component, E791 finds results consistent with earlier results from E691 and E687 with a nonresonant fit fraction of over 90%. Having reconstructed more events than the other experiments, E791 was led to include an extra low-mass S-wave $\overline{K}\pi$ resonance to account for the poor fit already seen by earlier experiments: A $\kappa(800)$ with $m = 797 \pm 19 \pm 43$ MeV/$c^2$ and $\Gamma = 410 \pm 43 \pm 87$ MeV/$c^2$ much improved the fits. The $\kappa(800)$ is now the dominant resonance and the nonresonant fit fraction is reduced from $90.9 \pm 2.6\%$ to $13.0 \pm 5.8 \pm 4.4\%$. As discussed with the $\sigma(500)$, the $K^-\pi^+$ S-wave result could be affected by modeling the dynamics of the $I = 2$ $\pi^+\pi^+$ interaction.

E791 also modeled the $K\pi$ S-wave phase variation as a function of $K\pi$ mass with the $K_0^*(1430)$ resonance only and a nonresonant component following the parameterization of LASS [33]. It was necessary to relax the unitarity constraint to describe the E791 data [34]. The $K\pi$ S-wave phase behavior in this model is consistent with the model that includes the $\kappa$ resonance.

CLEO allowed scalar $K\pi$ resonances in the fit to $D^0 \to K^-\pi^+\pi^0$ [6] and $D^0 \to K_S^0\pi^+\pi^-$ [4] and observed a significant contribution for only $K_0^*(1430)$ [35]. BABAR has analyzed the decay $D^0 \to K^0K^-\pi^+$ and $D^0 \to \overline{K}^0K^+\pi^-$ [27]. They fit the former Dalitz plot with both positively charged and neutral $\overline{K}^*(892)$, $\overline{K}_0^*(1430)$, $\overline{K}_2^*(1430)$, $\overline{K}^*(1680)$ and $a_0(980)^-$, $a_0(1450)^-$, $a_2(1310)^-$ resonances, and a nonresonant component. The second Dalitz plot is fit with the identical resonances except for the $a_2(1310)^-$. A good fit is obtained in both cases without including the $\kappa$.

***$f_0(980)$, $f_0(1370)$ and $f_0(1500)$*** — The proximity of the $K\overline{K}$ threshold requires a coupled-channel or Flatte parametrization [36] of the $f_0(980)$ in charm Dalitz-plot analyses. The width of the $f_0(980)$ is poorly known. E791 used a coupled-channel Breit-Wigner function, following the parametrization of

Ref. [37], to describe the $f_0(980)$ in $D_s^+ \to \pi^+\pi^+\pi^-$ [10], and measured $m_r = 977 \pm 3 \pm 2$ MeV/$c^2$, $g_{\pi\pi} = 0.09 \pm 0.01 \pm 0.01$, and $g_{KK} = 0.02 \pm 0.04 \pm 0.03$. Results similar to these are desirable for input to the analysis of the $D_s^+ \to K^+K^-\pi^+$ [29], which includes the $f_0(980)$ and $a_0(980)$.

The quark content of the $f_0(1370)$ and $f_0(1500)$ can perhaps be inferred from how they populate various Dalitz plots. The E791 analysis of $D^+ \to \pi^+\pi^+\pi^-$ [12] finds a contribution from the $f_0(1370)$ but not the $f_0(1500)$. The FOCUS analysis [11] of this decay does not find a significant contribution from the $f_0(1370)$. For the $D_s^+ \to \pi^+\pi^+\pi^-$, E687 [9] and FOCUS [11] do not see the $f_0(1370)$ but do see a resonance with parameters similar to the $f_0(1500)$, while E791 [10] observes a $\pi\pi$ resonance ($m = 1434 \pm 18 \pm 9$ MeV/$c^2$ and $\Gamma = 172 \pm 32 \pm 6$ MeV/$c^2$) that is not consistent with either meson. BABAR has found no evidence for either the $f_0(1370)$ or the $f_0(1500)$ in $D^0 \to \overline{K}^0 K^+K^-$ [27], while CLEO has observed the $f_0(1370)$ in $D^0 \to K_S^0\pi^+\pi^-$ [4]. Future analyses will present a clearer picture only if the same resonances and model of decay amplitudes are applied to all Dalitz-plot fits.

***Doubly Cabibbo-Suppressed Decays*** — There are two classes of multibody doubly Cabibbo-suppressed (DCS) decays of charm mesons. The first consists of those in which the DCS and corresponding Cabbibo-favored (CF) decays populate distinct Dalitz plots: the pairs $D^0 \to K^+\pi^-\pi^0$ and $D^0 \to K^-\pi^+\pi^0$, or $D^+ \to K^+\pi^+\pi^-$ and $D^+ \to K^-\pi^+\pi^+$, are examples. CLEO [5] has reported $\dfrac{\mathcal{B}(D^0 \to K^+\pi^-\pi^0)}{\mathcal{B}(D^0 \to K^-\pi^+\pi^0)} = (0.43^{+0.11}_{-0.10} \pm 0.07)\%$.

The second class consists of decays where the DCS and CF modes populate the same Dalitz plot: for example, $D^0 \to K^{*-}\pi^+$ and $D^0 \to K^{*+}\pi^-$ both contribute to $D^0 \to K_S^0\pi^+\pi^-$. In this case, the potential for interference of DCS and CF amplitudes increases the sensitivity to the DCS amplitude. CLEO [4] has reported the relative amplitudes and phases to be $(7.1 \pm 1.3^{+2.6}_{-0.6}{}^{+2.6}_{-0.6})\%$ and $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$, respectively, corresponding to $\dfrac{\mathcal{B}(D^0 \to K^*(892)^+\pi^-)}{\mathcal{B}(D^0 \to K^*(892)^-\pi^+)} = (0.5 \pm 0.2^{+0.5}_{-0.1}{}^{+0.4}_{-0.1})\%$.

***CP Violation*** — In the limit of $CP$ conservation, charge conjugate decays will have the same Dalitz-plot distribution. The $D^{*\pm}$ tag enables the discrimination between $D^0$ and $\overline{D}^0$. The integrated $CP$ violation across the Dalitz plot is determined from

$$\mathcal{A}_{CP} = \int \frac{|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2} \, dm_{ab}^2 \, dm_{bc}^2 \Big/ \int dm_{ab}^2 \, dm_{bc}^2 \, ,$$

where $\mathcal{M}$ and $\overline{\mathcal{M}}$ are the $D^0$ and $\overline{D}^0$ Dalitz-plot amplitudes. This expression is less sensitive to $CP$ violation than the individual resonant submodes reported in Ref. [7]. Table 5 reports the results for $CP$ violation. No evidence of $CP$ violation has been observed.

**Table 5:** Dalitz-plot-integrated $CP$ violation.

| Experiment | Decay mode | $\mathcal{A}_{CP}(\%)$ |
|---|---|---|
| CLEO [6] | $D^0 \to K^-\pi^+\pi^0$ | $-3.1 \pm 8.6$ |
| CLEO [5] | $D^0 \to K^+\pi^-\pi^0$ | $+9^{+22}_{-25}$ |
| CLEO [7] | $D^0 \to K^0_S\pi^+\pi^-$ | $-0.9 \pm 2.1^{+1.0+1.3}_{-4.3-3.7}$ |
| CLEO [8] | $D^0 \to \pi^+\pi^-\pi^0$ | $+1^{+9}_{-7} \pm 9$ |

The possibility of interference between $CP$–conserving and $CP$–violating amplitudes provides a more sensitive probe of $CP$ violation. The constraints on the square of the $CP$–violating amplitude obtained in the resonant submodes of $D^0 \to K^0_S\pi^+\pi^-$ range from (3.5 to 28.4)$\times10^{-4}$ at 95% confidence level [7].

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