$K^{\pm}$ $I(J^P) = \frac{1}{2}(0^-)$

## THE CHARGED KAON MASS

Revised 1994 by T.G. Trippe (LBNL).

The average of the six charged kaon mass measurements which we use in the Particle Listings is

$$m_{K^\pm} = 493.677 \pm 0.013 \text{ MeV (S = 2.4)} , \quad (1)$$

where the error has been increased by the scale factor S. The large scale factor indicates a serious disagreement between different input data. The average before scaling the error is

$$m_{K^\pm} = 493.677 \pm 0.005 \text{ MeV} ,$$
$$\chi^2 = 22.9 \text{ for 5 D.F., Prob.} = 0.04\% , \quad (2)$$

where the high $\chi^2$ and correspondingly low $\chi^2$ probability further quantify the disagreement.

The main disagreement is between the two most recent and precise results,

| | | |
|---|---|---|
| $m_{K^\pm}$ = | $493.696 \pm 0.007$ MeV | DENISOV 91 |
| $m_{K^\pm}$ = | $493.636 \pm 0.011$ MeV (S = 1.5) | GALL 88 |
| Average = | $493.679 \pm 0.006$ MeV | |

$$\chi^2 = 21.2 \text{ for 1 D.F., Prob.} = 0.0004\% , \quad (3)$$

both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall average in Eq. (2), it is clear that DENISOV 91 and GALL 88 dominate the overall average, and that their disagreement is responsible for most of the high $\chi^2$.

The GALL 88 measurement was made using four different kaonic atom transitions, $K^-$ Pb $(9 \to 8)$, $K^-$ Pb $(11 \to 10)$, $K^-$ W $(9 \to 8)$, and $K^-$ W $(11 \to 10)$. The $m_{K^\pm}$ values they obtain from each of these transitions is shown in the Particle Listings and in Fig. 1. Their $K^-$ Pb $(9 \to 8)$ $m_{K^\pm}$is below and somewhat inconsistent with their other three transitions. The average of their four measurements is

$$m_{K^\pm} = 493.636 \pm 0.007 \ ,$$
$$\chi^2 = 7.0 \text{ for 3 D.F., Prob. } = 7.2\% \ . \qquad (4)$$

This is a low but acceptable $\chi^2$ probability so, to be conservative, GALL 88 scaled up the error on their average by S=1.5 to obtain their published error $\pm 0.011$ shown in Eq. (3) above and used in the Particle Listings average.

The ideogram in Fig. 1 shows that the DENISOV 91 measurement and the GALL 88 $K^-$ Pb $(9 \to 8)$ measurement yield two well-separated peaks. One might suspect the GALL 88 $K^-$ Pb $(9 \to 8)$ measurement since it is responsible both for the internal inconsistency in the GALL 88 measurements and the disagreement with DENISOV 91.

To see if the disagreement could result from a systematic problem with the $K^-$ Pb $(9 \to 8)$ transition, we have separated the CHENG 75 data, which also used $K^-$ Pb, into its separate transitions. Figure 1 shows that the CHENG 75 and GALL 88 $K^-$ Pb $(9 \to 8)$ values are consistent, suggesting the possibility of a common effect such as contaminant nuclear $\gamma$ rays near the $K^-$ Pb $(9 \to 8)$ transition energy, although the CHENG 75 errors are too large to make a strong conclusion. The average of all 13 measurements has a $\chi^2$ of 52.6 as shown in Fig. 1 and the first line of Table 1, yielding an unacceptable $\chi^2$ probability of 0.00005%. The second line of Table 1 excludes

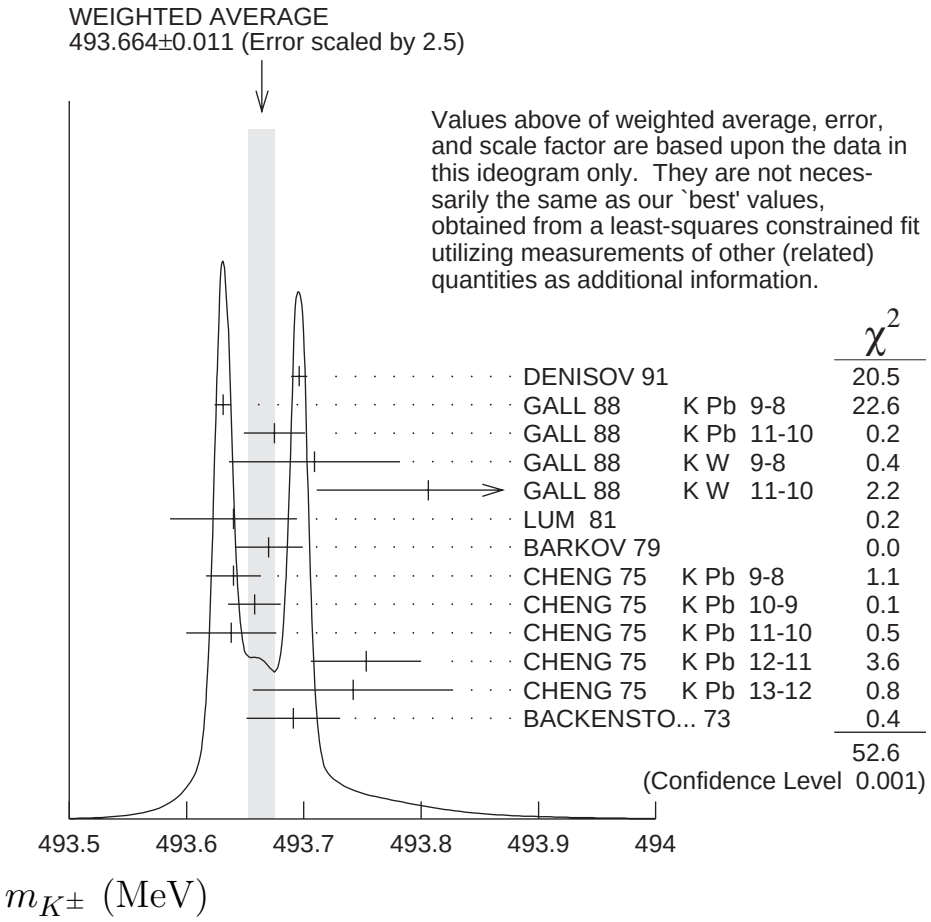



**Figure 1:** Ideogram of $m_{K^\pm}$ mass measurements. GALL 88 and CHENG 75 measurements are shown separately for each transition they measured.

both the GALL 88 and CHENG 75 measurements of the $K^-$ Pb $(9 \to 8)$ transition and yields a $\chi^2$ probability of 43%. The third [fourth] line of Table 1 excludes only the GALL 88 $K^-$ Pb $(9 \to 8)$ [DENISOV 91] measurement and yields a $\chi^2$ probability of 20% [8.6%]. Table 1 shows that removing both measurements of the $K^-$ Pb $(9 \to 8)$ transition produces the most consistent set of data, but that excluding only the

GALL 88 $K^-$ Pb $(9 \to 8)$ transition or DENISOV 91 also produces acceptable probabilities.

**Table 1:** $m_{K^\pm}$ averages for some combinations of Fig. 1 data.

| $m_{K^\pm}$ (MeV) | $\chi^2$ | D.F. | Prob. (%) | Measurements used |
|---|---|---|---|---|
| $493.664 \pm 0.004$ | 52.6 | 12 | 0.00005 | all 13 measurements |
| $493.690 \pm 0.006$ | 10.1 | 10 | 43 | no $K^-$ Pb$(9\to8)$ |
| $493.687 \pm 0.006$ | 14.6 | 11 | 20 | no GALL 88 $K^-$ Pb$(9\to8)$ |
| $493.642 \pm 0.006$ | 17.8 | 11 | 8.6 | no DENISOV 91 |

Yu.M. Ivanov, representing DENISOV 91, has estimated corrections needed for the older experiments because of improved $^{192}$Ir and $^{198}$Au calibration $\gamma$-ray energies. He estimates that CHENG 75 and BACKENSTOSS 73 $m_{K^\pm}$ values could be raised by about 15 keV and 22 keV, respectively. With these estimated corrections, Table 1 becomes Table 2. The last line of Table 2 shows that if such corrections are assumed, then GALL 88 $K^-$ Pb $(9 \to 8)$ is inconsistent with the rest of the data even when DENISOV 91 is excluded. Yu.M. Ivanov warns that these are rough estimates. Accordingly, we do not use Table 2 to reject the GALL 88 $K^-$ Pb $(9 \to 8)$ transition, but we note that a future reanalysis of the CHENG 75 data could be useful because it might provide supporting evidence for such a rejection.

The GALL 88 measurement uses a Ge semiconductor spectrometer which has a resolution of about 1 keV, so they run the risk of some contaminant nuclear $\gamma$ rays. Studies of $\gamma$ rays following stopped $\pi^-$ and $\Sigma^-$ absorption in nucleii (unpublished) do not show any evidence for contaminants according to GALL 88 spokesperson, B.L. Roberts. The DENISOV 91

**Table 2:** $m_{K^\pm}$ averages for some combinations of Fig. 1 data after raising CHENG 75 and BACKENSTOSS 73 values by 0.015 and 0.022 MeV respectively.

| $m_{K^\pm}$ (MeV) | $\chi^2$ | D.F. | Prob. (%) | Measurements used |
|---|---|---|---|---|
| $493.666 \pm 0.004$ | 53.9 | 12 | 0.00003 | all 13 measurements |
| $493.693 \pm 0.006$ | 9.0 | 10 | 53 | no $K^-$ Pb(9→8) |
| $493.690 \pm 0.006$ | 11.5 | 11 | 40 | no GALL 88 $K^-$ Pb(9→8) |
| $493.645 \pm 0.006$ | 23.0 | 11 | 1.8 | no DENISOV 91 |

measurement uses a crystal diffraction spectrometer with a resolution of 6.3 eV for radiation at 22.1 keV to measure the 4f-3d transition in $K^-$ $^{12}$C. The high resolution and the light nucleus reduce the probability for overlap by contaminant $\gamma$ rays, compared with the measurement of GALL 88. The DENISOV 91 measurement is supported by their high-precision measurement of the 4d-2p transition energy in $\pi^-$ $^{12}$C, which is good agreement with the calculated energy.

While we suspect that the GALL 88 $K^-$ Pb $(9 \rightarrow 8)$ measurements could be the problem, we are unable to find clear grounds for rejecting it. Therefore, we retain their measurement in the average and accept the large scale factor until further information can be obtained from new measurements and/or from reanalysis of GALL 88 and CHENG 75 data.

We thank B.L. Roberts (Boston Univ.) and Yu.M. Ivanov (Petersburg Nuclear Physics Inst.) for their extensive help in understanding this problem.

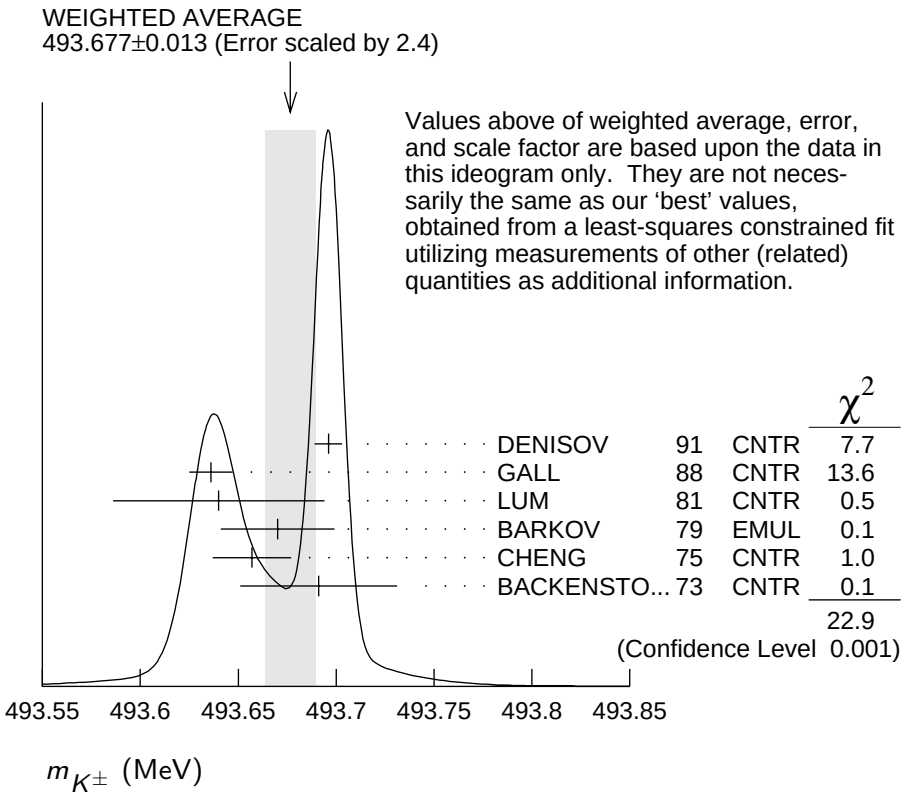
# $K^{\pm}$ MASS

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|
| **493.677±0.016 OUR FIT** Error includes scale factor of 2.8. | | | | | |
| **493.677±0.013 OUR AVERAGE** Error includes scale factor of 2.4. See the ideogram below. | | | | | |
| 493.696±0.007 | [1] DENISOV | 91 | CNTR | – | Kaonic atoms |
| 493.636±0.011 | [2] GALL | 88 | CNTR | – | Kaonic atoms |
| 493.640±0.054 | LUM | 81 | CNTR | – | Kaonic atoms |
| 493.670±0.029 | BARKOV | 79 | EMUL | ± | $e^+e^- \to K^+K^-$ |
| 493.657±0.020 | [2] CHENG | 75 | CNTR | – | Kaonic atoms |
| 493.691±0.040 | BACKENSTO...73 | | CNTR | – | Kaonic atoms |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 493.631±0.007 | GALL | 88 | CNTR | – | $K^-$ Pb (9→ 8) |
| 493.675±0.026 | GALL | 88 | CNTR | – | $K^-$ Pb (11→ 10) |
| 493.709±0.073 | GALL | 88 | CNTR | – | $K^-$ W (9→ 8) |
| 493.806±0.095 | GALL | 88 | CNTR | – | $K^-$ W (11→ 10) |
| 493.640±0.022±0.008 | [3] CHENG | 75 | CNTR | – | $K^-$ Pb (9→ 8) |
| 493.658±0.019±0.012 | [3] CHENG | 75 | CNTR | – | $K^-$ Pb (10→ 9) |
| 493.638±0.035±0.016 | [3] CHENG | 75 | CNTR | – | $K^-$ Pb (11→ 10) |
| 493.753±0.042±0.021 | [3] CHENG | 75 | CNTR | – | $K^-$ Pb (12→ 11) |
| 493.742±0.081±0.027 | [3] CHENG | 75 | CNTR | – | $K^-$ Pb (13→ 12) |

[1] Error increased from 0.0059 based on the error analysis in IVANOV 92.

[2] This value is the authors' combination of all of the separate transitions listed for this paper.

[3] The CHENG 75 values for separate transitions were calculated from their Table 7 transition energies. The first error includes a 20% systematic error in the noncircular contaminant shift. The second error is due to a ±5 eV uncertainty in the theoretical transition energies.




## $m_{K^+} - m_{K^-}$

Test of *CPT*.

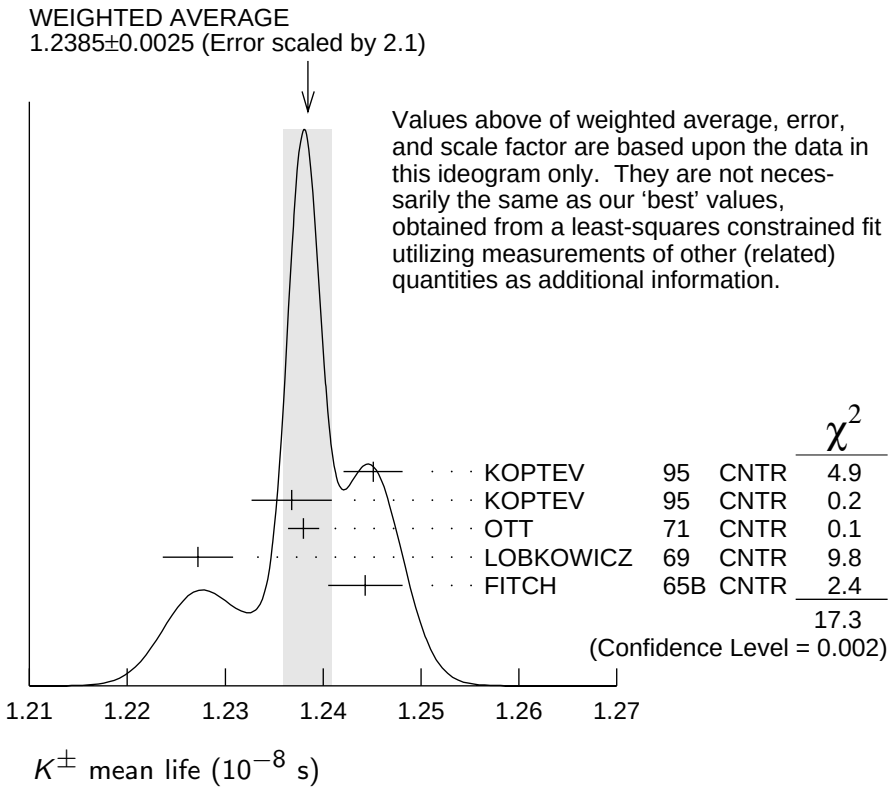
| *VALUE* (MeV) | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* |
|---|---|---|---|---|---|
| **−0.032±0.090** | 1.5M | [4] FORD | 72 | ASPK | ± |

[4] FORD 72 uses $m_{\pi^+} - m_{\pi^-} = +28 \pm 70$ keV.

## $K^\pm$ MEAN LIFE

| *VALUE* ($10^{-8}$ s) | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|
| **1.2384±0.0024 OUR FIT** Error includes scale factor of 2.0. | | | | | | |
| **1.2385±0.0025 OUR AVERAGE** Error includes scale factor of 2.1. See the ideogram below. | | | | | | |
| 1.2451±0.0030 | 250k | KOPTEV | 95 | CNTR | | *K* at rest, U target |
| 1.2368±0.0041 | 150k | KOPTEV | 95 | CNTR | | *K* at rest, Cu target |
| 1.2380±0.0016 | 3M | OTT | 71 | CNTR | + | *K* at rest |
| 1.2272±0.0036 | | LOBKOWICZ | 69 | CNTR | + | *K* in flight |
| 1.2443±0.0038 | | FITCH | 65B | CNTR | + | *K* at rest |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 1.2415±0.0024 | 400k | [5] KOPTEV | 95 | CNTR | | *K* at rest |
| 1.221 ±0.011 | | FORD | 67 | CNTR | ± | |
| 1.231 ±0.011 | | BOYARSKI | 62 | CNTR | + | |

[5] KOPTEV 95 report this weighted average of their U-target and Cu-target results, where they have weighted by $1/\sigma$ rather than $1/\sigma^2$.

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$(\tau_{K^+} - \tau_{K^-}) / \tau_{\text{average}}$

This quantity is a measure of *CPT* invariance in weak interactions.

| *VALUE* (%) | *DOCUMENT ID* | | *TECN* |
|---|---|---|---|
| **0.11 ±0.09 OUR AVERAGE** | Error includes scale factor of 1.2. | | |
| 0.090±0.078 | LOBKOWICZ | 69 | CNTR |
| 0.47 ±0.30 | FORD | 67 | CNTR |

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## RARE KAON DECAYS

(Revised November 2003 by L. Littenberg, BNL and G. Valencia, Iowa State University)

***A. Introduction***: There are several useful reviews on rare kaon decays and related topics [1–14]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters

3. Searches for $CP$ violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focused on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+ \ell^+ \ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0 \gamma\gamma$ and $K_L \to \ell^+ \ell^- \gamma$. The former is important in understanding a $CP$-conserving contribution to $K_L \to \pi^0 \ell^+ \ell^-$, whereas the latter could shed light on long distance contributions to $K_L \to \mu^+ \mu^-$.

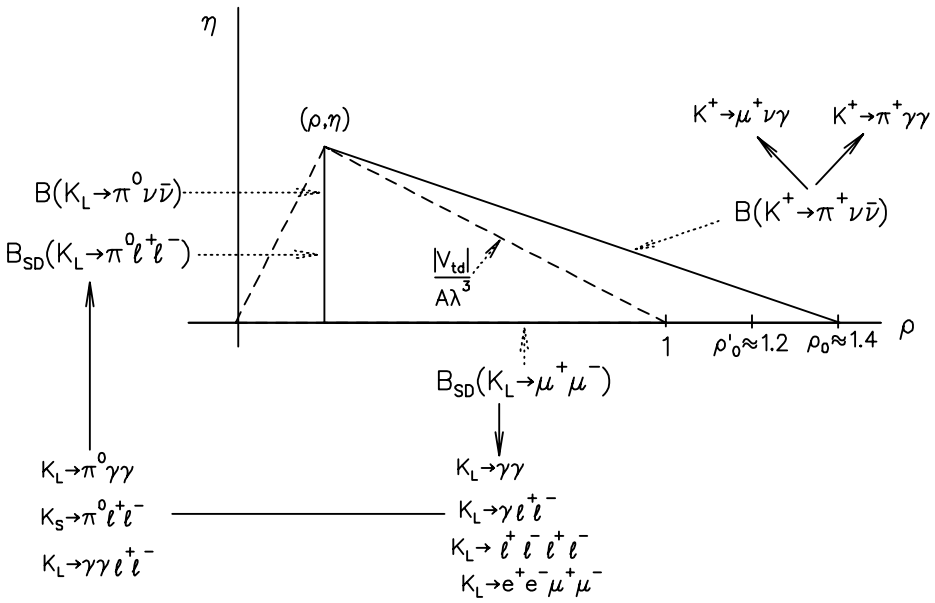


**Figure 1:** Role of rare kaon decays in determining the unitarity triangle. The solid arrows point to auxiliary modes needed to interpret the main results.

The interplay between Categories 2-4 can be illustrated in Fig. 1. The modes $K \to \pi\nu\overline{\nu}$ are the cleanest ones theoretically. They can provide accurate determinations of certain CKM parameters (shown in the figure). In combination with alternate determinations of these parameters they also constrain new interactions. The modes $K_L \to \pi^0 e^+ e^-$ and $K_L \to \mu^+\mu^-$ are also sensitive to CKM parameters. However, they suffer from a series of hadronic uncertainties that can be addressed, at least in part, through a systematic study of the additional modes indicated in the figure.

***B. Explicit violations of the Standard Model***: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass $M_X$ that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12}(148\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings. Limits on certain lepton-number violating kaon decays [15,16] also exist. Related searches in $\mu$ and $\tau$ processes are discussed in our section "Tests of Conservation Laws".

**Table 1:** Searches for lepton flavor violation in $K$ decay

| Mode | 90% CL upper limit | Exp't | Yr./Ref. |
|---|---|---|---|
| $K^+ \to \pi^+ e^- \mu^+$ | $1.2\times10^{-11}$ | BNL-865 (prelim.) | 2003/Ref. 17 |
| $K^+ \to \pi^+ e^+ \mu^-$ | $5.2\times10^{-10}$ | BNL-865 | 2001/Ref. 15 |
| $K_L \to \mu e$ | $4.7\times10^{-12}$ | BNL-871 | 1998/Ref. 18 |
| $K_L \to \pi^0 e\mu$ | $3.4\times10^{-10}$ | KTeV (prelim.) | 2003/Ref. 19 |

Physics beyond the SM is also pursued through the search for $K^+ \to \pi^+ X^0$, where $X^0$ is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, *etc.*). The 90% CL upper limit on this process is $5.9 \times 10^{-11}$ [20].

***C. Measurements of Standard Model parameters***: Until 1997, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ were motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [21] and long-distance contributions are known to be quite small [2,22]. Since then, BNL-787 has observed two candidate events [20,23], yielding a branching ratio of $(1.57^{+1.75}_{-0.82}) \times 10^{-10}$ [20]. At this level, this reaction becomes interesting from the point of view of constraining SM parameters. An upgrade to the experiment to collect roughly an order of magnitude more sensitivity is in progress [24]. A new experiment with a sensitivity goal of $\sim 10^{-12}$/event was given scientific approval at FNAL [25] in 2001, but as of the end of 2003 its outlook is less certain. In the future this mode may provide grounds for precision tests of the flavor structure of the Standard Model [26]. The branching

ratio can be written in terms of the very well-measured $K_{e3}$ rate as [2]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [27]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [28], and $X_{NL}^{\ell}$ is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which mainly affect $X_{NL}^{\ell}$, are known at next-to-leading order [12,29] and lead to a residual error of $< 10\%$ for the decay amplitude. Evaluating the constants in Eq. (1), one can cast this result in terms of the CKM parameters $A$, $\rho$ and $\eta$ (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [12]

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.0 \times 10^{-10} A^4 [\eta^2 + (\rho_o - \rho)^2] \qquad (2)$$

where $\rho_o \equiv 1 + (\frac{2}{3} X_{NL}^e + \frac{1}{3} X_{NL}^\tau)/(A^2 V_{us}^4 X(m_t)) \approx 1.4$. Thus, $\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the $\rho$, $\eta$ plane with center $(\rho_o, 0)$ and radius $\approx \frac{1}{A^2}\sqrt{\frac{\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})}{1.0\times 10^{-10}}}$. Current constraints on the CKM parameters lead to a predicted branching ratio $(7.2 \pm 2.1) \times 10^{-11}$ [30], near the lower end of the BNL-787 measurement.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter $\rho$, given by [12]:

$$\mathrm{B}_{\mathrm{SD}}(K_L \to \mu^+\mu^-) \approx 1.6 \times 10^{-9} A^4 (\rho_o' - \rho)^2 \qquad (3)$$

where $\rho'_o$ depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is determined by the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B}_{\mathrm{abs}}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$ [31]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain $\rho$ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present it is not possible to compute this long-distance component reliably, and therefore it is not possible to constrain $\rho$ from this mode in a model independent way [32]. Several hadronic models exist to estimate this long-distance component [33,34] and are commonly used to place rough bounds on new physics contributions to the $K_L \to \mu^+\mu^-$ rate [35,36]. The decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [32,34], is in good agreement with the BNL-871 measurement, $(8.7^{+5.7}_{-4.1}) \times 10^{-12}$ [37].

***D. Searches for direct CP violation***: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly $CP$-violating and free of hadronic uncertainties [2,38,39]. In the Standard Model this mode is dominated by an intermediate top-quark state and does not suffer from the small uncertainty associated with the charm-quark intermediate

state that affects the mode $K^+ \to \pi^+ \nu\overline{\nu}$. The branching ratio is given approximately by Ref. 12:

$$\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \approx 4.1 \times 10^{-10} A^4 \eta^2 \ . \tag{4}$$

With current constraints on the CKM parameters this leads to a predicted branching ratio $(2.8 \pm 1.0) \times 10^{-11}$ [30]. The current experimental upper bound is $\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \leq 5.9 \times 10^{-7}$ [40]. The 90% CL bound on $K^+ \to \pi^+ \nu\bar{\nu}$ provides a nearly model independent bound $B(K_L \to \pi^0 \nu\bar{\nu}) < 1.7 \times 10^{-9}$ [41]. A KEK experiment to reach the $3 \times 10^{-10}$/event level [42] is scheduled to begin data-taking in early 2004. The KOPIO [43] experiment aims to reach the $6 \times 10^{-13}$/event level for $K_L \to \pi^0 \nu\overline{\nu}$ at the BNL AGS. A Letter of Intent for an experiment of similar sensitivity has been submitted to the J-PARC PAC [44].

There has been much theoretical work on possible contributions to $\epsilon'/\epsilon$ and rare $K$ decays in supersymmetric extensions of the SM. While in the simplest case of the MSSM with no new sources of flavor or $CP$ violation the main effect is a suppression of the rare $K$ decays [45], substantial enhancements are possible in more general SUSY models [35,46]. Other recent predictions of the possible effects of new physics on rare kaon decays include those of large extra dimensions [47] and of effective theories with minimal flavor violation [36].

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the CKM parameter $\eta$ through its $CP$-violating component. There are both direct and indirect $CP$-violating amplitudes which can interfere. The direct $CP$-violating amplitude is short distance dominated and has been calculated in detail within the SM [9]. The indirect $CP$-violating amplitude can be inferred from

a measurement of $K_S \to \pi^0 e^+ e^-$. The complete $CP$-violating contribution to the rate can be written as [50]:

$$\mathrm{B_{CPV}} \approx \left(15.3 a_s^2 \pm 35 A^2 \eta |a_s| + 74.4 A^4 \eta^2\right) \times 10^{-12}. \qquad (5)$$

The parameter $a_s$ has recently been extracted by NA48 from a measurement of the decay $K_S \to \pi^0 e^+ e^-$ with the result $|a_s| = 1.06^{+0.26}_{-0.21} \pm 0.07$ [51]. With current constraints on the CKM parameters this implies that

$$\mathrm{B_{CPV}} \approx (17.2 \pm 9.4 + 4.7) \times 10^{-12}. \qquad (6)$$

The indirect $CP$ violation is larger than the direct $CP$ violation. While the sign of the interference is *a priori* unknown, arguments in favor of a positive sign have been put forward in Ref. 52.

This mode also has a $CP$-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0 \gamma\gamma$. To understand the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0 \gamma\gamma$ within chiral perturbation theory it is necessary to go beyond leading order, and this introduces three (a priori) unknown parameters [53]. Both KTeV and NA48 analyze their $K_L \to \pi^0 \gamma\gamma$ data assuming vector meson dominance and extract from this analysis a bound on the $CP$-conserving rate in $K_L \to \pi^0 e^+ e^-$ using the model of Ref. [54]. The two experiments report conflicting results. The more recent NA48 data finds a negligible rate in the low $m_{\gamma\gamma}$ region suggesting a very small $CP$-conserving component in $K_L \to \pi^0 e^+ e^-$, $B_{\mathrm{CP}}(K_L \to \pi^0 e^+ e^-) = (0.47^{+0.22}_{-0.18}) \times 10^{-12}$ [55]. On the other hand KTeV reported a larger fraction of $K_L \to \pi^0 \gamma\gamma$ in the low $m_{\gamma\gamma}$ region, favoring a larger $CP$-conserving rate

$B_{\rm CP}(K_L \to \pi^0 e^+ e^-)$ between $1 - 2 \times 10^{-12}$ [56]. In addition to this difference between the two experiments, there remain two other sources of uncertainty in the prediction for the $CP$-conserving rate in $K_L \to \pi^0 e^+ e^-$. The first one arises from the way the $K_L \to \pi^0 \gamma\gamma$ amplitude is parameterized [57] assuming vector meson dominance, and the second one reflects the model dependence of the form factor connecting the two modes [52]. For the $K_L \to \pi^0 \gamma\gamma$ rates, KTeV finds $B(K_L \to \pi^0 \gamma\gamma) = (1.68 \pm 0.07_{\rm stat} \pm 0.08_{\rm sys}) \times 10^{-6}$ [56], whereas NA48 finds $B(K_L \to \pi^0 \gamma\gamma) = (1.36 \pm 0.03_{\rm stat} \pm 0.03_{\rm sys} \pm 0.03_{\rm norm}) \times 10^{-6}$ [55].

The related process, $K_L \to \pi^0 \gamma e^+ e^-$, is potentially an additional background in some region of phase space [58]. This process has been observed with a branching ratio of $(2.34 \pm 0.35_{\rm stat} \pm 0.13_{\rm sys}) \times 10^{-8}$ [59].

The decay $K_L \to \gamma\gamma e^+ e^-$ constitutes the dominant background to $K_L \to \pi^0 e^+ e^-$. It was first observed by BNL-845 [60] and subsequently confirmed with a much larger sample by FNAL-799 [61]. It has been estimated that this background will enter at the level of $3 \times 10^{-10}$ [62,63], comparable to or larger than the signal level. Because of this, the observation of $K_L \to \pi^0 e^+ e^-$ will depend on background subtraction with good statistics. Possible alternative strategies are discussed in Ref. 52 and references cited therein.

The published 90% CL upper bound for the process $K_L \to \pi^0 e^+ e^-$ is $5.1 \times 10^{-10}$ [63]. There is now a preliminary 90% CL upper bound of $2.8 \times 10^{-10}$ [64]. For the closely related muonic process, the published upper bound is $\mathrm{B}(K_L \to \pi^0 \mu^+ \mu^-) \leq 3.8 \times 10^{-10}$ [65]. KTeV has additional data corresponding to about a factor 1.3 in sensitivity for the latter reaction that is still to be analyzed.

A recent study of $K_L \to \pi^0\mu^+\mu^-$ has indicated that it might be possible to extract the direct $CP$-violating contribution by a joint study of the Dalitz plot variables and the components of the $\mu^+$ polarization [66]. The latter tends to be quite substantial so that large statistics may not be necessary.

***E. Other long distance dominated modes***:

The decays $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e$ or $\mu$) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [67,68]. Ref. 50 has proposed a parameterization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections.

Much information has been recorded by KTeV and NA48 on the rates and spectrum for the Dalitz pair conversion modes $K_L \to \ell^+\ell^-\gamma$ [69], and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or $\mu$ [16,70–72]. All these results are used to test hadronic models and could further our understanding of the long distance component in $K_L \to \mu^+\mu^-$.

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## $K^+$ DECAY MODES

$K^-$ modes are charge conjugates of the modes below.

| | Mode | Fraction ($\Gamma_i/\Gamma$) | Scale factor/ Confidence level |
|---|---|---|---|
| | **Leptonic and semileptonic modes** | | |
| $\Gamma_1$ | $e^+\nu_e$ | ( 1.55 ±0.07 ) × $10^{-5}$ | |
| $\Gamma_2$ | $\mu^+\nu_\mu$ | (63.43 ±0.17 ) % | S=1.2 |
| $\Gamma_3$ | $\pi^0 e^+\nu_e$ | ( 4.87 ±0.06 ) % | S=1.2 |
| | Called $K^+_{e3}$. | | |
| $\Gamma_4$ | $\pi^0\mu^+\nu_\mu$ | ( 3.27 ±0.06 ) % | S=1.2 |
| | Called $K^+_{\mu3}$. | | |
| $\Gamma_5$ | $\pi^0\pi^0 e^+\nu_e$ | ( 2.1 ±0.4 ) × $10^{-5}$ | |
| $\Gamma_6$ | $\pi^+\pi^- e^+\nu_e$ | ( 4.08 ±0.09 ) × $10^{-5}$ | |
| $\Gamma_7$ | $\pi^+\pi^-\mu^+\nu_\mu$ | ( 1.4 ±0.9 ) × $10^{-5}$ | |
| $\Gamma_8$ | $\pi^0\pi^0\pi^0 e^+\nu_e$ | < 3.5 × $10^{-6}$ | CL=90% |
| | **Hadronic modes** | | |
| $\Gamma_9$ | $\pi^+\pi^0$ | (21.13 ±0.14 ) % | S=1.1 |
| $\Gamma_{10}$ | $\pi^+\pi^0\pi^0$ | ( 1.73 ±0.04 ) % | S=1.2 |
| $\Gamma_{11}$ | $\pi^+\pi^+\pi^-$ | ( 5.576±0.031) % | S=1.1 |

## Leptonic and semileptonic modes with photons

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Gamma_{12}$ | $\mu^+\nu_\mu\gamma$ | [a,b] | ( 5.50 | $\pm 0.28$ ) | $\times 10^{-3}$ | |
| $\Gamma_{13}$ | $\pi^0 e^+\nu_e\gamma$ | [a,b] | ( 2.65 | $\pm 0.20$ ) | $\times 10^{-4}$ | |
| $\Gamma_{14}$ | $\pi^0 e^+\nu_e\gamma$(SD) | [c] | $<$ 5.3 | | $\times 10^{-5}$ | CL=90% |
| $\Gamma_{15}$ | $\pi^0\mu^+\nu_\mu\gamma$ | [a,b] | $<$ 6.1 | | $\times 10^{-5}$ | CL=90% |
| $\Gamma_{16}$ | $\pi^0\pi^0 e^+\nu_e\gamma$ | | $<$ 5 | | $\times 10^{-6}$ | CL=90% |

## Hadronic modes with photons

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Gamma_{17}$ | $\pi^+\pi^0\gamma$ | [a,b] | ( 2.75 | $\pm 0.15$ ) | $\times 10^{-4}$ | |
| $\Gamma_{18}$ | $\pi^+\pi^0\gamma$(DE) | [b,d] | ( 4.4 | $\pm 0.8$ ) | $\times 10^{-6}$ | |
| $\Gamma_{19}$ | $\pi^+\pi^0\pi^0\gamma$ | [a,b] | ( 7.4 | $^{+5.5}_{-2.9}$ ) | $\times 10^{-6}$ | |
| $\Gamma_{20}$ | $\pi^+\pi^+\pi^-\gamma$ | [a,b] | ( 1.04 | $\pm 0.31$ ) | $\times 10^{-4}$ | |
| $\Gamma_{21}$ | $\pi^+\gamma\gamma$ | [b] | ( 1.10 | $\pm 0.32$ ) | $\times 10^{-6}$ | |
| $\Gamma_{22}$ | $\pi^+ 3\gamma$ | [b] | $<$ 1.0 | | $\times 10^{-4}$ | CL=90% |

## Leptonic modes with $\ell\overline{\ell}$ pairs

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Gamma_{23}$ | $e^+\nu_e\nu\overline{\nu}$ | | $<$ 6 | | $\times 10^{-5}$ | CL=90% |
| $\Gamma_{24}$ | $\mu^+\nu_\mu\nu\overline{\nu}$ | | $<$ 6.0 | | $\times 10^{-6}$ | CL=90% |
| $\Gamma_{25}$ | $e^+\nu_e e^+ e^-$ | | ( 2.48 | $\pm 0.20$ ) | $\times 10^{-8}$ | |
| $\Gamma_{26}$ | $\mu^+\nu_\mu e^+ e^-$ | | ( 7.06 | $\pm 0.31$ ) | $\times 10^{-8}$ | |
| $\Gamma_{27}$ | $e^+\nu_e\mu^+\mu^-$ | | $<$ 5 | | $\times 10^{-7}$ | CL=90% |
| $\Gamma_{28}$ | $\mu^+\nu_\mu\mu^+\mu^-$ | | $<$ 4.1 | | $\times 10^{-7}$ | CL=90% |

## Lepton Family number ($LF$), Lepton number ($L$), $\Delta S = \Delta Q$ ($SQ$) violating modes, or $\Delta S = 1$ weak neutral current ($S1$) modes

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Gamma_{29}$ | $\pi^+\pi^+ e^-\overline{\nu}_e$ | $SQ$ | $<$ 1.2 | | $\times 10^{-8}$ | CL=90% |
| $\Gamma_{30}$ | $\pi^+\pi^+\mu^-\overline{\nu}_\mu$ | $SQ$ | $<$ 3.0 | | $\times 10^{-6}$ | CL=95% |
| $\Gamma_{31}$ | $\pi^+ e^+ e^-$ | $S1$ | ( 2.88 | $\pm 0.13$ ) | $\times 10^{-7}$ | |
| $\Gamma_{32}$ | $\pi^+\mu^+\mu^-$ | $S1$ | ( 8.1 | $\pm 1.4$ ) | $\times 10^{-8}$ | S=2.7 |
| $\Gamma_{33}$ | $\pi^+\nu\overline{\nu}$ | $S1$ | ( 1.6 | $^{+1.8}_{-0.8}$ ) | $\times 10^{-10}$ | |
| $\Gamma_{34}$ | $\pi^+\pi^0\nu\overline{\nu}$ | $S1$ | $<$ 4.3 | | $\times 10^{-5}$ | CL=90% |
| $\Gamma_{35}$ | $\mu^-\nu e^+ e^+$ | $LF$ | $<$ 2.0 | | $\times 10^{-8}$ | CL=90% |
| $\Gamma_{36}$ | $\mu^+\nu_e$ | $LF$ | [e] $<$ 4 | | $\times 10^{-3}$ | CL=90% |
| $\Gamma_{37}$ | $\pi^+\mu^+ e^-$ | $LF$ | $<$ 2.8 | | $\times 10^{-11}$ | CL=90% |
| $\Gamma_{38}$ | $\pi^+\mu^- e^+$ | $LF$ | $<$ 5.2 | | $\times 10^{-10}$ | CL=90% |
| $\Gamma_{39}$ | $\pi^-\mu^+ e^+$ | $L$ | $<$ 5.0 | | $\times 10^{-10}$ | CL=90% |
| $\Gamma_{40}$ | $\pi^- e^+ e^+$ | $L$ | $<$ 6.4 | | $\times 10^{-10}$ | CL=90% |
| $\Gamma_{41}$ | $\pi^-\mu^+\mu^+$ | $L$ | [e] $<$ 3.0 | | $\times 10^{-9}$ | CL=90% |
| $\Gamma_{42}$ | $\mu^+\overline{\nu}_e$ | $L$ | [e] $<$ 3.3 | | $\times 10^{-3}$ | CL=90% |
| $\Gamma_{43}$ | $\pi^0 e^+\overline{\nu}_e$ | $L$ | $<$ 3 | | $\times 10^{-3}$ | CL=90% |
| $\Gamma_{44}$ | $\pi^+\gamma$ | | [f] $<$ 3.6 | | $\times 10^{-7}$ | CL=90% |

[*a*] Most of this radiative mode, the low-momentum $\gamma$ part, is also included in the parent mode listed without $\gamma$'s.

[*b*] See the Particle Listings below for the energy limits used in this measurement.

[*c*] Structure-dependent part.

[*d*] Direct-emission branching fraction.

[*e*] Derived from an analysis of neutrino-oscillation experiments.

[*f*] Violates angular-momentum conservation.

## CONSTRAINED FIT INFORMATION

An overall fit to the mean life, 2 decay rate, and 19 branching ratios uses 48 measurements and one constraint to determine 8 parameters. The overall fit has a $\chi^2 = 49.8$ for 41 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients $\left\langle \delta p_i \delta p_j \right\rangle / (\delta p_i \cdot \delta p_j)$, in percent, from the fit to parameters $p_i$, including the branching fractions, $x_i \equiv \Gamma_i/\Gamma_{\text{total}}$. The fit constrains the $x_i$ whose labels appear in this array to sum to one.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x_3$ | −51 | | | | | | |
| $x_4$ | −50 | 77 | | | | | |
| $x_5$ | −3 | 6 | 4 | | | | |
| $x_9$ | −67 | −17 | −16 | −1 | | | |
| $x_{10}$ | −23 | −1 | −1 | 0 | −2 | | |
| $x_{11}$ | −23 | 10 | 7 | 1 | −6 | 14 | |
| $\Gamma$ | 7 | −3 | −2 | 0 | 2 | −5 | −33 |
| | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_9$ | $x_{10}$ | $x_{11}$ |

| | Mode | Rate ($10^8$ s$^{-1}$) | Scale factor |
|---|---|---|---|
| $\Gamma_2$ | $\mu^+ \nu_\mu$ | 0.5122 ±0.0017 | 1.4 |
| $\Gamma_3$ | $\pi^0 e^+ \nu_e$ | 0.0393 ±0.0005 | 1.2 |
| | Called $K^+_{e3}$. | | |
| $\Gamma_4$ | $\pi^0 \mu^+ \nu_\mu$ | 0.0264 ±0.0005 | 1.2 |
| | Called $K^+_{\mu 3}$. | | |
| $\Gamma_5$ | $\pi^0 \pi^0 e^+ \nu_e$ | $(1.71\ {}^{+0.34}_{-0.30}) \times 10^{-5}$ | |
| $\Gamma_9$ | $\pi^+ \pi^0$ | 0.1706 ±0.0011 | 1.1 |
| $\Gamma_{10}$ | $\pi^+ \pi^0 \pi^0$ | 0.01395±0.00031 | 1.2 |
| $\Gamma_{11}$ | $\pi^+ \pi^+ \pi^-$ | 0.04503±0.00023 | 1.1 |

## $K^{\pm}$ DECAY RATES

### $\Gamma(\mu^+\nu_\mu)$ $\Gamma_2$

| VALUE ($10^6$ s$^{-1}$) | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|
| **51.22±0.17 OUR FIT** Error includes scale factor of 1.4. | | | | |
| **51.2 ±0.8** | FORD | 67 | CNTR | ± |

### $\Gamma(\pi^+\pi^+\pi^-)$ $\Gamma_{11}$

| VALUE ($10^6$ s$^{-1}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **4.503±0.023 OUR FIT** Error includes scale factor of 1.1. | | | | | |
| **4.511±0.024** | | [6] FORD | 70 | ASPK | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 4.529±0.032 | 3.2M | [6] FORD | 70 | ASPK | |
| 4.496±0.030 | | [6] FORD | 67 | CNTR | ± |

[6] First FORD 70 value is second FORD 70 combined with FORD 67.

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## $(\Gamma(K^+) - \Gamma(K^-)) / \Gamma(K)$

### $K^{\pm} \to \mu^{\pm}\nu_\mu$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

| VALUE (%) | DOCUMENT ID | | TECN |
|---|---|---|---|
| **−0.54±0.41** | FORD | 67 | CNTR |

### $K^{\pm} \to \pi^{\pm}\pi^+\pi^-$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

| VALUE (%) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **0.07±0.12 OUR AVERAGE** | | | | | |
| 0.08±0.12 | | [7] FORD | 70 | ASPK | |
| −0.50±0.90 | | FLETCHER | 67 | OSPK | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| −0.02±0.16 | | [8] SMITH | 73 | ASPK | ± |
| 0.10±0.14 | 3.2M | [7] FORD | 70 | ASPK | |
| −0.04±0.21 | | [7] FORD | 67 | CNTR | |

[7] First FORD 70 value is second FORD 70 combined with FORD 67.
[8] SMITH 73 value of $K^{\pm} \to \pi^{\pm}\pi^+\pi^-$ rate difference is derived from SMITH 73 value of $K^{\pm} \to \pi^{\pm}2\pi^0$ rate difference.

### $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

| VALUE (%) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **0.0 ±0.6 OUR AVERAGE** | | | | | |
| 0.08±0.58 | | SMITH | 73 | ASPK | ± |
| −1.1 ±1.8 | 1802 | HERZO | 69 | OSPK | |

### $K^{\pm} \to \pi^{\pm}\pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

| VALUE (%) | DOCUMENT ID | | TECN |
|---|---|---|---|
| **0.8±1.2** | HERZO | 69 | OSPK |

## $K^{\pm} \to \pi^{\pm}\pi^0\gamma$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

| VALUE (%) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.9±3.3 OUR AVERAGE** | | | | | | |
| 0.8±5.8 | 2461 | SMITH | 76 | WIRE | ± | $E_\pi$ 55–90 MeV |
| 1.0±4.0 | 4000 | ABRAMS | 73B | ASPK | ± | $E_\pi$ 51–100 MeV |

## $K^+$ BRANCHING RATIOS

### Leptonic and semileptonic modes

### $\Gamma(e^+\nu_e)/\Gamma(\mu^+\nu_\mu)$ $\Gamma_1/\Gamma_2$

| VALUE (units $10^{-5}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **2.45±0.11 OUR AVERAGE** | | | | | |
| 2.51±0.15 | 404 | HEINTZE | 76 | SPEC | + |
| 2.37±0.17 | 534 | HEARD | 75B | SPEC | + |
| 2.42±0.42 | 112 | CLARK | 72 | OSPK | + |

### $\Gamma(\mu^+\nu_\mu)/\Gamma_{\text{total}}$ $\Gamma_2/\Gamma$

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **63.43±0.17 OUR FIT** | Error includes scale factor of 1.2. | | | | | |
| **63.24±0.44** | 62k | CHIANG | 72 | OSPK | + | 1.84 GeV/$c$ $K^+$ |

### $\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$ $\Gamma_3/\Gamma$

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **4.87±0.06 OUR FIT** | Error includes scale factor of 1.2. | | | | | |
| **4.84±0.09 OUR AVERAGE** | | | | | | |
| 4.86±0.10 | 3516 | CHIANG | 72 | OSPK | + | 1.84 GeV/$c$ $K^+$ |
| 4.7 ±0.3 | 429 | SHAKLEE | 64 | HLBC | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 5.0 ±0.5 | | ROE | 61 | HLBC | + | |

### $\Gamma(\pi^0 e^+\nu_e)/\Gamma(\mu^+\nu_\mu)$ $\Gamma_3/\Gamma_2$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **0.0767±0.0011 OUR FIT** | Error includes scale factor of 1.3. | | | | |
| **0.0752±0.0024 OUR AVERAGE** | | | | | |
| 0.069 ±0.006 | 350 | ZELLER | 69 | ASPK | + |
| 0.0775±0.0033 | 960 | BOTTERILL | 68C | ASPK | + |
| 0.069 ±0.006 | 561 | GARLAND | 68 | OSPK | + |
| 0.0791±0.0054 | 295 | [9] AUERBACH | 67 | OSPK | + |

[9] AUERBACH 67 changed from 0.0797 ± 0.0054. See comment with ratio $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\mu^+\nu_\mu)$. The value 0.0785 ± 0.0025 given in AUERBACH 67 is an average of AUERBACH 67 $\Gamma(\pi^0 e^+\nu_e)/\Gamma(\mu^+\nu_\mu)$ and CESTER 66 $\Gamma(\pi^0 e^+\nu_e)/\left[\Gamma(\mu^+\nu_\mu) + \Gamma(\pi^+\pi^0)\right]$.

## $\Gamma(\pi^0 e^+ \nu_e)/[\Gamma(\mu^+ \nu_\mu) + \Gamma(\pi^+ \pi^0)]$ $\Gamma_3/(\Gamma_2+\Gamma_9)$

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **5.76±0.08 OUR FIT** Error includes scale factor of 1.2. | | | | | |
| **6.02±0.15 OUR AVERAGE** | | | | | |
| 6.16±0.22 | 5110 | ESCHSTRUTH | 68 | OSPK | + |
| 5.89±0.21 | 1679 | CESTER | 66 | OSPK | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 5.92±0.65 | | [10] WEISSENBE... | 76 | SPEC | + |

[10] Value calculated from WEISSENBERG 76 ($\pi^0 e\nu$), ($\mu\nu$), and ($\pi\pi^0$) values to eliminate dependence on our 1974 ($\pi 2\pi^0$) and ($\pi\pi^+\pi^-$) fractions.

## $\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\pi^+ \pi^0)$ $\Gamma_3/\Gamma_9$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.230±0.004 OUR FIT** Error includes scale factor of 1.2. | | | | | | |
| **0.221±0.012** | 786 | [11] LUCAS | 73B | HBC | − | Dalitz pairs only |

[11] LUCAS 73B gives N($K_{e3}$) = 786 ± 3.1%, N($2\pi$) = 3564 ± 3.1%. We divide.

## $\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\pi^+ \pi^+ \pi^-)$ $\Gamma_3/\Gamma_{11}$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **0.873±0.012 OUR FIT** Error includes scale factor of 1.2. | | | | | |
| **0.868±0.021 OUR AVERAGE** | | | | | |
| 0.867±0.027 | 2768 | BARMIN | 87 | XEBC | + |
| 0.856±0.040 | 2827 | BRAUN | 75 | HLBC | + |
| 0.90 ±0.06 | 230 | BORREANI | 64 | HBC | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 0.850±0.019 | 4385 | [12] HAIDT | 71 | HLBC | + |
| 0.846±0.021 | 4385 | [12] EICHTEN | 68 | HLBC | + |
| 0.94 ±0.09 | 854 | BELLOTTI | 67B | HLBC | |

[12] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in $\Gamma(\pi^0\mu^+\nu)/\Gamma(\pi^0 e^+\nu)$ with more precise results.

## $\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma_{\text{total}}$ $\Gamma_4/\Gamma$

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **3.27±0.06 OUR FIT** Error includes scale factor of 1.2. | | | | | | |
| **3.33±0.16** | 2345 | CHIANG | 72 | OSPK | + | 1.84 GeV/$c$ $K^+$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 2.8 ±0.4 | | [13] TAYLOR | 59 | EMUL | + | |

[13] Earlier experiments not averaged.

## $\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma(\mu^+ \nu_\mu)$ $\Gamma_4/\Gamma_2$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **$0.0515^{+0.0010}_{-0.0009}$ OUR FIT** Error includes scale factor of 1.2. | | | | | |
| **0.0483±0.0027 OUR AVERAGE** | | | | | |
| 0.0480±0.0037 | 424 | [14] GARLAND | 68 | OSPK | + |
| 0.0486±0.0040 | 307 | [15] AUERBACH | 67 | OSPK | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 0.054 ±0.009 | 240 | ZELLER | 69 | ASPK | + |

[14] GARLAND 68 changed from 0.055 ± 0.004 in agreement with $\mu$-spectrum calculation of GAILLARD 70 appendix B. L.G.Pondrom, (private communication 73).

[15] AUERBACH 67 changed from 0.0602 ± 0.0046 by erratum which brings the $\mu$-spectrum calculation into agreement with GAILLARD 70 appendix B.

## $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\pi^0 e^+\nu_e)$ $\Gamma_4/\Gamma_3$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.672±0.007 OUR FIT** | | | | | | |
| **0.672±0.007 OUR AVERAGE** | | | | | | |
| 0.671±0.007±0.008 | 24k | HORIE | 01 | SPEC | | |
| 0.670±0.014 | | [16] HEINTZE | 77 | SPEC | + | |
| 0.698±0.025 | 3480 | [17] CHIANG | 72 | OSPK | + | 1.84 GeV/$c$ $K^+$ |
| 0.667±0.017 | 5601 | BOTTERILL | 68B | ASPK | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 0.608±0.014 | 1585 | [18] BRAUN | 75 | HLBC | + | |
| 0.705±0.063 | 554 | [19] LUCAS | 73B | HBC | − | Dalitz pairs only |
| 0.596±0.025 | | [20] HAIDT | 71 | HLBC | + | |
| 0.604±0.022 | 1398 | [20] EICHTEN | 68 | HLBC | | |
| 0.703±0.056 | 1509 | CALLAHAN | 66B | HLBC | | |

[16] HEINTZE 77 value from fit to $\lambda_0$. Assumes $\mu$-$e$ universality.

[17] CHIANG 72 $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\pi^0 e^+\nu_e)$ is statistically independent of CHIANG 72 $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma_{\text{total}}$ and $\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$.

[18] BRAUN 75 value is from form factor fit. Assumes $\mu$-$e$ universality.

[19] LUCAS 73B gives N($K_{\mu 3}$) = 554 ± 7.6%, N($K_{e3}$) = 786 ± 3.1%. We divide.

[20] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy with more precise results.

## $\left[\Gamma(\pi^0\mu^+\nu_\mu)+\Gamma(\pi^+\pi^0)\right]/\Gamma_{\text{total}}$ $(\Gamma_4+\Gamma_9)/\Gamma$

We combine these two modes for experiments measuring them in xenon bubble chamber because of difficulties of separating them there.

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **24.40±0.14 OUR FIT** Error includes scale factor of 1.1. | | | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 25.4 ±0.9 | 886 | SHAKLEE | 64 | HLBC | + |
| 23.4 ±1.1 | | ROE | 61 | HLBC | + |

## $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\pi^+\pi^+\pi^-)$ $\Gamma_4/\Gamma_{11}$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.586±0.010 OUR FIT** Error includes scale factor of 1.2. | | | | | | |
| **0.63 ±0.07** | 2845 | [21] BISI | 65B | BC | + | HBC+HLBC |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 0.503±0.019 | 1505 | [22] HAIDT | 71 | HLBC | + | |
| 0.510±0.017 | 1505 | [22] EICHTEN | 68 | HLBC | + | |

[21] Error enlarged for background problems. See GAILLARD 70.

[22] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in $\Gamma(\pi^0\mu^+\nu)/\Gamma(\pi^0 e^+\nu)$ with more precise results.

## $\Gamma(\pi^0\pi^0 e^+\nu_e)/\Gamma_{total}$ $\Gamma_5/\Gamma$

| VALUE (units $10^{-5}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **2.1 ±0.4 OUR FIT** | | | | | |
| **2.54±0.89** | 10 | BARMIN | 88B | HLBC | + |

## $\Gamma(\pi^0\pi^0 e^+\nu_e)/\Gamma(\pi^0 e^+\nu_e)$ $\Gamma_5/\Gamma_3$

| VALUE (units $10^{-4}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **$4.3^{+0.9}_{-0.7}$ OUR FIT** | | | | | |
| **$4.1^{+1.0}_{-0.7}$ OUR AVERAGE** | | | | | |
| $4.2^{+1.0}_{-0.9}$ | 25 | BOLOTOV | 86B | CALO | − |
| $3.8^{+5.0}_{-1.2}$ | 2 | LJUNG | 73 | HLBC | + |

## $\Gamma(\pi^+\pi^- e^+\nu_e)/\Gamma(\pi^+\pi^+\pi^-)$ $\Gamma_6/\Gamma_{11}$

| VALUE (units $10^{-4}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **7.31±0.16 OUR AVERAGE** | | | | | |
| 7.35±0.01±0.19 | 388k | [23] PISLAK | 01 | B865 | |
| 7.21±0.32 | 30k | ROSSELET | 77 | SPEC | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 7.36±0.68 | 500 | BOURQUIN | 71 | ASPK | |
| 7.0 ±0.9 | 106 | SCHWEINB... | 71 | HLBC | + |
| 5.83±0.63 | 269 | ELY | 69 | HLBC | + |

[23] PISLAK 01 reports $\Gamma(\pi^+\pi^- e^+\nu_e)/\Gamma_{total} = (4.109 \pm 0.008 \pm 0.110) \times 10^{-5}$ using the PDG 00 value $\Gamma(\pi^+\pi^+\pi^-)/\Gamma_{total} = (5.59 \pm 0.05) \times 10^{-2}$. We divide by the PDG value and unfold its error from the systematic error. PISLAK 03 gives additional details on the branching ratio measurement and gives improved errors on the $S$-wave $\pi$-$\pi$ scattering length: $a_0^0 = 0.216 \pm 0.013$(stat.) $\pm 0.002$(syst.) $\pm 0.002$(theor.).

## $\Gamma(\pi^+\pi^-\mu^+\nu_\mu)/\Gamma_{total}$ $\Gamma_7/\Gamma$

| VALUE (units $10^{-5}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| $0.77^{+0.54}_{-0.50}$ | 1 | CLINE | 65 | FBC | + |

## $\Gamma(\pi^+\pi^-\mu^+\nu_\mu)/\Gamma(\pi^+\pi^+\pi^-)$ $\Gamma_7/\Gamma_{11}$

| VALUE (units $10^{-4}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **2.57±1.55** | 7 | BISI | 67 | DBC | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| ∼ 2.5 | 1 | GREINER | 64 | EMUL | + |

## $\Gamma(\pi^0\pi^0\pi^0 e^+\nu_e)/\Gamma_{total}$ $\Gamma_8/\Gamma$

| VALUE (units $10^{-6}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **<3.5** | 90 | 0 | BOLOTOV | 88 | SPEC | − |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| <9 | 90 | 0 | BARMIN | 92 | XEBC | + |

## Hadronic modes

### $\Gamma(\pi^+\pi^0)/\Gamma_{total}$ $\Gamma_9/\Gamma$

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **21.13±0.14 OUR FIT** | Error includes scale factor of 1.1. | | | | | |
| **21.18±0.28** | 16k | CHIANG | 72 | OSPK | + | 1.84 GeV/$c$ $K^+$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 21.0 ±0.6 | | CALLAHAN | 65 | HLBC | | See $\Gamma(\pi^+\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ |

### $\Gamma(\pi^+\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ $\Gamma_9/\Gamma_{11}$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **3.789±0.033 OUR FIT** | Error includes scale factor of 1.1. | | | | |
| **3.96 ±0.15** | 1045 | CALLAHAN | 66 | FBC | + |

### $\Gamma(\pi^+\pi^0)/\Gamma(\mu^+\nu_\mu)$ $\Gamma_9/\Gamma_2$

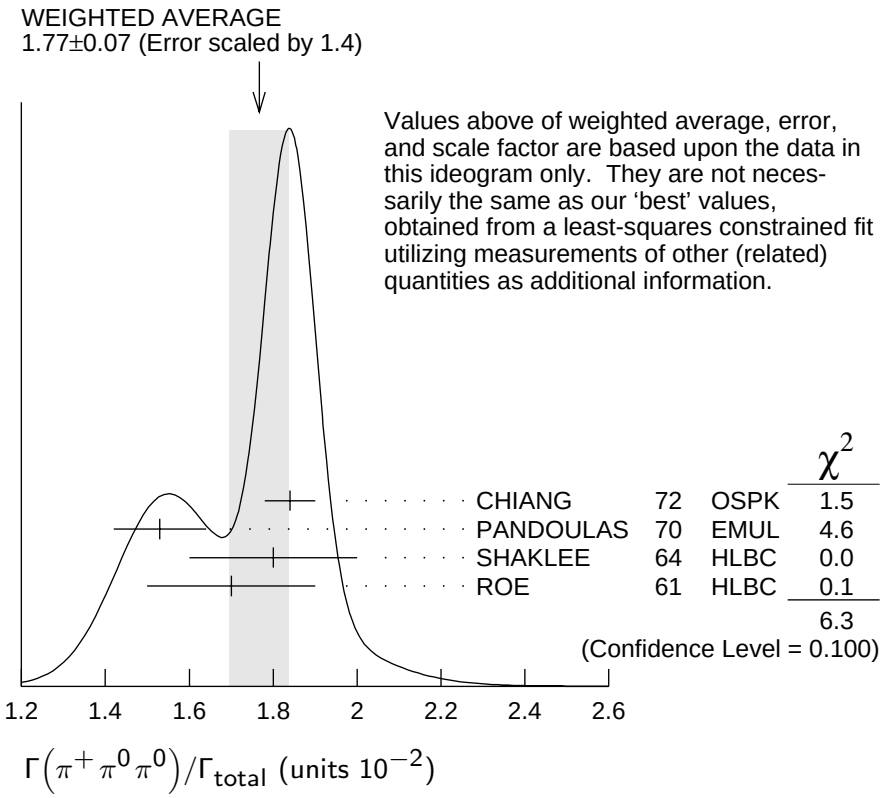
| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.3331±0.0028 OUR FIT** | Error includes scale factor of 1.1. | | | | | |
| **0.3316±0.0032 OUR AVERAGE** | | | | | | |
| 0.3329±0.0047±0.0010 | 45k | USHER | 92 | SPEC | + | $p\overline{p}$ at rest |
| 0.3355±0.0057 | | [24] WEISSENBE... | 76 | SPEC | + | |
| 0.305 ±0.018 | 1600 | ZELLER | 69 | ASPK | + | |
| 0.3277±0.0065 | 4517 | [25] AUERBACH | 67 | OSPK | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 0.328 ±0.005 | 25k | [24] WEISSENBE... | 74 | STRC | + | |

[24] WEISSENBERG 76 revises WEISSENBERG 74.
[25] AUERBACH 67 changed from 0.3253 ± 0.0065. See comment with ratio $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\mu^+\nu_\mu)$.

### $\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{total}$ $\Gamma_{10}/\Gamma$

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **1.73±0.04 OUR FIT** | Error includes scale factor of 1.2. | | | | | |
| **1.77±0.07 OUR AVERAGE** | Error includes scale factor of 1.4. See the ideogram below. | | | | | |
| 1.84±0.06 | 1307 | CHIANG | 72 | OSPK | + | 1.84 GeV/$c$ $K^+$ |
| 1.53±0.11 | 198 | [26] PANDOULAS | 70 | EMUL | + | |
| 1.8 ±0.2 | 108 | SHAKLEE | 64 | HLBC | + | |
| 1.7 ±0.2 | | ROE | 61 | HLBC | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 1.5 ±0.2 | | [27] TAYLOR | 59 | EMUL | + | |

[26] Includes events of TAYLOR 59.
[27] Earlier experiments not averaged.




## $\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^0)$ $\Gamma_{10}/\Gamma_9$

| *VALUE* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|
| **0.0818±0.0019 OUR FIT** Error includes scale factor of 1.2. | | | | | | |
| **0.081 ±0.005** | 574 | [28] LUCAS | 73B | HBC | − | Dalitz pairs only |

[28] LUCAS 73B gives $N(\pi 2\pi^0) = 574 \pm 5.9\%$, $N(2\pi) = 3564 \pm 3.1\%$. We quote $0.5N(\pi 2\pi^0)/N(2\pi)$ where 0.5 is because only Dalitz pair $\pi^0$'s were used.

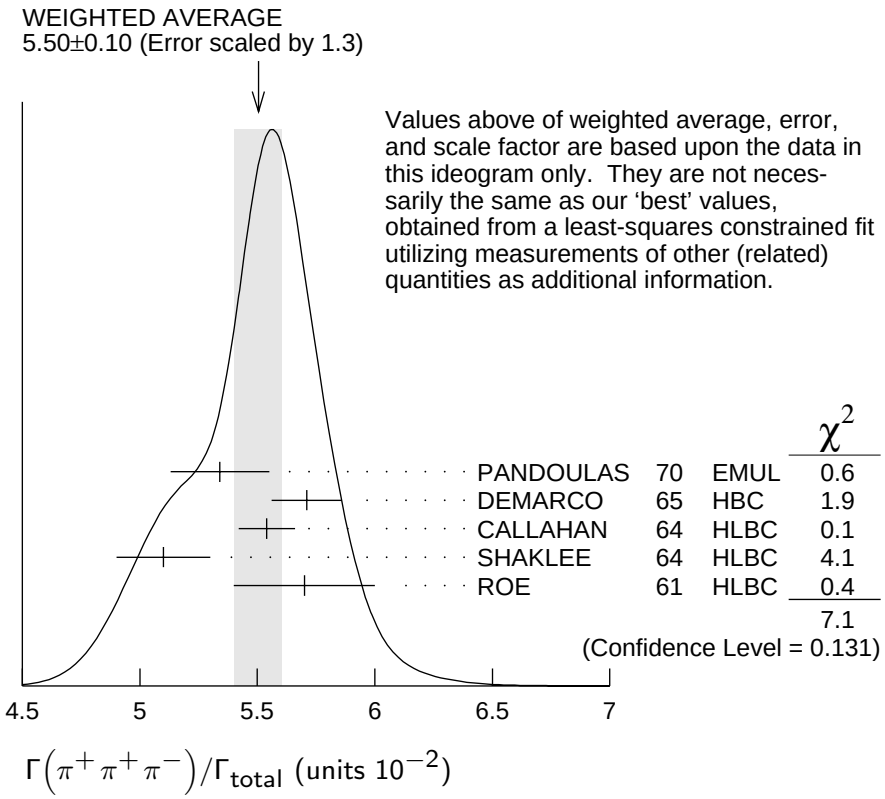
## $\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ $\Gamma_{10}/\Gamma_{11}$

| *VALUE* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|
| **0.310±0.007 OUR FIT** Error includes scale factor of 1.1. | | | | | | |
| **0.303±0.009** | 2027 | BISI | 65 | BC | + | HBC+HLBC |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 0.393±0.099 | 17 | YOUNG | 65 | EMUL | + | |

## $\Gamma(\pi^+\pi^+\pi^-)/\Gamma_{total}$ $\Gamma_{11}/\Gamma$

| *VALUE* (units $10^{-2}$) | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|
| **5.576±0.031 OUR FIT** Error includes scale factor of 1.1. | | | | | | |
| **5.50 ±0.10 OUR AVERAGE** Error includes scale factor of 1.3. See the ideogram below. | | | | | | |
| 5.34 ±0.21 | 693 | [29] PANDOULAS | 70 | EMUL | + | |
| 5.71 ±0.15 | | DEMARCO | 65 | HBC | | |
| 5.54 ±0.12 | 2332 | CALLAHAN | 64 | HLBC | + | |
| 5.1 ±0.2 | 540 | SHAKLEE | 64 | HLBC | + | |
| 5.7 ±0.3 | | ROE | 61 | HLBC | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 5.56 ±0.20 | 2330 | [30] CHIANG | 72 | OSPK | + | 1.84 GeV/$c$ $K^+$ |
| 6.0 ±0.4 | 44 | YOUNG | 65 | EMUL | + | |

[29] Includes events of TAYLOR 59.
[30] Value is not independent of CHIANG 72 $\Gamma(\mu^+\nu_\mu)/\Gamma_{\text{total}}$, $\Gamma(\pi^+\pi^0)/\Gamma_{\text{total}}$, $\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{\text{total}}$, $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma_{\text{total}}$, and $\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$.




## Leptonic and semileptonic modes with photons

### $\Gamma(\mu^+\nu_\mu\gamma)/\Gamma_{\text{total}}$ $\Gamma_{12}/\Gamma$

| *VALUE* (units $10^{-3}$) | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|
| **5.50±0.28 OUR AVERAGE** | | | | | | |
| 6.6 ±1.5 | | [31,32] DEMIDOV | 90 | XEBC | | P($\mu$) <231.5 MeV/$c$ |
| 6.0 ±0.9 | | BARMIN | 88 | HLBC | + | P($\mu$) <231.5 MeV/$c$ |
| 5.4 ±0.3 | | [33] AKIBA | 85 | SPEC | | P($\mu$) <231.5 MeV/$c$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 3.5 ±0.8 | | [32,34] DEMIDOV | 90 | XEBC | | E($\gamma$) > 20 MeV |
| 3.2 ±0.5 | 57 | [35] BARMIN | 88 | HLBC | + | $E(\gamma)$ >20 MeV |

[31] P($\mu$) cut given in DEMIDOV 90 paper, 235.1 MeV/$c$, is a misprint according to authors (private communication).
[32] DEMIDOV 90 quotes only inner bremsstrahlung (IB) part.
[33] Assumes $\mu$-$e$ universality and uses constraints from $K \to e\nu\gamma$.
[34] Not independent of above DEMIDOV 90 value. Cuts differ.
[35] Not independent of above BARMIN 88 value. Cuts differ.

## $\Gamma(\pi^0 e^+ \nu_e \gamma)/\Gamma(\pi^0 e^+ \nu_e)$ $\Gamma_{13}/\Gamma_3$

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.54±0.04 OUR AVERAGE** Error includes scale factor of 1.1. | | | | | | |
| 0.46±0.08 | 82 | [36] BARMIN | 91 | XEBC | | E($\gamma$) > 10 MeV, 0.6 < $\cos\theta_{e\,\gamma}$ < 0.9 |
| 0.56±0.04 | 192 | [37] BOLOTOV | 86B | CALO | − | $E(\gamma)$ >10 MeV |
| 0.76±0.28 | 13 | [38] ROMANO | 71 | HLBC | | $E(\gamma)$ >10 MeV |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 1.51±0.25 | 82 | [36] BARMIN | 91 | XEBC | | E($\gamma$) > 10 MeV, $\cos\theta_{e\,\gamma}$ < 0.98 |
| 0.48±0.20 | 16 | [39] LJUNG | 73 | HLBC | + | $E(\gamma)$ >30 MeV |
| $0.22^{+0.15}_{-0.10}$ | | [39] LJUNG | 73 | HLBC | + | $E(\gamma)$ >30 MeV |
| 0.53±0.22 | | [38] ROMANO | 71 | HLBC | + | $E(\gamma)$ >30 MeV |

[36] BARMIN 91 quotes branching ratio $\Gamma(K \to e\pi^0\nu\gamma)/\Gamma_{all}$. The measured normalization is $[\Gamma(K \to e\pi^0\nu) + \Gamma(K \to \pi^+\pi^+\pi^-)]$. For comparison with other experiments we used $\Gamma(K \to e\pi^0\nu)/\Gamma_{all} = 0.0482$ to calculate the values quoted here.

[37] $\cos\theta(e\gamma)$ between 0.6 and 0.9.

[38] Both ROMANO 71 values are for $\cos\theta(e\gamma)$ between 0.6 and 0.9. Second value is for comparison with second LJUNG 73 value. We use lowest $E(\gamma)$ cut for Summary Table value. See ROMANO 71 for $E_\gamma$ dependence.

[39] First LJUNG 73 value is for $\cos\theta(e\gamma)$ <0.9, second value is for $\cos\theta(e\gamma)$ between 0.6 and 0.9 for comparison with ROMANO 71.

## $\Gamma(\pi^0 e^+ \nu_e \gamma(\mathrm{SD}))/\Gamma_{total}$ $\Gamma_{14}/\Gamma$

Structure-dependent part.

| VALUE (units $10^{-5}$) | CL% | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **<5.3** | 90 | BOLOTOV | 86B | CALO | − |

## $\Gamma(\pi^0 \mu^+ \nu_\mu \gamma)/\Gamma_{total}$ $\Gamma_{15}/\Gamma$

| VALUE (units $10^{-5}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **<6.1** | 90 | 0 | LJUNG | 73 | HLBC | + | $E(\gamma)$ >30 MeV |

## $\Gamma(\pi^0 \pi^0 e^+ \nu_e \gamma)/\Gamma_{total}$ $\Gamma_{16}/\Gamma$

| VALUE (units $10^{-6}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **<5** | 90 | 0 | BARMIN | 92 | XEBC | + | $E_\gamma$ > 10 MeV |

## Hadronic modes with photons

## $\Gamma(\pi^+ \pi^0 \gamma)/\Gamma_{total}$ $\Gamma_{17}/\Gamma$

| VALUE (units $10^{-4}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **2.75±0.15 OUR AVERAGE** | | | | | | | |
| 2.71±0.45 | | 140 | BOLOTOV | 87 | WIRE | − | T$\pi^-$ 55–90 MeV |
| 2.87±0.32 | | 2461 | SMITH | 76 | WIRE | ± | T$\pi^\pm$ 55–90 MeV |
| 2.71±0.19 | | 2100 | ABRAMS | 72 | ASPK | ± | T$\pi^+$ 55–90 MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1.5 | $^{+1.1}_{-0.6}$ | | | [40] LJUNG | 73 | HLBC | + | $T\pi^+$ 55–80 MeV |
| 2.6 | $^{+1.5}_{-1.1}$ | | | [40] LJUNG | 73 | HLBC | + | $T\pi^+$ 55–90 MeV |
| 6.8 | $^{+3.7}_{-2.1}$ | | 17 | [40] LJUNG | 73 | HLBC | + | $T\pi^+$ 55–102 MeV |
| 2.4 | ±0.8 | | 24 | EDWARDS | 72 | OSPK | | $T\pi^+$ 58–90 MeV |
| <1.0 | | | 0 | [41] MALTSEV | 70 | HLBC | + | $T\pi^+$ <55 MeV |
| <1.9 | | 90 | 0 | EMMERSON | 69 | OSPK | | $T\pi^+$ 55–80 MeV |
| 2.2 | ±0.7 | | 18 | CLINE | 64 | FBC | + | $T\pi^+$ 55–80 MeV |

[40] The LJUNG 73 values are not independent.

[41] MALTSEV 70 selects low $\pi^+$ energy to enhance direct emission contribution.

## $\Gamma(\pi^+\pi^0\gamma(\mathrm{DE}))/\Gamma_{\mathrm{total}}$ $\Gamma_{18}/\Gamma$

Direct emission (DE) part of $\Gamma(\pi^+\pi^0\gamma)/\Gamma_{\mathrm{total}}$, assuming that interference (INT) component is zero.

| VALUE (units $10^{-6}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **4.4±0.8 OUR AVERAGE** | | | | | | |
| 3.2±1.3±1.0 | 4k | [42] ALIEV | 03 | K470 | + | $T\pi^+$ 55–90 MeV |
| 4.7±0.8±0.3 | 20k | [43] ADLER | 00C | B787 | + | $T\pi^+$ 55–90 MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | |
|---|---|---|---|---|---|---|
| 6.1±2.5±1.9 | 4k | [42] ALIEV | 03 | K470 | + | $T\pi^+$ full range |
| 20.5±4.6$^{+3.9}_{-2.3}$ | | BOLOTOV | 87 | WIRE | − | $T\pi^-$ 55–90 MeV |
| 15.6±3.5±5.0 | | ABRAMS | 72 | ASPK | ± | $T\pi^\pm$ 55–90 MeV |

[42] ALIEV 03 "$T\pi^+$ full range" result is extrapolated from their $T\pi$ >35 MeV measurement. They calculate the "$T\pi^+$ 55–90 MeV" result for comparison with other experiments. They measure the INT component to be $(-0.58^{+0.91}_{-0.83})$% of the inner bremsstrahlung (IB) component. The DE component is measured assuming INT=0.

[43] ADLER 00C measures the INT component to be $(-0.4 \pm 1.6)$% of the inner bremsstrahlung (IB) component. The DE component is measured assuming INT=0.

## $\Gamma(\pi^+\pi^0\pi^0\gamma)/\Gamma(\pi^+\pi^0\pi^0)$ $\Gamma_{19}/\Gamma_{10}$

| VALUE (units $10^{-4}$) | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|
| **4.3$^{+3.2}_{-1.7}$** | BOLOTOV | 85 | SPEC | − | $E(\gamma) > 10$ MeV |

## $\Gamma(\pi^+\pi^+\pi^-\gamma)/\Gamma_{\mathrm{total}}$ $\Gamma_{20}/\Gamma$

| VALUE (units $10^{-4}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **1.04±0.31 OUR AVERAGE** | | | | | | |
| 1.10±0.48 | 7 | BARMIN | 89 | XEBC | | $E(\gamma) > 5$ MeV |
| 1.0 ±0.4 | | STAMER | 65 | EMUL | + | $E(\gamma)$ >11 MeV |

## $\Gamma(\pi^+\gamma\gamma)/\Gamma_{\mathrm{total}}$ $\Gamma_{21}/\Gamma$

All values given here assume a phase space pion energy spectrum.

| VALUE (units $10^{-7}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **11 ± 3 ±1** | | 31 | [44] KITCHING | 97 | B787 | | |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| < 10 | | 90 | 0 | ATIYA | 90B | B787 | | Tπ 117–127 MeV |
| < 84 | | 90 | 0 | ASANO | 82 | CNTR | + | Tπ 117–127 MeV |
| −420 | ±520 | | 0 | ABRAMS | 77 | SPEC | + | Tπ <92 MeV |
| < 350 | | 90 | 0 | LJUNG | 73 | HLBC | + | 6–102, 114–127 MeV |
| < 500 | | 90 | 0 | KLEMS | 71 | OSPK | + | Tπ <117 MeV |
| −100 | ±600 | | | CHEN | 68 | OSPK | + | Tπ 60–90 MeV |

[44] KITCHING 97 is extrapolated from their model-independent branching fraction $(6.0 \pm 1.5 \pm 0.7) \times 10^{-7}$ for 100 MeV/$c$<$P_{\pi^+}$ < 180 MeV/$c$ using Chiral Perturbation Theory.

## $\Gamma(\pi^+ 3\gamma)/\Gamma_{\text{total}}$ $\Gamma_{22}/\Gamma$

Values given here assume a phase space pion energy spectrum.

| VALUE (units $10^{-4}$) | CL% | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **<1.0** | 90 | ASANO | 82 | CNTR | + | $T(\pi)$ 117–127 MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | |
|---|---|---|---|---|---|---|
| <3.0 | 90 | KLEMS | 71 | OSPK | + | $T(\pi)$ >117 MeV |

## Leptonic modes with $\ell\bar{\ell}$ pairs

## $\Gamma(e^+ \nu_e \nu\bar{\nu})/\Gamma(e^+ \nu_e)$ $\Gamma_{23}/\Gamma_1$

| VALUE | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **<3.8** | 90 | 0 | HEINTZE | 79 | SPEC | + |

## $\Gamma(\mu^+ \nu_\mu \nu\bar{\nu})/\Gamma_{\text{total}}$ $\Gamma_{24}/\Gamma$

| VALUE (units $10^{-6}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **<6.0** | 90 | 0 | [45] PANG | 73 | CNTR | + |

[45] PANG 73 assumes $\mu$ spectrum from $\nu$-$\nu$ interaction of BARDIN 70.

## $\Gamma(e^+ \nu_e e^+ e^-)/\Gamma_{\text{total}}$ $\Gamma_{25}/\Gamma$

| VALUE (units $10^{-8}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **2.48± 0.14±0.14** | 410 | POBLAGUEV | 02 | B865 | + | $m_{ee}$ >150 MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 20 | ±20 | 4 | DIAMANT-... | 76 | SPEC | + | $m_{e^+e^-}$ >140 MeV |

## $\Gamma(\mu^+ \nu_\mu e^+ e^-)/\Gamma_{\text{total}}$ $\Gamma_{26}/\Gamma$

| VALUE (units $10^{-8}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **7.06± 0.16±0.26** | 2.7k | POBLAGUEV | 02 | B865 | + | $m_{ee}$ >145 MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 100 | ±30 | 14 | DIAMANT-... | 76 | SPEC | + | $m_{e^+e^-}$ >140 MeV |

## $\Gamma(e^+ \nu_e \mu^+ \mu^-)/\Gamma_{\text{total}}$ $\Gamma_{27}/\Gamma$

| VALUE | CL% | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| **<5 × 10$^{-7}$** | 90 | ADLER | 98 | B787 |

### $\Gamma(\mu^+\nu_\mu\mu^+\mu^-)/\Gamma_{total}$ $\Gamma_{28}/\Gamma$

| VALUE (units $10^{-7}$) | CL% | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **<4.1** | 90 | ATIYA | 89 | B787 | + |

## Lepton Family number ($LF$), Lepton number ($L$), $\Delta S = \Delta Q$ ($SQ$) violating modes, or $\Delta S = 1$ weak neutral current ($S1$) modes

### $\Gamma(\pi^+\pi^+e^-\overline{\nu}_e)/\Gamma_{total}$ $\Gamma_{29}/\Gamma$

Test of $\Delta S = \Delta Q$ rule.

| VALUE (units $10^{-7}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| < 9.0 | 95 | 0 | SCHWEINB... | 71 | HLBC | + |
| < 6.9 | 95 | 0 | ELY | 69 | HLBC | + |
| <20. | 95 | | BIRGE | 65 | FBC | + |

### $\Gamma(\pi^+\pi^+e^-\overline{\nu}_e)/\Gamma(\pi^+\pi^-e^+\nu_e)$ $\Gamma_{29}/\Gamma_6$

Test of $\Delta S = \Delta Q$ rule.

| VALUE (units $10^{-4}$) | CL% | EVTS | DOCUMENT ID | | TECN |
|---|---|---|---|---|---|
| **< 3** | 90 | 3 | [46] BLOCH | 76 | SPEC |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| <130. | 95 | 0 | BOURQUIN | 71 | ASPK |

[46] BLOCH 76 quotes $3.6 \times 10^{-4}$ at CL = 95%, we convert.

### $\Gamma(\pi^+\pi^+\mu^-\overline{\nu}_\mu)/\Gamma_{total}$ $\Gamma_{30}/\Gamma$

Test of $\Delta S = \Delta Q$ rule.

| VALUE (units $10^{-6}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **<3.0** | 95 | 0 | BIRGE | 65 | FBC | + |

### $\Gamma(\pi^+e^+e^-)/\Gamma_{total}$ $\Gamma_{31}/\Gamma$

Test for $\Delta S = 1$ weak neutral current. Allowed by combined first-order weak and electromagnetic interactions.

| VALUE (units $10^{-7}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **2.88±0.13 OUR AVERAGE** | | | | | |
| 2.94±0.05±0.14 | 10300 | [47] APPEL | 99 | SPEC | + |
| 2.75±0.23±0.13 | 500 | [48] ALLIEGRO | 92 | SPEC | + |
| 2.7 ±0.5 | 41 | [49] BLOCH | 75 | SPEC | + |

[47] APPEL 99 establishes vector nature of this decay and determines form factor $f(Z)= f_0(1+\delta Z)$, $Z=M^2_{ee}/m^2_K$, $\delta$=2.14 ± 0.13 ± 0.15.

[48] ALLIEGRO 92 assumes a vector interaction with a form factor given by $\lambda = 0.105 \pm 0.035 \pm 0.015$ and a correlation coefficient of $-0.82$.

[49] BLOCH 75 assumes a vector interaction.

### $\Gamma(\pi^+\mu^+\mu^-)/\Gamma_{total}$ $\Gamma_{32}/\Gamma$

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions.

| VALUE (units $10^{-8}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **8.1 ±1.4 OUR AVERAGE** Error includes scale factor of 2.7. See the ideogram below. | | | | | | |
| 9.8 ±1.0 ±0.5 | | 110 | [50] PARK | 02 | HYCP | ± |
| 9.22±0.60±0.49 | | 402 | [51] MA | 00 | B865 | + |
| 5.0 ±0.4 ±0.9 | | 207 | [52] ADLER | 97C | B787 | + |

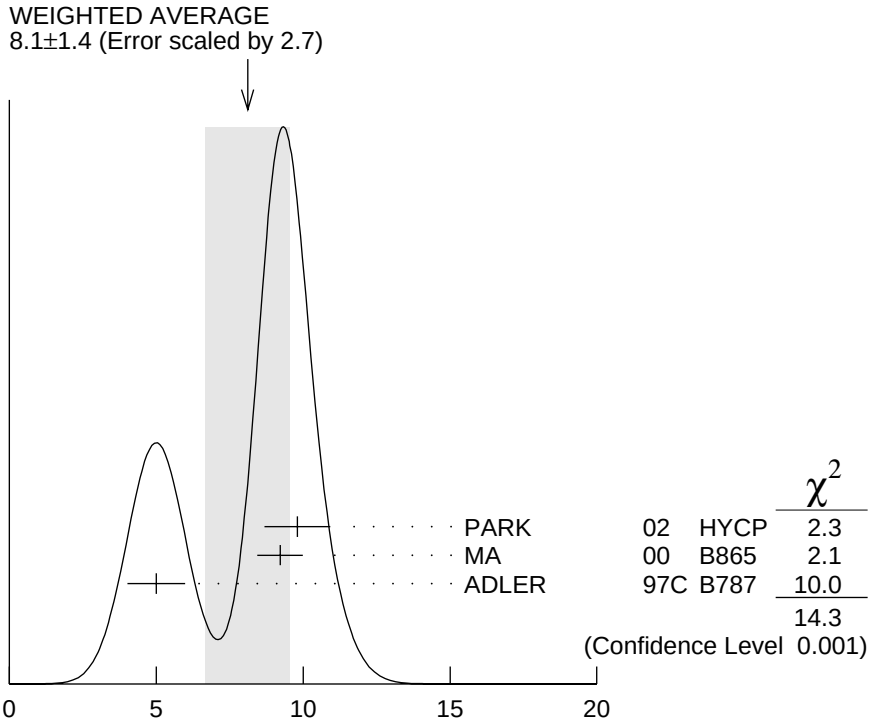
• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 9.7 | ±1.2 | ±0.4 | 65 | PARK | 02 | HYCP | + |
| 10.0 | ±1.9 | ±0.7 | 35 | PARK | 02 | HYCP | − |
| <23 | | | 90 | ATIYA | 89 | B787 | + |

[50] PARK 02 "±" result comes from combining $K^+ \to \pi^+\mu^+\mu^-$ and $K^- \to \pi^-\mu^+\mu^-$, assuming *CP* is conserved.

[51] MA 00 establishes vector nature of this decay and determines form factor $f(Z)=f_0(1+\delta Z)$, $Z=M^2_{\mu\mu}/m^2_K$, $\delta=2.45^{+1.30}_{-0.95}$.

[52] ADLER 97C gives systematic error $0.7\times10^{-8}$ and theoretical uncertainty $0.6\times10^{-8}$, which we combine in quadrature to obtain our second error.




$\Gamma\left(\pi^+\mu^+\mu^-\right)/\Gamma_{\text{total}}$ $\Gamma_{32}/\Gamma$

## $\Gamma\left(\pi^+\nu\overline{\nu}\right)/\Gamma_{\text{total}}$ $\Gamma_{33}/\Gamma$

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions. Branching ratio values are extrapolated from the momentum or energy regions shown in the comments assuming Standard Model phase space except for those labeled "Scalar" or "Tensor" to indicate the assumed non-Standard-Model interaction.

| *VALUE* (units $10^{-9}$) | *CL%* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|---|
| $\mathbf{0.157^{+0.175}_{-0.082}}$ | | 2 | ADLER | 02 | B787 | | $P_\pi$ >211 MeV/$c$ |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| < | 4.2 | 90 | 1 | ADLER | 02C | B787 | 140<$P_\pi$ <195 MeV/$c$ |
| < | 4.7 | 90 | | ADLER | 02C | B787 | Scalar |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| < 2.5 | | 90 | | ADLER | 02C | B787 | | Tensor |
| 0.15 | $^{+0.34}_{-0.12}$ | | 1 | ADLER | 00 | B787 | | In ADLER 02 |
| 0.42 | $^{+0.97}_{-0.35}$ | | 1 | ADLER | 97 | B787 | | |
| < 2.4 | | 90 | | ADLER | 96 | B787 | | |
| < 7.5 | | 90 | | ATIYA | 93 | B787 | + | $T(\pi)$ 115–127 MeV |
| < 5.2 | | 90 | | [53] ATIYA | 93 | B787 | + | |
| < 17 | | 90 | 0 | ATIYA | 93B | B787 | + | $T(\pi)$ 60–100 MeV |
| < 34 | | 90 | | ATIYA | 90 | B787 | + | |
| <140 | | 90 | | ASANO | 81B | CNTR | + | $T(\pi)$ 116–127 MeV |

[53] Combining ATIYA 93 and ATIYA 93B results. Superseded by ADLER 96.

## $\Gamma(\pi^+\pi^0\nu\overline{\nu})/\Gamma_{\text{total}}$ $\Gamma_{34}/\Gamma$

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions.

| *VALUE* (units $10^{-5}$) | *CL%* | *DOCUMENT ID* | | *TECN* |
|---|---|---|---|---|
| **<4.3** | 90 | [54] ADLER | 01 | SPEC |

[54] Search region defined by 90 MeV/$c$<$P_{\pi^+}$ <188 MeV/$c$ and 135 MeV<$E_{\pi^0}$ <180 MeV.

## $\Gamma(\mu^-\nu e^+ e^+)/\Gamma(\pi^+\pi^- e^+\nu_e)$ $\Gamma_{35}/\Gamma_6$

Test of lepton family number conservation.

| *VALUE* (units $10^{-3}$) | *CL%* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* |
|---|---|---|---|---|---|---|
| **<0.5** | 90 | 0 | [55] DIAMANT-... | 76 | SPEC | + |

[55] DIAMANT-BERGER 76 quotes this result times our 1975 $\pi^+\pi^- e\nu$ BR ratio.

## $\Gamma(\mu^+\nu_e)/\Gamma_{\text{total}}$ $\Gamma_{36}/\Gamma$

Forbidden by lepton family number conservation.

| *VALUE* | *CL%* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|---|
| **<0.004** | 90 | 0 | [56] LYONS | 81 | HLBC | 0 | 200 GeV $K^+$ narrow band $\nu$ beam |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | | |
| <0.012 | 90 | | [56] COOPER | 82 | HLBC | | Wideband $\nu$ beam |

[56] COOPER 82 and LYONS 81 limits on $\nu_e$ observation are here interpreted as limits on lepton family number violation in the absence of mixing.

## $\Gamma(\pi^+\mu^+ e^-)/\Gamma_{\text{total}}$ $\Gamma_{37}/\Gamma$

Test of lepton family number conservation.

| *VALUE* (units $10^{-10}$) | *CL%* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* |
|---|---|---|---|---|---|---|
| **<0.28** | 90 | | [57] APPEL | 00 | RVUE | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| <0.39 | 90 | | APPEL | 00 | B865 | + |
| <2.1 | 90 | 0 | LEE | 90 | SPEC | + |

[57] This result combines APPEL 00 BNL-E865 1996 data, BNL-E865 1995 data from BERGMAN 97 and PISLAK 97 theses, and LEE 90 BNL-E777 data.

## $\Gamma(\pi^+\mu^-e^+)/\Gamma_{total}$ $\Gamma_{38}/\Gamma$

Test of lepton family number conservation.

| VALUE (units $10^{-10}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **< 5.2** | 90 | 0 | APPEL | 00B | B865 | + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | |
|---|---|---|---|---|---|---|
| <70 | 90 | 0 | [58] DIAMANT-... | 76 | SPEC | + |

[58] Measurement actually applies to the sum of the $\pi^+\mu^-e^+$ and $\pi^-\mu^+e^+$ modes.

## $\Gamma(\pi^-\mu^+e^+)/\Gamma_{total}$ $\Gamma_{39}/\Gamma$

Test of total lepton number conservation.

| VALUE (units $10^{-10}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **< 5.0** | 90 | 0 | APPEL | 00B | B865 | + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | |
|---|---|---|---|---|---|---|
| <70 | 90 | 0 | [59] DIAMANT-... | 76 | SPEC | + |

[59] Measurement actually applies to the sum of the $\pi^+\mu^-e^+$ and $\pi^-\mu^+e^+$ modes.

## $\Gamma(\pi^-e^+e^+)/\Gamma_{total}$ $\Gamma_{40}/\Gamma$

Test of total lepton number conservation.

| VALUE | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **$<6.4\times10^{-10}$** | 90 | 0 | APPEL | 00B | B865 | + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | |
|---|---|---|---|---|---|---|
| $<9.2\times10^{-9}$ | 90 | 0 | DIAMANT-... | 76 | SPEC | + |
| $<1.5\times10^{-5}$ | | | CHANG | 68 | HBC | − |

## $\Gamma(\pi^-\mu^+\mu^+)/\Gamma_{total}$ $\Gamma_{41}/\Gamma$

Forbidden by total lepton number conservation.

| VALUE | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **$<3.0\times10^{-9}$** | 90 | 0 | APPEL | 00B | B865 | + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | |
|---|---|---|---|---|---|---|
| $<1.5\times10^{-4}$ | 90 | | [60] LITTENBERG | 92 | HBC | |

[60] LITTENBERG 92 is from retroactive data analysis of CHANG 68 bubble chamber data.

## $\Gamma(\mu^+\overline{\nu}_e)/\Gamma_{total}$ $\Gamma_{42}/\Gamma$

Forbidden by total lepton number conservation.

| VALUE (units $10^{-3}$) | CL% | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **<3.3** | 90 | [61] COOPER | 82 | HLBC | Wideband $\nu$ beam |

[61] COOPER 82 limit on $\overline{\nu}_e$ observation is here interpreted as a limit on lepton number violation in the absence of mixing.

## $\Gamma(\pi^0e^+\overline{\nu}_e)/\Gamma_{total}$ $\Gamma_{43}/\Gamma$

Forbidden by total lepton number conservation.

| VALUE | CL% | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **<0.003** | 90 | [62] COOPER | 82 | HLBC | Wideband $\nu$ beam |

[62] COOPER 82 limit on $\overline{\nu}_e$ observation is here interpreted as a limit on lepton number violation in the absence of mixing.

**$\Gamma(\pi^+\gamma)/\Gamma_{total}$** **$\Gamma_{44}/\Gamma$**

Violates angular momentum conservation. Current interest in this decay is as a search for exotic physics such as a vacuum expectation value of a new vector field, non-local Superstring effects, or departures from Lorentz invariance, as discussed in ADLER 02B.

| VALUE (units $10^{-7}$) | CL% | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **< 3.6** | 90 | ADLER | 02B | B787 | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| <14 | 90 | ASANO | 82 | CNTR | + |
| <40 | 90 | [63] KLEMS | 71 | OSPK | + |

[63] Test of model of Selleri, Nuovo Cimento **60A** 291 (1969).

## $K^+$ LONGITUDINAL POLARIZATION OF EMITTED $\mu^+$

| VALUE | CL% | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **<−0.990** | 90 | [64] AOKI | 94 | SPEC | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| <−0.990 | 90 | IMAZATO | 92 | SPEC | + | Repl. by AOKI 94 |
| −0.970±0.047 | | [65] YAMANAKA | 86 | SPEC | + | |
| −1.0 ±0.1 | | [65] CUTTS | 69 | SPRK | + | |
| −0.96 ±0.12 | | [65] COOMBES | 57 | CNTR | + | |

[64] AOKI 94 measures $\xi P_\mu = -0.9996 \pm 0.0030 \pm 0.0048$. The above limit is obtained by summing the statistical and systematic errors in quadrature, normalizing to the physically significant region ($|\xi P_\mu| < 1$) and assuming that $\xi$=1, its maximum value.

[65] Assumes $\xi$=1.

## DALITZ PLOT PARAMETERS FOR $K \to 3\pi$ DECAYS

Revised 1999 by T.G. Trippe (LBNL).

The Dalitz plot distribution for $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$, $K^\pm \to \pi^0\pi^0\pi^\pm$, and $K^0_L \to \pi^+\pi^-\pi^0$ can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

$$\left|M\right|^2 \propto 1 + g\frac{(s_3 - s_0)}{m^2_{\pi^+}} + h\left[\frac{s_3 - s_0}{m^2_{\pi^+}}\right]^2 + j\ \frac{(s_2 - s_1)}{m^2_{\pi^+}} + k\left[\frac{s_2 - s_1}{m^2_{\pi^+}}\right]^2 + f\ \frac{(s_2 - s_1)}{m^2_{\pi^+}}\frac{(s_3 - s_0)}{m^2_{\pi^+}} + \cdots\ , \quad (1)$$

where $m^2_{\pi^+}$ has been introduced to make the coefficients $g$, $h$, $j$, and $k$ dimensionless, and

$$s_i = (P_K - P_i)^2 = (m_K - m_i)^2 - 2m_K T_i \ , \ i = 1, 2, 3,$$
$$s_0 = \frac{1}{3}\sum_i s_i = \frac{1}{3}(m_K^2 + m_1^2 + m_2^2 + m_3^2) \quad .$$

Here the $P_i$ are four-vectors, $m_i$ and $T_i$ are the mass and kinetic energy of the $i^{th}$ pion, and the index 3 is used for the odd pion.

The coefficient $g$ is a measure of the slope in the variable $s_3$ (or $T_3$) of the Dalitz plot, while $h$ and $k$ measure the quadratic dependence on $s_3$ and $(s_2 - s_1)$, respectively. The coefficient $j$ is related to the asymmetry of the plot and must be zero if $CP$ invariance holds. Note also that if $CP$ is good, $g$, $h$, and $k$ must be the same for $K^+ \to \pi^+\pi^+\pi^-$ as for $K^- \to \pi^-\pi^-\pi^+$.

Since different experiments use different forms for $\left|M\right|^2$, in order to compare the experiments we have converted to $g$, $h$, $j$, and $k$ whatever coefficients have been measured. Where such conversions have been done, the measured coefficient $a_y$, $a_t$, $a_u$, or $a_v$ is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

## References

1. S. Weinberg, Phys. Rev. Lett. **4**, 87 (1960).
2. Particle Data Group, Phys. Lett. **111B**, 69 (1982).

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### ENERGY DEPENDENCE OF $K^\pm$ DALITZ PLOT

$|\text{matrix element}|^2 = 1 + gu + hu^2 + kv^2$
where $u = (s_3 - s_0) / m^2_\pi$ and $v = (s_2 - s_1) / m^2_\pi$

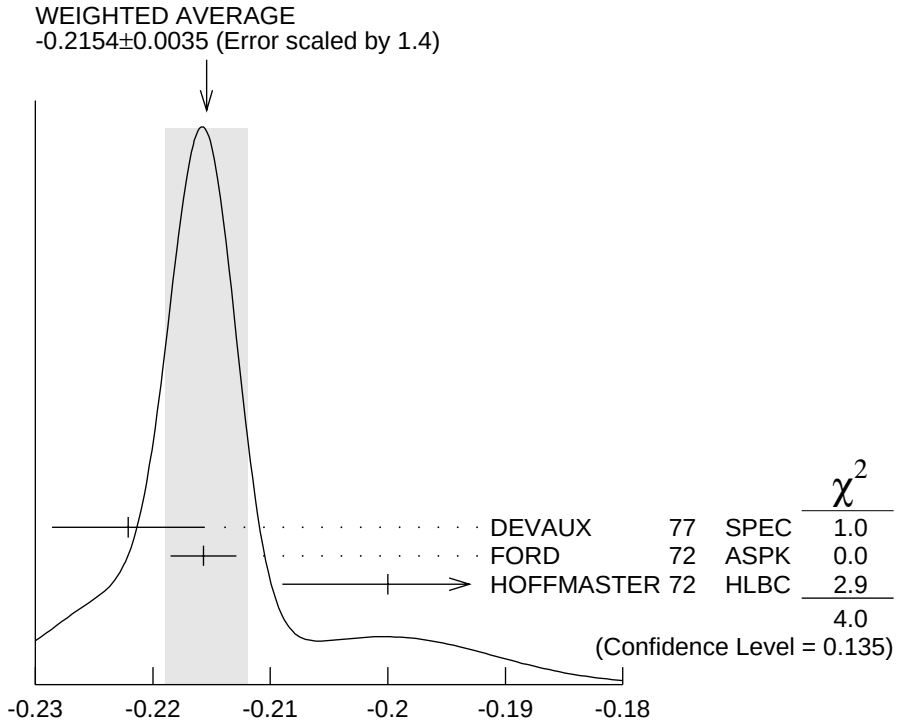
### LINEAR COEFFICIENT $g_{\tau^+}$ FOR $K^+ \to \pi^+\pi^+\pi^-$

Some experiments use Dalitz variables $x$ and $y$. In the comments we give $a_y$ = coefficient of $y$ term. See note above on "Dalitz Plot Parameters for $K \to 3\pi$

Decays." For discussion of the conversion of $a_y$ to $g$, see the earlier version of the same note in the *Review* published in Physics Letters **111B** 70 (1982).

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **−0.2154±0.0035 OUR AVERAGE** | | Error includes scale factor of 1.4. See the ideogram below. | | | | |
| −0.2221±0.0065 | 225k | DEVAUX | 77 | SPEC | + | $a_y$=.2814 ± .0082 |
| −0.2157±0.0028 | 750k | FORD | 72 | ASPK | + | $a_y$=.2734 ± .0035 |
| −0.200 ±0.009 | 39819 | [66] HOFFMASTER | 72 | HLBC | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| −0.196 ±0.012 | 17898 | [67] GRAUMAN | 70 | HLBC | + | $a_y$=0.228 ± 0.030 |
| −0.218 ±0.016 | 9994 | [68] BUTLER | 68 | HBC | + | $a_y$=0.277 ± 0.020 |
| −0.22 ±0.024 | 5428 | [68,69] ZINCHENKO | 67 | HBC | + | $a_y$=0.28 ± 0.03 |

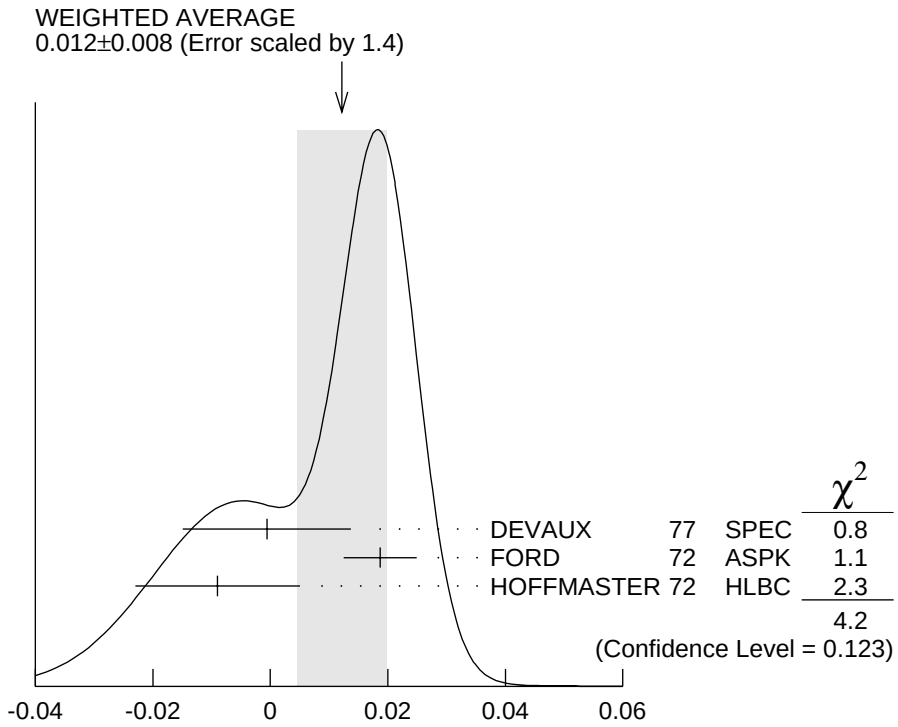
[66] HOFFMASTER 72 includes GRAUMAN 70 data.
[67] Emulsion data added — all events included by HOFFMASTER 72.
[68] Experiments with large errors not included in average.
[69] Also includes DBC events.




Linear energy dependence for $K^+ \to \pi^+\pi^+\pi^-$

## QUADRATIC COEFFICIENT *h* FOR $K^+ \to \pi^+\pi^+\pi^-$

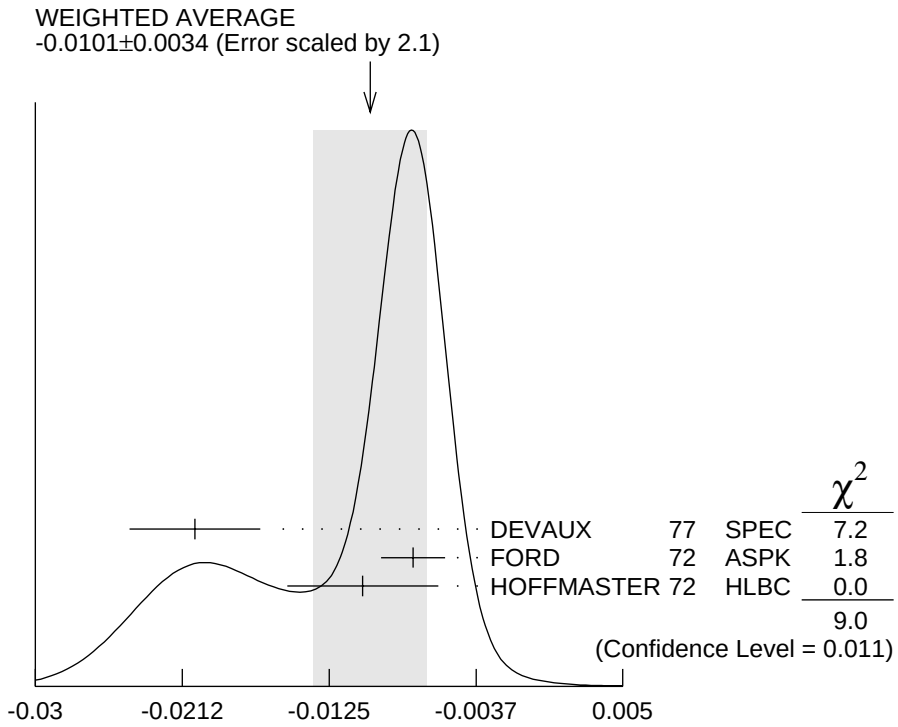
| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **0.012 ±0.008 OUR AVERAGE** | | Error includes scale factor of 1.4. See the ideogram below. | | | |
| −0.0006±0.0143 | 225k | DEVAUX | 77 | SPEC | + |
| 0.0187±0.0062 | 750k | FORD | 72 | ASPK | + |
| −0.009 ±0.014 | 39819 | HOFFMASTER | 72 | HLBC | + |




Quadratic coefficient $h$ for $K^+ \to \pi^+\pi^+\pi^-$

## QUADRATIC COEFFICIENT $k$ FOR $K^+ \to \pi^+\pi^+\pi^-$

| *VALUE* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* |
|---|---|---|---|---|---|
| **$-0.0101 \pm 0.0034$ OUR AVERAGE** Error includes scale factor of 2.1. See the ideogram below. | | | | | |
| $-0.0205 \pm 0.0039$ | 225k | DEVAUX | 77 | SPEC | + |
| $-0.0075 \pm 0.0019$ | 750k | FORD | 72 | ASPK | + |
| $-0.0105 \pm 0.0045$ | 39819 | HOFFMASTER | 72 | HLBC | + |




Quadratic coefficient $k$ for $K^+ \to \pi^+\pi^+\pi^-$

## LINEAR COEFFICIENT $g_{\tau-}$ FOR $K^- \to \pi^-\pi^-\pi^+$

Some experiments use Dalitz variables $x$ and $y$. In the comments we give $a_y$ = coefficient of $y$ term. See note above on "Dalitz Plot Parameters for $K \to 3\pi$ Decays." For discussion of the conversion of $a_y$ to $g$, see the earlier version of the same note in the *Review* published in Physics Letters **111B** 70 (1982).

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **−0.217 ±0.007 OUR AVERAGE** | | Error includes scale factor of 2.5. | | | | |
| −0.2186±0.0028 | 750k | FORD | 72 | ASPK | − | $a_y$=.2770 ± .0035 |
| −0.193 ±0.010 | 50919 | MAST | 69 | HBC | − | $a_y$=0.244 ± 0.013 |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| −0.199 ±0.008 | 81k | [70] LUCAS | 73 | HBC | − | $a_y$=0.252 ± 0.011 |
| −0.190 ±0.023 | 5778 | [71,72] MOSCOSO | 68 | HBC | − | $a_y$=0.242 ± 0.029 |
| −0.220 ±0.035 | 1347 | [73] FERRO-LUZZI | 61 | HBC | − | $a_y$=0.28 ± 0.045 |

[70] Quadratic dependence is required by $K^0_L$ experiments. For comparison we average only those $K^\pm$ experiments which quote quadratic fit values.
[71] Experiments with large errors not included in average.
[72] Also includes DBC events.
[73] No radiative corrections included.

## QUADRATIC COEFFICIENT $h$ FOR $K^- \to \pi^-\pi^-\pi^+$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **0.010 ±0.006 OUR AVERAGE** | | | | | |
| 0.0125±0.0062 | 750k | FORD | 72 | ASPK | − |
| −0.001 ±0.012 | 50919 | MAST | 69 | HBC | − |

## QUADRATIC COEFFICIENT $k$ FOR $K^- \to \pi^-\pi^-\pi^+$

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **$-0.0084 \pm 0.0019$ OUR AVERAGE** | | | | | |
| $-0.0083 \pm 0.0019$ | 750k | FORD | 72 | ASPK | − |
| $-0.014 \pm 0.012$ | 50919 | MAST | 69 | HBC | − |

## $(g_{\tau^+} - g_{\tau^-}) / (g_{\tau^+} + g_{\tau^-})$ FOR $K^\pm \to \pi^\pm\pi^+\pi^-$

A nonzero value for this quantity indicates *CP* violation.

| VALUE (%) | EVTS | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| **$-0.70 \pm 0.53$** | 3.2M | FORD | 70 | ASPK |

## LINEAR COEFFICIENT $g$ FOR $K^\pm \to \pi^\pm\pi^0\pi^0$

Unless otherwise stated, all experiments include terms quadratic in $(s_3 - s_0) / m^2_{\pi^+}$. See note above on "Dalitz Plot Parameters for $K \to 3\pi$ Decays."

See BATUSOV 98 for a discussion of the discrepancy between their result and others, especially BOLOTOV 86. At this time we have no way to resolve the discrepancy so we depend on the large scale factor as a warning.

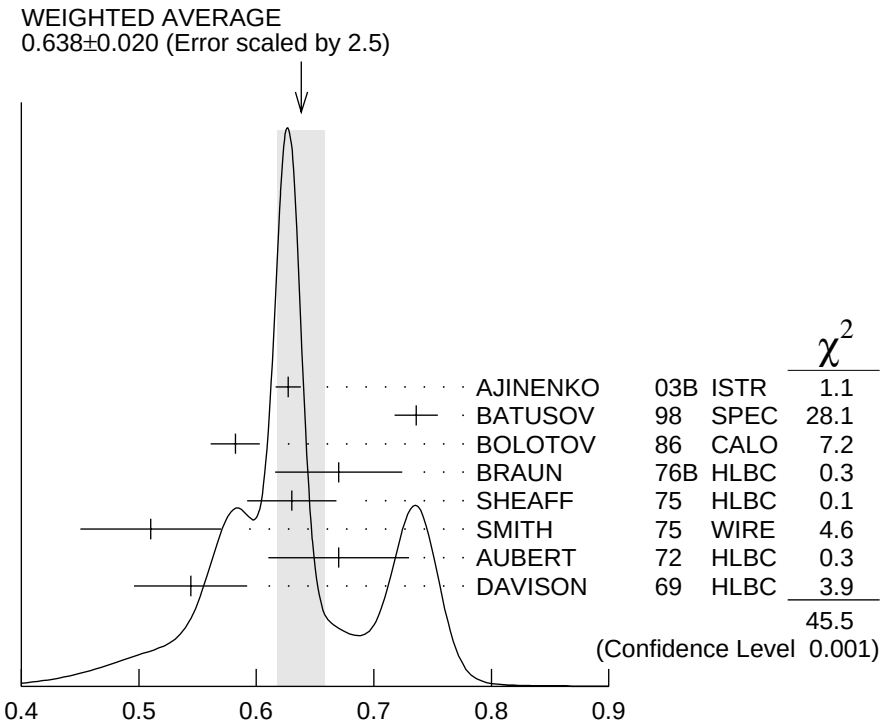
| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **$0.638 \pm 0.020$ OUR AVERAGE** | | Error includes scale factor of 2.5. See the ideogram below. | | | | |
| $0.627 \pm 0.004 \pm 0.010$ | 252k | [74,75] AJINENKO | 03B | ISTR | − | |
| $0.736 \pm 0.014 \pm 0.012$ | 33k | BATUSOV | 98 | SPEC | + | |
| $0.582 \pm 0.021$ | 43k | BOLOTOV | 86 | CALO | − | |
| $0.670 \pm 0.054$ | 3263 | BRAUN | 76B | HLBC | + | |
| $0.630 \pm 0.038$ | 5635 | SHEAFF | 75 | HLBC | + | |
| $0.510 \pm 0.060$ | 27k | SMITH | 75 | WIRE | + | |
| $0.67 \pm 0.06$ | 1365 | AUBERT | 72 | HLBC | + | |
| $0.544 \pm 0.048$ | 4048 | DAVISON | 69 | HLBC | + | Also emulsion |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| $0.518 \pm 0.039$ | 815 | [76] SHIN | 00 | SPEC | + | |
| $0.806 \pm 0.220$ | 4639 | [77] BERTRAND | 76 | EMUL | + | |
| $0.484 \pm 0.084$ | 574 | [76] LUCAS | 73B | HBC | − | Dalitz pairs only |
| $0.527 \pm 0.102$ | 198 | [77] PANDOULAS | 70 | EMUL | + | |
| $0.586 \pm 0.098$ | 1874 | [76] BISI | 65 | HLBC | + | Also HBC |
| $0.48 \pm 0.04$ | 1792 | [76] KALMUS | 64 | HLBC | + | |

[74] Measured using in-flight decays of the 25 GeV negative secondary beam.

[75] They form new world averages $g_- = (0.617 \pm 0.018)$ and $g_+ = (0.684 \pm 0.033)$ which give $\Delta g_{\tau'} = 0.051 \pm 0.028$.

[76] Authors give linear fit only.

[77] Experiments with large errors not included in average.

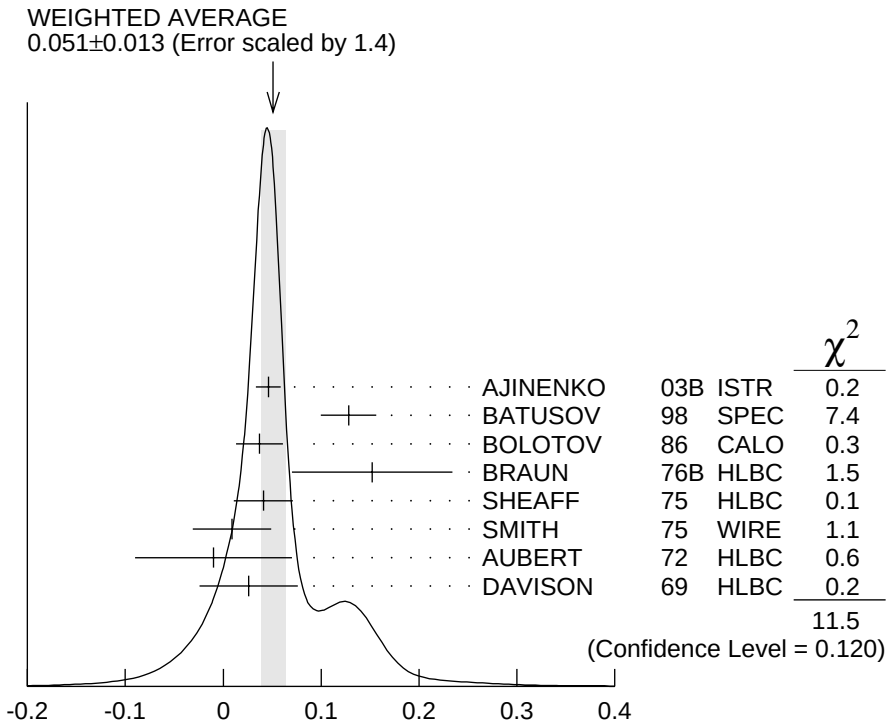



Linear energy dependence for $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$

## QUADRATIC COEFFICIENT *h* FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$

| *VALUE* | *EVTS* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|---|
| **0.051±0.013 OUR AVERAGE** | | Error includes scale factor of 1.4. See the ideogram below. | | | | |
| 0.046±0.004±0.012 | 252k | [78] AJINENKO | 03B | ISTR | − | |
| 0.128±0.015±0.024 | 33k | BATUSOV | 98 | SPEC | + | |
| 0.037±0.024 | 43k | BOLOTOV | 86 | CALO | − | |
| 0.152±0.082 | 3263 | BRAUN | 76B | HLBC | + | |
| 0.041±0.030 | 5635 | SHEAFF | 75 | HLBC | + | |
| 0.009±0.040 | 27k | SMITH | 75 | WIRE | + | |
| −0.01 ±0.08 | 1365 | AUBERT | 72 | HLBC | + | |
| 0.026±0.050 | 4048 | DAVISON | 69 | HLBC | + | Also emulsion |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 0.164±0.121 | 4639 | [79] BERTRAND | 76 | EMUL | + | |
| 0.018±0.124 | 198 | [79] PANDOULAS | 70 | EMUL | + | |

[78] Measured using in-flight decays of the 25 GeV negative secondary beam.
[79] Experiments with large errors not included in average.




Quadratic coefficient $h$ FOR $K^\pm \to \pi^\pm\pi^0\pi^0$

**QUADRATIC COEFFICIENT $k$ FOR $K^\pm \to \pi^\pm\pi^0\pi^0$**

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **0.004 ±0.007 OUR AVERAGE** | | Error includes scale factor of 3.2. | | | |
| 0.001 ±0.001 ±0.002 | 252k | [80] AJINENKO | 03B | ISTR | − |
| 0.0197±0.0045±0.0029 | 33k | BATUSOV | 98 | SPEC | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 0.043 ±0.020 | 815 | SHIN | 00 | SPEC | + |

[80] Measured using in-flight decays of the 25 GeV negative secondary beam.

## $K^\pm_{\ell 3}$ AND $K^0_{\ell 3}$ FORM FACTORS

Revised January 2004 by T.G. Trippe (LBNL).

Assuming that only the vector current contributes to $K \to \pi\ell\nu$ decays, we write the matrix element as

$$M \propto f_+(t)\left[(P_K + P_\pi)_\mu \overline{\ell}\gamma_\mu(1+\gamma_5)\nu\right] + f_-(t)\left[m_\ell\overline{\ell}(1+\gamma_5)\nu\right] \ , \quad (1)$$

where $P_K$ and $P_\pi$ are the four-momenta of the $K$ and $\pi$ mesons, $m_\ell$ is the lepton mass, and $f_+$ and $f_-$ are dimensionless

form factors which can depend only on $t = (P_K - P_\pi)^2$, the square of the four-momentum transfer to the leptons. If time-reversal invariance holds, $f_+$ and $f_-$ are relatively real. $K_{\mu 3}$ experiments, discussed immediately below, measure $f_+$ and $f_-$, while $K_{e3}$ experiments, discussed further below, are sensitive only to $f_+$ because the small electron mass makes the $f_-$ term negligible.

For this edition, fits to the form factor data have been made with and without the assumption of $\mu$-$e$ universality, as described near the end of the note.

***$K_{\mu 3}$ Experiments***. Analyses of $K_{\mu 3}$ data frequently assume a linear dependence of $f_+$ and $f_-$ on $t$, *i.e.*,

$$f_\pm(t) = f_\pm(0)\left[1 + \lambda_\pm(t/m_\pi^2)\right] \tag{2}$$

Most $K_{\mu 3}$ data are adequately described by Eq. (2) for $f_+$ and a constant $f_-$ (*i.e.*, $\lambda_- = 0$). There are two equivalent parametrizations commonly used in these analyses:

***(1) $\lambda_+, \xi(0)$ parametrization***. Analyses of $K_{\mu 3}$ data often introduce the ratio of the two form factors

$$\xi(t) = f_-(t)/f_+(t)\ . \tag{3}$$

The $K_{\mu 3}$ decay distribution is then described by the two parameters $\lambda_+$ and $\xi(0)$ (assuming time reversal invariance and $\lambda_- = 0$). These parameters can be determined by three different methods:

***Method A***. By studying the Dalitz plot or the pion spectrum of $K_{\mu 3}$ decay. The Dalitz plot density is (see, *e.g.*, Chounet *et al.* [1]):

$$\rho(E_\pi, E_\mu) \propto f_+^2(t)\left[A + B\xi(t) + C\xi(t)^2\right]\ ,$$

where

$$A = m_K \left(2E_\mu E_\nu - m_K E'_\pi\right) + m_\mu^2 \left(\frac{1}{4}E'_\pi - E_\nu\right) \ ,$$

$$B = m_\mu^2 \left(E_\nu - \frac{1}{2}E'_\pi\right) \ ,$$

$$C = \frac{1}{4}m_\mu^2 E'_\pi \ ,$$

$$E'_\pi = E_\pi^{\rm max} - E_\pi = \left(m_K^2 + m_\pi^2 - m_\mu^2\right)/2m_K - E_\pi \ . \quad (4)$$

Here $E_\pi$, $E_\mu$, and $E_\nu$ are, respectively, the pion, muon, and neutrino energies in the kaon center of mass. The density $\rho$ is fit to the data to determine the values of $\lambda_+, \xi(0)$, and their correlation.

***Method B***. By measuring the $K_{\mu 3}/K_{e3}$ branching ratio and comparing it with the theoretical ratio (see, *e.g.*, Fearing *et al.* [2]) as given in terms of $\lambda_+$ and $\xi(0)$, assuming $\mu$-$e$ universality:

$$\Gamma(K^\pm_{\mu 3})/\Gamma(K^\pm_{e3}) = 0.6457 + 1.4115\lambda_+ + 0.1264\xi(0) + 0.0192\xi(0)^2 + 0.0080\lambda_+\xi(0) \ ,$$

$$\Gamma(K^0_{\mu 3})/\Gamma(K^0_{e3}) = 0.6452 + 1.3162\lambda_+ + 0.1264\xi(0) + 0.0186\xi(0)^2 + 0.0064\lambda_+\xi(0) \ . \quad (5)$$

This cannot determine $\lambda_+$ and $\xi(0)$ simultaneously but simply fixes a relationship between them.

***Method C***. By measuring the muon polarization in $K_{\mu 3}$ decay. In the rest frame of the $K$, the $\mu$ is expected to be polarized in

the direction $\mathbf{A}$ with $\mathbf{P} = \mathbf{A}/\left|\,\mathbf{A}\,\right|$, where $\mathbf{A}$ is given (Cabibbo and Maksymowicz [3]) by

$$\begin{aligned}\mathbf{A} &= a_1(\xi)\mathbf{p}_\mu \\ &- a_2(\xi)\left[\frac{\mathbf{p}_\mu}{m_\mu}\left(m_K - E_\pi + \frac{\mathbf{p}_\pi\cdot\mathbf{p}_\mu}{\left|\,\mathbf{p}_\mu\,\right|^2}(E_\mu - m_\mu)\right) + \mathbf{p}_\pi\right] \\ &+ m_K \mathrm{Im}\xi(t)(\mathbf{p}_\pi\times\mathbf{p}_\mu)\ . \end{aligned} \tag{6}$$

If time-reversal invariance holds, $\xi$ is real, and thus there is no polarization perpendicular to the $K$-decay plane. Polarization experiments measure the weighted average of $\xi(t)$ over the $t$ range of the experiment, where the weighting accounts for the variation with $t$ of the sensitivity to $\xi(t)$.

***(2) $\lambda_+, \lambda_0$ parametrization.*** Most of the more recent $K_{\mu 3}$ analyses have parameterized in terms of the form factors $f_+$ and $f_0$ which are associated with vector and scalar exchange, respectively, to the lepton pair. $f_0$ is related to $f_+$ and $f_-$ by

$$f_0(t) = f_+(t) + \left[t/(m_K^2 - m_\pi^2)\right] f_-(t)\ . \tag{7}$$

Here $f_0(0)$ must equal $f_+(0)$ unless $f_-(t)$ diverges at $t = 0$. The earlier assumption that $f_+$ is linear in $t$ and $f_-$ is constant leads to $f_0$ linear in $t$:

$$f_0(t) = f_0(0)\left[1 + \lambda_0(t/m_\pi^2)\right]\ . \tag{8}$$

With the assumption that $f_0(0) = f_+(0)$, the two parametrizations, $(\lambda_+, \xi(0))$ and $(\lambda_+, \lambda_0)$ are equivalent as long as correlation information is retained. $(\lambda_+, \lambda_0)$ correlations tend to be less strong than $(\lambda_+, \xi(0))$ correlations.

The experimental results for $\xi(0)$ and its correlation with $\lambda_+$ are listed in the $K^\pm$ and $K_L^0$ sections of the Particle Listings

in section $\xi_{\rm A}$, $\xi_{\rm B}$, or $\xi_{\rm C}$ depending on whether method A, B, or C discussed above was used. The corresponding values of $\lambda_+$ are also listed.

Because recent experiments tend to use the $(\lambda_+, \lambda_0)$ parametrization, we include a subsection for $\lambda_0$ results. Wherever possible we have converted $\xi(0)$ results into $\lambda_0$ results and vice versa.

See the 1982 version of this note [4] for additional discussion of the $K^0_{\mu 3}$ parameters, correlations, and conversion between parametrizations.

***$K_{e3}$ Experiments***. Analysis of $K_{e3}$ data is simpler than that of $K_{\mu 3}$ because the second term of the matrix element assuming a pure vector current [Eq. (1) above] can be neglected. Here $f_+$ is usually assumed to be linear in $t$, and the linear coefficient $\lambda_+$ of Eq. (2) is determined.

If we remove the assumption of a pure vector current, then the matrix element for the decay, in addition to the terms in Eq. (1), would contain

$$+2m_K\, f_S\, \overline{\ell}(1+\gamma_5)\nu$$
$$+(2f_T/m_K)(P_K)_\lambda (P_\pi)_\mu\, \overline{\ell}\, \sigma_{\lambda\mu}(1+\gamma_5)\nu\ , \qquad (9)$$

where $f_S$ is the scalar form factor, and $f_T$ is the tensor form factor. In the case of the $K_{e3}$ decays where the $f_-$ term can be neglected, experiments have yielded limits on $|f_S/f_+|$ and $|f_T/f_+|$.

***Fits for $K_{\ell 3}$ Form Factors***. Fits are made for the two parameters $\lambda_+$ and $\lambda_0$ and their correlation, or equivalently for $\lambda_+$ and $\xi(0)$ and their correlation. The fits are done both with and without assuming $\mu$-$e$ universality. This assumption sets $\lambda_+(K_{\mu 3}) = \lambda_+(K_{e3})$ so that the more precise $\lambda_+$ values from $K_{e3}$ can be included with the $K_{\mu 3}$ data in the fit. In addition

it allows the $K_{\mu 3}/K_{e3}$ branching ratio to be used in the determination of $\xi(0)$ or $\lambda_0$ as a function of $\lambda_+$. Without this assumption, $\lambda_+(K_{e3})$ values are averaged separately and excluded from the fit to $K_{\mu 3}$ data, and branching ratio data are not used. The Kaon Particle Listings show the results with and without assuming $\mu$-$e$ universality.

## References


1. L.M. Chounet, J.M. Gaillard, and M.K. Gaillard, Phys. Reports **4C**, 199 (1972).
2. H.W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. **D2**, 542 (1970).
3. N. Cabibbo and A. Maksymowicz, Phys. Lett. **9**, 352 (1964).
4. Particle Data Group, Phys. Lett. **111B**, 73 (1982).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### $K^{\pm}_{\ell 3}$ FORM FACTORS

In the form factor comments, the following symbols are used.

$f_+$ and $f_-$ are form factors for the vector matrix element.

$f_S$ and $f_T$ refer to the scalar and tensor term.

$f_0 = f_+ + f_- t/(m_K^2 - m_\pi^2)$.

$\lambda_+$, $\lambda_-$, and $\lambda_0$ are the linear expansion coefficients of $f_+$, $f_-$, and $f_0$.

$\lambda_+$ refers to the $K^{\pm}_{\mu 3}$ value except in the $K^{\pm}_{e3}$ sections.

$d\xi(0)/d\lambda_+$ is the correlation between $\xi(0)$ and $\lambda_+$ in $K^{\pm}_{\mu 3}$.

$d\lambda_0/d\lambda_+$ is the correlation between $\lambda_0$ and $\lambda_+$ in $K^{\pm}_{\mu 3}$.

$t$ = momentum transfer to the $\pi$ in units of $m_\pi^2$.

DP = Dalitz plot analysis.

PI = $\pi$ spectrum analysis.

MU = $\mu$ spectrum analysis.

POL= $\mu$ polarization analysis.

BR = $K^{\pm}_{\mu 3}/K^{\pm}_{e3}$ branching ratio analysis.

E = positron or electron spectrum analysis.

RC = radiative corrections.

## $\lambda_+$ (LINEAR ENERGY DEPENDENCE OF $f_+$ IN $K^\pm_{e3}$ DECAY)

For radiative correction of $K^\pm_{e3}$ Dalitz plot, see GINSBERG 67 and BECHERRAWY 70.

Results labeled OUR FIT are discussed in the review "$K^\pm_{\ell 3}$ and $K^0_{\ell 3}$ Form Factors" above.

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **2.78 ±0.07 OUR FIT** | Error includes scale factor of 1.5. Assumes $\mu$-$e$ universality. | | | | | |
| **2.77 ±0.05 OUR AVERAGE** | | | | | | |
| 2.774±0.047±0.032 | 919k | [81] YUSHCHENKO | 04B | ISTR | − | |
| 2.78 ±0.26 ±0.30 | 41k | SHIMIZU | 00 | SPEC | + | DP, uses RC |
| 1.8 ±0.7 | 3000 | ARTEMOV | 97B | SPEC | − | DP |
| 2.84 ±0.27 ±0.20 | 32k | [82] AKIMENKO | 91 | SPEC | | PI, no RC |
| 2.9 ±0.4 | 62k | [83] BOLOTOV | 88 | SPEC | | PI, no RC |
| 2.7 ±0.8 | | [84] BRAUN | 73B | HLBC | + | DP, no RC |
| 2.9 ±1.1 | 4017 | CHIANG | 72 | OSPK | + | DP, RC negligible |
| 2.7 ±1.0 | 2707 | STEINER | 71 | HLBC | + | DP, uses RC |
| 4.5 ±1.5 | 1458 | BOTTERILL | 70 | OSPK | | PI, uses RC |
| 4.5 $^{+1.7}_{-1.8}$ | 854 | BELLOTTI | 67B | FBC | + | DP, uses RC |
| 1.6 ±1.6 | 1393 | IMLAY | 67 | OSPK | + | DP, no RC |
| 2.8 $^{+1.3}_{-1.4}$ | 515 | KALMUS | 67 | FBC | + | $e^+$, PI, no RC |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 2.93 ±0.15 ±0.2 | 130k | [85] AJINENKO | 02 | SPEC | | DP, uses RC |
| 2.5 ±0.7 | | [86] BRAUN | 74 | HLBC | + | $K_{\mu 3}/K_{e3}$ vs. $t$ |

[81] This should be considered a mean slope. A significant quadratic non-linearity is observed, with linear and quadratic form factor parameters $\lambda_+ = 0.02324 \pm 0.00152 \pm 0.00032$ and $\lambda'_+ = 0.00084 \pm 0.00027 \pm 0.00031$.

[82] AKIMENKO 91 state that radiative corrections would raise $\lambda_+$ by 0.0013.

[83] BOLOTOV 88 state radiative corrections of GINSBERG 67 would raise $\lambda_+$ by 0.002.

[84] BRAUN 73B states that radiative corrections of GINSBERG 67 would lower $\lambda^e_+$ by 0.002 but that radiative corrections of BECHERRAWY 70 disagrees and would raise $\lambda^e_+$ by 0.005.

[85] Superseded by YUSHCHENKO 04B.

[86] BRAUN 74 is a combined $K_{\mu 3}$-$K_{e3}$ result. It is not independent of BRAUN 73C ($K_{\mu 3}$) and BRAUN 73B ($K_{e3}$) form factor results.

## $\xi_A = f_-/f_+$ (determined from $K^\pm_{\mu 3}$ spectra)

Results labeled OUR FIT are discussed in the review "$K^\pm_{\ell 3}$ and $K^0_{\ell 3}$ Form Factors" above. $\xi_A$ is $\xi(0)$ determined by Method $A$ of that review. The parameter $\xi(0)$ is redundant with $\lambda_0$ below and is not put into the Meson Summary Table.

| VALUE | $d\xi(0)/d\lambda_+$ | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **−0.125±0.023 OUR FIT** | Error includes scale factor of 1.5. Correlation is $d\xi(0)/d\lambda_+ = -16.3$. Assumes $\mu$-$e$ universality. | | | | | | |
| **−0.14 ±0.05 OUR FIT** | Error includes scale factor of 1.8. Correlation is $d\xi(0)/d\lambda_+ = -16.6$. | | | | | | |
| −0.116±0.030 | −17 | 540k | [87] YUSHCHENKO | 04 | ISTR | − | DP |
| +0.54 ±0.39 | −12 | 3000 | [87] ARTEMOV | 97B | SPEC | − | DP |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $-0.27$ | $\pm0.25$ | $-17$ | 3973 | | WHITMAN | 80 | SPEC | + | DP |
| $-0.8$ | $\pm0.8$ | $-20$ | 490 | [88] | ARNOLD | 74 | HLBC | + | DP |
| $-0.57$ | $\pm0.24$ | $-9$ | 6527 | [89] | MERLAN | 74 | ASPK | + | DP |
| $-0.36$ | $\pm0.40$ | $-19$ | 1897 | [90] | BRAUN | 73C | HLBC | + | DP |
| $-0.62$ | $\pm0.28$ | $-12$ | 4025 | [91] | ANKENBRA... | 72 | ASPK | + | PI |
| $+0.45$ | $\pm0.28$ | $-15$ | 3480 | [92] | CHIANG | 72 | OSPK | + | DP |
| $-0.5$ | $\pm0.8$ | $-26$ | 2041 | [93] | KIJEWSKI | 69 | OSPK | + | PI |
| $+0.72$ | $\pm0.93$ | $-17$ | 444 | | CALLAHAN | 66B | FBC | + | PI |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | | | | |
| $-0.14$ | $\pm0.10$ | $-18$ | 112k | [87,94] | AJINENKO | 03 | ISTR | − | DP |
| $-1.1$ | $\pm0.56$ | $-29$ | 3240 | [95] | HAIDT | 71 | HLBC | + | DP |
| $-0.5$ | $\pm0.9$ | none | 78 | | EISLER | 68 | HLBC | + | PI, $\lambda_+$=0 |
| 0.0 | $^{+1.1}_{-0.9}$ | | 2648 | [96] | CALLAHAN | 66B | FBC | + | $\mu$, $\lambda_+$=0 |
| $+0.7$ | $\pm0.5$ | | 87 | | GIACOMELLI | 64 | EMUL | + | MU+BR,$\lambda_+$=0 |
| $-0.08$ | $\pm0.7$ | | | [97] | JENSEN | 64 | XEBC | + | DP+BR |
| $+1.8$ | $\pm0.6$ | | 76 | | BROWN | 62B | XEBC | + | DP+BR, $\lambda_+$=0 |

[87] Calculated from $\lambda_+$, $\lambda_0$, and $d\lambda_0/d\lambda_+$.

[88] ARNOLD 74 figure 4 was used to obtain $\xi_A$ and $d\xi(0)/d\lambda_+$.

[89] MERLAN 74 figure 5 was used to obtain $d\xi(0)/d\lambda_+$.

[90] BRAUN 73C gives $\xi(t) = -0.34 \pm 0.20$, $d\xi(t)/d\lambda_+ = -14$ for $\lambda_+ = 0.027$, $t = 6.6$. We calculate above $\xi(0)$ and $d\xi(0)/d\lambda_+$ for their $\lambda_+ = 0.025 \pm 0.017$.

[91] ANKENBRANDT 72 figure 3 was used to obtain $d\xi(0)/d\lambda_+$.

[92] CHIANG 72 figure 10 was used to obtain $d\xi(0)/d\lambda_+$. Fit had $\lambda_- = \lambda_+$ but would not change for $\lambda_- = 0$. L.Pondrom, (private communication 74).

[93] KIJEWSKI 69 figure 17 was used to obtain $d\xi(0)/d\lambda_+$ and errors.

[94] Superseded by YUSHCHENKO 04.

[95] HAIDT 71 table 8 (Dalitz plot analysis) gives $d\xi(0)/d\lambda_+ = (-1.1+0.5)/(0.050-0.029) = -29$, error raised from 0.50 to agree with $d\xi(0) = 0.20$ for fixed $\lambda_+$. Not included in fit because of large disagreement with more precise $K_{\mu3}/K_{e3}$ branching ratio measurement.

[96] CALLAHAN 66 table 1 ($\pi$ analysis) gives $d\xi(0)/d\lambda_+ = (0.72–0.05)/(0–0.04) = -17$, error raised from 0.80 to agree with $d\xi(0) = 0.37$ for fixed $\lambda_+$. $t$ unknown.

[97] JENSEN 64 gives $\lambda_+^\mu = \lambda_+^e = -0.020 \pm 0.027$. $d\xi(0)/d\lambda_+$ unknown. Includes SHAKLEE 64 $\xi_B(K_{\mu3}/K_{e3})$.

## $\xi_B = f_-/f_+$ (determined from $K^\pm_{\mu3}/K^\pm_{e3}$)

The $K^\pm_{\mu3}/K^\pm_{e3}$ branching ratio fixes a relationship between $\xi(0)$ and $\lambda_+$ if $\mu$-$e$ universality is assumed. We quote the author's $\xi(0)$ and associated $\lambda_+$ but do not average because the $\lambda_+$ values differ. The result labeled OUR FIT below does not use these $\xi_B$ values. Instead it uses the authors $K^+_{\mu3}/K^+_{e3}$ branching ratios to obtain the fitted $K^\pm_{\mu3}/K^\pm_{e3}$ ratio which is then converted to KL3FIT value below, as discussed in the review "$K^\pm_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above. $\xi_B$ is $\xi(0)$ determined by Method $B$ of

that review. The parameter $\xi(0)$ is redundant with $\lambda_0$ below and is not put into the Meson Summary Table.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **$-0.125\pm0.023$ OUR FIT** Error includes scale factor of 1.5. Correlation is $d\xi(0)/d\lambda_+ = -16.3$. Assumes $\mu$-$e$ universality. | | | | | | |
| $-0.13\ \pm0.06$ | | [98] KL3FIT | 04 | RVUE | | $\lambda_+ = 0.030$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| $-0.12\ \pm0.12$ | 55k | [99] HEINTZE | 77 | CNTR | + | $\lambda_+ = 0.029$ |
| $0.0\ \pm0.15$ | 5825 | CHIANG | 72 | OSPK | + | $\lambda_+ = 0.03$, fig.10 |
| $-0.81\ \pm0.27$ | 1505 | [100] HAIDT | 71 | HLBC | + | $\lambda_+ = 0.028$, fig.8 |
| $-0.35\ \pm0.22$ | | [101] BOTTERILL | 70 | OSPK | + | $\lambda_+ = 0.045 \pm 0.015$ |
| $+0.91\ \pm0.82$ | | ZELLER | 69 | ASPK | + | $\lambda_+ = 0.023$ |
| $-0.08\ \pm0.15$ | 5601 | [101] BOTTERILL | 68B | ASPK | + | $\lambda_+ = 0.023 \pm 0.008$ |
| $-0.60\ \pm0.20$ | 1398 | [100] EICHTEN | 68 | HLBC | + | See note |
| $+1.0\ \pm0.6$ | 986 | GARLAND | 68 | OSPK | + | $\lambda_+ = 0$ |
| $+0.75\ \pm0.50$ | 306 | AUERBACH | 67 | OSPK | + | $\lambda_+ = 0$ |
| $+0.4\ \pm0.4$ | 636 | CALLAHAN | 66B | FBC | + | $\lambda_+ = 0$ |
| $+0.6\ \pm0.5$ | | BISI | 65B | HBC | + | $\lambda_+ = 0$ |
| $+0.8\ \pm0.6$ | 500 | CUTTS | 65 | OSPK | + | $\lambda_+ = 0$ |
| $-0.17\ ^{+0.75}_{-0.99}$ | | SHAKLEE | 64 | XEBC | + | $\lambda_+ = 0$ |

[98] KL3FIT value is from fitted $K^{\pm}_{\mu3}/K^{\pm}_{e3}$ branching ratio. $d\xi(0)/d\lambda_+ = -11.6$.

[99] Calculated by us from $\lambda_0$ and $\lambda_+$ given below.

[100] EICHTEN 68 has $\lambda_+ = 0.023 \pm 0.008$, $t = 4$, independent of $\lambda_-$. Replaced by HAIDT 71.

[101] BOTTERILL 70 is re-evaluation of BOTTERILL 68B with different $\lambda_+$.

## $\xi_C = f_-/f_+$ (determined from $\mu$ polarization in $K^{\pm}_{\mu3}$)

The $\mu$ polarization is a measure of $\xi(t)$. No assumptions on $\lambda_{+-}$ are necessary, but $t$ (weighted by sensitivity to $\xi(t)$) should be specified. In $\lambda_+$, $\xi(0)$ parametrization this is $\xi(0)$ for $\lambda_+ = 0$. $d\xi/d\lambda = \xi t$. For radiative correction to muon polarization in $K^{\pm}_{\mu3}$, see GINSBERG 71. Results labeled OUR FIT are discussed in the review "$K^{\pm}_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above. $\xi_C$ is $\xi(0)$ determined by Method $C$ of that review. The parameter $\xi(0)$ is redundant with $\lambda_0$ below and is not put into the Meson Summary Table.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **$-0.125\pm0.023$ OUR FIT** Error includes scale factor of 1.5. Correlation is $d\xi(0)/d\lambda_+ = -16.3$. Assumes $\mu$-$e$ universality. | | | | | | |
| **$-0.14\ \pm0.05$ OUR FIT** Error includes scale factor of 1.8. Correlation is $d\xi(0)/d\lambda_+ = -16.6$. | | | | | | |
| $-0.95\ \pm0.3$ | 3133 | [102] CUTTS | 69 | OSPK | + | Total pol. $t$=4.0 |
| $-1.0\ \pm0.3$ | 6000 | [103] BETTELS | 68 | HLBC | + | Total pol. $t$=4.9 |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| $-0.25\ \pm1.20$ | 1585 | [104] BRAUN | 75 | HLBC | + | POL, $t$=4.2 |
| $-0.64\ \pm0.27$ | 40k | [105] MERLAN | 74 | ASPK | + | POL, $d\xi(0)/d\lambda_+ = +1.7$ |

[102] CUTTS 69 $t = 4.0$ was calculated from figure 8. $d\xi(0)/d\lambda_+ = \xi t = -0.95\times4 = -3.8$.

[103] BETTELS 68 $d\xi(0)/d\lambda_+ = \xi t = -1.0\times4.9 = -4.9$.

[104] BRAUN 75 $d\xi(0)/d\lambda_+ = \xi t = -0.25\times4.2 = -1.0$.

[105] MERLAN 74 polarization result (figure 5) not possible. See discussion of polarization experiments in note on "$K_{\ell3}$ Form Factors" in the 1982 edition of this *Review* [Physics Letters **111B** (1982)].

## $\lambda_+$ (LINEAR ENERGY DEPENDENCE OF $f_+$ IN $K^\pm_{\mu3}$ DECAY)

See also the corresponding entries and footnotes in sections $\xi_A$, $\xi_C$, and $\lambda_0$. For radiative correction of $K^\pm_{\mu3}$ Dalitz plot, see GINSBERG 70 and BECHERRAWY 70. Results labeled OUR FIT are discussed in the review "$K^\pm_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above.

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **2.78±0.07 OUR FIT** | Error includes scale factor of 1.5. Assumes $\mu$-$e$ universality. | | | | | |
| **2.84±0.27 OUR FIT** | Error includes scale factor of 1.8. | | | | | |
| 2.77±0.13±0.09 | 540k | YUSHCHENKO | 04 | ISTR | − | DP |
| 1.4 ±2.4 | 3000 | ARTEMOV | 97B | SPEC | − | DP |
| 5.0 ±1.3 | 3973 | WHITMAN | 80 | SPEC | + | DP |
| 2.5 ±3.0 | 490 | ARNOLD | 74 | HLBC | + | DP |
| 2.7 ±1.9 | 6527 | MERLAN | 74 | ASPK | + | DP |
| 2.5 ±1.7 | 1897 | BRAUN | 73C | HLBC | + | DP |
| 2.4 ±1.9 | 4025 | [106] ANKENBRA... | 72 | ASPK | + | PI |
| −0.6 ±1.5 | 3480 | CHIANG | 72 | OSPK | + | DP |
| 0.9 ±2.6 | 2041 | KIJEWSKI | 69 | OSPK | + | PI |
| 0.0 ±5.0 | 444 | CALLAHAN | 66B | FBC | + | PI |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 3.21±0.45 | 112k | [107] AJINENKO | 03 | ISTR | − | DP |
| 2.9 ±2.4 | 3000 | [108] ARTEMOV | 97 | SPEC | − | DP |
| 5.0 ±1.8 | 3240 | [109] HAIDT | 71 | HLBC | + | DP |

[106] ANKENBRANDT 72 $\lambda_+$ from figure 3 to match $d\xi(0)/d\lambda_+$. Text gives $0.024 \pm 0.022$.

[107] Superseded by YUSHCHENKO 04.

[108] Superseded by ARTEMOV 97B.

[109] Not included in fit because of large discrepancy in $K_{\mu3}/K_{e3}$ branching ratio with more precise experiments.

## $\lambda_0$ (LINEAR ENERGY DEPENDENCE OF $f_0$ IN $K^\pm_{\mu3}$ DECAY)

Wherever possible, we have converted the above values of $\xi(0)$ into values of $\lambda_0$ using the associated $\lambda^\mu_+$ and $d\xi/d\lambda$ . Results labeled OUR FIT are discussed in the review "$K^\pm_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above.

| VALUE (units $10^{-2}$) | $d\lambda_0/d\lambda_+$ | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **1.74±0.22 OUR FIT** | Error includes scale factor of 1.8. Correlation is $d\lambda_0/d\lambda_+ = -0.34$. | | | | | | |
| **1.77±0.16 OUR FIT** | Error includes scale factor of 1.5. Correlation is $d\lambda_0/d\lambda_+ = -0.32$. Assumes $\mu$-$e$ universality. | | | | | | |
| +2.0 ±0.5 | +0.06 | | [110] KL3FIT | 04 | RVUE | | $\lambda_+ = 0.030$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $+1.83 \pm 0.11 \pm 0.06$ | $-0.348$ | 540k | | YUSHCHENKO | 04 | ISTR | $-$ | DP |
| $+5.8 \pm 2.0$ | $+0.0$ | 3000 | [111] | ARTEMOV | 97B | SPEC | $-$ | DP |
| $+2.9 \pm 1.1$ | $-0.37$ | 3973 | | WHITMAN | 80 | SPEC | $+$ | DP |
| $-4.0 \pm 4.0$ | $-0.62$ | 490 | | ARNOLD | 74 | HLBC | $+$ | DP |
| $-1.9 \pm 1.5$ | $+0.27$ | 6527 | [112] | MERLAN | 74 | ASPK | $+$ | DP |
| $-0.8 \pm 2.0$ | $-0.53$ | 1897 | [113] | BRAUN | 73C | HLBC | $+$ | DP |
| $-2.6 \pm 1.3$ | $+0.03$ | 4025 | [114] | ANKENBRA... | 72 | ASPK | $+$ | PI |
| $+3.0 \pm 1.4$ | $-0.21$ | 3480 | [114] | CHIANG | 72 | OSPK | $+$ | DP |
| $-5.6 \pm 2.4$ | $+0.69$ | 3133 | [115] | CUTTS | 69 | OSPK | $+$ | POL |
| $-3.1 \pm 4.5$ | $-1.10$ | 2041 | [114] | KIJEWSKI | 69 | OSPK | $+$ | PI |
| $-6.3 \pm 2.4$ | $+0.60$ | 6000 | [115] | BETTELS | 68 | HLBC | $+$ | POL |
| $+5.8 \pm 3.6$ | $-0.37$ | 444 | [114] | CALLAHAN | 66B | FBC | $+$ | PI |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | | | |
| $+2.09 \pm 0.45$ | $-0.46$ | 112k | [116] | AJINENKO | 03 | ISTR | $-$ | DP |
| $+1.9 \pm 0.64$ | | 24k | [117] | HORIE | 01 | SPEC | $+$ | BR |
| $+6.2 \pm 2.4$ | $+0.0$ | 3000 | [118] | ARTEMOV | 97 | SPEC | $-$ | DP |
| $+1.9 \pm 1.0$ | $+0.03$ | 55k | [119] | HEINTZE | 77 | SPEC | $+$ | BR |
| $+0.8 \pm 9.7$ | $+0.92$ | 1585 | [115] | BRAUN | 75 | HLBC | $+$ | POL |
| $-1.7 \pm 1.1$ | | | [120] | BRAUN | 74 | HLBC | $+$ | $K_{\mu 3}/K_{e3}$ vs. $t$ |
| $-3.9 \pm 2.9$ | $-1.34$ | 3240 | [114] | HAIDT | 71 | HLBC | $+$ | DP |

[110] KL3FIT 04 value is from our fitted value of the $K^{\pm}_{\mu 3}/K^{\pm}_{e3}$ branching ratio. Assumes $\mu - e$ universality.

[111] ARTEMOV 97B does not give $d\lambda_0/d\lambda_+$ so we take it to be zero.

[112] MERLAN 74 $\lambda_0$ and $d\lambda_0/d\lambda_+$ were calculated by us from $\xi_A$, $\lambda^{\mu}_{+}$, and $d\xi(0)/d\lambda_+$. Their figure 6 gives $\lambda_0 = -0.025 \pm 0.012$ and no $d\lambda_0/d\lambda_+$.

[113] This value and error are taken from BRAUN 75 but correspond to the BRAUN 73C $\lambda^{\mu}_{+}$ result. $d\lambda_0/d\lambda_+$ is from BRAUN 73C $d\xi(0)/d\lambda_+$ in $\xi_A$ above.

[114] $\lambda_0$ calculated by us from $\xi(0)$, $\lambda^{\mu}_{+}$, and $d\xi(0)/d\lambda_+$.

[115] $\lambda_0$ value is for $\lambda_+ = 0.03$ calculated by us from $\xi(0)$ and $d\xi(0)/d\lambda_+$.

[116] Superseded by YUSHCHENKO 04.

[117] HORIE 01 assumes $\mu$-$e$ universality in $K^{+}_{\ell 3}$ decay and uses SHIMIZU 00 value $\lambda$=0.0278$\pm$ 0.0040 from $K^{\pm}_{e3}$ decay. Enters fit via $K_{\mu 3}/K_{e3}$ branching ratio.

[118] ARTEMOV 97 does not give $d\lambda_0/d\lambda_+$ so we take it to be zero. Superseded by ARTEMOV 97B.

[119] HEINTZE 77 uses $\lambda_+ = 0.029 \pm 0.003$. $d\lambda_0/d\lambda_+$ estimated by us. Enters fit via $K_{\mu 3}/K_{e3}$ branching ratio.

[120] BRAUN 74 is a combined $K_{\mu 3}$-$K_{e3}$ result. It is not independent of BRAUN 73C ($K_{\mu 3}$) and BRAUN 73B ($K_{e3}$) form factor results.

## $|f_S/f_+|$ FOR $K^{\pm}_{e3}$ DECAY

Ratio of scalar to $f_+$ couplings.

| VALUE (units $10^{-2}$) | CL% | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **$-0.3 \pm {}^{0.8}_{0.7}$ OUR AVERAGE** | | | | | | | |
| $-0.37 \pm {}^{0.66}_{0.56} \pm 0.41$ | | 919k | YUSHCHENKO | 04B | ISTR | $-$ | |
| $0.2 \pm 2.6 \pm 1.4$ | | 41k | SHIMIZU | 00 | SPEC | $+$ | $\lambda_+$, $f_S$, $f_T$ fit |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| – | 1.9 | $^{+\ 2.5}_{-\ 1.6}$ | | | 130k | [121] AJINENKO | 02 | SPEC | | $\lambda_+$, $f_S$ fit |
| | 7.0 | ± 1.6 | ±1.6 | | 32k | AKIMENKO | 91 | SPEC | | $\lambda_+$, $f_S$, $f_T$, $\phi$ fit |
| | 0 | ±10 | | | 2827 | [122] BRAUN | 75 | HLBC | + | |
| < | 13 | | | 90 | 4017 | CHIANG | 72 | OSPK | + | |
| | 14 | $^{+\ 3}_{-\ 4}$ | | | 2707 | [122] STEINER | 71 | HLBC | + | $\lambda_+$, $f_S$, $f_T$, $\phi$ fit |
| < | 23 | | | 90 | | BOTTERILL | 68C | ASPK | | |
| < | 18 | | | 90 | | BELLOTTI | 67B | HLBC | | |
| < | 30 | | | 95 | | KALMUS | 67 | HLBC | + | |

[121] Superseded by YUSHCHENKO 04B.
[122] Statistical errors only.

## $|f_T/f_+|$ FOR $K^\pm_{e3}$ DECAY

Ratio of tensor to $f_+$ couplings.

| VALUE (units $10^{-2}$) | CL% | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **– 1.2± 2.3 OUR AVERAGE** | | | | | | |
| – 1.2± 2.1± 1.1 | | 919k | YUSHCHENKO 04B | ISTR | – | |
| 1 ±14 ± 9 | | 41k | SHIMIZU 00 | SPEC | + | $\lambda_+$, $f_S$, $f_T$ fit |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| – | 4.5 | $^{+\ 6.0}_{-\ 5.7}$ | | | 130k | [123] AJINENKO | 02 | SPEC | | $\lambda_+$, $f_T$ fit |
| | 53 | $^{+\ 9}_{-10}$ | ±10 | | 32k | AKIMENKO | 91 | SPEC | | $\lambda_+$, $f_S$, $f_T$, $\phi$ fit |
| | 7 | ±37 | | | 2827 | [124] BRAUN | 75 | HLBC | + | |
| < | 75. | | | 90 | 4017 | CHIANG | 72 | OSPK | + | |
| | 24 | $^{+16}_{-14}$ | | | 2707 | [124] STEINER | 71 | HLBC | + | $\lambda_+$, $f_S$, $f_T$, $\phi$ fit |
| < | 58. | | | 90 | | BOTTERILL | 68C | ASPK | | |
| < | 58. | | | 90 | | BELLOTTI | 67B | HLBC | | |
| < | 110. | | | 95 | | KALMUS | 67 | HLBC | + | |

[123] Superseded by YUSHCHENKO 04B.
[124] Statistical errors only.

## $f_S/f_+$ FOR $K^\pm_{\mu3}$ DECAY

Ratio of scalar to $f_+$ couplings.

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|---|---|---|---|---|---|
| **0.17±0.14±0.54** | 540k | [125] YUSHCHENKO 04 | ISTR | – | DP |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.4 | ±0.5 | ±0.5 | 112k | [126] AJINENKO | 03 | ISTR | – | DP |

[125] The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for $\lambda_0$, ±0.0053, combined in quadrature with the systematic error ±0.0009.
[126] The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for $\lambda_0$. Superseded by YUSHCHENKO 04.

## $f_T/f_+$ FOR $K^\pm_{\mu 3}$ DECAY

Ratio of tensor to $f_+$ couplings.

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **$-0.1 \pm 0.7$ OUR AVERAGE** | | | | | | |
| $-0.07 \pm 0.71 \pm 0.20$ | 540k | YUSHCHENKO | 04 | ISTR | − | DP |
| 2 $\pm 12$ | 1585 | BRAUN | 75 | HLBC | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| $-2.1 \pm 2.8 \pm 1.4$ | 112k | [127] AJINENKO | 03 | ISTR | − | DP |

[127] The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for $\lambda_0$. Superseded by YUSHCHENKO 04.

## DECAY FORM FACTORS FOR $K^\pm \to \pi^+\pi^- e^\pm \nu_e$

Given in PISLAK 01, ROSSELET 77, BEIER 73, and BASILE 71C.

## DECAY FORM FACTOR FOR $K^\pm \to \pi^0\pi^0 e^\pm \nu$

Given in BOLOTOV 86B and BARMIN 88B.

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# $K^\pm \to \ell^\pm \nu\gamma$ FORM FACTORS

For definitions of the axial-vector $F_A$ and vector $F_V$ form factor, see the "Note on $\pi^\pm \to \ell^\pm \nu\gamma$ and $K^\pm \to \ell^\pm \nu\gamma$ Form Factors" in the $\pi^\pm$ section. In the kaon literature, often different definitions $a_K = F_A/m_K$ and $v_K = F_V/m_K$ are used.

## $F_A + F_V$, SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to e\nu_e\gamma$

| VALUE | EVTS | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| **$0.148 \pm 0.010$ OUR AVERAGE** | | | | |
| $0.147 \pm 0.011$ | 51 | [128] HEINTZE | 79 | SPEC |
| $0.150^{+0.018}_{-0.023}$ | 56 | [129] HEARD | 75 | SPEC |

[128] HEINTZE 79 quotes absolute value of $|F_A + F_V| \sin\theta_c$. We use $\sin\theta_c = V_{us} = 0.2205$.
[129] HEARD 75 quotes absolute value of $|F_A + F_V| \sin\theta_c$. We use $\sin\theta_c = V_{us} = 0.2205$.

## $F_A + F_V$, SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to \mu\nu_\mu\gamma$

| VALUE | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **$0.165 \pm 0.007 \pm 0.011$** | | 2588 | [130] ADLER | 00B | B787 | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| $-1.2$ to 1.1 | | 90 | DEMIDOV | 90 | XEBC | |
| $< 0.23$ | | 90 | [130] AKIBA | 85 | SPEC | |

[130] Quotes absolute value. Sign not determined.

## $F_A - F_V$, DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to e\nu_e\gamma$

| VALUE | EVTS | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| **$<0.49$** | 90 | [131] HEINTZE | 79 | SPEC |

[131] HEINTZE 79 quotes $|F_A - F_V| < \sqrt{11}\,|F_A + F_V|$.

## $F_A - F_V$, DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to \mu\nu_\mu\gamma$

| VALUE | CL% | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|---|
| **−0.24 to 0.04** | 90 | 2588 | ADLER | 00B | B787 | + |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| −2.2 to 0.6 | 90 | | DEMIDOV | 90 | XEBC | |
| −2.5 to 0.3 | 90 | | AKIBA | 85 | SPEC | |

## $K^\pm$ CHARGE RADIUS

| VALUE (fm) | DOCUMENT ID | | COMMENT |
|---|---|---|---|
| **0.560±0.031 OUR AVERAGE** | | | |
| 0.580±0.040 | AMENDOLIA | 86B | $Ke \to Ke$ |
| 0.530±0.050 | DALLY | 80 | $Ke \to Ke$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| 0.620±0.037 | BLATNIK | 79 | VMD + dispersion relations |

## CP VIOLATION TESTS IN $K^+$ AND $K^-$ DECAYS

$$\Delta(K^\pm_{\pi\mu\mu}) = \frac{\Gamma(K^+_{\pi\mu\mu}) - \Gamma(K^-_{\pi\mu\mu})}{\Gamma(K^+_{\pi\mu\mu}) + \Gamma(K^-_{\pi\mu\mu})}$$

| VALUE | DOCUMENT ID | | TECN |
|---|---|---|---|
| **−0.02±0.11±0.04** | PARK | 02 | HYCP |

## T VIOLATION TESTS IN $K^+$ AND $K^-$ DECAYS

### $P_T$ in $K^+ \to \pi^0\mu^+\nu_\mu$

T-violating muon polarization. Sensitive to new sources of CP violation beyond the Standard Model.

| VALUE (units $10^{-3}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **−4.2±4.9±0.9** | 3.9M | ABE | 99S | K246 | + |

### $P_T$ in $K^+ \to \mu^+\nu_\mu\gamma$

T-violating muon polarization. Sensitive to new sources of CP violation beyond the Standard Model.

| VALUE (units $10^{-2}$) | EVTS | DOCUMENT ID | | TECN | CHG |
|---|---|---|---|---|---|
| **−0.64±1.85±0.10** | 114k | [132] ANISIMOVSKY03 | | K246 | + |

[132] Muons stopped and polarization measured from decay to positrons.

### Im($\xi$) in $K^+ \to \pi^0\mu^+\nu_\mu$ DECAY (from transverse $\mu$ pol.)

Test of $T$ reversal invariance.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **−0.014±0.014 OUR AVERAGE** | | | | | | |
| −0.013±0.016±0.003 | 3.9M | ABE | 99S | CNTR | + | $p_T$ $K^+$ at rest |
| −0.016±0.025 | 20M | CAMPBELL | 81 | CNTR | + | Pol. |

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