DETERMINATION OF $|V_{cb}|$

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I. Introduction

In the framework of the Standard Model, the quark sector is characterized by a rich pattern of flavor-changing transitions, described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix(See CKM review [1]). This report focuses on the quark mixing parameter $|V_{cb}|$.

Two different methods have been used to extract this parameter from data: the **exclusive** measurement, where $|V_{cb}|$ is extracted by studying exclusive $B \to D^\star \ell\nu$ and $B \to D\ell\nu$ decay processes; and the **inclusive** measurement, which uses the semileptonic width of b-hadron decays. Theoretical estimates play a crucial role in extracting $|V_{cb}|$, and an understanding of their uncertainties is very important.

II. Exclusive $|V_{cb}|$ determination

The exclusive $|V_{cb}|$ determination is obtained studying the $B \to D^\star \ell\nu$ and $B \to D\ell\nu$ decays, using Heavy Quark Effective Theory (HQET), an exact theory in the limit of infinite quark masses. Presently the $B \to D\ell\nu$ transition provides a less precise value, and is used as a check.

The decay $B \to D^\star \ell\nu$ in HQET: HQET predicts that the differential partial decay width for this process, $d\Gamma/dw$, is related to $|V_{cb}|$ through:

$$\frac{d\Gamma}{dw}(B \to D^\star \ell\nu) = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} \mathcal{K}(w)\mathcal{F}(w)^2, \qquad (1)$$

where w is the inner product of the B and $D^\star$ meson 4-velocities, $\mathcal{K}(w)$ is a known phase-space factor, and the form factor $\mathcal{F}(w)$ is generally expressed as the product of a normalization constant, $\mathcal{F}(1)$, and a function, $g(w)$, constrained by dispersion relations [2].

There are several different corrections to the infinite mass value $\mathcal{F}(1) = 1$ [3]:

$$\mathcal{F}(1) = \eta_{\rm QED}\eta_A \left[1 + \delta_{1/m_Q^2} + ...\right] \qquad (2)$$

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where $Q = c$ or b. By virtue of Luke's theorem [4], the first term in the non-perturbative expansion in powers of $1/m_Q$ vanishes. QED corrections up to leading-logarithmic order give $\eta_{\rm QED} \approx 1.007$ [3] and QCD radiative corrections to two loops give $\eta_A = 0.960 \pm 0.007$ [5]. Different estimates of the $1/m_Q^2$ corrections, involving terms proportional to $1/m_b^2$, $1/m_c^2$, and $1/(m_b m_c)$, have been performed in a quark model [6,7], with OPE sum rules [8], and, more recently, with an HQET based lattice gauge calculation [9]. The value from this quenched lattice HQET calculation is $\mathcal{F}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}{}^{+0.000}_{-0.016}{}^{+0.006}_{-0.014}$. The errors quoted reflect the statistical accuracy, the matching error, the lattice finite size, the uncertainty in the quark masses, and an estimate of the error induced by the quenched approximation, respectively. The central value obtained with OPE sum rules is similar, with an error of ± 0.04 [10]. Consequently, $\mathcal{F}(1) = 0.91 \pm 0.04$ [10] will be used here.

The analytical expression of $\mathcal{F}(w)$ is not known a-priori, and this introduces an additional uncertainty in the determination of $\mathcal{F}(1)|V_{cb}|$. First measurements of $|V_{cb}|$ were performed assuming a linear approximation for $\mathcal{F}(w)$. It has been shown [11] that this assumption is not justified, and that linear fits systematically underestimate the extrapolation at zero recoil ($w = 1$) by about 3%. Most of this effect is related to the curvature of the form factor, and does not depend strongly upon the details of the non-linear shape chosen [11]. All recent published results use a non-linear shape for $\mathcal{F}(w)$, approximated with an expansion near $w = 1$ [12]. $\mathcal{F}(w)$ is parameterized in terms of the variable ρ^2, which is the slope of the form factor at zero recoil given in Ref. 12.

Experimental techniques to study the decay $B \to D^\star \ell \nu$: The decay $B \to D^\star \ell \nu$ has been studied in experiments performed at center-of-mass energies equal to the $\Upsilon(4S)$ mass and the Z^0 mass. At the $\Upsilon(4S)$, experiments have the advantage that the w resolution is quite good. However, they have more limited statistics near $w = 1$ in the decay $\overline{B}^o \to D^{\star +} \ell \nu$, because of the lower reconstruction efficiency of the slow pion, from the $D^{\star +} \to \pi^+ D^0$ decay. The decay $B^- \to D^{\star 0} \ell \overline{\nu}$ is not affected by this problem, and CLEO [13] studies both channels. In addition, kinematic constraints enable $\Upsilon(4S)$ experiments to identify the final state, including the $D^\star$, without a large contamination from the poorly known semileptonic decays including a hadronic system heavier than $D^\star$, commonly identified as '$D^{\star\star}$.' At LEP, B's are produced with a large momentum (about 30 GeV on average). This makes the determination of w dependent upon the neutrino four-momentum reconstruction, thus giving a relatively poor resolution and limited physics background rejection capabilities. By contrast, LEP experiments benefit from an efficiency that is only mildly dependent upon w.

Experiments determine the product $(\mathcal{F}(1) \cdot |V_{cb}|)^2$ by fitting the measured $d\Gamma/dw$ distribution. Measurements have been performed by CLEO [13], Belle [14], DELPHI [15], ALEPH [16], and OPAL [17]. At LEP, the dominant source of systematic error is the uncertainty on the contribution to $d\Gamma/dw$ from semileptonic B decays with final states including a hadron system heavier than the $D^\star$. This component includes both narrow orbitally excited charmed mesons and non-resonant or broad species. The existence of narrow resonant states is well established [1], and a signal of a broad resonance has been seen by CLEO [18], but the decay characteristics of these states in b-hadron semileptonic decays have large uncertainties. The average of ALEPH [19], CLEO [20], and DELPHI [21] narrow state branching fractions show that the ratio $R_{\star\star} = \dfrac{\mathrm{B}(\overline{B} \to D_2^\star \ell \overline{\nu})}{\mathrm{B}(\overline{B} \to D_1 \ell \overline{\nu})}$ is smaller than one (< 0.6 at 95% C.L. [22]), in disagreement with HQET models where an infinite quark mass is assumed [23], but in agreement with models which take into account finite quark mass corrections [24]. Hence, LEP experiments use the treatment of narrow $D^{\star\star}$ proposed in Ref. 24, which accounts for $\mathcal{O}(1/m_c)$ corrections. Ref. 24 provides several possible approximations of the form factors that depend on five different expansion schemes, and on three input parameters. To calculate the systematic errors, each proposed scheme is tested, with the relevant input parameters varied over a range consistent with the experimental limit on $R_{\star\star}$. The quoted systematic error is the maximal difference from the central value obtained with this method. Broad resonances or other non-resonant terms may not be modelled correctly with this approach.

To combine the published data, the central values and the errors of $\mathcal{F}(1)|V_{cb}|$ and ρ^2 are re-scaled to the same set of input parameters and their quoted uncertainties. The $\mathcal{F}(1)|V_{cb}|$ values used for this average are extracted using the parametrization in Ref. 13, based on the experimental determinations of the vector and axial form factor ratios R_1 and R_2 [26]. The LEP data, which originally used theoretical values for these ratios, are re-scaled accordingly [25]. Table 1 summarizes the corrected data. The averaging procedure [25] takes into account statistical and systematic correlations between $\mathcal{F}(1)|V_{cb}|$ and ρ^2. Averaging the measurements in Table 1, we get:

$$\mathcal{F}(1)|V_{cb}| = (38.3 \pm 1.0) \times 10^{-3}$$

and

$$\rho^2 = 1.5 \pm 0.13 \ , \tag{3}$$

with a confidence level* of 5.1%. The error ellipses for the corrected measurements and for the world average are shown in Figure 1.

The main contributions to the $\mathcal{F}(1)|V_{cb}|$ systematic error are from the uncertainty on the $B \to D^{\star\star}\ell\nu$ shape and $\mathrm{B}(b \to B_d)$, (0.57×10^{-3}), fully correlated among the LEP experiments, the branching fraction of D and $D^\star$ decays, (0.4×10^{-3}), fully correlated among all the experiments, and the slow pion reconstruction from Belle and CLEO which are uncorrelated, $(0.28{\times}10^{-3})$. The main contribution to the ρ^2 systematic error is from the uncertainties in the measured values of R_1 and R_2 (0.13), fully correlated among experiments. Because of the large contribution of this uncertainty to the non-diagonal terms of the covariance matrix, the averaged ρ^2 is higher than one would naively expect.

Using $\mathcal{F}(1) = 0.91 \pm 0.04$ [10], we get $|V_{cb}| = (42.1 \pm 1.1_{\mathrm{exp}} \pm 1.9_{\mathrm{theo}}) \times 10^{-3}$. The dominant error is theoretical, but there are good prospects that lattice gauge calculations will improve significantly the accuracy of their estimate.

* The χ^2 per degree of freedom is less than 2, and we do not scale the error.

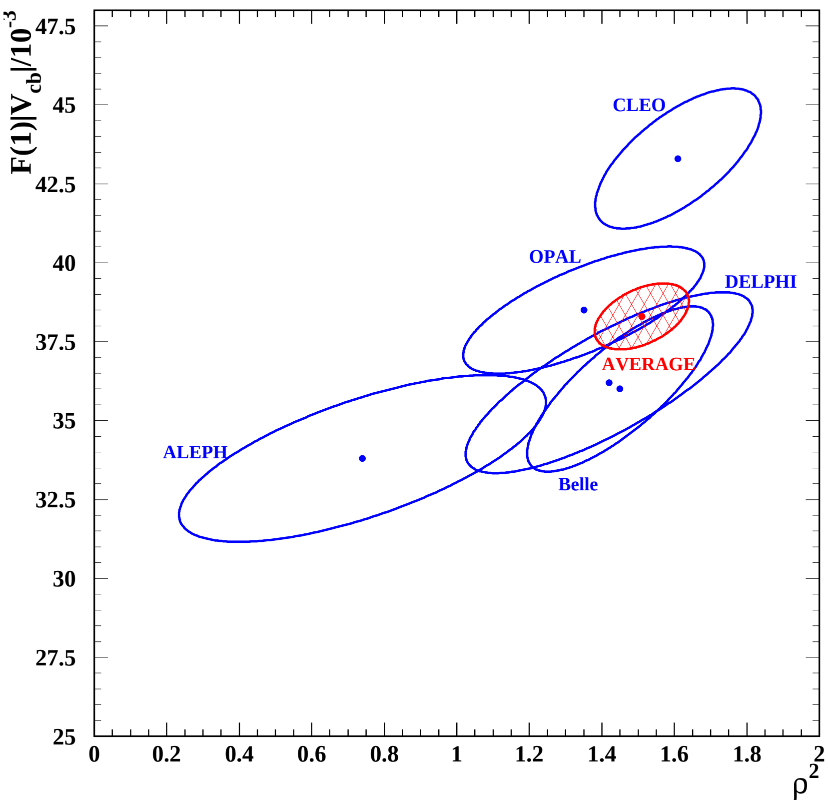

Figure 1: The error ellipses for the corrected measurements and world average for $\mathcal{F}(1)|V_{cb}|$ vs ρ^2. The ellipses are the product between the 1 σ error of $\mathcal{F}(1)|V_{cb}|$, ρ^2, and the correlation between the two. Consequently the ellipses correspond to about 37% CL.

The decay $B \longrightarrow D\ell\nu$: The study of the decay $B \to D\ell\nu$ poses new challenges both from the theoretical and experimental point of view.

The differential decay rate for $B \to D\ell\nu$ can be expressed as:

$$\frac{d\Gamma_D}{dw}(B \to D\ell\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\mathcal{K}_{\mathcal{D}}(w)\mathcal{G}(w)^2, \tag{4}$$

where w is the inner product of the B and D meson 4-velocities, $\mathcal{K}_{\mathcal{D}}(w)$ is the phase space, and the form factor $\mathcal{G}(w)$ is generally

Table 1: Experimental results (from $B \to D^{\star}\ell\nu$ analyses) after the correction to common inputs and world average. The LEP numbers are corrected to use R_1 and R_2 from CLEO data. ρ^2 is the slope of the form factor at zero recoil as defined in Ref. 12.

Exp.	$\mathcal{F}(1)\lvert V_{cb}\rvert(\times 10^3)$	ρ^2	$\mathrm{Corr}_{\mathrm{stat}}$
ALEPH	$33.8 \pm 2.1 \pm 1.6$	$0.74 \pm 0.25 \pm 0.41$	94%
DELPHI	$36.1 \pm 1.4 \pm 2.5$	$1.42 \pm 0.14 \pm 0.37$	94%
OPAL	$38.5 \pm 0.9 \pm 1.8$	$1.35 \pm 0.12 \pm 0.31$	89%
Belle	$36.0 \pm 1.9 \pm 1.8$	$1.45 \pm 0.16 \pm 0.20$	90%
CLEO	$43.3 \pm 1.3 \pm 1.8$	$1.61 \pm 0.09 \pm 0.21$	86%
World average	$38.3 \pm 0.5 \pm 0.9$	$1.51 \pm 0.05 \pm 0.12$	86%

expressed as the product of a normalization factor, $\mathcal{G}(1)$, and a function, $g_D(w)$, constrained by dispersion relations [2].

The strategy to extract $\mathcal{G}(1)|V_{cb}|$ is identical to that used for the $B \to D^{\star}\ell\nu$ decay. However, in this case there is no suppression of $1/m_Q$ (*i.e.*, no Luke theorem) and corrections and QCD effects on $\mathcal{G}(1)$ are calculated with less accuracy than $\mathcal{F}(1)$ [27,28]. Moreover, $d\Gamma_D/dw$ is more heavily suppressed near $w = 1$ than $d\Gamma_{D^*}/dw$, due to the helicity mismatch between initial and final states. This channel is also much more challenging from the experimental point of view as it is hard to isolate from the dominant background $B \to D^{\star}\ell\nu$, as well as from fake D–ℓ combinations. Thus, the extraction of $|V_{cb}|$ from this channel is less precise than the one from the $B \to D^{\star}\ell\nu$ decay. Nevertheless, the $B \to D\ell\nu$ channel provides a consistency check, and allows a test of heavy-quark symmetry [28] through the measurement of the form factor $\mathcal{G}(w)$, as HQET predicts the ratio $\mathcal{G}(w)/\mathcal{F}(w)$ to be very close to one.

Belle [29] and ALEPH [16] studied the $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ channel, while CLEO [30] studied both $B^+ \to D^0\ell^+\overline{\nu}$ and $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ decays. Averaging the data in Table 2 [25], we get $\mathcal{G}(1)|V_{cb}| = (41.3 \pm 4.0) \times 10^{-3}$ and $\rho_D^2 = 1.19 \pm 0.19$, where ρ_D^2 is the slope of the form factor at zero recoil given in Ref. 12.

Table 2: Experimental results after the correction to common inputs and world average. ρ_D^2 is the slope of the form factor at zero recoil given in Ref. 12.

Exp.	$\mathcal{G}(1)|V_{cb}|(\times 10^3)$	ρ_D^2
ALEPH	$37.7 \pm 9.9 \pm 6.5$	$0.90 \pm 0.98 \pm 0.38$
Belle	$41.2 \pm 4.4 \pm 5.1$	$1.12 \pm 0.22 \pm 0.14$
CLEO	$44.6 \pm 5.8 \pm 3.5$	$1.27 \pm 0.25 \pm 0.14$
World average	$41.3 \pm 2.9 \pm 2.7$	$1.19 \pm 0.15 \pm 0.12$

The theoretical predictions for $\mathcal{G}(1)$ are consistent: 1.03 ± 0.07 [31], and 0.98 ± 0.07 [28]. A quenched lattice calculation gives $\mathcal{G}(1) = 1.058^{+0.020}_{-0.017}$ [32], where the errors do not include the uncertainties induced by the quenching approximation and lattice spacing. Using $\mathcal{G}(1) = 1.0 \pm\ 0.07$, we get $|V_{cb}| = (41.3 \pm 4.0_{\rm exp} \pm 2.9_{\rm theo}) \times 10^{-3}$, consistent with the value extracted from $B \to D^\star \ell \nu$ decay, but with a larger uncertainty.

The experiments have also measured the differential decay rate distribution to extract the ratio $\mathcal{G}(w)/\mathcal{F}(w)$. The data are compatible with a universal from factor as predicted by HQET. From the measured values of $\mathcal{G}(1)|V_{cb}|$ and $\mathcal{F}(1)|V_{cb}|$, we get $\mathcal{G}(1)/\mathcal{F}(1) = 1.08 \pm 0.09$, consistent with the form-factor values we used.

III. $|V_{cb}|$ determination from inclusive B semileptonic decays

Alternatively, $|V_{cb}|$ can be extracted from the inclusive branching fraction for semileptonic b hadron decays $\mathrm{B}(B \to X_c \ell \nu)$ [33,34]. Several studies have shown that the spectator model decay rate is the leading term in a well-defined expansion controlled by the parameter $\Lambda_{\rm QCD}/m_b$. Non-perturbative corrections to this leading approximation arise only to order $1/m_b^2$. The key issue in this approach is the ability to separate non-perturbative corrections, that can be expressed as a series in powers of $1/m_b$, and perturbative corrections, expressed in powers of α_s. Quark-hadron duality is an important *ab initio* assumption in these calculations. While several authors [35] argue that this ansatz does not introduce appreciable errors, as they expect that duality violations affect the semileptonic width only in high powers in the non-perturbative expansion, other authors recognize that a presently unknown correction may be associated with this assumption [36]. Arguments supporting a possible sizeable source of errors related to the assumption of quark-hadron duality have been proposed [37]. This issue needs to be resolved with further experimental studies. At present, no explicit additional error has been added to account for possible quark-hadron duality violation.

The coefficients of the $1/m_b$ power terms are expectations values of operators that include non-perturbative physics. Relationships that are valid up to $1/m_b^2$ include four such parameters: the expectation value of the kinetic operator, corresponding to the average of the square of the heavy-quark momentum inside the hadron, the expectation value of the chromomagnetic operator, and the heavy-quark masses (m_b and m_c). The expectation value of the kinetic operator is introduced in the literature as $-\lambda_1$ [38,39] or μ_π^2 [33,34], whereas the expectation value of the chromomagnetic operator is defined as λ_2 [38,39] or μ_G^2 [33,34]. The two notations reflect a difference in the approach used to handle the energy scale μ used to separate long-distance from short-distance physics. HQET is most commonly renormalized in a mass-independent scheme, thus making the quark masses the pole masses of the underlying theory (QCD). The second group of authors prefer the definition of the non-perturbative operators using a mass scale $\mu \approx 1$ GeV.

The corresponding equations for the semileptonic width can be found in Refs. 33,40. Ref. 40 has been used to extract $|V_{cb}|$ from the semileptonic branching fraction measured by CLEO, and to measure the heavy-quark expansion (HQE) parameters $\overline{\Lambda}$ and λ_1, as discussed below.

The quark masses are related to the corresponding meson masses through [6]:

$$m_b = \overline{M}_B - \overline{\Lambda} + \frac{\lambda_1}{2\overline{M}_B}, \tag{5}$$

where $\overline{M}_B$ is the spin averaged $B-B^\star$ mass ($\overline{M}_B = 5.3134$ GeV/c^2). A similar equation relates m_c and $\overline{M}_D$. The parameter $\overline{\Lambda}$ represents the energy of the light quark and gluons.

HQE and moments in semileptonic decays: Experimental determinations of the HQE parameters are important in several respects. In particular, redundant determinations of these parameters may uncover inconsistencies, or point to violation of some important assumptions inherent in these calculations. The parameter λ_2 can be extracted from the $B^\star-B$ mass splitting, whereas the other parameters need more elaborate measurements.

The first stage of this experimental program has been completed recently. The CLEO collaboration has measured the shape of the photon spectrum in $b \to s\gamma$ inclusive decays. Its first moment, giving the average energy of the γ emitted in this transition, is related to the b quark mass. In the formalism of Ref. 40, this corresponds to the measurement of the parameter $\overline{\Lambda} = 0.35 \pm 0.07 \pm 0.10$ GeV [41].

The parameter λ_1 is determined experimentally through a measurement of the first moment of the mass M_X of the hadronic system recoiling against the $\ell-\overline{\nu}$ pair. The relationship between the first moment of $M_1 =< M_X^2 - M_D^2 > /M_B^2$ and the parameters $\overline{\Lambda}$ and λ_1 is given in Ref. 42.

The measured value for $< M_X^2 - M_D^2 >$ [42] is 0.251 ± 0.066 GeV^2. This constraint, combined with the measurement of the mean photon energy in $b \to s\gamma$, implies a value of $\lambda_1 = -0.24 \pm 0.11$ GeV^2, to order $1/M_B^3$ and $\beta_0\alpha_s^2$ in ($\overline{\rm MS}$). The quoted theoretical uncertainty of 2% accounts for the $1/M_B^3$ and α_s uncertainties, but not for possible violations of quark-hadron duality.

Experimental determination of the semileptonic branching fraction: The value of B($B \to X_c\ell\nu$) has been measured both at the $\Upsilon(4S)$ and LEP.

The most recent CLEO data, published in 1996 and based on a subset of the data sample accumulated now, obtains this branching fraction using a lepton tagged sample [43]. In this approach, a di-lepton sample is studied, and the charge correlation between the two leptons is used to disentangle leptons coming from the direct decay $B \to X_c \ell \nu$ and the dominant background at low lepton momenta, the cascade decay $B \to X_c \to X_s \ell \nu$. This method was pioneered by the ARGUS collaboration [44] to measure the electron spectrum from $B \to X_c \ell \nu$ down to 0.6 GeV/c. Thus, it reduces the model dependence of the extracted semileptonic branching fraction very substantially. They obtain $\mathrm{B}(B \to X_c e \nu) = (10.49 \pm 0.17 \pm 0.43)\%$. The systematic error (4%) is dominated by experimental uncertainties. Lepton identification efficiency, fake rate determination, and tracking efficiencies contribute to 3% of this overall error. The remaining error is a sum of several small corrections associated with the uncertainty in the mixing parameter, and additional background estimates [43].

Combining $\Upsilon(4S)$ results [1], we obtain: $\mathrm{B}(b \to X \ell \nu) = (10.38 \pm 0.32)\%$. Using τ_{B^+}, τ_{B^0} [1], $f_{+-}/f_{00} = 1.04 \pm 0.08$ [45], and subtracting $\mathrm{B}(b \to u \ell \nu) = (0.17 \pm 0.05)\%$, we get: $\mathrm{B}(b \to X_c \ell \nu) = (10.21 \pm 0.32)\%$ and $\Gamma(b \to X_c \ell \nu) = (0.419 \pm 0.013 \pm 0.003) \times 10^{-10}$ MeV, where 0.003×10^{-10} MeV includes the uncertainties from $\mathrm{B}(b \to u \ell \nu)$, and the model dependence correlated with LEP.

At LEP, B^0, B^-, B_s, and b baryon are produced, so the measured inclusive semileptonic branching ratio is an average over the different hadron species. Assuming that the semileptonic widths of all b hadrons are equal, the following relation holds:

$$
\begin{aligned}
\mathrm{B}(b \to X_c l \nu)_{\mathrm{LEP}} = & \\
& f_{B^0} \frac{\Gamma(B^0 \to X_c l \nu)}{\Gamma(B^0)} + f_{B^-} \frac{\Gamma(B^- \to X_c l \nu)}{\Gamma(B^-)} \\
& + f_{B_s} \frac{\Gamma(B_s \to X_c l \nu)}{\Gamma(B_s)} + f_{\Lambda_b} \frac{\Gamma(\Lambda_b \to X_c l \nu)}{\Gamma(\Lambda_b)} \\
& = \Gamma(B \to X_c \ell \nu) \tau_b \ , \qquad (6)
\end{aligned}
$$

where τ_b is the average b-hadron lifetime. Taking into account the present precision of LEP measurements of b-baryon semileptonic branching ratios and lifetimes, the estimate uncertainty for a possible difference for the width of b baryons is 0.13%.

At LEP, B($b \to X\ell\nu$) is measured with dedicated analyses [47–50](Table 3). The average LEP value for B($b \to X\ell\nu$) = $(10.59 \pm 0.09 \pm 0.30)\%$ is taken from a fit [46], which combines the semileptonic branching ratios, the $B^0-\overline{B}^0$ mixing parameter $\overline{\chi}_b$, and $R_b = \Gamma(Z \to b\overline{b})/\Gamma(Z \to \text{had})$. Ref. 47 shows that the main contribution to the modelling error is the uncertainty in the composition of the semileptonic width, including the narrow, wide and non-resonant $D^{\star\star}$ states. B_s and b baryons are about 20% of the total signal, and their contribution to the uncertainty of the spectrum is small. In this average, we use the modelling error quoted by Ref. 47, rather than the error from the combined fit, as the ALEPH procedure is based on more recent information. The dominant errors in the combined branching fraction are the modelling of semileptonic decays (2.6%) and the detector related items (1.3%).

Table 3: B($b \to \ell$) measurement from LEP and their average. The errors quoted reflect statistical, systematic, and modelling uncertainties respectively.

Experiment	B($b \to \ell\nu$)%
ALEPH	$10.70 \pm 0.10 \pm 0.23 \pm 0.26$
DELPHI	$10.70 \pm 0.08 \pm 0.21 \pm^{+0.44}_{-0.30}$
L3	$10.85 \pm 0.12 \pm 0.38 \pm 0.26$
L3 (double-tag)	$10.16 \pm 0.13 \pm 0.20 \pm 0.22$
OPAL	$10.83 \pm 0.10 \pm 0.20 \pm^{+0.20}_{-0.13}$
LEP Average	$10.59 \pm 0.09 \pm 0.15 \pm 0.26$

Subtracting B($b \to u\ell\nu$) from the LEP semileptonic branching fraction, we get: B($b \to X_c\ell\nu$) = $(10.42 \pm 0.34)\%$, and using τ_b [1]: $\Gamma(b \to X_c\ell\nu) = (0.439 \pm 0.010 \pm 0.011) \times 10^{-10}$ MeV, where the systematic error 0.011×10^{-10}MeV reflects the

$B(b \to u\ell\nu)$ uncertainty and the model dependence, correlated with the $\Upsilon(4S)$ result.

Combining the LEP and the $\Upsilon(4S)$ semileptonic widths, we get: $\Gamma(b \to X_c\ell\nu) = (0.43 \pm 0.01) \times 10^{-10}$ MeV, which is used in the formula of Ref. 42 to get:

$$|V_{cb}|_{\rm incl} = (40.4 \pm 0.5_{\rm exp} \pm 0.5_{\lambda_1,\overline{\Lambda}} \pm 0.8_{\rm theo}) \times 10^{-3} \ , \qquad (7)$$

where the first error is experimental, and the second is from the measured value of λ_1 and $\overline{\Lambda}$, assumed to be universal up to higher orders. The third error is from $1/m_b^3$ corrections and from the ambiguity in the α_s scale definition. The error on the average b-hadron lifetime is assumed to be uncorrelated with the error on the semileptonic branching ratio.

IV. Conclusions

The values of $|V_{cb}|$ obtained both from the inclusive and exclusive method agree within errors. The value of $|V_{cb}|$ obtained from the analysis of the $B \to D^\star\ell\nu$ decay is:

$$|V_{cb}|_{\rm exclusive} = (42.1 \pm 1.1_{\rm exp} \pm 1.9_{\rm theo}) \times 10^{-3} \ , \qquad (8)$$

where the first error is experimental and the second error is from the $1/m_Q^2$ corrections to $\mathcal{F}(1)$. The value of $|V_{cb}|$, obtained from inclusive semileptonic branching fractions is:

$$|V_{cb}|_{\rm incl} = (40.4 \pm 0.5_{\rm exp} \pm 0.5_{\lambda_1,\overline{\Lambda}} \pm 0.8_{\rm theo}) \times 10^{-3}, \qquad (9)$$

where the first error is experimental, the second error is from the measured values of λ_1 and $\overline{\Lambda}$, assumed to be universal up to higher orders, and the last is from $1/m_b^3$ corrections and α_s. Non-quantified uncertainties are associated with a possible quark-hadron duality violation. For this reason, we chose not to average the two numbers.

While experimental errors have reached 2.7% and 1.2% levels respectively, the dominant uncertainties remain of theoretical origin. The theoretical errors are difficult to assign and may not correspond to a Gaussian probability distribution function. High precision tests of HQET, checks on possible violations of quark-hadron duality in semileptonic decays, and experimental determination of m_b, $m_b - m_c$, and μ_π^2 are needed to complete this challenging experimental program.

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