B^0–$\overline{B}^0$ MIXING

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There are two neutral B^0–$\overline{B}^0$ meson systems, B^0_d–$\overline{B}^0_d$ and B^0_s–$\overline{B}^0_s$ (generically denoted B^0_q–$\overline{B}^0_q$, $q = s, d$), which exhibit particle-antiparticle mixing [1]. This mixing phenomenon is described in Ref. 2. In the following, we adopt the notation introduced in Ref. 2, and assume CPT conservation throughout. In each system, the light (L) and heavy (H) mass eigenstates,

$$|B_{\rm L,H}\rangle = p|B^0_q\rangle \pm q|\overline{B}^0_q\rangle \,, \tag{1}$$

have a mass difference $\Delta m_q = m_{\rm H} - m_{\rm L} > 0$, and a total decay width difference $\Delta\Gamma_q = \Gamma_{\rm H} - \Gamma_{\rm L}$. In the absence of CP violation in the mixing, $|q/p| = 1$, these differences are given by $\Delta m_q = 2|M_{12}|$ and $|\Delta\Gamma_q| = 2|\Gamma_{12}|$, where M_{12} and Γ_{12} are the off-diagonal elements of the mass and decay matrices [2]. The evolution of a pure $|B^0_q\rangle$ or $|\overline{B}^0_q\rangle$ state at $t = 0$ is given by

$$|B^0_q(t)\rangle = g_+(t)\,|B^0_q\rangle + \frac{q}{p}\,g_-(t)\,|\overline{B}^0_q\rangle \,, \tag{2}$$

$$|\overline{B}^0_q(t)\rangle = g_+(t)\,|\overline{B}^0_q\rangle + \frac{p}{q}g_-(t)\,|B^0_q\rangle \,, \tag{3}$$

which means that the flavor states remain unchanged (+) or oscillate into each other (−) with time-dependent probabilities proportional to

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}\,t\right) \pm \cos(\Delta m_q\, t)\right] \,, \tag{4}$$

where $\Gamma_q = (\Gamma_{\rm H} + \Gamma_{\rm L})/2$. In the absence of CP violation, the time-integrated mixing probability $\int |g_-(t)|^2\,dt/(\int |g_-(t)|^2\,dt + \int |g_+(t)|^2\,dt)$ is given by

$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2+1)}\,, \qquad \text{where} \qquad x_q = \frac{\Delta m_q}{\Gamma_q}\,, \qquad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}\,. \tag{5}$$

Standard Model predictions and phenomenology

In the Standard Model, the transitions $B^0_q \to \overline{B}^0_q$ and $\overline{B}^0_q \to B^0_q$ are due to the weak interaction. They are described, at the lowest order, by box diagrams involving two W bosons and two

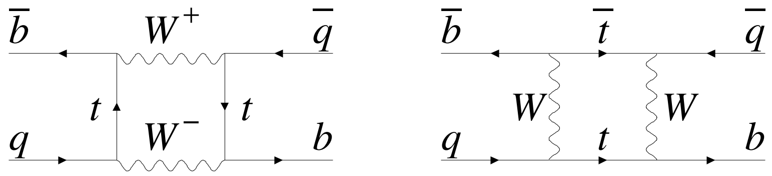

Figure 1: Dominant box diagrams for the $B_q^0 \to \overline{B}_q^0$ transitions ($q = d$ or s). Similar diagrams exist where one or both t quarks are replaced with c or u quarks.

up-type quarks (see Fig. 1), as is the case for K^0–$\overline{K}^0$ mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral B meson systems, because the large B mass is off the region of hadronic resonances. The calculation of the dispersive and absorptive parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} S_0(m_t^2/m_W^2)\,(V_{tq}^* V_{tb})^2 \,, \qquad (6)$$

$$\begin{aligned} \Gamma_{12} = {} & \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi} \\ & \times \left[(V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb}\ \mathcal{O}\!\left(\frac{m_c^2}{m_b^2}\right) \right. \\ & \left. \quad + (V_{cq}^* V_{cb})^2\ \mathcal{O}\!\left(\frac{m_c^4}{m_b^4}\right) \right] , \end{aligned} \qquad (7)$$

where G_F is the Fermi constant, m_W the W boson mass, and m_i the mass of quark i; m_{B_q}, f_{B_q} and B_{B_q} are the B_q^0 mass, weak decay constant and bag parameter, respectively. The known function $S_0(x_t)$ can be approximated very well by $0.784\,x_t^{0.76}$ [4], and V_{ij} are the elements of the CKM matrix [5]. The QCD corrections η_B and η'_B are of order unity. The only non-negligible contributions to M_{12} are from box diagrams involving two top quarks. The phases of M_{12} and Γ_{12} satisfy

$$\phi_M - \phi_\Gamma = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) , \qquad (8)$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the K^0–$\overline{K}^0$

system, the heavy state has a smaller decay width than that of the light state. Hence, $\Delta\Gamma$ is expected to be negative in the Standard Model.

Furthermore, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2}\frac{m_b^2}{m_W^2}\frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) \tag{9}$$

is small, and a power expansion of $|q/p|^2$ yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\phi_M - \phi_\Gamma) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right) . \tag{10}$$

Therefore, considering both Eqs. (8) and (9), the CP-violating parameter

$$1 - \left|\frac{q}{p}\right|^2 \simeq \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \tag{11}$$

is expected to be very small: $\sim \mathcal{O}(10^{-3})$ for the B_d^0–$\overline{B}_d^0$ system and $\lesssim \mathcal{O}(10^{-4})$ for the B_s^0–$\overline{B}_s^0$ system [6].

In the approximation of negligible CP violation in mixing, the ratio $\Delta\Gamma_q/\Delta m_q$ is equal to the small quantity $|\Gamma_{12}/M_{12}|$ of Eq. (9); it is hence independent of CKM matrix elements, *i.e.*, the same for the B_d^0–$\overline{B}_d^0$ and B_s^0–$\overline{B}_s^0$ systems. It can be calculated with lattice QCD techniques; typical results are $\sim 5 \times 10^{-3}$ with quoted uncertainties of $\sim 30\%$. Given the current experimental knowledge (discussed below) on the mixing parameter x_q,

$$\begin{cases} x_d = 0.771 \pm 0.012 & (B_d^0\text{–}\overline{B}_d^0 \text{ system}) \\ x_s > 20.6 \text{ at } 95\% \text{ CL} & (B_s^0\text{–}\overline{B}_s^0 \text{ system}) \end{cases} , \tag{12}$$

the Standard Model thus predicts that $\Delta\Gamma_d/\Gamma_d$ is very small (below 1%), but $\Delta\Gamma_s/\Gamma_s$ considerably larger ($\sim 10\%$). These width differences are caused by the existence of final states to which both the B_q^0 and $\overline{B}_q^0$ mesons can decay. Such decays involve $b \to c\bar{c}q$ quark-level transitions, which are Cabibbo-suppressed if $q = d$ and Cabibbo-allowed if $q = s$.

Experimental issues and methods for oscillation analyses

Time-integrated measurements of B^0–$\overline{B}^0$ mixing were published for the first time in 1987 by UA1 [7] and ARGUS [8], and since then by many other experiments. These measurements are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced $b\overline{b}$ pairs. Such analyses cannot easily separate the contributions from the different b-hadron species, therefore, the clean environment of $\Upsilon(4S)$ machines (where only B^0_d and charged B_u mesons are produced) is in principle best suited to measure χ_d.

However, better sensitivity is obtained from time-dependent analyses aimed at the direct measurement of the oscillation frequencies Δm_d and Δm_s, from the proper time distributions of B^0_d or B^0_s candidates identified through their decay in (mostly) flavor-specific modes, and suitably tagged as mixed or unmixed. (This is particularly true for the B^0_s–$\overline{B}^0_s$ system, where the large value of x_s implies maximal mixing, *i.e.*, $\chi_s \simeq 1/2$.) In such analyses, the B^0_d or B^0_s mesons are either fully reconstructed, partially reconstructed from a charm meson, selected from a lepton with the characteristics of a $b \to \ell^-$ decay, or selected from a reconstructed displaced vertex. At high-energy colliders (LEP, SLC, Tevatron), the proper time $t = \dfrac{m_B}{p} L$ is measured from the distance L between the production vertex and the B decay vertex, and from an estimate of the B momentum p. At asymmetric B factories (KEKB, PEP-II), producing $e^+e^- \to \Upsilon(4S) \to B^0_d\overline{B}^0_d$ events with a boost $\beta\gamma$ (= 0.425, 0.55), the proper time difference between the two B candidates is estimated as $\Delta t \simeq \dfrac{\Delta z}{\beta\gamma c}$, where Δz is the spatial separation between the two B decay vertices along the boost direction. In all cases, the good resolution needed on the vertex positions is obtained with silicon detectors.

The average statistical significance $\mathcal{S}$ of a B^0_d or B^0_s oscillation signal can be approximated as [9]

$$\mathcal{S} \approx \sqrt{N/2}\, f_{\rm sig}\,(1-2\eta)\, e^{-(\Delta m\, \sigma_t)^2/2}\,, \tag{13}$$

where N and $f_{\rm sig}$ are the number of candidates and the fraction of signal in the selected sample, η is the total mistag probability, and σ_t is the resolution on proper time (or proper time difference). The quantity $\mathcal{S}$ decreases very quickly as Δm increases; this dependence is controlled by σ_t, which is therefore a critical parameter for Δm_s analyses. At high-energy colliders, the proper time resolution $\sigma_t \sim \frac{m_B}{\langle p \rangle}\sigma_L \oplus t\frac{\sigma_p}{p}$ includes a constant contribution due to the decay length resolution σ_L (typically 0.05–0.3 ps), and a term due to the relative momentum resolution σ_p/p (typically 10–20% for partially reconstructed decays), which increases with proper time. At B factories, the boost of the B mesons is estimated from the known beam energies, and the term due to the spatial resolution dominates (typically 1–1.5 ps because of the much smaller B boost).

In order to tag a B candidate as mixed or unmixed, it is necessary to determine its flavor both in the initial state and in the final state. The initial and final state mistag probabilities, η_i and η_f, degrade $\mathcal{S}$ by a total factor $(1-2\eta) = (1-2\eta_i)(1-2\eta_f)$. In lepton-based analyses, the final state is tagged by the charge of the lepton from $b \to \ell^-$ decays; the biggest contribution to η_f is then due to $\overline{b} \to \overline{c} \to \ell^-$ decays. Alternatively, the charge of a reconstructed charm meson (D^{*-} from B^0_d or D^-_s from B^0_s), or that of a kaon thought to come from a $b \to c \to s$ decay [10], can be used. For fully inclusive analyses based on topological vertexing, final state tagging techniques include jet charge [11] and charge dipole [12,13] methods.

At high-energy colliders, the methods to tag the initial state (*i.e.*, the state at production), can be divided in two groups: the ones that tag the initial charge of the $\overline{b}$ quark contained in the B candidate itself (same-side tag), and the ones that tag the initial charge of the other b quark produced in the event (opposite-side tag). On the same side, the charge of a track from the primary vertex is correlated with the production state of the B if that track is a decay product of a B^{**} state or the first particle in the fragmentation chain [14,15]. Jet- and vertex-charge techniques work on both sides and on the opposite side, respectively. Finally, the charge of a lepton from $b \to \ell^-$ or of a kaon from $b \to c \to s$ can be used as opposite side tags, keeping in mind that their performance is degraded due to integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the $Z \to b\overline{b}$ decays, and provided another very interesting and effective initial state tag based on the polar angle of the B candidate [12]. Initial state tags have also been combined to reach $\eta_i \sim 26\%$ at

LEP [15,16], or even 22% at SLD [12] with full efficiency. In the case $\eta_f = 0$, this corresponds to an effective tagging efficiency (defined as $Q = \epsilon(1-2\eta)^2$, where ϵ is the tagging efficiency) in the range $23-31\%$. The equivalent figure at CDF is $\sim 3.5\%$ for Tevatron Run I [17] (expected to reach $\sim 5\%$ for Run II [18]), reflecting the fact that tagging is very challenging at hadron colliders.

At B factories, the flavor of a B_d^0 meson at production cannot be determined, since the two neutral B mesons produced in a $\Upsilon(4S)$ decay evolve in a coherent P-wave state where they keep opposite flavors at any time. However, as soon as one of them decays, the other follows a time-evolution given by Eqs. (2) or (3), where t is replaced with Δt (which will take negative values half of the time). Hence, the "initial state" tag of a B can be taken as the final state tag of the other B. Effective tagging efficiencies $Q = 28-29\%$ are achieved by BABAR and Belle [19], using different techniques including $b \to \ell^-$ and $b \to c \to s$ tags. It is interesting to note that, in this case, mixing of this other B (*i.e.*, the coherent mixing occurring before the first B decay) does not contribute to the mistag probability.

In the absence of experimental evidence for a decay-width difference, oscillation analyses typically neglect $\Delta\Gamma$ in Eq. (4), and describe the data with the physics functions $\Gamma e^{-\Gamma t}(1 \pm \cos(\Delta m t))/2$ (high-energy colliders) or $\Gamma e^{-\Gamma|\Delta t|}(1 \pm \cos(\Delta m \Delta t))/4$ (asymmetric $\Upsilon(4S)$ machines). As can be seen from Eq. (4), a non-zero value of $\Delta\Gamma$ would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Measurements of Δm_d are usually extracted from the data using a maximum likelihood fit. No significant B_s^0–$\overline{B}_s^0$ oscillations have been seen so far. To extract information useful to set lower limits on Δm_s, B_s^0 analyses follow a method [9] in which a B_s^0 oscillation amplitude $\mathcal{A}$ is measured as a function of a fixed test value of Δm_s, using a maximum likelihood fit based on the functions $\Gamma_s e^{-\Gamma_s t}(1 \pm \mathcal{A}\cos(\Delta m_s t))/2$. To a very good approximation, the statistical uncertainty on $\mathcal{A}$ is Gaussian and equal to $1/\mathcal{S}$ from Eq. (13). If $\Delta m_s = \Delta m_s^{\rm true}$, one expects $\mathcal{A} = 1$ within the total uncertainty $\sigma_{\mathcal{A}}$; however, if Δm_s is (far) below its true value, a measurement consistent with $\mathcal{A} = 0$ is expected. A value of Δm_s can be excluded at 95% CL if $\mathcal{A} + 1.645\,\sigma_{\mathcal{A}} \leq 1$. If $\Delta m_s^{\rm true}$ is very large, one expects $\mathcal{A} = 0$, and all values of Δm_s such that $1.645\,\sigma_{\mathcal{A}}(\Delta m_s) < 1$ are expected to be excluded at 95% CL. Because of the proper time resolution, the quantity $\sigma_{\mathcal{A}}(\Delta m_s)$ is an increasing function of Δm_s, and one therefore expects to be able to exclude individual Δm_s values up to $\Delta m_s^{\rm sens}$, where $\Delta m_s^{\rm sens}$, called here the sensitivity of the analysis, is defined by $1.645\,\sigma_{\mathcal{A}}(\Delta m_s^{\rm sens}) = 1$.

B_d^0 mixing studies

Many B_d^0–$\overline{B}_d^0$ oscillations analyses have been published by the ALEPH [20], BABAR [21], Belle [22], CDF [14], DELPHI [13,23], L3 [24], and OPAL [25] collaborations. Although a variety of different techniques have been used, the individual Δm_d results obtained at high-energy colliders have remarkably similar precision. Their average is compatible with the recent and more precise measurements from asymmetric B factories. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or b-hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the b-hadron lifetimes and fractions published in this *Review*. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of b hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all published measurements [13,14,20–25] and accounting for all identified correlations as described in Ref. 26, yields $\Delta m_d = 0.502 \pm 0.004({\rm stat}) \pm 0.005({\rm syst})\ \ {\rm ps}^{-1}$.

On the other hand, ARGUS and CLEO have published time-integrated measurements [27–29], which average to $\chi_d = 0.182 \pm 0.015$. Following Ref. 29, the width difference $\Delta\Gamma_d$ could in principle be extracted from the measured value of Γ_d and the above averages for Δm_d and χ_d (see Eq. (5)), provided that $\Delta\Gamma_d$ has a negligible impact on the Δm_d measurements. However, direct time-dependent studies yield stronger constraints: DELPHI published the result $|\Delta\Gamma_d|/\Gamma_d < 18\%$ at 95% CL [13], while BABAR recently obtained $-8.4\% < \mathrm{sign}(\mathrm{Re}\lambda_{\mathrm{CP}})\Delta\Gamma_{\mathrm{d}}/\Gamma_{\mathrm{d}} < 6.8\%$ at 90% CL [30].

Assuming $\Delta\Gamma_d = 0$ and no CP violation in mixing, and using the measured B^0_d lifetime, the Δm_d and χ_d results are combined to yield the world average

$$\Delta m_d = 0.502 \pm 0.007 \ \ \mathrm{ps}^{-1} \tag{14}$$

or, equivalently,

$$\chi_d = 0.186 \pm 0.004\,. \tag{15}$$

Evidence for CP violation in B^0_d mixing has been searched for, both with flavor-specific and inclusive B^0_d decays, in samples where the initial flavor state is tagged. In the case of semileptonic (or other flavor-specific) decays, where the final state tag is also available, the following asymmetry [2]

$$\mathcal{A}_{\mathrm{SL}} = \frac{N(\overline{B}^0_d(t) \to \ell^+\nu_\ell X) - N(B^0_d(t) \to \ell^-\overline{\nu}_\ell X)}{N(\overline{B}^0_d(t) \to \ell^+\nu_\ell X) + N(B^0_d(t) \to \ell^-\overline{\nu}_\ell X)} \simeq 1 - |q/p|^2_d \tag{16}$$

has been measured, either in time-integrated analyses at CLEO [28,29,31] and CDF [32], or in time-dependent analyses at LEP [33–35] and BABAR [30,36]. In the inclusive case, also investigated at LEP [34,35,37], no final state tag is used, and the asymmetry [38]

$$\begin{aligned} &\frac{N(B^0_d(t) \to \mathrm{all}) - N(\overline{B}^0_d(t) \to \mathrm{all})}{N(B^0_d(t) \to \mathrm{all}) + N(\overline{B}^0_d(t) \to \mathrm{all})} \\ &\simeq \mathcal{A}_{\mathrm{SL}} \left[\frac{x_d}{2} \sin(\Delta m_d\, t) - \sin^2\left(\frac{\Delta m_d\, t}{2}\right)\right] \end{aligned} \tag{17}$$

must be measured as a function of the proper time to extract information on CP violation. In all cases, asymmetries compatible with zero have been found, with a precision limited by the available statistics. A simple average of all published results for the B^0_d meson [29,31,33,35–37] yields $\mathcal{A}_{\mathrm{SL}} = +0.002 \pm 0.013$, or $|q/p|_d = 0.999 \pm 0.006$, a result which does not yet constrain the Standard Model.

The Δm_d result of Eq. (14) provides an estimate of $2|M_{12}|$, and can be used, together with Eq. (6), to extract the magnitude of the CKM matrix element V_{td} within the Standard Model [39]. The main experimental uncertainties on the resulting estimate of $|V_{td}|$ come from m_t and Δm_d; however, the extraction is at present completely dominated by the uncertainty on the hadronic matrix element $f_{B_d}\sqrt{B_{B_d}} = 221 \pm 28^{+0}_{-22}$ MeV obtained from lattice QCD calculations [40].

B^0_s mixing studies

B^0_s–$\overline{B}^0_s$ oscillations have been the subject of many studies from ALEPH [15,41], CDF [42], DELPHI [13,16,43,44], OPAL [45], and SLD [12,46,47]. No oscillation signal has been found so far. The most sensitive analyses appear to be the ones based on inclusive lepton samples at LEP. Because of their better proper time resolution, the small data samples analyzed inclusively at SLD, as well as the few fully reconstructed B_s decays at LEP, turn out to be also very useful to explore the high Δm_s region.

All results are limited by the available statistics. They can easily be combined, since all experiments provide measurements of the B^0_s oscillation amplitude. All published results [12,13,16,41,42,43,45,46] are averaged using the procedure of Ref. 26 to yield the combined amplitudes $\mathcal{A}$ shown in Fig. 2 as a function of Δm_s. The individual results have been adjusted to common physics inputs, and all known correlations have been accounted for; the sensitivities of the inclusive analyses, which depend directly through Eq. (13) on the assumed fraction f_s of B^0_s mesons in an unbiased sample of weakly-decaying b hadrons, have also been rescaled to a common average of $f_s = 0.107 \pm 0.011$. The combined sensitivity for 95% CL exclusion of Δm_s values is found to be 17.8 ps^{-1}. All values of Δm_s below 14.4 ps^{-1} are excluded at 95% CL, which we express as

$$\Delta m_s > 14.4 \ \text{ps}^{-1} \quad \text{at } 95\% \text{ CL}. \tag{18}$$

The values between 14.4 and 21.8 ps^{-1} cannot be excluded, because the data is compatible with a signal in this region.

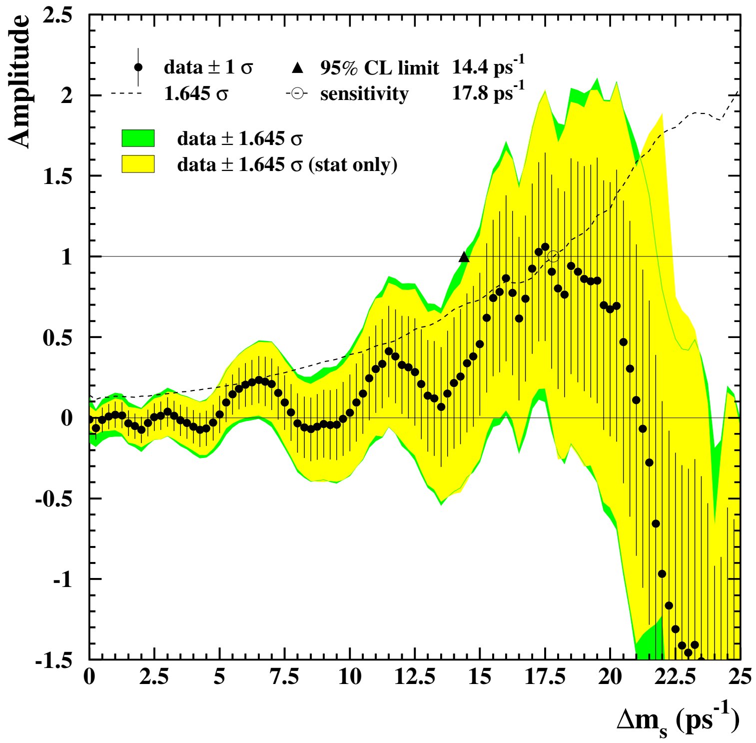

Figure 2: Combined measurements of the B_s^0 oscillation amplitude as a function of Δm_s, including all results published by November 2003. The measurements are dominated by statistical uncertainties. Neighboring points are statistically correlated. See full-color version on color pages at end of book.

However, no deviation from $\mathcal{A} = 0$ is seen in Fig. 2 that would indicate the observation of a signal.

Some Δm_s analyses are still unpublished [44,47]. Including these in the above combination would yield $\Delta m_s > 14.6$ ps^{-1} at 95% CL with a sensitivity of 19.3 ps^{-1}.

The information on $|V_{ts}|$ obtained, in the framework of the Standard Model, from the combined amplitude spectrum, is hampered by the hadronic uncertainty, as in the B_d^0 case. However, several uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}}\,\xi^2 \left|\frac{V_{ts}}{V_{td}}\right|^2 , \qquad (19)$$

where $\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}}) = 1.15 \pm 0.05^{+0.12}_{-0.00}$ is an SU(3) flavor-symmetry breaking factor obtained from lattice QCD calculations [40]. The CKM matrix can be constrained using the experimental results on Δm_d, Δm_s, $|V_{ub}/V_{cb}|$, ϵ_K, and $\sin(2\beta)$ together with theoretical inputs and unitarity conditions [39,48]. Given all measurements other than Δm_d and Δm_s, the constraint from our knowledge on the ratio $\Delta m_s/\Delta m_d$ is presently more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the Δm_d measurements alone, due to the reduced hadronic uncertainty in Eq. (19). We note also that it would be difficult for the Standard Model to accommodate values of Δm_s above $\sim 25\ \mathrm{ps}^{-1}$ [48].

Information on $\Delta\Gamma_s$ can be obtained by studying the proper time distribution of untagged data samples enriched in B_s^0 mesons [49]. In the case of an inclusive B_s^0 selection [50], or a semileptonic B_s^0 decay selection [16,51], both the short- and long-lived components are present, and the proper time distribution is a superposition of two exponentials with decay constants $\Gamma_s \pm \Delta\Gamma_s/2$. In principle, this provides sensitivity to both Γ_s and $(\Delta\Gamma_s/\Gamma_s)^2$. Ignoring $\Delta\Gamma_s$ and fitting for a single exponential leads to an estimate of Γ_s with a relative bias proportional to $(\Delta\Gamma_s/\Gamma_s)^2$. An alternative approach, which is directly sensitive to first order in $\Delta\Gamma_s/\Gamma_s$, is to determine the lifetime of B_s^0 candidates decaying to CP eigenstates; measurements exist for $B_s^0 \to J/\psi\phi$ [52] and $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ [53], which are mostly CP-even states [54]. An estimate of $\Delta\Gamma_s/\Gamma_s$ has also been obtained directly from a measurement of the $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ branching ratio [53], under the assumption that these decays account for all the CP-even final states (however, no systematic uncertainty due to this assumption is given, so the average quoted below will not include this estimate).

Present data is not precise enough to efficiently constrain both Γ_s and $\Delta\Gamma_s/\Gamma_s$; since the B_s^0 and B_d^0 lifetimes are predicted to be equal within less than a percent [55], an expectation compatible with the current experimental data [56], the constraint $\Gamma_s = \Gamma_d$ can also be used to improve the extraction of $\Delta\Gamma_s/\Gamma_s$. Applying the combination procedure of Ref. 26 on the published results [16,51–53,57], yields

$$|\Delta\Gamma_s|/\Gamma_s < 0.54 \qquad \text{at } 95\%\ \text{CL} \tag{20}$$

without external constraint, or

$$|\Delta\Gamma_s|/\Gamma_s < 0.29 \quad \text{at } 95\% \text{ CL} \tag{21}$$

when constraining $1/\Gamma_s$ to the measured B_d^0 lifetime. These results are not yet precise enough to test Standard Model predictions.

Average b-hadron mixing and b-hadron production fractions at high energy

Let f_u, f_d, f_s and f_{baryon} be the fractions of B_u, B_d^0, B_s^0 and b-baryon composing an unbiased sample of weakly decaying b hadrons produced in high-energy colliders. LEP experiments have measured $f_s \times \text{BR}(B_s^0 \to D_s^- \ell^+ \nu_\ell X)$ [58], $\text{BR}(b \to \Lambda_b^0) \times \text{BR}(\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu}_\ell X)$ [59], and $\text{BR}(b \to \Xi_b^-) \times \text{BR}(\Xi_b^- \to \Xi^- \ell^- \overline{\nu}_\ell X)$ [60] from partially reconstructed final states, including a lepton, f_{baryon} from protons identified in b events [61], and the production rate of charged b hadrons [62]. The various b-hadron fractions have also been measured at CDF from electron-charm final states [63]. All these published results have been combined following the procedure and assumptions described in Ref. 26, to yield $f_u = f_d = (40.3 \pm 1.1)\%$, $f_s = (8.8 \pm 2.1)\%$, and $f_{\text{baryon}} = (10.7 \pm 1.8)\%$ under the constraints

$$f_u = f_d \qquad \text{and} \qquad f_u + f_d + f_s + f_{\text{baryon}} = 1\,. \tag{22}$$

Time-integrated mixing analyses performed with lepton pairs from $b\overline{b}$ events produced at high-energy colliders measure the quantity

$$\overline{\chi} = f'_d\,\chi_d + f'_s\,\chi_s\,, \tag{23}$$

where f'_d and f'_s are the fractions of B_d^0 and B_s^0 hadrons in a sample of semileptonic b-hadron decays. Assuming that all b hadrons have the same semileptonic decay width implies $f'_q = f_q/(\Gamma_q \tau_b)$ $(q = s, d)$, where τ_b is the average b-hadron lifetime. Hence $\overline{\chi}$ measurements can be used to improve our knowledge on the fractions f_u, f_d, f_s and f_{baryon}.

Combining the above estimates of these fractions with the average $\overline{\chi} = 0.1257 \pm 0.0042$ (published in this *Review*), χ_d from

Eq. (15), and $\chi_s = 1/2$ yields, under the constraints of Eq. (22),

$$f_u = f_d = (39.7 \pm 1.0)\%\,, \tag{24}$$

$$f_s = (10.7 \pm 1.1)\%\,, \tag{25}$$

$$f_{\rm baryon} = (9.9 \pm 1.7)\%\,, \tag{26}$$

showing that mixing information substantially reduces the uncertainty on f_s. These results and the averages quoted in Eqs. (14) and (15) for χ_d and Δm_d have been obtained in a consistent way by the B oscillations working group [26], taking into account the fact that many individual measurements of Δm_d depend on the assumed values for the b-hadron fractions.

Summary and prospects

B^0–$\overline{B}^0$ mixing has been and still is a field of intense study. The mass difference in the B^0_d–$\overline{B}^0_d$ system is very well measured (with an accuracy of 1.3%) but, despite an impressive theoretical effort, the hadronic uncertainty still limits the precision of the extracted estimate of $|V_{td}|$. The mass difference in the B^0_s–$\overline{B}^0_s$ system is much larger and still unmeasured. However, the current experimental lower limit on Δm_s already provides, together with Δm_d, a significant constraint on the CKM matrix within the Standard Model. No strong experimental evidence exists yet for the rather large decay width difference expected in the B^0_s–$\overline{B}^0_s$ system. It is interesting to recall that the ratio $\Delta\Gamma_s/\Delta m_s$ does not depend on CKM matrix elements in the Standard Model (see Eq. (9)), and that a measurement of either Δm_s or $\Delta\Gamma_s$ could be turned into a Standard Model prediction of the other one.

In the near future, the most promising prospects for B^0_s mixing are from Run II at the Tevatron, where both Δm_s and $\Delta\Gamma_s$ are expected to be measured with fully reconstructed B^0_s decays. The CDF and D0 collaborations expect to be able to observe B^0_s oscillations with $2-3$ fb^{-1} of data, if Δm_s is consistent with the current Standard Model prediction [18]. Should this not be the case, then the discovery of B^0_s oscillations will most likely be made at CERN's Large Hadron Collider scheduled to come into operation in 2007, where the LHC collaboration claims to have the potential to cover a Δm_s

range up to $\sim 68\ \mathrm{ps}^{-1}$ after 2 fb^{-1} of data (10^7 s) have been analyzed [64]. The BTeV experiment at Fermilab should have a comparable sensitivity to Δm_s and is expected to turn on in 2009 [65].

CP violation in B mixing, which has not been seen yet, as well as the phases involved in B mixing, will be further investigated with the large statistics that will become available at the B factories, the Tevatron, and the LHC.

B mixing may not have delivered all its secrets yet, because it is one of the phenomena where new physics might very well reveal itself (for example, new particles involved in the box diagrams). Theoretical calculations in lattice QCD are becoming more reliable, and further progress in reducing hadronic uncertainties is expected. In the long term, a stringent check of the consistency, within the Standard Model, of the B^0_d and B^0_s mixing measurements, with all other measured observables in B physics (including CP asymmetries in B decays), will be possible, allowing to place limits on new physics or, better, discover new physics.

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