DETERMINATION OF $|V_{ub}|$

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The precise determination of the magnitude of $|V_{ub}|$ with a robust, well-understood uncertainty remains one of the key goals of the heavy flavor physics programs, both experimentally and theoretically. Because $|V_{ub}|$, the smallest element in the CKM mixing matrix, provides a bound on the upper vertex of one of the triangles representing the unitarity property of the CKM matrix, it plays a crucial role in the examination of the unitarity constraints and the fundamental questions on which the constraints can bear [1–2]. Investigation of these issues requires measurements that are precise and that have well-understood uncertainties.

Since the initial observation of the $b \to u$ transition by CLEO [3] and ARGUS [4] over a decade ago, we have made great strides both in defining and performing new measurements, and in evaluating the related uncertainties in the extraction of $|V_{ub}|$.

The charmless semi-leptonic (S.L.) decay channel $b \to u\ell\overline{\nu}$ provides the cleanest path for the determination of $|V_{ub}|$. However, the theory for the heavy-to-light $b \to u$ transition cannot be as well-constrained as that for the heavy-to-heavy $b \to c$ transition used in the determination of $|V_{cb}|$ [5]. The extraction of $|V_{ub}|$ and the interplay between experimental measurements and their theoretical interpretation are further complicated by the large background from $b \to c\ell\overline{\nu}$ decay, which has a rate about 60 times higher than that for charmless S.L. decay. Measurements based both on exclusive decay channels and on inclusive techniques have been, and are being, pursued.

We will review the current determinations of $|V_{ub}|$ by CLEO and the LEP experiments within this overall context, and outline the potential for precise determinations of $|V_{ub}|$ at the B factories.

Exclusive determinations: Reconstruction of exclusive $b \to u\ell\overline{\nu}$ channels provides powerful kinematic constraints for suppression of the $b \to c\ell\overline{\nu}$ background. For this suppression to be effective, an estimate of the four momenta of the undetected

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neutrino must be provided. The measurements to date have made use of detector hermeticity and the well-determined beam parameters to define a missing momentum that is defined as the neutrino momentum. Signal-to-background ratios (S/B) of order one have been obtained in these channels.

To extract $|V_{ub}|$ from an exclusive channel, the form factors for that channel must be known. The form factor normalization dominates the uncertainty on $|V_{ub}|$. The q^2 dependence of the form factors, which is needed to determine the experimental efficiency, also contributes to the uncertainty, but at a much reduced level. For example, the requirement of a stiff lepton for background reduction in these analyses introduces a q^2 dependence to the efficiency. In the limit of a massless charged lepton (a reasonable limit for the electron and muon decay channels), the $B \to \pi\ell\nu$ decay depends on one form factor $f_1(q^2)$:

$$\frac{d\Gamma(B^0 \to \pi^-\ell^+\nu)}{dy\, d\cos\theta_\ell} = |V_{ub}|^2 \frac{G_F^2 p_\pi^3 M_B^2}{32\pi^3} \sin^2\theta_\ell |f_1(q^2)|^2, \qquad (1)$$

where $y = q^2/M_B^2$, and θ_ℓ is the angle between the charged lepton direction in the virtual W $(\ell + \nu)$ rest frame and the direction of the virtual W. For the vector meson final states ρ and ω, three form factors, A_1, A_2 and V, are necessary (see *e.g.*, Ref. 6).

Calculation of these form factors constitutes a considerable theoretical industry, with a variety of techniques now being employed. Form factors based on lattice calculations [11–23], and on light cone sum rules [24–32], currently have uncertainties in the 15% to 20% range. A variety of quark model calculations exist [33–47]. Finally, a number of other approaches [48–53], such as dispersive bounds and experimentally-constrained models based on Heavy Quark Symmetry, seek to improve the q^2 range over which the form factors can be estimated without introduction of a significant model dependence. Unfortunately, all these calculations currently have contributions to the uncertainty that remain uncontrolled. The light cone sum rules calculations assume quark-hadron duality, offering a "canonical" contribution to the uncertainty of 10%, but with no known means of rigorously limiting that uncertainty. The lattice calculations to date remain in the "quenched" approximation (no light quark loops in the propagators), which limits the ultimate precision to the 15% to 20% range. For the quark model calculations, there exists no means for systematic evaluation of the uncertainties.

There have been two exclusive $|V_{ub}|$ analyses by the CLEO Collaboration: a simultaneous measurement of the $B \to \pi\ell\overline{\nu}$ and the $B \to \rho\ell\overline{\nu}$ transitions [9], and a second measurement of the $B \to \rho\ell\overline{\nu}$ rate [10]. Both measurements employ the missing energy and momentum to estimate the neutrino momentum. With that technique, the major background results from $b \to c\ell\overline{\nu}$ decays in events that cannot be properly reconstructed (for example, because of additional neutrinos in the event). Both measurements also employ the isospin relations

$$\Gamma(B^0 \to \pi^-\ell^+\nu) = 2\Gamma(B^+ \to \pi^0\ell^+\nu)$$

and

$$\Gamma(B^0 \to \rho^-\ell^+\nu) = 2\Gamma(B^+ \to \rho^0\ell^+\nu) \tag{2}$$

to combine the charged and neutral decays. In the original method, strict event quality requirements were made that resulted in a low efficiency, but a relatively low background to signal ratio over a fairly broad lepton momentum range. The branching fractions obtained were

$$\mathrm{B}(B^0 \to \pi^-\ell^+\nu) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}$$

and

$$\mathrm{B}(B^0 \to \rho^-\ell^+\nu) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4}\ . \tag{3}$$

The second analysis loosened the event cleanliness requirements, resulting in a much higher efficiency. The efficiency gain comes at the price of an increased background, and the analysis was primarily sensitive to signal with lepton momenta above 2.3 GeV/c, which is near (and beyond) the kinematic endpoint for $b \to c\ell\overline{\nu}$ decays, which are therefore highly suppressed. This analysis obtained

$$\mathrm{B}(B^0 \to \rho^-\ell^+\nu) = (2.69 \pm 0.41^{+0.35}_{-0.40} \pm 0.50) \times 10^{-4}\ . \tag{4}$$

The results of the two analyses are largely statistically independent, and they have been combined, accounting for correlated uncertainties, to obtain:

$$|V_{ub}| = (3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55) \times 10^{-3} \ , \qquad (5)$$

where the errors arise from statistical, experimental systematic, and form factor uncertainties, respectively. The last term has been estimated by comparing a large number of available models and, for this average, the earlier analysis was updated to the set of models used in the later analysis. A potential non-resonant $\pi\pi\ell\nu$ contribution (assumed to be zero in the analyses) results in the asymmetric systematic uncertainty. The model dependence uncertainty is dominated by the overall normalization. Evaluation of the systematic considered both the spread among individual models and calculations, as well as the uncertainties claimed for the calculations. The central value spread and the estimated uncertainties were of the same order (roughly 15%).

The B factories have recently released very preliminary results of their first analyses of these exclusive modes. Belle has produced a $B \to \pi\ell\nu$ analysis [54] that is very similar to the original CLEO exclusive analysis [9]. They find

$$\mathrm{B}(B^0 \to \pi^- \ell^+ \nu) = (1.28 \pm 0.20 \pm 0.26 \pm \sigma_{\mathrm{model}}) \times 10^{-4} \ . \quad (6)$$

BABAR has recently presented preliminary results for a $B \to \rho\ell\nu$ analysis [55] that is quite similar to the second CLEO analysis [10], for which they obtained

$$\mathrm{B}(B^0 \to \rho^- \ell^+ \nu) = (2.97 \pm 0.56^{+0.48}_{-0.56} \pm \sigma_{\mathrm{model}}) \times 10^{-4} \ . \quad (7)$$

Both experiments use neutrino reconstruction, but have not yet advanced the detailed event cleanup (see Ref. 56) to the level of CLEO. The uncertainties, which are comparable to the original CLEO errors despite the much larger integrated luminosity, reflect this preliminary situation. This situation will certainly improve as the experiments mature.

The future for exclusive determinations of $|V_{ub}|$ appears promising. Unquenched lattice calculations begin to be feasible, and this will eliminate the primary source of uncontrolled uncertainty in these calculations. Simultaneously, the B factories are performing very well, and very large samples of events in which one B meson has been fully reconstructed will be available. This will allow a more robust determination of the neutrino momentum, and should allow a significant reduction of backgrounds and experimental systematic uncertainties. The high statistics should also allow detailed measurements of $d\Gamma/dq^2$, which will provide a sorely-needed litmus test for the form factor calculations, and will make the form factor shape contribution to the uncertainty on $|V_{ub}|$ negligible. Should theory allow use of the full range of q^2 in the extraction of $|V_{ub}|$, the B factories have already logged data sufficient for a 5% statistical determination of $|V_{ub}|$. If the data must be restricted to low hadronic recoil momentum (large q^2), an order of magnitude more data would be necessary.

For both lattice and the B factories, $\pi\ell\nu$ appears to be a golden mode for future precise determination of $|V_{ub}|$. The one caveat is management of contributions from the B^* pole, but recent work [21] suggests that this problem can be successfully overcome. $B \to \eta\ell\nu$ will provide a valuable cross-check. The $\rho\ell\nu$ mode will be more problematic for high precision: the broad width introduces both experimental and theoretical difficulties. Experiments must, for example, assess potential nonresonant $\pi\pi$ contributions, but only crude arguments based on isospin and quark-popping have been brought to bear to date. Theoretically, no calculation, including lattice, has dealt with the width of the ρ. Even worse, when the lattice calculations become unquenched, the ρ will become unstable, and the $\pi\pi$ final state must be faced by the calculations. The methodology for accommodation of two particle final states on the lattice remains quite primitive, very costly, and works only for low-energy states, so it may be unsuitable for ρ decay. One could put the ρ “in a box” to prevent its decay, but this introduces uncertainties of order width/mass [57]. Fortunately, the $\omega\ell\nu$ mode provides an excellent alternative to the ρ mode, though it has remained elusive to date. Agreement between accurate

$|V_{ub}|$ determinations from $\pi\ell\nu$ and from $\omega\ell\nu$ will provide added confidence in both.

Inclusive determinations: In principle, the fully inclusive rate BR($b \to X_u\ell\overline{\nu}$) can be calculated quite reliably within the OPE framework [58–61], with a $\simeq 6\%$ theoretical uncertainty in $|V_{ub}|$ attainable. The calculations find

$$|V_{ub}| = 0.00445 \times \left(\frac{\mathrm{B}(b \to u\ell\overline{\nu})}{0.002} \frac{1.55\mathrm{ps}}{\tau_b} \right)^{1/2} \times (1 \pm 0.020 \pm 0.052)\,, \tag{8}$$

where the first error arises from uncertainties in the OPE expansion, and the second from uncertainty in the b quark mass, for which $m_b^{\mathrm{kin}}(1\ \mathrm{GeV}) = (4.58 \pm 0.09)$ GeV has been assumed. With the large number of final states available over a broad mass range, deviations from global quark-hadron duality are expected to be small for the total charmless S.L. rate.

Unfortunately, realizing this accuracy is extremely difficult in practice. The background from $b \to c\ell\overline{\nu}$ decays forces experiments to limit their sensitivity within some restricted region of the total phase space. These regions are the lepton energy endpoint $E_\ell > (M_B^2 - M_D^2)/(2M_B)$, the low hadronic mass region $M_X < M_D$, and the large dilepton mass $q^2 > (M_B - M_D)^2$. They select $\simeq 15\%$, 70%, and 20% of the charmless S.L. rate, respectively. The typical S/B ratios achieved within these regions are a factor of 5–10 smaller than those for the exclusive analyses. These restrictions introduce additional uncertainties in the calculation of the total charmless branching fraction that are difficult to quantify. In addition, they may end up moving the quark-duality assumption away from the well-grounded global assumption towards a local assumption. While the limitations of the quark-hadron duality assumption are expected to be quite small for fully integrated rates, they may become more pronounced in partially integrated rates. The applicability of general results for the inclusive OPE approach, and the control of the theoretical uncertainties for measurements restricted within these limited regions, remain the subject of ongoing debate within the community.

The original observations of the $b \to u$ transition at the $\Upsilon(4S)$ [3,4] were inclusive analyses that focused on leptons in the endpoint region of the single lepton spectrum, beyond the kinematical limit for compatibility with $b \to c\ell\overline{\nu}$ transitions. The ACCMM [7] and ISGW [35] models were used to estimate the rate into the endpoint, from which $|V_{ub}/V_{cb}| = (0.08 \pm 0.02)$ was obtained, where the 25% error is dominated by the theoretical uncertainty. That the error "guesstimated" with those models happened to give an error that was reasonably appropriate was a historical accident. Had the ISGW II model [36] been available at that time (and used), the model dependence would have been significantly underestimated [8]. The theoretical uncertainty for an endpoint analysis has been very difficult to quantify. Because the endpoint region extends beyond the partonic endpoint, and the size of the endpoint is of order Λ_{QCD}, an infinite series of terms in the OPE rate calculation become equally important. The leading twist singularities can be resummed into a structure function [75–78]. This shape function encapsulates the "Fermi motion" of the b quark within the hadron, and must be evaluated when experimental determinations of $|V_{ub}|$ are forced near the boundary of a kinematic distribution [73].

A number of authors observed that selection of S.L. decays $b \to X\ell\overline{\nu}$, with hadronic mass M_X below that of the D meson, provides a separation of the charmless $X_u\ell\overline{\nu}$ signal from the $X_c\ell\overline{\nu}$ background, with an efficiency that can be reliably estimated [62,68,69,73]. These observations motivated an intense effort into such inclusive analyses at LEP. The significant B-hadron boost, and the containment of its decay products into a narrow jet in $Z^0 \to b\overline{b}$ events, make these measurements at the Z^0 pole interestingly complementary to those performed at the $\Upsilon(4S)$ peak. Over the past several years, the ALEPH [63], DELPHI [64], L3 [65], and, most recently, OPAL [66] collaborations, have published inclusive measurements of the $b \to u\ell\overline{\nu}$ rate. Three separate methods have been employed. The ALEPH and OPAL analyses use neural networks that take as input a large number (twenty in the case of ALEPH, and seven in the OPAL analysis) of kinematic variables which provide separation between $b \to c\ell\nu$ and $b \to u\ell\nu$ decays. The signal is extracted in both cases from a fit to the network output discriminant, restricted to a region enriched in signal decays. L3 applies a sequential-cut analysis, using the kinematics of the lepton and of the leading hadron in the same jet for discrimination of the signal events. The DELPHI analysis performs an inclusive reconstruction of the hadronic mass of the system emitted together with the lepton in the b-hadron decay. The S.L. B sample is split into $b \to u$-enriched and -depleted samples, based on the separation between tertiary and secondary vertices (making use of the finite charm lifetime), and on the presence of tagged kaons in the final state. The mass of the hadronic system M_X is used to further subdivide the sample into a $b \to X_u \ell \nu$–favored region ($M_X < 1.6$ GeV) and a $b \to X_c \ell \nu$–dominated region. The signal is extracted from a simultaneous fit to the number of decays classified according to the four different categories and the distributions of the lepton energy in the reconstructed B-rest frame. While the approaches of the various LEP analyses differ, they tend to be sensitive to $b \to u \ell \overline{\nu}$, primarily when the mass of the hadronic system is in the region $M_X \lesssim M_D$: DELPHI explicitly so, ALEPH and OPAL implicitly, in that after their neural net requirement, the efficiency falls noticeably with increasing hadronic mass (they gain some additional sensitivity at higher masses when stiff leptons or hadrons are present in the event). These analyses are sensitive to a significantly larger portion of the phase space than the endpoint analyses, but at the cost of larger backgrounds from $b \to c \ell \overline{\nu}$ decays (see Table 1).

The uncertainty in the determination of the fraction of the total charmless rate selected by a given cut in M_X has been studied by several authors. A major source of uncertainty is represented by the value of the b-quark mass m_b. A relative error of ±15–30% on the inclusive charmless branching fraction, obtained with M_X cut values from M_D down to 1.5 GeV, has been estimated from the uncertainty on the mass (assuming ± 150 MeV) and the kinetic energy of the b quark [73]. Other estimates are compatible with this range [68,69]. For restrictions in the range $M_X^2 < \ m_b \Lambda_{\rm QCD} \approx M_D^2$, the calculation of the inclusive charmless branching fraction is also

Table 1: Summary of $|V_{ub}|$ determinations by experiment. The method, the S/B ratio of the analyses, the result with the statistical+experimental, the $b \to c$ and the published $b \to u$ uncertainties, the fractional systematic uncertainty for the non $b \to u$ contributions, and our evaluation for the range of the estimated theoretical uncertainty (including uncertainty in the shape functions from $b \to s\gamma$) are given.

Exp.	Method	S/B	$\|V_{ub}\|$ $[10^{-3}]$	$\sigma_{b\to c}$ $(\|V_{ub}\|)$	σ_{th} $(\|V_{ub}\|)$
ALEPH	Neural Net	0.07	$4.12 \pm .67 \pm .62 \pm 0.35$	15%	9%
OPAL	Neural Net	0.05	$4.00 \pm .71 \pm .59 \pm 0.40$	15%	10%
DELPHI	M_X	0.10	$4.07 \pm .65 \pm .47 \pm 0.39$	12%	10%
L3	$\pi - \ell$ Cut	0.22	$5.7 \pm 1.0 \pm 1.3 \pm 0.5$	22%	10%
LEP	**Average**		$\mathbf{4.09 \pm 0.37 \pm 0.44 \pm 0.34}$		**9–15%**
CLEO	$\boldsymbol{E_\ell}$ **endpoint**	**0.39**	$\mathbf{4.12 \pm 0.34 \pm 0.44 \pm 0.33}$	**7%**	**10–15%**
CLEO	$\pi(\rho)\ell\overline{\nu}+$ strict ν-rec	2.1 (0.6)	$3.30 \pm 0.4 \pm 0.7$	8%	
CLEO	$\rho\ell\overline{\nu}+$ loose ν-rec	0.7–1.5	$3.23 \pm 0.35 \pm 0.58$	5%	
CLEO	$\boldsymbol{\pi + \rho\ell\overline{\nu}}$ **Average**		$\mathbf{3.25^{+0.25}_{-0.32} \pm 0.55}$		**15–20%**

sensitive to the shape function uncertainties that affect the endpoint region, particularly as the cut is lowered much below M_D [68,69,73](though model-dependent studies suggest that the importance of the shape function may be reduced in the M_X case). Higher-twist contributions and unknown power corrections of order $\Lambda_{\rm QCD}/M_B \approx 10\%$ (for example, corrections to the method of convolution of the parton-level spectra with a shape function [70,71]) also contribute to the uncertainty on $|V_{ub}|$. This leads to an estimate of the overall $b \to u$ systematics on $|V_{ub}|$ extracted with these methods of order 10–15%, which still allows for a largely model-independent determination.

The DELPHI analysis follows this framework with the theoretical uncertainties evaluated within the framework outlined, for example, in Ref. 69. At that time, however, no detailed information regarding the shape function existed, and the experiment relied on models. The neural net analyses are somewhat more difficult to interpret directly, and the experiments rely more heavily on model estimates to gauge the uncertainty. Given that those measurements tend to have sensitivity primarily in the regions affected by the shape functions, a theoretical uncertainty within the 10%–15% window seems likely. An average by the LEP Heavy Flavour Group [72] results in

$$V_{ub} = (4.09^{+0.36}_{-0.39}{}^{+0.42}_{-0.47} \pm 0.25 \pm 0.23) \times 10^{-3}\ . \qquad (9)$$

The first error includes the statistical and detector-level systematic uncertainties, and the second the systematics from the $b \to c$ background. The third error reflects the uncertainty in the extrapolation of the yields measured in the restricted kinematic region to the total charmless S.L. branching fraction determination. The above was obtained based on model studies for each analysis, accounting for the contributions that could be quantified. It profits from the partially uncorrelated sources of systematics that result from the different techniques adopted by the four experiments. The discussion above suggests the more conservative range of ±10–15%. The final error reflects the ±6% uncertainty for extraction of $|V_{ub}|$ from the total charmless S.L. branching fraction.

Observation of semileptonic $b \to u$ decays at LEP has been an experimental tour de force. The successful realization of those analyses is due to some advantages offered by the kinematics at the Z^0 pole, and to the performance of the detectors. While these studies have pioneered a path towards new approaches for extracting $|V_{ub}|$, they have exposed the drawbacks of analyses with S/B ratios that require control of the background level to better than 5% of itself. Some areas of concern here, discussed within the community, include the modeling uncertainties of the non-D and D^* components of the background from B decay, the modeling of the B_s and b-baryon S.L. decays, and the estimate of the $b \to u$ modeling uncertainties due to the uneven sampling of the decay phase space.

CLEO has recently submitted for publication [74] a new measurement based on the lepton endpoint fraction that makes a significant step away from reliance on models for the theoretical uncertainty. It has been known for some time [77,76] that, at leading twist, the same shape function corrects the parton level $b \to s\gamma$ photon spectrum and the $b \to u\ell\nu$ lepton spectrum. While measurements of the non-perturbative $1/m_b^2$ parameters have been made for the heavy-to-heavy $b \to c\ell\overline{\nu}$ transition [79], differences in the higher order corrections in the OPE spoil their application to heavy-to-light transitions. Since both $b \to s\gamma$ and $b \to u\ell\overline{\nu}$ are heavy-to-light transitions, however, one can relate the parameters determined in one system to the other, up to power corrections of order $\Lambda_{\rm QCD}/M_B$ arising from nonlocal operators [71]. It has been suggested that the stability of the extracted $|V_{ub}|$ under variation of the lepton momentum endpoint region can limit the uncertainty due to these corrections [71].

Because the CLEO experiment must account for the distortion of the endpoint spectrum due to the motion of the B mesons, initial-state radiation, and experimental resolution, it fits the observed data using a theoretical momentum spectrum to which the distortions are applied. Several ansatz [82,83] for the form of the shape function were employed. CLEO finds

$$|V_{ub}| = (4.12 \pm 0.34 \pm 0.44 \pm 0.23 \pm 0.24) \times 10^{-3} \ , \qquad (10)$$

based on the lepton momentum range 2.2–2.6 GeV/c. The first error is the combined statistical and experimental uncertainty on the rate into the accepted momentum range. The second error is the uncertainty on the fraction of leptons expected to lie within this range based on the uncertainty, statistical and systematic, in the parameters derived from the $b \to s\gamma$ photon spectrum. The third uncertainty is the same HQE uncertainty on the extraction of $|V_{ub}|$ from the total rate as above. The final error is an estimate of the scale of the uncertainty that results from the unknown power corrections in applying the $b \to s\gamma$ shape function to $b \to u\ell\nu$. In the limit of integration over the full lepton spectrum, this uncertainty would vanish: in fact, as one moves away from the phase space boundary, the importance of the shape function diminishes. To evaluate this uncertainty, the parameters of the shape function were varied by the expected order of the corrections: $\Lambda_{\rm QCD}/M_B \approx 10\%$. This sets the *scale* of the uncertainty, but is not a precise evaluation of the uncertainty—we do not know if the true uncertainty is a factor of two larger or smaller, for example. Variation of the size of the endpoint region results in consistent determinations of $|V_{ub}|$, but the experimental uncertainties limit our ability to make more precise statements regarding the power corrections at this time.

Finally, a method of extracting $|V_{ub}|$ inclusively in a restricted region of q^2, the mass of the leptonic system, has been proposed [84]. This region is free from uncertainty due to the shape functions, but does receive order $1/m_c^3$ corrections [85]. Given the orthogonal uncertainties, this method will provide a valuable crosscheck to other inclusive determinations of $|V_{ub}|$. While this method has been found to be unsuitable for experiments at higher energy, application at the $\Upsilon(4S)$ facilities should be feasible, where resolutions on q^2 of approximately 1 GeV^2 can already be obtained, though with large tails from mis-reconstructed events.

As the B factories bring their full data samples to bear on inclusive measurements of $|V_{ub}|$, there is a potential for considerable progress in $|V_{ub}|$. More precise evaluation of the $b \to s\gamma$ photon spectrum will lead to more precisely determined effective shape functions. With the potential for large samples of events with one fully reconstructed B, reconstruction of the hadronic recoil mass with much reduced contamination from $b \to c\ell\overline{\nu}$ decays and of q^2 should be possible. Hence, statistical and systematic experimental uncertainties should be reduced as well. As long as the various determinations remain in agreement, while their precision improves, we can enhance our confidence that the uncontrolled theoretical uncertainties are not biasing $|V_{ub}|$ beyond whatever level of precision has been reached in the individual measurements.

Summary and outlook: There is considerable debate (even between the authors) regarding the soundest use of the various measurements. While our knowledge regarding $|V_{ub}|$ is far more robust than it was ten years ago, the uncertainty on $|V_{ub}|$ from each method still receives contributions from some uncontrolled sources. To validate a given level of precision in this situation, measurements based on complementary techniques that agree within that precision are needed. The present results from inclusive and exclusive determinations display a promising agreement (see Table 1). However, there is a fairly widespread consensus that the inclusive and exclusive measurements cannot be reliably combined until we can quantify all of their yet-uncontrolled uncertainties.

Restricting to the inclusive determinations, several results have been already obtained, all within the same HQE framework, and with comparable estimated accuracies. Their averaging would improve the overall $|V_{ub}|$ accuracy, since the uncorrelated uncertainties are sizeable. The LEP Heavy Flavour Group has already performed the exercise of such averaging for the four LEP measurements. The result has been obtained in the framework of OPE, and no additional error has been added to account for additional corrections to the $1/m_b$ expansion, and of the quark-hadron duality assumption [72]. These issues need to be addressed and tested by further experimental studies.

As discussed above, there remain uncertainties that cannot be precisely quantified in the different analyses. Furthermore, potential violation of local quark-hadron duality might affect each kinematic region differently, though future study of the end-point spectrum for B_u and B_d separately may help in constraining these effects. Hence, there exist sources of uncertainty that we cannot yet quantify that may be as large as the current statistical uncertainties, though, in the end, they could also be found to be small. The agreement among the current results limits the uncontrolled uncertainties to the order of 20%, the current precision of the comparison. Without some independent means of controlling such uncertainties, an average of the different methods may underestimate the uncertainty with which we have truly determined $|V_{ub}|$.

Currently, we have a variety of measurements that individually approach a 15% uncertainty, and that all agree within that uncertainty. Any of the individual measurements can, therefore, be used as representative of $|V_{ub}|$.

The prospects for improved precision on $|V_{ub}|$ are excellent. With the large data samples now becoming available at the

$\Upsilon(4S)$ facilities, the experimental uncertainties will continue to shrink. With these improved uncertainties will come more stringent comparisons of techniques to "stress test" the theory, and either continued confidence in the increased precision on $|V_{ub}|$, or an indication of where the shortcomings lie. With the continued advancement of lattice QCD, exclusive determinations of $|V_{ub}|$ from $B \to \pi\ell\nu$ and $B \to \omega\ell\nu$ well below 10%, appear feasible within the decade. We now have event samples that allow model independent extractions of $|V_{ub}|$ from a variety of inclusive techniques. As the event samples from the B factories increase, the precision of these techniques will continue to improve, and agreement among them can limit the uncontrolled uncertainties. Barring disagreements as the measurements improve, sub 10% precisions also appear feasible from the inclusive techniques.

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