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$K^{\pm}$

$I(J^P) = \frac{1}{2}(0^-)$

THE CHARGED KAON MASS

Revised 1994 by T.G. Trippe (LBNL).

The average of the six charged kaon mass measurements which we use in the Particle Listings is

$$m_{K^{\pm}} = 493.677 \pm 0.013 \text{ MeV (S = 2.4)} , \qquad (1)$$

where the error has been increased by the scale factor S. The large scale factor indicates a serious disagreement between different input data. The average before scaling the error is

$$m_{K^{\pm}} = 493.677 \pm 0.005 \text{ MeV} ,$$

$$\chi^2 = 22.9 \text{ for 5 D.F., Prob. } = 0.04\% , \qquad (2)$$

where the high χ^2 and correspondingly low χ^2 probability further quantify the disagreement.

The main disagreement is between the two most recent and precise results,

$$\begin{aligned} m_{K^{\pm}} &= 493.696 \pm 0.007 \text{ MeV} && \text{DENISOV 91} \\ m_{K^{\pm}} &= 493.636 \pm 0.011 \text{ MeV (S = 1.5)} && \text{GALL 88} \\ \hline \text{Average} &= 493.679 \pm 0.006 \text{ MeV} \end{aligned}$$

$$\chi^2 = 21.2 \text{ for 1 D.F., Prob. } = 0.0004\% , \qquad (3)$$

both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall average in Eq. (2), it is clear that DENISOV 91 and GALL 88 dominate the overall average, and that their disagreement is responsible for most of the high χ^2.

The GALL 88 measurement was made using four different kaonic atom transitions, K^- Pb (9 → 8), K^- Pb (11 → 10), K^- W (9 → 8), and K^- W (11 → 10). The $m_{K^\pm}$ values they obtain from each of these transitions is shown in the Particle Listings and in Fig. 1. Their K^- Pb (9 → 8) $m_{K^\pm}$is below and somewhat inconsistent with their other three transitions. The average of their four measurements is

$$m_{K^\pm} = 493.636 \pm 0.007 \; , \qquad \chi^2 = 7.0 \text{ for 3 D.F., Prob. } = 7.2\% \; . \tag{4}$$

This is a low but acceptable χ^2 probability so, to be conservative, GALL 88 scaled up the error on their average by S=1.5 to obtain their published error ±0.011 shown in Eq. (3) above and used in the Particle Listings average.

The ideogram in Fig. 1 shows that the DENISOV 91 measurement and the GALL 88 K^- Pb (9 → 8) measurement yield two well-separated peaks. One might suspect the GALL 88 K^- Pb (9 → 8) measurement since it is responsible both for the internal inconsistency in the GALL 88 measurements and the disagreement with DENISOV 91.

To see if the disagreement could result from a systematic problem with the K^- Pb (9 → 8) transition, we have separated the CHENG 75 data, which also used K^- Pb, into its separate transitions. Figure 1 shows that the CHENG 75 and GALL 88 K^- Pb (9 → 8) values are consistent, suggesting the possibility of a common effect such as contaminant nuclear γ rays near the K^- Pb (9 → 8) transition energy, although the CHENG 75 errors are too large to make a strong conclusion. The average of all 13 measurements has a χ^2 of 52.6 as shown in Fig. 1 and the first line of Table 1, yielding an unacceptable χ^2 probability of 0.00005%. The second line of Table 1 excludes

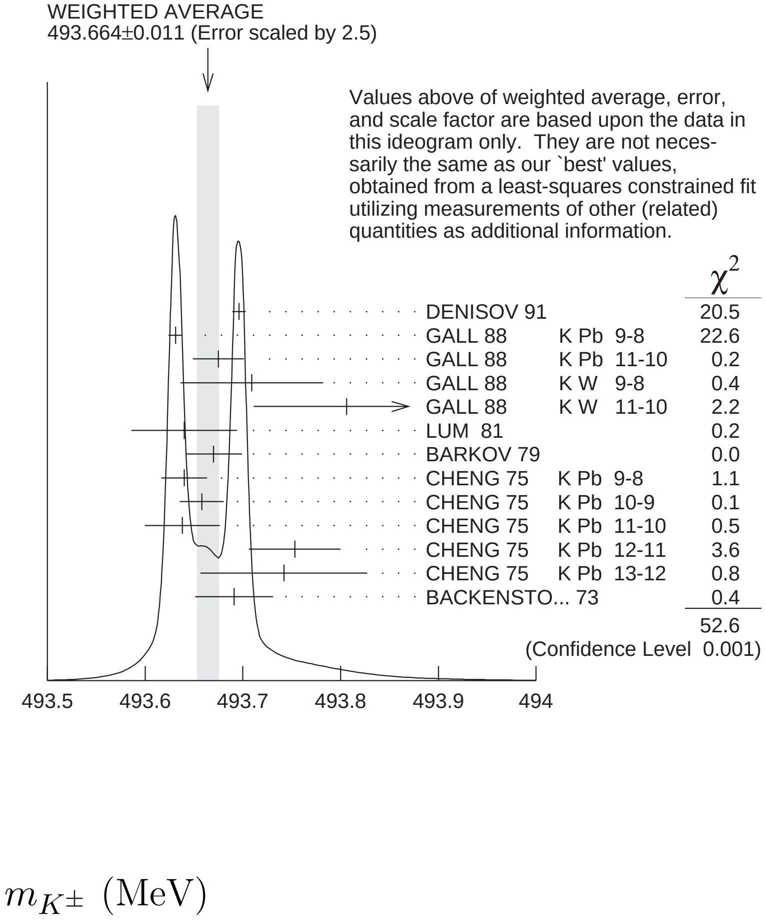


Figure 1: Ideogram of $m_{K^\pm}$ mass measurements. GALL 88 and CHENG 75 measurements are shown separately for each transition they measured.

both the GALL 88 and CHENG 75 measurements of the K^-Pb ($9 \rightarrow 8$) transition and yields a χ^2 probability of 43%. The third [fourth] line of Table 1 excludes only the GALL 88 K^-Pb ($9 \rightarrow 8$) [DENISOV 91] measurement and yields a χ^2 probability of 20% [8.6%]. Table 1 shows that removing both measurements of the K^-Pb ($9 \rightarrow 8$) transition produces the most consistent set of data, but that excluding only the

GALL 88 K^-Pb (9 → 8) transition or DENISOV 91 also produces acceptable probabilities.

Table 1: $m_{K^\pm}$ averages for some combinations of Fig. 1 data.

$m_{K^\pm}$ (MeV)	χ^2	D.F.	Prob. (%)	Measurements used
493.664 ± 0.004	52.6	12	0.00005	all 13 measurements
493.690 ± 0.006	10.1	10	43	no K^-Pb(9→8)
493.687 ± 0.006	14.6	11	20	no GALL 88 K^-Pb(9→8)
493.642 ± 0.006	17.8	11	8.6	no DENISOV 91

Yu.M. Ivanov, representing DENISOV 91, has estimated corrections needed for the older experiments because of improved ^{192}Ir and ^{198}Au calibration γ-ray energies. He estimates that CHENG 75 and BACKENSTOSS 73 $m_{K^\pm}$ values could be raised by about 15 keV and 22 keV, respectively. With these estimated corrections, Table 1 becomes Table 2. The last line of Table 2 shows that if such corrections are assumed, then GALL 88 K^-Pb (9 → 8) is inconsistent with the rest of the data even when DENISOV 91 is excluded. Yu.M. Ivanov warns that these are rough estimates. Accordingly, we do not use Table 2 to reject the GALL 88 K^-Pb (9 → 8) transition, but we note that a future reanalysis of the CHENG 75 data could be useful because it might provide supporting evidence for such a rejection.

The GALL 88 measurement uses a Ge semiconductor spectrometer which has a resolution of about 1 keV, so they run the risk of some contaminant nuclear γ rays. Studies of γ rays following stopped π^- and Σ^- absorption in nucleii (unpublished) do not show any evidence for contaminants according to GALL 88 spokesperson, B.L. Roberts. The DENISOV 91

Table 2: $m_{K^\pm}$ averages for some combinations of Fig. 1 data after raising CHENG 75 and BACKENSTOSS 73 values by 0.015 and 0.022 MeV respectively.

$m_{K^\pm}$ (MeV)	χ^2	D.F.	Prob. (%)	Measurements used
493.666 ± 0.004	53.9	12	0.00003	all 13 measurements
493.693 ± 0.006	9.0	10	53	no K^- Pb(9→8)
493.690 ± 0.006	11.5	11	40	no GALL 88 K^- Pb(9→8)
493.645 ± 0.006	23.0	11	1.8	no DENISOV 91

measurement uses a crystal diffraction spectrometer with a resolution of 6.3 eV for radiation at 22.1 keV to measure the 4f-3d transition in $K^-\ ^{12}$C. The high resolution and the light nucleus reduce the probability for overlap by contaminant γ rays, compared with the measurement of GALL 88. The DENISOV 91 measurement is supported by their high-precision measurement of the 4d-2p transition energy in $\pi^-\ ^{12}$C, which is good agreement with the calculated energy.

While we suspect that the GALL 88 K^- Pb $(9 \to 8)$ measurements could be the problem, we are unable to find clear grounds for rejecting it. Therefore, we retain their measurement in the average and accept the large scale factor until further information can be obtained from new measurements and/or from reanalysis of GALL 88 and CHENG 75 data.

We thank B.L. Roberts (Boston Univ.) and Yu.M. Ivanov (Petersburg Nuclear Physics Inst.) for their extensive help in understanding this problem.

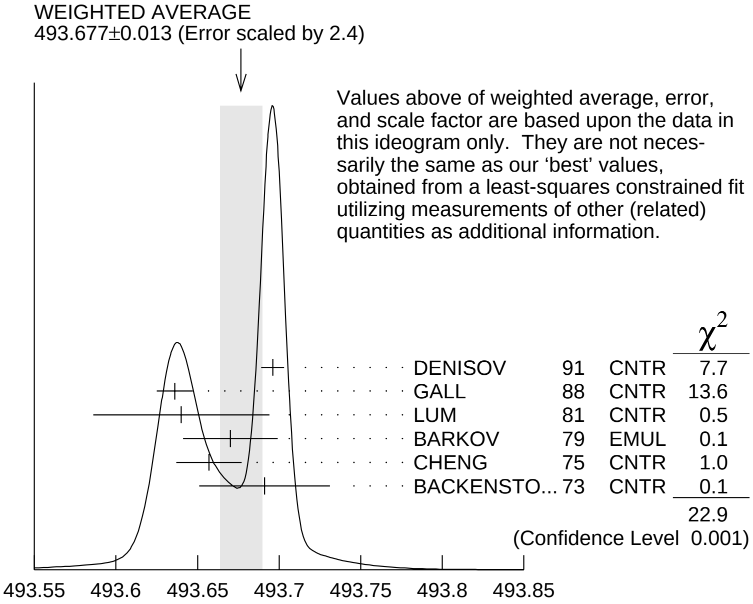
$K^\pm$ MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
493.677±0.016 OUR FIT Error includes scale factor of 2.8.					
493.677±0.013 OUR AVERAGE Error includes scale factor of 2.4. See the ideogram below.					
493.696±0.007	[1] DENISOV	91	CNTR	−	Kaonic atoms
493.636±0.011	[2] GALL	88	CNTR	−	Kaonic atoms
493.640±0.054	LUM	81	CNTR	−	Kaonic atoms
493.670±0.029	BARKOV	79	EMUL	±	$e^+ e^- \rightarrow K^+ K^-$
493.657±0.020	[2] CHENG	75	CNTR	−	Kaonic atoms
493.691±0.040	BACKENSTO...	73	CNTR	−	Kaonic atoms
• • • We do not use the following data for averages, fits, limits, etc. • • •					
493.631±0.007	GALL	88	CNTR	−	K^- Pb (9→ 8)
493.675±0.026	GALL	88	CNTR	−	K^- Pb (11→ 10)
493.709±0.073	GALL	88	CNTR	−	K^- W (9→ 8)
493.806±0.095	GALL	88	CNTR	−	K^- W (11→ 10)
493.640±0.022±0.008	[3] CHENG	75	CNTR	−	K^- Pb (9→ 8)
493.658±0.019±0.012	[3] CHENG	75	CNTR	−	K^- Pb (10→ 9)
493.638±0.035±0.016	[3] CHENG	75	CNTR	−	K^- Pb (11→ 10)
493.753±0.042±0.021	[3] CHENG	75	CNTR	−	K^- Pb (12→ 11)
493.742±0.081±0.027	[3] CHENG	75	CNTR	−	K^- Pb (13→ 12)

[1] Error increased from 0.0059 based on the error analysis in IVANOV 92.

[2] This value is the authors' combination of all of the separate transitions listed for this paper.

[3] The CHENG 75 values for separate transitions were calculated from their Table 7 transition energies. The first error includes a 20% systematic error in the noncircular contaminant shift. The second error is due to a ±5 eV uncertainty in the theoretical transition energies.



$m_{K^\pm}$ (MeV)

$m_{K^+} - m_{K^-}$

Test of *CPT*.

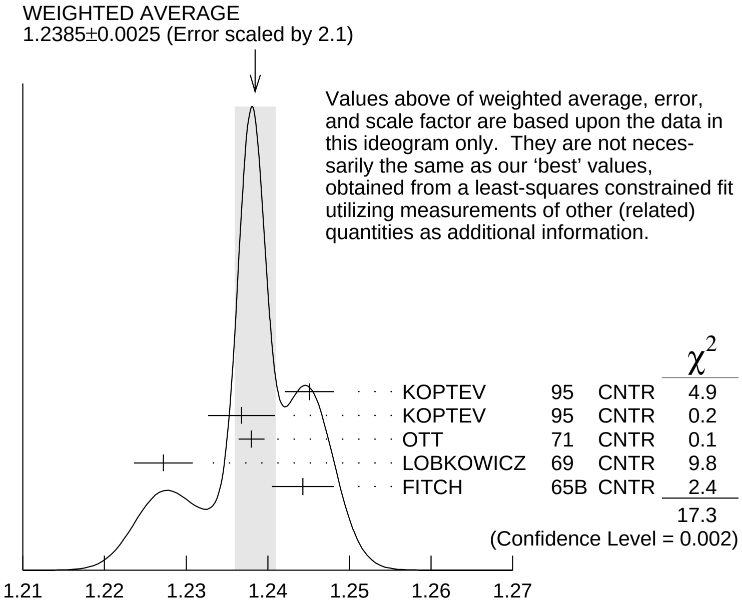
VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
−0.032±0.090	1.5M	[4] FORD	72	ASPK	±

[4] FORD 72 uses $m_{\pi^+} - m_{\pi^-} = +28 \pm 70$ keV.

$K^\pm$ MEAN LIFE

VALUE (10^{-8} s)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
1.2384±0.0024 OUR FIT Error includes scale factor of 2.0.						
1.2385±0.0025 OUR AVERAGE Error includes scale factor of 2.1. See the ideogram below.						
1.2451±0.0030	250k	KOPTEV	95	CNTR		*K* at rest, U target
1.2368±0.0041	150k	KOPTEV	95	CNTR		*K* at rest, Cu target
1.2380±0.0016	3M	OTT	71	CNTR	+	*K* at rest
1.2272±0.0036		LOBKOWICZ	69	CNTR	+	*K* in flight
1.2443±0.0038		FITCH	65B	CNTR	+	*K* at rest
• • • We do not use the following data for averages, fits, limits, etc. • • •						
1.2415±0.0024	400k	[5] KOPTEV	95	CNTR		*K* at rest
1.221 ±0.011		FORD	67	CNTR	±	
1.231 ±0.011		BOYARSKI	62	CNTR	+	

[5] KOPTEV 95 report this weighted average of their U-target and Cu-target results, where they have weighted by $1/\sigma$ rather than $1/\sigma^2$.



$K^\pm$ mean life (10^{-8} s)

$(\tau_{K^+} - \tau_{K^-}) / \tau_{\text{average}}$

This quantity is a measure of *CPT* invariance in weak interactions.

VALUE (%)	*DOCUMENT ID*	*TECN*
0.11 ±0.09 OUR AVERAGE	Error includes scale factor of 1.2.	
0.090±0.078	LOBKOWICZ 69	CNTR
0.47 ±0.30	FORD 67	CNTR

RARE KAON DECAYS

Revised March 2002 by L. Littenberg (BNL) and G. Valencia, (Iowa State University).

A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–14]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters

3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focused on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+ \ell^+ \ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0 \gamma\gamma$ and $K_L \to \ell^+ \ell^- \gamma$. The former is important in understanding a CP-conserving contribution to $K_L \to \pi^0 \ell^+ \ell^-$, whereas the latter could shed light on long distance contributions to $K_L \to \mu^+ \mu^-$.

B. Explicit violations of the Standard Model: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12} (148\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV, along with the expected near-future progress. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings. There have also been recent limits placed on lepton-number violating kaon decays [15,16]. Related

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.	(Near-) future aim
$K^+ \to \pi^+ e^- \mu^+$	2.8×10^{-11}	BNL-865	01/17	9×10^{-12}
$K^+ \to \pi^+ e^+ \mu^-$	5.2×10^{-10}	BNL-865	01/15	
$K_L \to \mu e$	4.7×10^{-12}	BNL-871	98/18	
$K_L \to \pi^0 e \mu$	4.4×10^{-10}	KTeV	01/19	3×10^{-10}

searches in μ and τ processes are discussed in our section "Tests of Conservation Laws".

Physics beyond the SM is also pursued through the search for $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, *etc.*). The 90% CL upper limit on this process has recently been improved to 5.9×10^{-11} [20].

C. Measurements of Standard Model parameters: Until 1997, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ were motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [21] and long-distance contributions are known to be quite small [2,22]. Since then, BNL-787 has observed two candidate events [23,20], yielding a branching ratio of $(1.57^{+1.75}_{-0.82}) \times 10^{-10}$ [20]. At this level, this reaction becomes interesting from the point of view of constraining SM parameters. An upgrade to the experiment to collect roughly an order of magnitude more sensitivity is in progress [24], and a new experiment with a sensitivity goal of $\sim 10^{-12}$/event has recently been given scientific approval at FNAL [25]. In the future this mode may provide grounds for precision tests of the flavor structure of the standard model [26].

The branching ratio can be written in terms of the very well-measured K_{e3} rate as [2]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [27]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [28], and X_{NL}^{ℓ} is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which mainly affect X_{NL}^{ℓ}, are known at next-to-leading order [12,29] and lead to a residual error of $< 10\%$ for the decay amplitude. Evaluating the constants in Eq. (1), one can cast this result in terms of the CKM parameters A, ρ and η (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [12]

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.0 \times 10^{-10} A^4 [\eta^2 + (\rho_o - \rho)^2] \qquad (2)$$

where $\rho_o \equiv 1 + (\frac{2}{3} X_{NL}^{e} + \frac{1}{3} X_{NL}^{\tau})/(A^2 V_{us}^4 X(m_t)) \approx 1.4$. Thus, $\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the ρ, η plane with center $(\rho_o, 0)$ and radius $\approx \frac{1}{A^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.0\times 10^{-10}}}$.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter ρ, given by [12]:

$$\mathrm{B}_{\mathrm{SD}}(K_L \to \mu^+\mu^-) \approx 1.6 \times 10^{-9} A^4 (\rho_o' - \rho)^2 \qquad (3)$$

where ρ_o' depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state.

The absorptive (imaginary) part of the long-distance component is calculated in terms of the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B}_{\mathrm{abs}}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$ [30]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain ρ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present it is not possible to compute this long-distance component reliably, and therefore it is not possible to constrain ρ from this mode in a model independent way [31]. Several models exist to estimate this long-distance component [32,33] that are sufficient to place rough bounds on new physics from the measured rate for $K_L \to \mu^+\mu^-$ [34]. The decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [31,33], is in good agreement with the recent measurement [35]. It is expected that studies of the reactions $K_L \to \ell^+\ell^-\gamma$ [36], and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ [37,16], currently under active study by the KTeV and NA48 experiments, will improve our understanding of the long distance effects in $K_L \to \mu^+\mu^-$ (the current data is often parameterized in terms of α_K^*, discussed at the end of the Form Factors section of the K_L^0 Particle Properties Listing in this edition).

D. Searches for direct CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,38,39]. In the Standard Model this mode is dominated by an intermediate top-quark state and does not suffer from the uncertainty associated with the charm-quark intermediate state that affects the mode $K^+ \to \pi^+ \nu\overline{\nu}$. The branching ratio is given approximately by Ref. 12:

$$\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \approx 4.1 \times 10^{-10} A^4 \eta^2 \ . \tag{4}$$

With current constraints on the CKM parameters this leads to a predicted branching ratio $(2.6 \pm 1.2) \times 10^{-11}$. The current experimental upper bound is $\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \leq 5.9 \times 10^{-7}$ [40]. The 90% CL bound on $K^+ \to \pi^+ \nu\bar{\nu}$ provides a nearly model independent bound $B(K_L \to \pi^0 \nu\bar{\nu}) < 1.7 \times 10^{-9}$ [41]. A KEK experiment to reach the 3×10^{-10}/event level is in preparation [42]. The KOPIO [43] proposal aims to reach the 6×10^{-13}/event level for $K_L \to \pi^0 \nu\overline{\nu}$ at the BNL AGS.

There has been much theoretical work on possible contributions to ϵ'/ϵ and rare K decays in supersymmetric extensions of the SM. While in the simplest case of the MSSM with no new sources of flavor or CP-violation the main effect is a suppression of the rare K decays [44], substantial enhancements are possible in more general SUSY models [34,45].

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the product $A^4 \eta^2$. It has a direct CP-violating component given by [12]:

$$\mathrm{B}_{\mathrm{dir}}(K_L \to \pi^0 e^+ e^-) \approx 6.7 \times 10^{-11} A^4 \eta^2 \ . \tag{5}$$

However, like $K_L \to \mu^+ \mu^-$ this mode suffers from large theoretical uncertainties due to long distance strong interaction effects.

The CP-violating component also receives an indirect contribution which is given by:

$$\mathrm{B}_{\mathrm{ind}}(K_L \to \pi^0 e^+ e^-) = |\epsilon|^2 \frac{\tau_{K_L}}{\tau_{K_S}} \mathrm{B}(K_S \to \pi^0 e^+ e^-) \ , \tag{6}$$

when interference between the direct and indirect contributions is neglected. Certain models that relate the processes $K_S \to \pi^0 e^+ e^-$ and $K^+ \to \pi^+ e^+ e^-$ have been used to predict that $B_{\rm ind}(K_L \to \pi^0 e^+ e^-)$ is less than 10^{-12} [46]. However, precise knowledge of this component awaits measurement of $K_S \to \pi^0 e^+ e^-$ [4,47]. The 90% CL upper limit, $B(K_S \to \pi^0 e^+ e^-) < 1.4 \times 10^{-7}$, recently obtained by NA48 [48] is about two orders of magnitude short of the expected level. NA48 proposes to reach $\sim 10^{-9}$/event sensitivity for this mode in their upcoming K_S run [49].

There is also a CP-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0 \gamma\gamma$. To understand the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0 \gamma\gamma$ within chiral perturbation theory it is necessary to go beyond leading order. It is possible to accommodate the existing measurements in terms of one parameter, a_V [50]. A fit to the distribution by the KTeV collaboration [51] has found $a_V = -0.72 \pm 0.05 \pm 0.06$. This value suggests that the absorptive part of the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$ could be comparable to the direct CP-violating component [47,51]. However, a new result from NA48, $a_V = -0.46 \pm 0.03 \pm 0.03 \pm 0.02$ [52] would suggest that this contribution is smaller. A model independent prediction for the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$ is not possible in terms of a_V alone [53]. The related process, $K_L \to \pi^0 \gamma e^+ e^-$, is potentially an additional background in some region of phase space [54]. This process has recently been observed with a branching ratio of $(2.34 \pm 0.35_{\rm stat} \pm 0.13_{\rm sys}) \times 10^{-8}$ [55].

Finally, BNL-845 observed a potential background to $K_L \to \pi^0 e^+ e^-$ from the decay $K_L \to \gamma\gamma e^+ e^-$ [56]. This has recently been confirmed with a 500-fold larger sample by FNAL-799 [57], which measured additional kinematic quantities. It has been estimated that this background will enter at the level of 3×10^{-10} [58,59], comparable to or larger than the signal level. Because of this, the observation of $K_L \to \pi^0 e^+ e^-$ will depend on background subtraction with good statistics.

The current 90% CL preliminary upper bound for the process $K_L \to \pi^0 e^+ e^-$ is 5.1×10^{-10} [59]. For the closely related muonic process, the corresponding upper bound is $\mathrm{B}(K_L \to \pi^0 \mu^+ \mu^-) \leq 3.8 \times 10^{-10}$ [60]. KTeV has collected data corresponding to about a factor 1.3 in sensitivity for both reactions which is still to be analyzed [61].

Recently, a new study of $K_L \to \pi^0 \mu^+ \mu^-$ has indicated that it might be possible to extract the direct CP-violating contribution by a joint study of the Dalitz plot variables and the components of the μ^+ polarization [62]. The latter tend to be quite substantial so that large statistics may not be necessary.

E. Other long distance dominated modes:

The decays $K^+ \to \pi^+ \ell^+ \ell^-$ ($\ell = e$ or μ) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [63,64]. Ref. 65 has proposed a parameterization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections.

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K^+ DECAY MODES

K^- modes are charge conjugates of the modes below.

Leptonic and semileptonic modes

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$e^+\nu_e$		$(1.55 \pm 0.07) \times 10^{-5}$	
Γ_2	$\mu^+\nu_\mu$		(63.43 ± 0.17) %	S=1.2
Γ_3	$\pi^0 e^+\nu_e$		(4.87 ± 0.06) %	S=1.2
	Called K^+_{e3}.			
Γ_4	$\pi^0\mu^+\nu_\mu$		(3.27 ± 0.06) %	S=1.2
	Called $K^+_{\mu3}$.			
Γ_5	$\pi^0\pi^0 e^+\nu_e$		$(2.1 \pm 0.4) \times 10^{-5}$	
Γ_6	$\pi^+\pi^- e^+\nu_e$		$(4.08 \pm 0.09) \times 10^{-5}$	
Γ_7	$\pi^+\pi^-\mu^+\nu_\mu$		$(1.4 \pm 0.9) \times 10^{-5}$	
Γ_8	$\pi^0\pi^0\pi^0 e^+\nu_e$		$< 3.5 \times 10^{-6}$	CL=90%

Hadronic modes

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_9	$\pi^+\pi^0$		(21.13 ± 0.14) %	S=1.1
Γ_{10}	$\pi^+\pi^0\pi^0$		(1.73 ± 0.04) %	S=1.2
Γ_{11}	$\pi^+\pi^+\pi^-$		(5.576 ± 0.031) %	S=1.1

Leptonic and semileptonic modes with photons

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_{12}	$\mu^+\nu_\mu\gamma$	[*a*,*b*]	$(5.50 \pm 0.28) \times 10^{-3}$	
Γ_{13}	$\pi^0 e^+\nu_e\gamma$	[*a*,*b*]	$(2.65 \pm 0.20) \times 10^{-4}$	
Γ_{14}	$\pi^0 e^+\nu_e\gamma(\text{SD})$	[*c*]	$< 5.3 \times 10^{-5}$	CL=90%
Γ_{15}	$\pi^0\mu^+\nu_\mu\gamma$	[*a*,*b*]	$< 6.1 \times 10^{-5}$	CL=90%
Γ_{16}	$\pi^0\pi^0 e^+\nu_e\gamma$		$< 5 \times 10^{-6}$	CL=90%

Hadronic modes with photons

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_{17}	$\pi^+\pi^0\gamma$	[*a*,*b*]	$(2.75 \pm 0.15) \times 10^{-4}$	
Γ_{18}	$\pi^+\pi^0\gamma(\text{DE})$	[*b*,*d*]	$(4.7 \pm 0.9) \times 10^{-6}$	
Γ_{19}	$\pi^+\pi^0\pi^0\gamma$	[*a*,*b*]	$(7.4\,^{+5.5}_{-2.9}) \times 10^{-6}$	
Γ_{20}	$\pi^+\pi^+\pi^-\gamma$	[*a*,*b*]	$(1.04 \pm 0.31) \times 10^{-4}$	
Γ_{21}	$\pi^+\gamma\gamma$	[*b*]	$(1.10 \pm 0.32) \times 10^{-6}$	
Γ_{22}	$\pi^+3\gamma$	[*b*]	$< 1.0 \times 10^{-4}$	CL=90%

Leptonic modes with $\ell \overline{\ell}$ pairs

Γ_{23}	$e^+ \nu_e \nu \overline{\nu}$		<	6		$\times 10^{-5}$	CL=90%
Γ_{24}	$\mu^+ \nu_\mu \nu \overline{\nu}$		<	6.0		$\times 10^{-6}$	CL=90%
Γ_{25}	$e^+ \nu_e e^+ e^-$		(	3.1	$^{+3.1}_{-1.6}$	) $\times 10^{-8}$	
Γ_{26}	$\mu^+ \nu_\mu e^+ e^-$		(	1.3	± 0.4	) $\times 10^{-7}$	
Γ_{27}	$e^+ \nu_e \mu^+ \mu^-$		<	5		$\times 10^{-7}$	CL=90%
Γ_{28}	$\mu^+ \nu_\mu \mu^+ \mu^-$		<	4.1		$\times 10^{-7}$	CL=90%

Lepton Family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes

Γ_{29}	$\pi^+ \pi^+ e^- \overline{\nu}_e$	*SQ*	<	1.2		$\times 10^{-8}$	CL=90%
Γ_{30}	$\pi^+ \pi^+ \mu^- \overline{\nu}_\mu$	*SQ*	<	3.0		$\times 10^{-6}$	CL=95%
Γ_{31}	$\pi^+ e^+ e^-$	*S1*	(	2.88	± 0.13	) $\times 10^{-7}$	
Γ_{32}	$\pi^+ \mu^+ \mu^-$	*S1*	(	7.6	± 2.1	) $\times 10^{-8}$	S=3.4
Γ_{33}	$\pi^+ \nu \overline{\nu}$	*S1*	(	1.6	$^{+1.8}_{-0.8}$	) $\times 10^{-10}$	
Γ_{34}	$\pi^+ \pi^0 \nu \overline{\nu}$	*S1*	<	4.3		$\times 10^{-5}$	CL=90%
Γ_{35}	$\mu^- \nu e^+ e^+$	*LF*	<	2.0		$\times 10^{-8}$	CL=90%
Γ_{36}	$\mu^+ \nu_e$	*LF*	[*e*] <	4		$\times 10^{-3}$	CL=90%
Γ_{37}	$\pi^+ \mu^+ e^-$	*LF*	<	2.8		$\times 10^{-11}$	CL=90%
Γ_{38}	$\pi^+ \mu^- e^+$	*LF*	<	5.2		$\times 10^{-10}$	CL=90%
Γ_{39}	$\pi^- \mu^+ e^+$	*L*	<	5.0		$\times 10^{-10}$	CL=90%
Γ_{40}	$\pi^- e^+ e^+$	*L*	<	6.4		$\times 10^{-10}$	CL=90%
Γ_{41}	$\pi^- \mu^+ \mu^+$	*L*	[*e*] <	3.0		$\times 10^{-9}$	CL=90%
Γ_{42}	$\mu^+ \overline{\nu}_e$	*L*	[*e*] <	3.3		$\times 10^{-3}$	CL=90%
Γ_{43}	$\pi^0 e^+ \overline{\nu}_e$	*L*	<	3		$\times 10^{-3}$	CL=90%
Γ_{44}	$\pi^+ \gamma$		[*f*] <	3.6		$\times 10^{-7}$	CL=90%

[*a*] Most of this radiative mode, the low-momentum γ part, is also included in the parent mode listed without γ's.

[*b*] See the Particle Listings below for the energy limits used in this measurement.

[*c*] Structure-dependent part.

[*d*] Direct-emission branching fraction.

[*e*] Derived from an analysis of neutrino-oscillation experiments.

[*f*] Violates angular-momentum conservation.

CONSTRAINED FIT INFORMATION

An overall fit to the mean life, 2 decay rate, and 19 branching ratios uses 48 measurements and one constraint to determine 8 parameters. The overall fit has a χ^2 = 49.8 for 41 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients $\left\langle\delta p_i \delta p_j\right\rangle/(\delta p_i{\cdot}\delta p_j)$, in percent, from the fit to parameters p_i, including the branching fractions, $x_i \equiv \Gamma_i/\Gamma_{\text{total}}$. The fit constrains the x_i whose labels appear in this array to sum to one.

x_3	−51						
x_4	−50	77					
x_5	−3	6	4				
x_9	−67	−17	−16	−1			
x_{10}	−23	−1	−1	0	−2		
x_{11}	−23	10	7	1	−6	14	
Γ	7	−3	−2	0	2	−5	−33
	x_2	x_3	x_4	x_5	x_9	x_{10}	x_{11}

	Mode	Rate (10^8 s^{-1})	Scale factor
Γ_2	$\mu^+ \nu_\mu$	0.5122 ±0.0017	1.4
Γ_3	$\pi^0 e^+ \nu_e$ Called K^+_{e3}.	0.0393 ±0.0005	1.2
Γ_4	$\pi^0 \mu^+ \nu_\mu$ Called $K^+_{\mu3}$.	0.0264 ±0.0005	1.2
Γ_5	$\pi^0 \pi^0 e^+ \nu_e$	$(1.71\ ^{+0.34}_{-0.30}) \times 10^{-5}$	
Γ_9	$\pi^+ \pi^0$	0.1706 ±0.0011	1.1
Γ_{10}	$\pi^+ \pi^0 \pi^0$	0.01395±0.00031	1.2
Γ_{11}	$\pi^+ \pi^+ \pi^-$	0.04503±0.00023	1.1

$K^\pm$ DECAY RATES

$\Gamma(\mu^+ \nu_\mu)$ Γ_2

VALUE (10^6 s^{-1})	*DOCUMENT ID*		*TECN*	*CHG*
51.22±0.17 OUR FIT Error includes scale factor of 1.4.				
51.2 ±0.8	FORD	67	CNTR	±

$\Gamma(\pi^+ \pi^+ \pi^-)$ Γ_{11}

VALUE (10^6 s^{-1})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
4.503±0.023 OUR FIT Error includes scale factor of 1.1.					
4.511±0.024		[6] FORD	70	ASPK	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
4.529±0.032	3.2M	[6] FORD	70	ASPK	
4.496±0.030		[6] FORD	67	CNTR	±

[6] First FORD 70 value is second FORD 70 combined with FORD 67.

$(\Gamma(K^+) - \Gamma(K^-)) / \Gamma(K)$

$K^\pm \to \mu^\pm \nu_\mu$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

VALUE (%)	*DOCUMENT ID*		*TECN*
−0.54±0.41	FORD	67	CNTR

$K^\pm \to \pi^\pm \pi^+ \pi^-$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.07±0.12 OUR AVERAGE					
0.08±0.12		[7] FORD	70	ASPK	
−0.50±0.90		FLETCHER	67	OSPK	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.02±0.16		[8] SMITH	73	ASPK	±
0.10±0.14	3.2M	[7] FORD	70	ASPK	
−0.04±0.21		[7] FORD	67	CNTR	

[7] First FORD 70 value is second FORD 70 combined with FORD 67.

[8] SMITH 73 value of $K^\pm \to \pi^\pm \pi^+ \pi^-$ rate difference is derived from SMITH 73 value of $K^\pm \to \pi^\pm 2\pi^0$ rate difference.

$K^\pm \to \pi^\pm \pi^0 \pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.0 ±0.6 OUR AVERAGE					
0.08±0.58		SMITH	73	ASPK	±
−1.1 ±1.8	1802	HERZO	69	OSPK	

$K^\pm \to \pi^\pm \pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

VALUE (%)	*DOCUMENT ID*		*TECN*
0.8±1.2	HERZO	69	OSPK

$K^\pm \to \pi^\pm \pi^0 \gamma$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.9±3.3 OUR AVERAGE						
0.8±5.8	2461	SMITH	76	WIRE	±	E_π 55–90 MeV
1.0±4.0	4000	ABRAMS	73B	ASPK	±	E_π 51–100 MeV

K^+ BRANCHING RATIOS

Leptonic and semileptonic modes

$\Gamma(e^+ \nu_e)/\Gamma(\mu^+ \nu_\mu)$ Γ_1/Γ_2

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
2.45±0.11 OUR AVERAGE					
2.51±0.15	404	HEINTZE	76	SPEC	+
2.37±0.17	534	HEARD	75B	SPEC	+
2.42±0.42	112	CLARK	72	OSPK	+

$\Gamma(\mu^+\nu_\mu)/\Gamma_{\rm total}$ Γ_2/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
63.43±0.17 OUR FIT	Error includes scale factor of 1.2.				
63.24±0.44	62k	CHIANG 72	OSPK	+	1.84 GeV/c K^+

$\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\rm total}$ Γ_3/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
4.87±0.06 OUR FIT	Error includes scale factor of 1.2.				
4.84±0.09 OUR AVERAGE					
4.86±0.10	3516	CHIANG 72	OSPK	+	1.84 GeV/c K^+
4.7 ±0.3	429	SHAKLEE 64	HLBC	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
5.0 ±0.5		ROE 61	HLBC	+	

$\Gamma(\pi^0 e^+\nu_e)/\Gamma(\mu^+\nu_\mu)$ Γ_3/Γ_2

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
0.0767±0.0011 OUR FIT	Error includes scale factor of 1.3.			
0.0752±0.0024 OUR AVERAGE				
0.069 ±0.006	350	ZELLER 69	ASPK	+
0.0775±0.0033	960	BOTTERILL 68C	ASPK	+
0.069 ±0.006	561	GARLAND 68	OSPK	+
0.0791±0.0054	295	[9] AUERBACH 67	OSPK	+

[9] AUERBACH 67 changed from 0.0797 ± 0.0054. See comment with ratio $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\mu^+\nu_\mu)$. The value 0.0785 ± 0.0025 given in AUERBACH 67 is an average of AUERBACH 67 $\Gamma(\pi^0 e^+\nu_e)/\Gamma(\mu^+\nu_\mu)$ and CESTER 66 $\Gamma(\pi^0 e^+\nu_e)/\big[\Gamma(\mu^+\nu_\mu) + \Gamma(\pi^+\pi^0)\big]$.

$\Gamma(\pi^0 e^+\nu_e)/\big[\Gamma(\mu^+\nu_\mu)+\Gamma(\pi^+\pi^0)\big]$ $\Gamma_3/(\Gamma_2+\Gamma_9)$

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
5.76±0.08 OUR FIT	Error includes scale factor of 1.2.			
6.02±0.15 OUR AVERAGE				
6.16±0.22	5110	ESCHSTRUTH 68	OSPK	+
5.89±0.21	1679	CESTER 66	OSPK	+
• • • We do not use the following data for averages, fits, limits, etc. • • •				
5.92±0.65		[10] WEISSENBE... 76	SPEC	+

[10] Value calculated from WEISSENBERG 76 ($\pi^0 e\nu$), ($\mu\nu$), and ($\pi\pi^0$) values to eliminate dependence on our 1974 ($\pi 2\pi^0$) and ($\pi\pi^+\pi^-$) fractions.

$\Gamma(\pi^0 e^+\nu_e)/\Gamma(\pi^+\pi^0)$ Γ_3/Γ_9

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.230±0.004 OUR FIT	Error includes scale factor of 1.2.				
0.221±0.012	786	[11] LUCAS 73B	HBC	−	Dalitz pairs only

[11] LUCAS 73B gives N(K_{e3}) = 786 ± 3.1%, N(2π) = 3564 ± 3.1%. We divide.

$\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\pi^+\pi^+\pi^-)$ Γ_3/Γ_{11}

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
0.873±0.012 OUR FIT Error includes scale factor of 1.2.				
0.868±0.021 OUR AVERAGE				
0.867±0.027	2768	BARMIN 87	XEBC	+
0.856±0.040	2827	BRAUN 75	HLBC	+
0.90 ±0.06	230	BORREANI 64	HBC	+
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.850±0.019	4385	[12] HAIDT 71	HLBC	+
0.846±0.021	4385	[12] EICHTEN 68	HLBC	+
0.94 ±0.09	854	BELLOTTI 67B	HLBC	

[12] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in $\Gamma(\pi^0 \mu^+ \nu)/\Gamma(\pi^0 e^+ \nu)$ with more precise results.

$\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma_{\text{total}}$ Γ_4/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
3.27±0.06 OUR FIT Error includes scale factor of 1.2.					
3.33±0.16	2345	CHIANG 72	OSPK	+	1.84 GeV/c K^+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
2.8 ±0.4		[13] TAYLOR 59	EMUL	+	

[13] Earlier experiments not averaged.

$\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma(\mu^+ \nu_\mu)$ Γ_4/Γ_2

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
$\mathbf{0.0515^{+0.0010}_{-0.0009}}$ **OUR FIT** Error includes scale factor of 1.2.				
0.0483±0.0027 OUR AVERAGE				
0.0480±0.0037	424	[14] GARLAND 68	OSPK	+
0.0486±0.0040	307	[15] AUERBACH 67	OSPK	+
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.054 ±0.009	240	ZELLER 69	ASPK	+

[14] GARLAND 68 changed from 0.055 ± 0.004 in agreement with μ-spectrum calculation of GAILLARD 70 appendix B. L.G.Pondrom, (private communication 73).

[15] AUERBACH 67 changed from 0.0602 ± 0.0046 by erratum which brings the μ-spectrum calculation into agreement with GAILLARD 70 appendix B.

$\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma(\pi^0 e^+ \nu_e)$ Γ_4/Γ_3

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.672±0.007 OUR FIT					
0.672±0.007 OUR AVERAGE					
0.671±0.007±0.008	24k	HORIE 01	SPEC		
0.670±0.014		[16] HEINTZE 77	SPEC	+	
0.698±0.025	3480	[17] CHIANG 72	OSPK	+	1.84 GeV/c K^+
0.667±0.017	5601	BOTTERILL 68B	ASPK	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.608±0.014	1585	[18] BRAUN 75	HLBC	+	
0.705±0.063	554	[19] LUCAS 73B	HBC	−	Dalitz pairs only
0.596±0.025		[20] HAIDT 71	HLBC	+	
0.604±0.022	1398	[20] EICHTEN 68	HLBC		
0.703±0.056	1509	CALLAHAN 66B	HLBC		

[16] HEINTZE 77 value from fit to λ_0. Assumes μ-e universality.

[17] CHIANG 72 $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\pi^0 e^+\nu_e)$ is statistically independent of CHIANG 72 $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma_{\text{total}}$ and $\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$.

[18] BRAUN 75 value is from form factor fit. Assumes μ-e universality.

[19] LUCAS 73B gives N($K_{\mu3}$) = 554 ± 7.6%, N(K_{e3}) = 786 ± 3.1%. We divide.

[20] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy with more precise results.

$[\Gamma(\pi^0\mu^+\nu_\mu)+\Gamma(\pi^+\pi^0)]/\Gamma_{\text{total}}$ $(\Gamma_4+\Gamma_9)/\Gamma$

We combine these two modes for experiments measuring them in xenon bubble chamber because of difficulties of separating them there.

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
24.40±0.14 OUR FIT	Error includes scale factor of 1.1.				
• • • We do not use the following data for averages, fits, limits, etc. • • •					
25.4 ±0.9	886	SHAKLEE	64	HLBC	+
23.4 ±1.1		ROE	61	HLBC	+

$\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\pi^+\pi^+\pi^-)$ Γ_4/Γ_{11}

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.586±0.010 OUR FIT	Error includes scale factor of 1.2.					
0.63 ±0.07	2845	[21] BISI	65B	BC	+	HBC+HLBC
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.503±0.019	1505	[22] HAIDT	71	HLBC	+	
0.510±0.017	1505	[22] EICHTEN	68	HLBC	+	

[21] Error enlarged for background problems. See GAILLARD 70.

[22] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in $\Gamma(\pi^0\mu^+\nu)/\Gamma(\pi^0 e^+\nu)$ with more precise results.

$\Gamma(\pi^0\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$ Γ_5/Γ

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
2.1 ±0.4 OUR FIT					
2.54±0.89	10	BARMIN	88B	HLBC	+

$\Gamma(\pi^0\pi^0 e^+\nu_e)/\Gamma(\pi^0 e^+\nu_e)$ Γ_5/Γ_3

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
$\mathbf{4.3^{+0.9}_{-0.7}}$ **OUR FIT**					
$\mathbf{4.1^{+1.0}_{-0.7}}$ **OUR AVERAGE**					
$4.2^{+1.0}_{-0.9}$	25	BOLOTOV	86B	CALO	−
$3.8^{+5.0}_{-1.2}$	2	LJUNG	73	HLBC	+

$\Gamma(\pi^+\pi^- e^+\nu_e)/\Gamma(\pi^+\pi^+\pi^-)$ — Γ_6/Γ_{11}

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
7.31±0.16 OUR AVERAGE					
7.35±0.01±0.19	388k	[23] PISLAK	01	B865	
7.21±0.32	30k	ROSSELET	77	SPEC	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
7.36±0.68	500	BOURQUIN	71	ASPK	
7.0 ±0.9	106	SCHWEINB...	71	HLBC	+
5.83±0.63	269	ELY	69	HLBC	+

[23] PISLAK 01 reports $\Gamma(\pi^+\pi^- e^+\nu_e)/\Gamma_{\text{total}} = (4.109 \pm 0.008 \pm 0.110) \times 10^{-5}$ using the PDG 00 value $\Gamma(\pi^+\pi^+\pi^-)/\Gamma_{\text{total}} = (5.59 \pm 0.05) \times 10^{-2}$. We divide by the PDG value and unfold its error from the systematic error.

$\Gamma(\pi^+\pi^-\mu^+\nu_\mu)/\Gamma_{\text{total}}$ — Γ_7/Γ

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$0.77^{+0.54}_{-0.50}$	1	CLINE	65	FBC	+

$\Gamma(\pi^+\pi^-\mu^+\nu_\mu)/\Gamma(\pi^+\pi^+\pi^-)$ — Γ_7/Γ_{11}

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
2.57±1.55	7	BISI	67	DBC	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
~ 2.5	1	GREINER	64	EMUL	+

$\Gamma(\pi^0\pi^0\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$ — Γ_8/Γ

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<3.5	90	0	BOLOTOV	88	SPEC	−
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<9	90	0	BARMIN	92	XEBC	+

Hadronic modes

$\Gamma(\pi^+\pi^0)/\Gamma_{\text{total}}$ — Γ_9/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
21.13±0.14 OUR FIT	Error includes scale factor of 1.1.					
21.18±0.28	16k	CHIANG	72	OSPK	+	1.84 GeV/*c* K^+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
21.0 ±0.6		CALLAHAN	65	HLBC		See $\Gamma(\pi^+\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$

$\Gamma(\pi^+\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ — Γ_9/Γ_{11}

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
3.789±0.033 OUR FIT	Error includes scale factor of 1.1.				
3.96 ±0.15	1045	CALLAHAN	66	FBC	+

$\Gamma(\pi^+\pi^0)/\Gamma(\mu^+\nu_\mu)$ Γ_9/Γ_2

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.3331±0.0028 OUR FIT Error includes scale factor of 1.1.					
0.3316±0.0032 OUR AVERAGE					
0.3329±0.0047±0.0010	45k	USHER 92	SPEC	+	$p\overline{p}$ at rest
0.3355±0.0057		[24] WEISSENBE... 76	SPEC	+	
0.305 ±0.018	1600	ZELLER 69	ASPK	+	
0.3277±0.0065	4517	[25] AUERBACH 67	OSPK	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.328 ±0.005	25k	[24] WEISSENBE... 74	STRC	+	

[24] WEISSENBERG 76 revises WEISSENBERG 74.

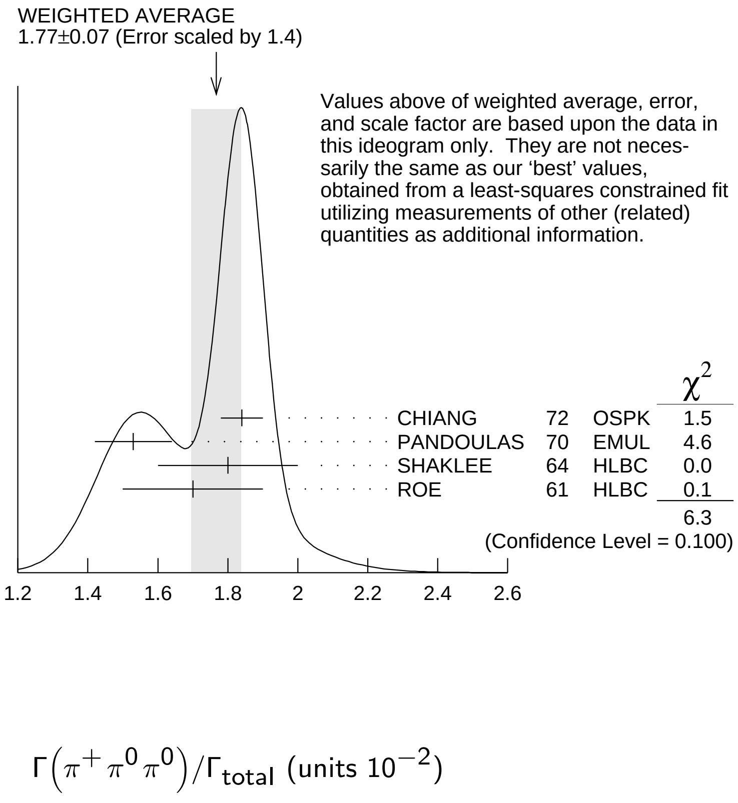
[25] AUERBACH 67 changed from 0.3253 ± 0.0065. See comment with ratio $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\mu^+\nu_\mu)$.

$\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{\rm total}$ Γ_{10}/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1.73±0.04 OUR FIT Error includes scale factor of 1.2.					
1.77±0.07 OUR AVERAGE Error includes scale factor of 1.4. See the ideogram below.					
1.84±0.06	1307	CHIANG 72	OSPK	+	1.84 GeV/c K^+
1.53±0.11	198	[26] PANDOULAS 70	EMUL	+	
1.8 ±0.2	108	SHAKLEE 64	HLBC	+	
1.7 ±0.2		ROE 61	HLBC	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
1.5 ±0.2		[27] TAYLOR 59	EMUL	+	

[26] Includes events of TAYLOR 59.

[27] Earlier experiments not averaged.



Values above of weighted average, error, and scale factor are based upon the data in this ideogram only. They are not necessarily the same as our ‘best’ values, obtained from a least-squares constrained fit utilizing measurements of other (related) quantities as additional information.

			χ^2
CHIANG	72	OSPK	1.5
PANDOULAS	70	EMUL	4.6
SHAKLEE	64	HLBC	0.0
ROE	61	HLBC	0.1
			6.3

(Confidence Level = 0.100)

$\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^0)$ Γ_{10}/Γ_9

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.0818±0.0019 OUR FIT Error includes scale factor of 1.2.						
0.081 ±0.005	574	[28] LUCAS	73B	HBC	−	Dalitz pairs only

[28] LUCAS 73B gives $N(\pi 2\pi^0) = 574 \pm 5.9\%$, $N(2\pi) = 3564 \pm 3.1\%$. We quote $0.5N(\pi 2\pi^0)/N(2\pi)$ where 0.5 is because only Dalitz pair π^0's were used.

$\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ Γ_{10}/Γ_{11}

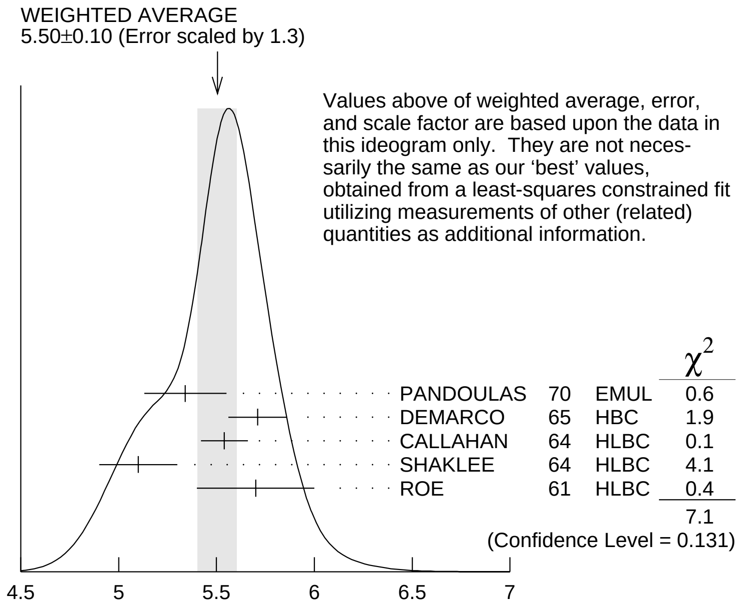
VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.310±0.007 OUR FIT Error includes scale factor of 1.1.						
0.303±0.009	2027	BISI	65	BC	+	HBC+HLBC
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.393±0.099	17	YOUNG	65	EMUL	+	

$\Gamma(\pi^+\pi^+\pi^-)/\Gamma_{\text{total}}$ Γ_{11}/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
5.576±0.031 OUR FIT Error includes scale factor of 1.1.						
5.50 ±0.10 OUR AVERAGE Error includes scale factor of 1.3. See the ideogram below.						
5.34 ±0.21	693	[29] PANDOULAS	70	EMUL	+	
5.71 ±0.15		DEMARCO	65	HBC		
5.54 ±0.12	2332	CALLAHAN	64	HLBC	+	
5.1 ±0.2	540	SHAKLEE	64	HLBC	+	
5.7 ±0.3		ROE	61	HLBC	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •						
5.56 ±0.20	2330	[30] CHIANG	72	OSPK	+	1.84 GeV/c K^+
6.0 ±0.4	44	YOUNG	65	EMUL	+	

[29] Includes events of TAYLOR 59.

[30] Value is not independent of CHIANG 72 $\Gamma(\mu^+\nu_\mu)/\Gamma_{\text{total}}$, $\Gamma(\pi^+\pi^0)/\Gamma_{\text{total}}$, $\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{\text{total}}$, $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma_{\text{total}}$, and $\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$.



$\Gamma\left(\pi^+\pi^+\pi^-\right)/\Gamma_{\text{total}}$ (units 10^{-2})

Leptonic and semileptonic modes with photons

$\Gamma(\mu^+\nu_\mu\gamma)/\Gamma_{\text{total}}$ Γ_{12}/Γ

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
5.50±0.28 OUR AVERAGE						
6.6 ±1.5		[31,32] DEMIDOV	90	XEBC		P(μ) <231.5 MeV/c
6.0 ±0.9		BARMIN	88	HLBC	+	P(μ) <231.5 MeV/c
5.4 ±0.3		[33] AKIBA	85	SPEC		P(μ) <231.5 MeV/c
• • • We do not use the following data for averages, fits, limits, etc. • • •						
3.5 ±0.8		[32,34] DEMIDOV	90	XEBC		E(γ) > 20 MeV
3.2 ±0.5	57	[35] BARMIN	88	HLBC	+	*E*(γ) >20 MeV

[31] P(μ) cut given in DEMIDOV 90 paper, 235.1 MeV/c, is a misprint according to authors (private communication).

[32] DEMIDOV 90 quotes only inner bremsstrahlung (IB) part.

[33] Assumes μ-e universality and uses constraints from $K \rightarrow e\nu\gamma$.

[34] Not independent of above DEMIDOV 90 value. Cuts differ.

[35] Not independent of above BARMIN 88 value. Cuts differ.

$\Gamma(\pi^0 e^+ \nu_e \gamma)/\Gamma(\pi^0 e^+ \nu_e)$ Γ_{13}/Γ_3

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.54±0.04 OUR AVERAGE Error includes scale factor of 1.1.						
0.46±0.08	82	[36] BARMIN	91	XEBC		$E(\gamma) > 10$ MeV, $0.6 < \cos\theta_{e\,\gamma} < 0.9$
0.56±0.04	192	[37] BOLOTOV	86B	CALO	−	$E(\gamma) > 10$ MeV
0.76±0.28	13	[38] ROMANO	71	HLBC		$E(\gamma) > 10$ MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •						
1.51±0.25	82	[36] BARMIN	91	XEBC		$E(\gamma) > 10$ MeV, $\cos\theta_{e\,\gamma} < 0.98$
0.48±0.20	16	[39] LJUNG	73	HLBC	+	$E(\gamma) > 30$ MeV
$0.22^{+0.15}_{-0.10}$		[39] LJUNG	73	HLBC	+	$E(\gamma) > 30$ MeV
0.53±0.22		[38] ROMANO	71	HLBC	+	$E(\gamma) > 30$ MeV

[36] BARMIN 91 quotes branching ratio $\Gamma(K \to e\pi^0\nu\gamma)/\Gamma_{\text{all}}$. The measured normalization is $[\Gamma(K \to e\pi^0\nu) + \Gamma(K \to \pi^+\pi^+\pi^-)]$. For comparison with other experiments we used $\Gamma(K \to e\pi^0\nu)/\Gamma_{\text{all}} = 0.0482$ to calculate the values quoted here.

[37] $\cos\theta(e\gamma)$ between 0.6 and 0.9.

[38] Both ROMANO 71 values are for $\cos\theta(e\gamma)$ between 0.6 and 0.9. Second value is for comparison with second LJUNG 73 value. We use lowest $E(\gamma)$ cut for Summary Table value. See ROMANO 71 for E_γ dependence.

[39] First LJUNG 73 value is for $\cos\theta(e\gamma) < 0.9$, second value is for $\cos\theta(e\gamma)$ between 0.6 and 0.9 for comparison with ROMANO 71.

$\Gamma(\pi^0 e^+ \nu_e \gamma(\text{SD}))/\Gamma_{\text{total}}$ Γ_{14}/Γ

Structure-dependent part.

VALUE (units 10^{-5})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*
<5.3	90	BOLOTOV	86B	CALO	−

$\Gamma(\pi^0 \mu^+ \nu_\mu \gamma)/\Gamma_{\text{total}}$ Γ_{15}/Γ

VALUE (units 10^{-5})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<6.1	90	0	LJUNG	73	HLBC	+	$E(\gamma) > 30$ MeV

$\Gamma(\pi^0 \pi^0 e^+ \nu_e \gamma)/\Gamma_{\text{total}}$ Γ_{16}/Γ

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<5	90	0	BARMIN	92	XEBC	+	$E_\gamma > 10$ MeV

Hadronic modes with photons

$\Gamma(\pi^+ \pi^0 \gamma)/\Gamma_{\text{total}}$ Γ_{17}/Γ

VALUE (units 10^{-4})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
2.75±0.15 OUR AVERAGE							
2.71±0.45		140	BOLOTOV	87	WIRE	−	$T\pi^-$ 55–90 MeV
2.87±0.32		2461	SMITH	76	WIRE	±	$T\pi^\pm$ 55–90 MeV
2.71±0.19		2100	ABRAMS	72	ASPK	±	$T\pi^+$ 55–90 MeV

• • • We do not use the following data for averages, fits, limits, etc. • • •

$1.5^{+1.1}_{-0.6}$			[40] LJUNG 73	HLBC	+	Tπ^+ 55–80 MeV
$2.6^{+1.5}_{-1.1}$			[40] LJUNG 73	HLBC	+	Tπ^+ 55–90 MeV
$6.8^{+3.7}_{-2.1}$		17	[40] LJUNG 73	HLBC	+	Tπ^+ 55–102 MeV
2.4 ± 0.8		24	EDWARDS 72	OSPK		Tπ^+ 58–90 MeV
<1.0		0	[41] MALTSEV 70	HLBC	+	Tπ^+ <55 MeV
<1.9	90	0	EMMERSON 69	OSPK		Tπ^+ 55–80 MeV
2.2 ± 0.7		18	CLINE 64	FBC	+	Tπ^+ 55–80 MeV

[40] The LJUNG 73 values are not independent.

[41] MALTSEV 70 selects low π^+ energy to enhance direct emission contribution.

$\Gamma(\pi^+\pi^0\gamma(\text{DE}))/\Gamma_{\text{total}}$ Γ_{18}/Γ

Direct emission part of $\Gamma(\pi^+\pi^0\gamma)/\Gamma_{\text{total}}$.

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
$0.47 \pm 0.08 \pm 0.03$	20k	ADLER 00C	B787	+	Tπ^+ 55–90 MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$2.05 \pm 0.46^{+0.39}_{-0.23}$		BOLOTOV 87	WIRE	−	Tπ^- 55–90 MeV
$1.56 \pm 0.35 \pm 0.5$		ABRAMS 72	ASPK	±	T$\pi^\pm$ 55–90 MeV

$\Gamma(\pi^+\pi^0\pi^0\gamma)/\Gamma(\pi^+\pi^0\pi^0)$ Γ_{19}/Γ_{10}

VALUE (units 10^{-4})	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
$4.3^{+3.2}_{-1.7}$	BOLOTOV 85	SPEC	−	$E(\gamma) > 10$ MeV

$\Gamma(\pi^+\pi^+\pi^-\gamma)/\Gamma_{\text{total}}$ Γ_{20}/Γ

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1.04 ± 0.31 OUR AVERAGE					
1.10 ± 0.48	7	BARMIN 89	XEBC		$E(\gamma) > 5$ MeV
1.0 ± 0.4		STAMER 65	EMUL	+	$E(\gamma) > 11$ MeV

$\Gamma(\pi^+\gamma\gamma)/\Gamma_{\text{total}}$ Γ_{21}/Γ

All values given here assume a phase space pion energy spectrum.

VALUE (units 10^{-7})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
$11 \pm 3 \pm 1$		31	[42] KITCHING 97	B787		
• • • We do not use the following data for averages, fits, limits, etc. • • •						
< 10	90	0	ATIYA 90B	B787		Tπ 117–127 MeV
< 84	90	0	ASANO 82	CNTR	+	Tπ 117–127 MeV
-420 ± 520		0	ABRAMS 77	SPEC	+	Tπ <92 MeV
< 350	90	0	LJUNG 73	HLBC	+	6–102, 114–127 MeV
< 500	90	0	KLEMS 71	OSPK	+	Tπ <117 MeV
-100 ± 600			CHEN 68	OSPK	+	Tπ 60–90 MeV

[42] KITCHING 97 is extrapolated from their model-independent branching fraction $(6.0 \pm 1.5 \pm 0.7) \times 10^{-7}$ for 100 MeV/c<P_{π^+} < 180 MeV/c using Chiral Perturbation Theory.

$\Gamma(\pi^+ 3\gamma)/\Gamma_{\text{total}}$ Γ_{22}/Γ

Values given here assume a phase space pion energy spectrum.

VALUE (units 10^{-4})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<1.0	90	ASANO	82	CNTR	+	$T(\pi)$ 117–127 MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<3.0	90	KLEMS	71	OSPK	+	$T(\pi)$ >117 MeV

Leptonic modes with $\ell\overline{\ell}$ pairs

$\Gamma(e^+ \nu_e \nu\overline{\nu})/\Gamma(e^+ \nu_e)$ Γ_{23}/Γ_1

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<3.8	90	0	HEINTZE	79	SPEC	+

$\Gamma(\mu^+ \nu_\mu \nu\overline{\nu})/\Gamma_{\text{total}}$ Γ_{24}/Γ

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<6.0	90	0	[43] PANG	73	CNTR	+

[43] PANG 73 assumes μ spectrum from ν-ν interaction of BARDIN 70.

$\Gamma(e^+ \nu_e e^+ e^-)/\Gamma(\pi^+ \pi^- e^+ \nu_e)$ Γ_{25}/Γ_6

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
$\mathbf{0.76^{+0.76}_{-0.38}}$	4	[44] DIAMANT-...	76	SPEC	+	$m_{e^+ e^-}$ >140 MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$5.4\ ^{+5.4}_{-2.7}$	4	[44] DIAMANT-...	76	SPEC	+	Extrapolated BR

[44] DIAMANT-BERGER 76 gives this result times our 1975 $\pi^+ \pi^- e \nu$ BR ratio. The second DIAMANT-BERGER 76 value is the first value extrapolated to 0 to include low mass $e^+ e^-$ pairs. More recent calculations (BIJNENS 93) of this extrapolation disagree with those of DIAMANT-BERGER 76.

$\Gamma(\mu^+ \nu_\mu e^+ e^-)/\Gamma(\pi^+ \pi^- e^+ \nu_e)$ Γ_{26}/Γ_6

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
3.3±0.9	14	[45] DIAMANT-...	76	SPEC	+	$m_{e^+ e^-}$ >140 MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •						
27. ±8.	14	[45] DIAMANT-...	76	SPEC	+	Extrapolated BR

[45] DIAMANT-BERGER 76 gives this result times our 1975 $\pi^+ \pi^- e \nu$ BR ratio. The second DIAMANT-BERGER 76 value is the first value extrapolated to 0 to include low mass $e^+ e^-$ pairs. More recent calculations (BIJNENS 93) of this extrapolation disagree with those of DIAMANT-BERGER 76.

$\Gamma(e^+ \nu_e \mu^+ \mu^-)/\Gamma_{\text{total}}$ Γ_{27}/Γ

VALUE	*CL%*	*DOCUMENT ID*		*TECN*
$\mathbf{<5 \times 10^{-7}}$	90	ADLER	98	B787

$\Gamma(\mu^+ \nu_\mu \mu^+ \mu^-)/\Gamma_{\text{total}}$ Γ_{28}/Γ

VALUE (units 10^{-7})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*
<4.1	90	ATIYA	89	B787	+

Lepton Family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes

$\Gamma(\pi^+\pi^+e^-\overline{\nu}_e)/\Gamma_{\text{total}}$ Γ_{29}/Γ

Test of $\Delta S = \Delta Q$ rule.

VALUE (units 10^{-7})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<$ 9.0	95	0	SCHWEINB...	71	HLBC	+
$<$ 6.9	95	0	ELY	69	HLBC	+
$<$20.	95		BIRGE	65	FBC	+

$\Gamma(\pi^+\pi^+e^-\overline{\nu}_e)/\Gamma(\pi^+\pi^-e^+\nu_e)$ Γ_{29}/Γ_6

Test of $\Delta S = \Delta Q$ rule.

VALUE (units 10^{-4})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*
< 3	90	3	[46] BLOCH	76	SPEC

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<$130.	95	0	BOURQUIN	71	ASPK

[46] BLOCH 76 quotes 3.6×10^{-4} at CL = 95%, we convert.

$\Gamma(\pi^+\pi^+\mu^-\overline{\nu}_\mu)/\Gamma_{\text{total}}$ Γ_{30}/Γ

Test of $\Delta S = \Delta Q$ rule.

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<3.0	95	0	BIRGE	65	FBC	+

$\Gamma(\pi^+e^+e^-)/\Gamma_{\text{total}}$ Γ_{31}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by combined first-order weak and electromagnetic interactions.

VALUE (units 10^{-7})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
2.88±0.13 OUR AVERAGE					
2.94±0.05±0.14	10300	[47] APPEL	99	SPEC	+
2.75±0.23±0.13	500	[48] ALLIEGRO	92	SPEC	+
2.7 ±0.5	41	[49] BLOCH	75	SPEC	+

[47] APPEL 99 establishes vector nature of this decay and determines form factor $f(Z)= f_0(1+\delta Z)$, $Z=M^2_{ee}/m^2_K$, $\delta=2.14 \pm 0.13 \pm 0.15$.

[48] ALLIEGRO 92 assumes a vector interaction with a form factor given by $\lambda = 0.105 \pm 0.035 \pm 0.015$ and a correlation coefficient of -0.82.

[49] BLOCH 75 assumes a vector interaction.

$\Gamma(\pi^+\mu^+\mu^-)/\Gamma_{\text{total}}$ Γ_{32}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions.

VALUE (units 10^{-8})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
7.6 ±2.1 OUR AVERAGE	Error includes scale factor of 3.4.					
9.22±0.60±0.49		402	[50] MA	00	B865	+
5.0 ±0.4 ±0.9		207	[51] ADLER	97C	B787	+

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<$23	90		ATIYA	89	B787	+

[50] MA 00 establishes vector nature of this decay and determines form factor $f(Z)= f_0(1+\delta Z)$, $Z=M^2_{\mu\mu}/m^2_K$, $\delta=2.45^{+1.30}_{-0.95}$.

[51] ADLER 97C gives systematic error 0.7×10^{-8} and theoretical uncertainty 0.6×10^{-8}, which we combine in quadrature to obtain our second error.

$\Gamma(\pi^+ \nu\overline{\nu})/\Gamma_{\text{total}}$ Γ_{33}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions.

VALUE (units 10^{-9})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
$\mathbf{0.157^{+0.175}_{-0.082}}$		2	ADLER	02	B787		

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (units 10^{-9})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
$0.15\ ^{+0.34}_{-0.12}$		1	ADLER	00	B787		In ADLER 02
$0.42\ ^{+0.97}_{-0.35}$		1	ADLER	97	B787		
$<\ 2.4$	90		ADLER	96	B787		
$<\ 7.5$	90		ATIYA	93	B787	+	$T(\pi)$ 115–127 MeV
$<\ 5.2$	90		[52] ATIYA	93	B787	+	
$<\ 17$	90	0	ATIYA	93B	B787	+	$T(\pi)$ 60–100 MeV
$<\ 34$	90		ATIYA	90	B787	+	
<140	90		ASANO	81B	CNTR	+	$T(\pi)$ 116–127 MeV

[52] Combining ATIYA 93 and ATIYA 93B results. Superseded by ADLER 96.

$\Gamma(\pi^+ \pi^0 \nu\overline{\nu})/\Gamma_{\text{total}}$ Γ_{34}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions.

VALUE (units 10^{-5})	*CL%*	*DOCUMENT ID*		*TECN*
<4.3	90	[53] ADLER	01	SPEC

[53] Search region defined by 90 MeV/c<P_{π^+} <188 MeV/c and 135 MeV<E_{π^0} <180 MeV.

$\Gamma(\mu^- \nu e^+ e^+)/\Gamma(\pi^+ \pi^- e^+ \nu_e)$ Γ_{35}/Γ_6

Test of lepton family number conservation.

VALUE (units 10^{-3})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<0.5	90	0	[54] DIAMANT-...	76	SPEC	+

[54] DIAMANT-BERGER 76 quotes this result times our 1975 $\pi^+ \pi^- e \nu$ BR ratio.

$\Gamma(\mu^+ \nu_e)/\Gamma_{\text{total}}$ Γ_{36}/Γ

Forbidden by lepton family number conservation.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<0.004	90	0	[55] LYONS	81	HLBC	0	200 GeV K^+ narrow band ν beam

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<0.012	90		[55] COOPER	82	HLBC		Wideband ν beam

[55] COOPER 82 and LYONS 81 limits on ν_e observation are here interpreted as limits on lepton family number violation in the absence of mixing.

$\Gamma(\pi^+ \mu^+ e^-)/\Gamma_{\text{total}}$ Γ_{37}/Γ

Test of lepton family number conservation.

VALUE (units 10^{-10})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<0.28	90		[56] APPEL	00	RVUE	+

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (units 10^{-10})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<0.39	90		APPEL	00	B865	+
<2.1	90	0	LEE	90	SPEC	+

[56] This result combines APPEL 00 BNL-E865 1996 data, BNL-E865 1995 data from BERGMAN 97 and PISLAK 97 theses, and LEE 90 BNL-E777 data.

$\Gamma(\pi^+ \mu^- e^+)/\Gamma_{\text{total}}$ Γ_{38}/Γ

Test of lepton family number conservation.

VALUE (units 10^{-10})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
< 5.2	90	0	APPEL	00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<70	90	0	[57] DIAMANT-...	76	SPEC	+

[57] Measurement actually applies to the sum of the $\pi^+ \mu^- e^+$ and $\pi^- \mu^+ e^+$ modes.

$\Gamma(\pi^- \mu^+ e^+)/\Gamma_{\text{total}}$ Γ_{39}/Γ

Test of total lepton number conservation.

VALUE (units 10^{-10})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
< 5.0	90	0	APPEL	00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<70	90	0	[58] DIAMANT-...	76	SPEC	+

[58] Measurement actually applies to the sum of the $\pi^+ \mu^- e^+$ and $\pi^- \mu^+ e^+$ modes.

$\Gamma(\pi^- e^+ e^+)/\Gamma_{\text{total}}$ Γ_{40}/Γ

Test of total lepton number conservation.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
$<6.4 \times 10^{-10}$	90	0	APPEL	00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$<9.2 \times 10^{-9}$	90	0	DIAMANT-...	76	SPEC	+
$<1.5 \times 10^{-5}$			CHANG	68	HBC	−

$\Gamma(\pi^- \mu^+ \mu^+)/\Gamma_{\text{total}}$ Γ_{41}/Γ

Forbidden by total lepton number conservation.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
$<3.0 \times 10^{-9}$	90	0	APPEL	00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$<1.5 \times 10^{-4}$	90		[59] LITTENBERG	92	HBC	

[59] LITTENBERG 92 is from retroactive data analysis of CHANG 68 bubble chamber data.

$\Gamma(\mu^+ \overline{\nu}_e)/\Gamma_{\text{total}}$ Γ_{42}/Γ

Forbidden by total lepton number conservation.

VALUE (units 10^{-3})	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
<3.3	90	[60] COOPER	82	HLBC	Wideband ν beam

[60] COOPER 82 limit on $\overline{\nu}_e$ observation is here interpreted as a limit on lepton number violation in the absence of mixing.

$\Gamma(\pi^0 e^+ \overline{\nu}_e)/\Gamma_{\text{total}}$ Γ_{43}/Γ

Forbidden by total lepton number conservation.

VALUE	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
<0.003	90	[61] COOPER	82	HLBC	Wideband ν beam

[61] COOPER 82 limit on $\overline{\nu}_e$ observation is here interpreted as a limit on lepton number violation in the absence of mixing.

$\Gamma(\pi^+\gamma)/\Gamma_{\text{total}}$ Γ_{44}/Γ

Violates angular momentum conservation. Current interest in this decay is as a search for exotic physics such as a vacuum expectation value of a new vector field, non-local Superstring effects, or departures from Lorentz invariance, as discussed in ADLER 02B.

VALUE (units 10^{-7})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*
< 3.6	90	ADLER	02B	B787	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
<14	90	ASANO	82	CNTR	+
<40	90	[62] KLEMS	71	OSPK	+

[62] Test of model of Selleri, Nuovo Cimento **60A** 291 (1969).

K^+ LONGITUDINAL POLARIZATION OF EMITTED μ^+

VALUE	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<−0.990	90	[63] AOKI	94	SPEC	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<−0.990	90	IMAZATO	92	SPEC	+	Repl. by AOKI 94
−0.970±0.047		[64] YAMANAKA	86	SPEC	+	
−1.0 ±0.1		[64] CUTTS	69	SPRK	+	
−0.96 ±0.12		[64] COOMBES	57	CNTR	+	

[63] AOKI 94 measures $\xi P_\mu = -0.9996 \pm 0.0030 \pm 0.0048$. The above limit is obtained by summing the statistical and systematic errors in quadrature, normalizing to the physically significant region ($|\xi P_\mu| < 1$) and assuming that ξ=1, its maximum value.

[64] Assumes ξ=1.

DALITZ PLOT PARAMETERS FOR $K \rightarrow 3\pi$ DECAYS

Revised 1999 by T.G. Trippe (LBNL).

The Dalitz plot distribution for $K^\pm \rightarrow \pi^\pm\pi^\pm\pi^\mp$, $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$, and $K^0_L \rightarrow \pi^+\pi^-\pi^0$ can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

$$\begin{aligned}\left|M\right|^2 \propto 1 &+ g\frac{(s_3-s_0)}{am^2_{\pi^+}} + h\left[\frac{s_3-s_0}{m^2_{\pi^+}}\right]^2 \\ &+j\ \frac{(s_2-s_1)}{m^2_{\pi^+}} + k\left[\frac{s_2-s_1}{m^2_{\pi^+}}\right]^2 \\ &+f\ \frac{(s_2-s_1)}{m^2_{\pi^+}}\frac{(s_3-s_0)}{m^2_{\pi^+}} + \cdots\ , \qquad (1)\end{aligned}$$

where $m_{\pi^+}^2$ has been introduced to make the coefficients g, h, j, and k dimensionless, and

$$s_i = (P_K - P_i)^2 = (m_K - m_i)^2 - 2m_K T_i \ , \ i = 1, 2, 3,$$
$$s_0 = \frac{1}{3}\sum_i s_i = \frac{1}{3}(m_K^2 + m_1^2 + m_2^2 + m_3^2)$$

Here the P_i are four-vectors, m_i and T_i are the mass and kinetic energy of the i^{th} pion, and the index 3 is used for the odd pion.

The coefficient g is a measure of the slope in the variable s_3 (or T_3) of the Dalitz plot, while h and k measure the quadratic dependence on s_3 and $(s_2 - s_1)$, respectively. The coefficient j is related to the asymmetry of the plot and must be zero if CP invariance holds. Note also that if CP is good, g, h, and k must be the same for $K^+ \to \pi^+\pi^+\pi^-$ as for $K^- \to \pi^-\pi^-\pi^+$.

Since different experiments use different forms for $\left|M\right|^2$, in order to compare the experiments we have converted to g, h, j, and k whatever coefficients have been measured. Where such conversions have been done, the measured coefficient a_y, a_t, a_u, or a_v is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

References

1. S. Weinberg, Phys. Rev. Lett. **4**, 87 (1960).
2. Particle Data Group, Phys. Lett. **111B**, 69 (1982).

ENERGY DEPENDENCE OF $K^{\pm}$ DALITZ PLOT

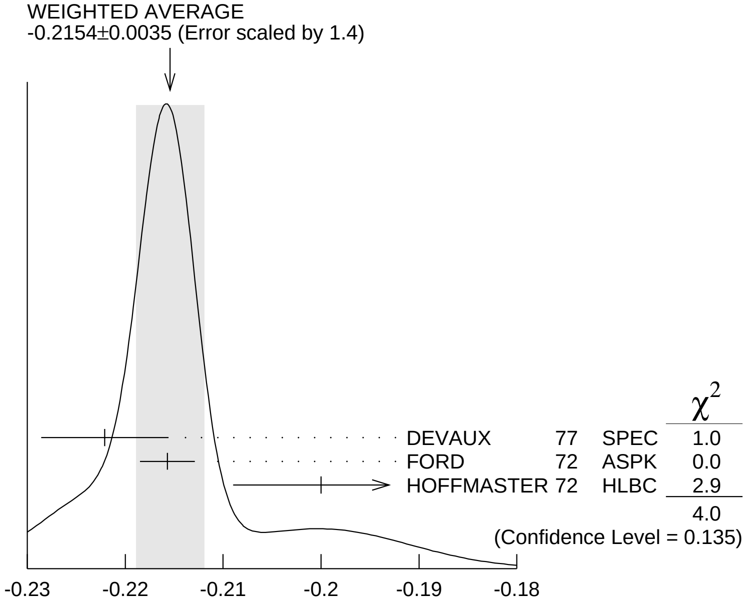
$|\text{matrix element}|^2 = 1 + gu + hu^2 + kv^2$
where $u = (s_3 - s_0) / m_\pi^2$ and $v = (s_1 - s_2) / m_\pi^2$

LINEAR COEFFICIENT g_{τ^+} FOR $K^+ \rightarrow \pi^+\pi^+\pi^-$

Some experiments use Dalitz variables x and y. In the comments we give a_y = coefficient of y term. See note above on "Dalitz Plot Parameters for $K \rightarrow 3\pi$ Decays." For discussion of the conversion of a_y to g, see the earlier version of the same note in the *Review* published in Physics Letters **111B** 70 (1982).

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
−0.2154±0.0035 OUR AVERAGE	Error includes scale factor of 1.4. See the ideogram below.				
−0.2221±0.0065	225k	DEVAUX 77	SPEC	+	a_y=.2814 ± .0082
−0.2157±0.0028	750k	FORD 72	ASPK	+	a_y=.2734 ± .0035
−0.200 ±0.009	39819	[65] HOFFMASTER 72	HLBC	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.196 ±0.012	17898	[66] GRAUMAN 70	HLBC	+	a_y=0.228 ± 0.030
−0.218 ±0.016	9994	[67] BUTLER 68	HBC	+	a_y=0.277 ± 0.020
−0.22 ±0.024	5428	[67,68] ZINCHENKO 67	HBC	+	a_y=0.28 ± 0.03

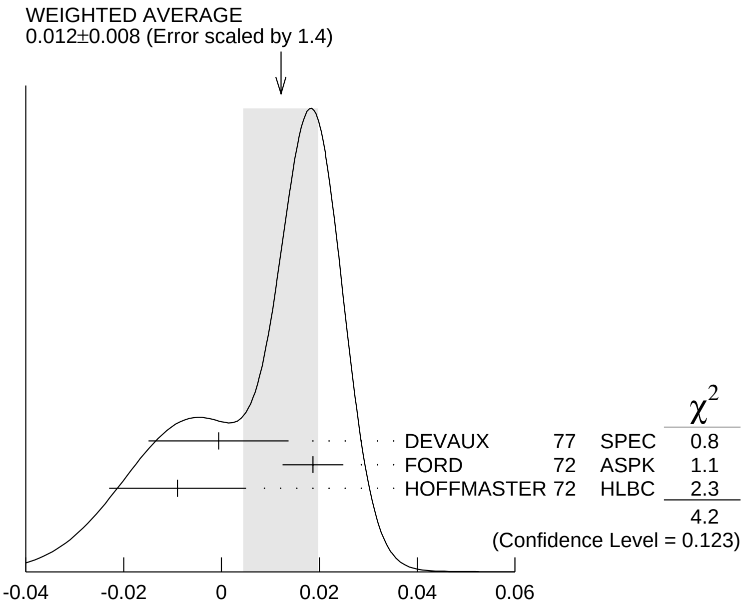
[65] HOFFMASTER 72 includes GRAUMAN 70 data.
[66] Emulsion data added — all events included by HOFFMASTER 72.
[67] Experiments with large errors not included in average.
[68] Also includes DBC events.



Linear energy dependence for $K^+ \rightarrow \pi^+\pi^+\pi^-$

QUADRATIC COEFFICIENT *h* FOR $K^+ \rightarrow \pi^+\pi^+\pi^-$

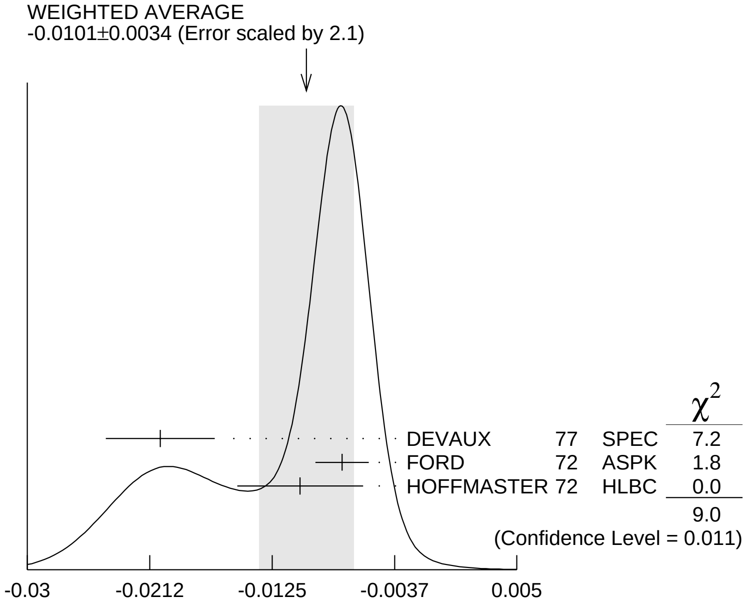
VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.012 ±0.008 OUR AVERAGE	Error includes scale factor of 1.4. See the ideogram below.				
−0.0006±0.0143	225k	DEVAUX	77	SPEC	+
0.0187±0.0062	750k	FORD	72	ASPK	+
−0.009 ±0.014	39819	HOFFMASTER	72	HLBC	+



Quadratic coefficient *h* for $K^+ \rightarrow \pi^+\pi^+\pi^-$

QUADRATIC COEFFICIENT *k* FOR $K^+ \rightarrow \pi^+\pi^+\pi^-$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
−0.0101±0.0034 OUR AVERAGE	Error includes scale factor of 2.1. See the ideogram below.				
−0.0205±0.0039	225k	DEVAUX	77	SPEC	+
−0.0075±0.0019	750k	FORD	72	ASPK	+
−0.0105±0.0045	39819	HOFFMASTER	72	HLBC	+



Quadratic coefficient k for $K^+ \rightarrow \pi^+\pi^+\pi^-$

LINEAR COEFFICIENT g_{τ^-} FOR $K^- \rightarrow \pi^-\pi^-\pi^+$

Some experiments use Dalitz variables x and y. In the comments we give a_y = coefficient of y term. See note above on "Dalitz Plot Parameters for $K \rightarrow 3\pi$ Decays." For discussion of the conversion of a_y to g, see the earlier version of the same note in the *Review* published in Physics Letters **111B** 70 (1982).

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
−0.217 ±0.007 OUR AVERAGE		Error includes scale factor of 2.5.				
−0.2186±0.0028	750k	FORD	72	ASPK	−	a_y=.2770 ± .0035
−0.193 ±0.010	50919	MAST	69	HBC	−	a_y=0.244 ± 0.013
• • • We do not use the following data for averages, fits, limits, etc. • • •						
−0.199 ±0.008	81k	[69] LUCAS	73	HBC	−	a_y=0.252 ± 0.011
−0.190 ±0.023	5778	[70,71] MOSCOSO	68	HBC	−	a_y=0.242 ± 0.029
−0.220 ±0.035	1347	[72] FERRO-LUZZI	61	HBC	−	a_y=0.28 ± 0.045

[69] Quadratic dependence is required by K^0_L experiments. For comparison we average only those $K^\pm$ experiments which quote quadratic fit values.

[70] Experiments with large errors not included in average.

[71] Also includes DBC events.

[72] No radiative corrections included.

QUADRATIC COEFFICIENT h FOR $K^- \rightarrow \pi^-\pi^-\pi^+$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.010 ±0.006 OUR AVERAGE					
0.0125±0.0062	750k	FORD	72	ASPK	−
−0.001 ±0.012	50919	MAST	69	HBC	−

QUADRATIC COEFFICIENT k FOR $K^- \to \pi^-\pi^-\pi^+$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
$-$0.0084$\pm$0.0019 OUR AVERAGE				
$-$0.0083$\pm$0.0019	750k	FORD 72	ASPK	$-$
$-$0.014 $\pm$0.012	50919	MAST 69	HBC	$-$

$(g_{\tau^+} - g_{\tau^-}) / (g_{\tau^+} + g_{\tau^-})$ FOR $K^\pm \to \pi^\pm\pi^+\pi^-$

A nonzero value for this quantity indicates *CP* violation.

VALUE (%)	*EVTS*	*DOCUMENT ID*	*TECN*
$-$0.70$\pm$0.53	3.2M	FORD 70	ASPK

LINEAR COEFFICIENT g FOR $K^\pm \to \pi^\pm\pi^0\pi^0$

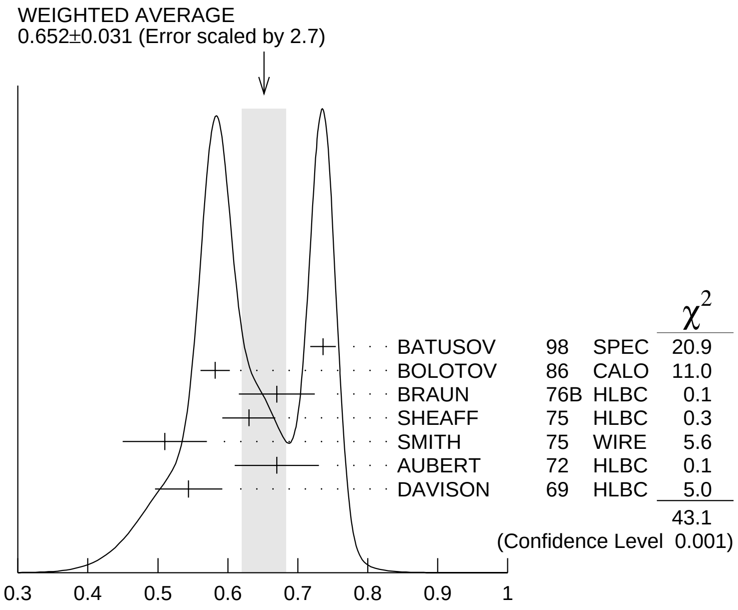
Unless otherwise stated, all experiments include terms quadratic in $(s_3 - s_0) / m^2_{\pi^+}$. See note above on "Dalitz Plot Parameters for $K \to 3\pi$ Decays."

See BATUSOV 98 for a discussion of the discrepancy between their result and others, especially BOLOTOV 86. At this time we have no way to resolve the discrepancy so we depend on the large scale factor as a warning.

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.652$\pm$0.031 OUR AVERAGE	Error includes scale factor of 2.7. See the ideogram below.				
0.736$\pm$0.014$\pm$0.012	33k	BATUSOV 98	SPEC	+	
0.582$\pm$0.021	43k	BOLOTOV 86	CALO	$-$	
0.670$\pm$0.054	3263	BRAUN 76B	HLBC	+	
0.630$\pm$0.038	5635	SHEAFF 75	HLBC	+	
0.510$\pm$0.060	27k	SMITH 75	WIRE	+	
0.67 $\pm$0.06	1365	AUBERT 72	HLBC	+	
0.544$\pm$0.048	4048	DAVISON 69	HLBC	+	Also emulsion
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.518$\pm$0.039	815	[73] SHIN 00	SPEC	+	
0.806$\pm$0.220	4639	[74] BERTRAND 76	EMUL	+	
0.484$\pm$0.084	574	[73] LUCAS 73B	HBC	$-$	Dalitz pairs only
0.527$\pm$0.102	198	[74] PANDOULAS 70	EMUL	+	
0.586$\pm$0.098	1874	[73] BISI 65	HLBC	+	Also HBC
0.48 $\pm$0.04	1792	[73] KALMUS 64	HLBC	+	

[73] Authors give linear fit only.

[74] Experiments with large errors not included in average.

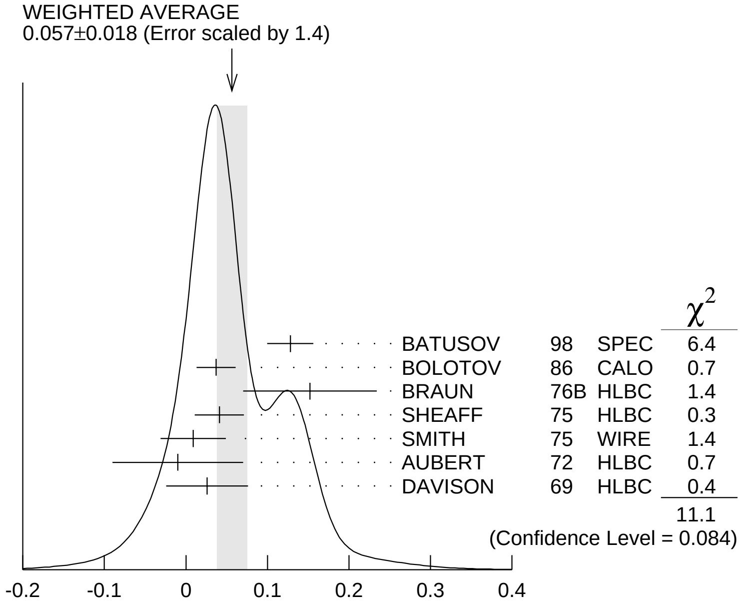


Linear energy dependence for $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

QUADRATIC COEFFICIENT *h* FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.057±0.018 OUR AVERAGE	Error includes scale factor of 1.4. See the ideogram below.				
0.128±0.015±0.024	33k	BATUSOV 98	SPEC	+	
0.037±0.024	43k	BOLOTOV 86	CALO	−	
0.152±0.082	3263	BRAUN 76B	HLBC	+	
0.041±0.030	5635	SHEAFF 75	HLBC	+	
0.009±0.040	27k	SMITH 75	WIRE	+	
−0.01 ±0.08	1365	AUBERT 72	HLBC	+	
0.026±0.050	4048	DAVISON 69	HLBC	+	Also emulsion
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.164±0.121	4639	[75] BERTRAND 76	EMUL	+	
0.018±0.124	198	[75] PANDOULAS 70	EMUL	+	

[75] Experiments with large errors not included in average.



Quadratic coefficient *h* FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$

QUADRATIC COEFFICIENT *k* FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.0197±0.0045±0.0029	33k	BATUSOV	98	SPEC	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.043 ±0.020	815	SHIN	00	SPEC	+

$K^{\pm}_{\ell 3}$ AND $K^{0}_{\ell 3}$ FORM FACTORS

Revised May 2002 by T.G. Trippe (LBNL).

Assuming that only the vector current contributes to $K \rightarrow \pi\ell\nu$ decays, we write the matrix element as

$$M \propto f_+(t)\left[(P_K + P_\pi)_\mu \overline{\ell}\gamma_\mu(1+\gamma_5)\nu\right] + f_-(t)\left[m_\ell \overline{\ell}(1+\gamma_5)\nu\right] \;, \qquad (1)$$

where P_K and P_π are the four-momenta of the K and π mesons, m_ℓ is the lepton mass, and f_+ and f_- are dimensionless form factors which can depend only on $t = (P_K - P_\pi)^2$, the square of the four-momentum transfer to the leptons. If time-reversal invariance holds, f_+ and f_- are relatively real. $K_{\mu 3}$

experiments, discussed immediately below, measure f_+ and f_-, while K_{e3} experiments, discussed further below, are sensitive only to f_+ because the small electron mass makes the f_- term negligible.

For this edition, fits to the form factor data have been made with and without the assumption of μ-e universality, as described near the end of the note.

$K_{\mu 3}$ Experiments. Analyses of $K_{\mu 3}$ data frequently assume a linear dependence of f_+ and f_- on t, *i.e.*,

$$f_{\pm}(t) = f_{\pm}(0)\left[1 + \lambda_{\pm}(t/m_{\pi}^2)\right] \tag{2}$$

Most $K_{\mu 3}$ data are adequately described by Eq. (2) for f_+ and a constant f_- (*i.e.*, $\lambda_- = 0$). There are two equivalent parametrizations commonly used in these analyses:

(1) $\lambda_+, \xi(0)$ parametrization. Analyses of $K_{\mu 3}$ data often introduce the ratio of the two form factors

$$\xi(t) = f_-(t)/f_+(t)\ . \tag{3}$$

The $K_{\mu 3}$ decay distribution is then described by the two parameters λ_+ and $\xi(0)$ (assuming time reversal invariance and $\lambda_- = 0$). These parameters can be determined by three different methods:

Method A. By studying the Dalitz plot or the pion spectrum of $K_{\mu 3}$ decay. The Dalitz plot density is (see, *e.g.*, Chounet *et al.* [1]):

$$\rho(E_\pi, E_\mu) \propto f_+^2(t)\left[A + B\xi(t) + C\xi(t)^2\right]\ ,$$

where

$$\begin{aligned} A &= m_K\left(2E_\mu E_\nu - m_K E'_\pi\right) + m_\mu^2\left(\frac{1}{4}E'_\pi - E_\nu\right) \ , \\ B &= m_\mu^2\left(E_\nu - \frac{1}{2}E'_\pi\right) \ , \\ C &= \frac{1}{4}m_\mu^2 E'_\pi \ , \\ E'_\pi &= E_\pi^{\max} - E_\pi = \left(m_K^2 + m_\pi^2 - m_\mu^2\right)/2m_K - E_\pi \ . \end{aligned} \quad (4)$$

Here E_π, E_μ, and E_ν are, respectively, the pion, muon, and neutrino energies in the kaon center of mass. The density ρ is fit to the data to determine the values of $\lambda_+, \xi(0)$, and their correlation.

Method B. By measuring the $K_{\mu3}/K_{e3}$ branching ratio and comparing it with the theoretical ratio (see, *e.g.*, Fearing *et al.* [2]) as given in terms of λ_+ and $\xi(0)$, assuming μ-e universality:

$$\begin{aligned} \Gamma(K_{\mu3}^{\pm})/\Gamma(K_{e3}^{\pm}) &= 0.6457 + 1.4115\lambda_+ + 0.1264\xi(0) \\ &\quad + 0.0192\xi(0)^2 + 0.0080\lambda_+\xi(0) \ , \\ \Gamma(K_{\mu3}^{0})/\Gamma(K_{e3}^{0}) &= 0.6452 + 1.3162\lambda_+ + 0.1264\xi(0) \\ &\quad + 0.0186\xi(0)^2 + 0.0064\lambda_+\xi(0) \ . \end{aligned} \quad (5)$$

This cannot determine λ_+ and $\xi(0)$ simultaneously but simply fixes a relationship between them.

Method C. By measuring the muon polarization in $K_{\mu3}$ decay. In the rest frame of the K, the μ is expected to be polarized in the direction $\mathbf{A}$ with $\mathbf{P} = \mathbf{A}/\left|\mathbf{A}\right|$, where $\mathbf{A}$ is given (Cabined and Maksymowicz [3]) by

$$\begin{aligned}\mathbf{A} = {} & a_1(\xi)\mathbf{p}_\mu \\ & - a_2(\xi)\left[\frac{\mathbf{p}_\mu}{m_\mu}\left(m_K - E_\pi + \frac{\mathbf{p}_\pi \cdot \mathbf{p}_\mu}{\left|\mathbf{p}_\mu\right|^2}(E_\mu - m_\mu)\right) + \mathbf{p}_\pi\right] \\ & + m_K \mathrm{Im}\xi(t)(\mathbf{p}_\pi \times \mathbf{p}_\mu) \ . \end{aligned} \tag{6}$$

If time-reversal invariance holds, ξ is real, and thus there is no polarization perpendicular to the K-decay plane. Polarization experiments measure the weighted average of $\xi(t)$ over the t range of the experiment, where the weighting accounts for the variation with t of the sensitivity to $\xi(t)$.

(2) $\boldsymbol{\lambda_+, \lambda_0}$ parametrization. Most of the more recent $K_{\mu 3}$ analyses have parameterized in terms of the form factors f_+ and f_0 which are associated with vector and scalar exchange, respectively, to the lepton pair. f_0 is related to f_+ and f_- by

$$f_0(t) = f_+(t) + \left[t/(m_K^2 - m_\pi^2)\right] f_-(t) \ . \tag{7}$$

Here $f_0(0)$ must equal $f_+(0)$ unless $f_-(t)$ diverges at $t = 0$. The earlier assumption that f_+ is linear in t and f_- is constant leads to f_0 linear in t:

$$f_0(t) = f_0(0)\left[1 + \lambda_0(t/m_\pi^2)\right] \ . \tag{8}$$

With the assumption that $f_0(0) = f_+(0)$, the two parametrizations, $(\lambda_+, \xi(0))$ and (λ_+, λ_0) are equivalent as long as correlation information is retained. (λ_+, λ_0) correlations tend to be less strong than $(\lambda_+, \xi(0))$ correlations.

The experimental results for $\xi(0)$ and its correlation with λ_+ are listed in the $K^\pm$ and K_L^0 sections of the Particle Listings in section ξ_A, ξ_B, or ξ_C depending on whether method A, B, or C discussed above was used. The corresponding values of λ_+ are also listed.

Because recent experiments tend to use the (λ_+, λ_0) parametrization, we include a subsection for λ_0 results. Wherever possible we have converted $\xi(0)$ results into λ_0 results and vice versa.

See the 1982 version of this note [4] for additional discussion of the $K^0_{\mu 3}$ parameters, correlations, and conversion between parametrizations.

K_{e3} Experiments. Analysis of K_{e3} data is simpler than that of $K_{\mu 3}$ because the second term of the matrix element assuming a pure vector current [Eq. (1) above] can be neglected. Here f_+ is usually assumed to be linear in t, and the linear coefficient λ_+ of Eq. (2) is determined.

If we remove the assumption of a pure vector current, then the matrix element for the decay, in addition to the terms in Eq. (1), would contain

$$\begin{aligned}&+2m_K\, f_S\, \overline{\ell}(1+\gamma_5)\nu \\ &\qquad +(2f_T/m_K)(P_K)_\lambda (P_\pi)_\mu\, \overline{\ell}\, \sigma_{\lambda\mu}(1+\gamma_5)\nu\ , \qquad (9)\end{aligned}$$

where f_S is the scalar form factor, and f_T is the tensor form factor. In the case of the K_{e3} decays where the f_- term can be neglected, experiments have yielded limits on $|f_S/f_+|$ and $|f_T/f_+|$.

Fits for $K_{\ell 3}$FormFactors. Fits are made for the two parameters λ_+ and λ_0 and their correlation, or equivalently for λ_+ and $\xi(0)$ and their correlation. The fits are done both with and without assuming μ-e universality. This assumption sets $\lambda_+(K_{\mu 3}) = \lambda_+(K_{e3})$ so that the more precise λ_+ values from K_{e3} can be included with the $K_{\mu 3}$ data in the fit. In addition it allows the $K_{\mu 3}/K_{e3}$ branching ratio to be used in the determination of $\xi(0)$ or λ_0 as a function of λ_+. Without this assumption, $\lambda_+(K_{e3})$ values are averaged separately and excluded from the fit to $K_{\mu 3}$ data, and branching ratio data are not used. The Kaon Particle Listings show the results with and without assuming μ-e universality.

References

1. L.M. Chounet, J.M. Gaillard, and M.K. Gaillard, Phys. Reports **4C**, 199 (1972).
2. H.W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. **D2**, 542 (1970).
3. N. Cabined and A. Mathematica, Phys. Lett. **9**, 352 (1964).
4. Particle Data Group, Phys. Lett. **111B**, 73 (1982).

$K^{\pm}_{\ell 3}$ FORM FACTORS

In the form factor comments, the following symbols are used.

f_+ and f_- are form factors for the vector matrix element.

f_S and f_T refer to the scalar and tensor term.

$f_0 = f_+ + f_- t/(m^2_K - m^2_\pi)$.

λ_+, λ_-, and λ_0 are the linear expansion coefficients of f_+, f_-, and f_0.

λ_+ refers to the $K^{\pm}_{\mu 3}$ value except in the $K^{\pm}_{e3}$ sections.

$d\xi(0)/d\lambda_+$ is the correlation between $\xi(0)$ and λ_+ in $K^{\pm}_{\mu 3}$.

$d\lambda_0/d\lambda_+$ is the correlation between λ_0 and λ_+ in $K^{\pm}_{\mu 3}$.

t = momentum transfer to the π in units of m^2_π.

DP = Dalitz plot analysis.

PI = π spectrum analysis.

MU = μ spectrum analysis.

POL= μ polarization analysis.

BR = $K^{\pm}_{\mu 3}/K^{\pm}_{e3}$ branching ratio analysis.

E = positron or electron spectrum analysis.

RC = radiative corrections.

λ_+ (LINEAR ENERGY DEPENDENCE OF f_+ IN $K^{\pm}_{e3}$ DECAY)

For radiative correction of $K^{\pm}_{e3}$ Dalitz plot, see GINSBERG 67 and BECHERRAWY 70.

Results labeled OUR FIT are discussed in the review "$K^{\pm}_{\ell 3}$ and $K^0_{\ell 3}$ Form Factors" above.

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.0282±0.0027 OUR FIT	Error includes scale factor of 1.5. Assumes μ-e universality.					
0.0278±0.0019 OUR AVERAGE						
0.0278±0.0026±0.0030	41k	SHIMIZU	00	SPEC	+	DP, uses RC
0.018 ±0.007	3000	ARTEMOV	97B	SPEC	−	DP
0.0284±0.0027±0.0020	32k	[76] AKIMENKO	91	SPEC		PI, no RC
0.029 ±0.004	62k	[77] BOLOTOV	88	SPEC		PI, no RC
0.027 ±0.008		[78] BRAUN	73B	HLBC	+	DP, no RC
0.029 ±0.011	4017	CHIANG	72	OSPK	+	DP, RC negligible
0.027 ±0.010	2707	STEINER	71	HLBC	+	DP, uses RC
0.045 ±0.015	1458	BOTTERILL	70	OSPK		PI, uses RC
0.045 $^{+0.017}_{-0.018}$	854	BELLOTTI	67B	FBC	+	DP, uses RC
+0.016 ±0.016	1393	IMLAY	67	OSPK	+	DP, no RC
+0.028 $^{+0.013}_{-0.014}$	515	KALMUS	67	FBC	+	e^+, PI, no RC
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.025 ±0.007		[79] BRAUN	74	HLBC	+	$K_{\mu 3}/K_{e3}$ vs. t

[76] AKIMENKO 91 state that radiative corrections would raise λ_+ by 0.0013.

[77] BOLOTOV 88 state radiative corrections of GINSBERG 67 would raise λ_+ by 0.002.

[78] BRAUN 73B states that radiative corrections of GINSBERG 67 would lower λ^e_+ by 0.002 but that radiative corrections of BECHERRAWY 70 disagrees and would raise λ^e_+ by 0.005.

[79] BRAUN 74 is a combined $K_{\mu3}$-K_{e3} result. It is not independent of BRAUN 73C ($K_{\mu3}$) and BRAUN 73B (K_{e3}) form factor results.

$\xi_A = f_-/f_+$ (determined from $K^\pm_{\mu3}$ spectra)

Results labeled OUR FIT are discussed in the review "$K^\pm_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above. ξ_A is $\xi(0)$ determined by Method *A* of that review. The parameter $\xi(0)$ is redundant with λ_0 below and is not put into the Meson Summary Table.

VALUE	*$d\xi(0)/d\lambda_+$*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
−0.19±0.07 OUR FIT	Error includes scale factor of 1.5. Correlation is $d\xi(0)/d\lambda_+ = -12.6$. Assumes μ-e universality.						
−0.36±0.18 OUR FIT	Error includes scale factor of 1.8. Correlation is $d\xi(0)/d\lambda_+ = -13.8$.						
+0.54±0.39	−12	3000	[80] ARTEMOV	97B	SPEC	−	DP
−0.27±0.25	−17	3973	WHITMAN	80	SPEC	+	DP
−0.8 ±0.8	−20	490	[81] ARNOLD	74	HLBC	+	DP
−0.57±0.24	−9	6527	[82] MERLAN	74	ASPK	+	DP
−0.36±0.40	−19	1897	[83] BRAUN	73C	HLBC	+	DP
−0.62±0.28	−12	4025	[84] ANKENBRA...	72	ASPK	+	PI
+0.45±0.28	−15	3480	[85] CHIANG	72	OSPK	+	DP
−0.5 ±0.8	−26	2041	[86] KIJEWSKI	69	OSPK	+	PI
+0.72±0.93	−17	444	CALLAHAN	66B	FBC	+	PI
• • • We do not use the following data for averages, fits, limits, etc. • • •							
−1.1 ±0.56	−29	3240	[87] HAIDT	71	HLBC	+	DP
−0.5 ±0.9	none	78	EISLER	68	HLBC	+	PI, $\lambda_+=0$
$0.0\ ^{+1.1}_{-0.9}$		2648	[88] CALLAHAN	66B	FBC	+	μ, $\lambda_+=0$
+0.7 ±0.5		87	GIACOMELLI	64	EMUL	+	MU+BR,$\lambda_+=0$
−0.08±0.7			[89] JENSEN	64	XEBC	+	DP+BR
+1.8 ±0.6		76	BROWN	62B	XEBC	+	DP+BR, $\lambda_+=0$

[80] Calculated from ARTEMOV 97B λ_+, λ_0, and $d\lambda_0/d\lambda_+$.

[81] ARNOLD 74 figure 4 was used to obtain ξ_A and $d\xi(0)/d\lambda_+$.

[82] MERLAN 74 figure 5 was used to obtain $d\xi(0)/d\lambda_+$.

[83] BRAUN 73C gives $\xi(t) = -0.34 \pm 0.20$, $d\xi(t)/d\lambda_+ = -14$ for $\lambda_+ = 0.027$, $t = 6.6$. We calculate above $\xi(0)$ and $d\xi(0)/d\lambda_+$ for their $\lambda_+ = 0.025 \pm 0.017$.

[84] ANKENBRANDT 72 figure 3 was used to obtain $d\xi(0)/d\lambda_+$.

[85] CHIANG 72 figure 10 was used to obtain $d\xi(0)/d\lambda_+$. Fit had $\lambda_- = \lambda_+$ but would not change for $\lambda_- = 0$. L.Pondrom, (private communication 74).

[86] KIJEWSKI 69 figure 17 was used to obtain $d\xi(0)/d\lambda_+$ and errors.

[87] HAIDT 71 table 8 (Dalitz plot analysis) gives $d\xi(0)/d\lambda_+ = (-1.1+0.5)/(0.050-0.029) = -29$, error raised from 0.50 to agree with $d\xi(0) = 0.20$ for fixed λ_+. Not included in fit because of large disagreement with more precise $K_{\mu3}/K_{e3}$ branching ratio measurement.

[88] CALLAHAN 66 table 1 (π analysis) gives $d\xi(0)/d\lambda_+ = (0.72\text{–}0.05)/(0\text{–}0.04) = -17$, error raised from 0.80 to agree with $d\xi(0) = 0.37$ for fixed λ_+. t unknown.

[89] JENSEN 64 gives $\lambda^\mu_+ = \lambda^e_+ = -0.020 \pm 0.027$. $d\xi(0)/d\lambda_+$ unknown. Includes SHAKLEE 64 $\xi_B(K_{\mu3}/K_{e3})$.

$\xi_B = f_-/f_+$ (determined from $K^\pm_{\mu3}/K^\pm_{e3}$)

The $K^\pm_{\mu3}/K^\pm_{e3}$ branching ratio fixes a relationship between $\xi(0)$ and λ_+ if μ-e universality is assumed. We quote the author's $\xi(0)$ and associated λ_+ but do not average because the λ_+ values differ. The result labeled OUR FIT below does not use these ξ_B values. Instead it uses the authors $K^+_{\mu3}/K^+_{e3}$ branching ratios to obtain the fitted $K^\pm_{\mu3}/K^\pm_{e3}$ ratio which is then converted to KL3FIT value below, as discussed in the review "$K^\pm_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above. ξ_B is $\xi(0)$ determined by Method B of that review. The parameter $\xi(0)$ is redundant with λ_0 below and is not put into the Meson Summary Table.

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
−0.19±0.07 OUR FIT Error includes scale factor of 1.5. Correlation is $d\xi(0)/d\lambda_+=-12.6$. Assumes μ-e universality.					
−0.13±0.06		[90] KL3FIT 02	RVUE		$\lambda_+ = 0.030$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.12±0.12	55k	[91] HEINTZE 77	CNTR	+	$\lambda_+=0.029$
0.0 ±0.15	5825	CHIANG 72	OSPK	+	$\lambda_+=0.03$, fig.10
−0.81±0.27	1505	[92] HAIDT 71	HLBC	+	$\lambda_+=0.028$, fig.8
−0.35±0.22		[93] BOTTERILL 70	OSPK	+	$\lambda_+=0.045\pm0.015$
+0.91±0.82		ZELLER 69	ASPK	+	$\lambda_+=0.023$
−0.08±0.15	5601	[93] BOTTERILL 68B	ASPK	+	$\lambda_+=0.023\pm0.008$
−0.60±0.20	1398	[92] EICHTEN 68	HLBC	+	See note
+1.0 ±0.6	986	GARLAND 68	OSPK	+	$\lambda_+=0$
+0.75±0.50	306	AUERBACH 67	OSPK	+	$\lambda_+=0$
+0.4 ±0.4	636	CALLAHAN 66B	FBC	+	$\lambda_+=0$
+0.6 ±0.5		BISI 65B	HBC	+	$\lambda_+=0$
+0.8 ±0.6	500	CUTTS 65	OSPK	+	$\lambda_+=0$
$-0.17^{+0.75}_{-0.99}$		SHAKLEE 64	XEBC	+	$\lambda_+=0$

[90] KL3FIT value is from fitted $K^\pm_{\mu3}/K^\pm_{e3}$ branching ratio. $d\xi(0)/d\lambda_+ = -11.6$.

[91] Calculated by us from λ_0 and λ_+ given below.

[92] EICHTEN 68 has $\lambda_+ = 0.023 \pm 0.008$, $t = 4$, independent of λ_-. Replaced by HAIDT 71.

[93] BOTTERILL 70 is re-evaluation of BOTTERILL 68B with different λ_+.

$\xi_C = f_-/f_+$ (determined from μ polarization in $K^\pm_{\mu3}$)

The μ polarization is a measure of $\xi(t)$. No assumptions on λ_{+-} are necessary, but t (weighted by sensitivity to $\xi(t)$) should be specified. In λ_+, $\xi(0)$ parametrization this is $\xi(0)$ for $\lambda_+=0$. $d\xi/d\lambda = \xi t$. For radiative correction to muon polarization in $K^\pm_{\mu3}$, see GINSBERG 71. Results labeled OUR FIT are discussed in the review "$K^\pm_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above. ξ_C is $\xi(0)$ determined by Method C of that review. The parameter $\xi(0)$ is redundant with λ_0 below and is not put into the Meson Summary Table.

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
−0.19±0.07 OUR FIT Error includes scale factor of 1.5. Correlation is $d\xi(0)/d\lambda_+ = -12.6$. Assumes μ-e universality.					
−0.36±0.18 OUR FIT Error includes scale factor of 1.8. Correlation is $d\xi(0)/d\lambda_+ = -13.8$.					
−0.95±0.3	3133	[94] CUTTS 69	OSPK	+	Total pol. t=4.0
−1.0 ±0.3	6000	[95] BETTELS 68	HLBC	+	Total pol. t=4.9

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
−0.25±1.20	1585	[96] BRAUN 75	HLBC	+	POL, t=4.2
−0.64±0.27	40k	[97] MERLAN 74	ASPK	+	POL, $d\xi(0)/d\lambda_+ = +1.7$

[94] CUTTS 69 $t = 4.0$ was calculated from figure 8. $d\xi(0)/d\lambda_+ = \xi t = -0.95\times4 = -3.8$.

[95] BETTELS 68 $d\xi(0)/d\lambda_+ = \xi t = -1.0\times4.9 = -4.9$.

[96] BRAUN 75 $d\xi(0)/d\lambda_+ = \xi t = -0.25\times4.2 = -1.0$.

[97] MERLAN 74 polarization result (figure 5) not possible. See discussion of polarization experiments in note on "$K_{\ell3}$ Form Factors" in the 1982 edition of this *Review* [Physics Letters **111B** (1982)].

Im(ξ) in $K^{\pm}_{\mu3}$ DECAY (from transverse μ pol.)

Test of T reversal invariance.

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
−0.014±0.014 OUR AVERAGE					
−0.013±0.016±0.003	3.9M	ABE 99S	CNTR	+	p_T K^+ at rest
−0.016±0.025	20M	CAMPBELL 81	CNTR	+	Pol.

λ_+ (LINEAR ENERGY DEPENDENCE OF f_+ IN $K^{\pm}_{\mu3}$ DECAY)

See also the corresponding entries and footnotes in sections ξ_A, ξ_C, and λ_0. For radiative correction of $K^{\pm}_{\mu3}$ Dalitz plot, see GINSBERG 70 and BECHERRAWY 70. Results labeled OUR FIT are discussed in the review "$K^{\pm}_{\ell3}$ and $K^0_{\ell3}$ Form Factors" above.

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.0282±0.0027 OUR FIT Error includes scale factor of 1.5. Assumes μ-e universality.					
0.033 ±0.010 OUR FIT Error includes scale factor of 1.8.					
0.014 ±0.024	3000	ARTEMOV 97B	SPEC	−	DP
+0.050 ±0.013	3973	WHITMAN 80	SPEC	+	DP
0.025 ±0.030	490	ARNOLD 74	HLBC	+	DP
0.027 ±0.019	6527	MERLAN 74	ASPK	+	DP
0.025 ±0.017	1897	BRAUN 73C	HLBC	+	DP
0.024 ±0.019	4025	[98] ANKENBRA... 72	ASPK	+	PI
−0.006 ±0.015	3480	CHIANG 72	OSPK	+	DP
0.009 ±0.026	2041	KIJEWSKI 69	OSPK	+	PI
0.0 ±0.05	444	CALLAHAN 66B	FBC	+	PI

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.029 ±0.024	3000	[99] ARTEMOV 97	SPEC	−	DP
0.050 ±0.018	3240	[100] HAIDT 71	HLBC	+	DP

[98] ANKENBRANDT 72 λ_+ from figure 3 to match $d\xi(0)/d\lambda_+$. Text gives 0.024 ± 0.022.

[99] Superseded by ARTEMOV 97B.

[100] Not included in fit because of large discrepancy in $K_{\mu3}/K_{e3}$ branching ratio with more precise experiements.

λ_0 (LINEAR ENERGY DEPENDENCE OF f_0 IN $K^{\pm}_{\mu3}$ DECAY)

Wherever possible, we have converted the above values of $\xi(0)$ into values of λ_0 using the associated λ^{μ}_{+} and $d\xi/d\lambda$. Results labeled OUR FIT are discussed in the review "$K^{\pm}_{\ell3}$ and $K^{0}_{\ell3}$ Form Factors" above.

0.004±0.009 OUR FIT Error includes scale factor of 1.8. Correlation is $d\lambda_0/d\lambda_+ = -0.12$.

0.013±0.005 OUR FIT Error includes scale factor of 1.5. Correlation is $d\lambda_0/d\lambda_+ = -0.02$. Assumes μ-e universality.

VALUE	*$d\lambda_0/d\lambda_+$*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
$+0.020\pm0.005$	$+0.06$		[101] KL3FIT 02	RVUE		$\lambda_+ = 0.030$
$+0.058\pm0.020$	0.0	3000	[102] ARTEMOV 97B	SPEC	$-$	DP
$+0.029\pm0.011$	-0.37	3973	WHITMAN 80	SPEC	$+$	DP
-0.040 ± 0.040	-0.62	490	ARNOLD 74	HLBC	$+$	DP
-0.019 ± 0.015	$+0.27$	6527	[103] MERLAN 74	ASPK	$+$	DP
-0.008 ± 0.020	-0.53	1897	[104] BRAUN 73C	HLBC	$+$	DP
-0.026 ± 0.013	$+0.03$	4025	[105] ANKENBRA... 72	ASPK	$+$	PI
$+0.030\pm0.014$	-0.21	3480	[105] CHIANG 72	OSPK	$+$	DP
-0.056 ± 0.024	$+0.69$	3133	[106] CUTTS 69	OSPK	$+$	POL
-0.031 ± 0.045	-1.10	2041	[105] KIJEWSKI 69	OSPK	$+$	PI
-0.063 ± 0.024	$+0.60$	6000	[106] BETTELS 68	HLBC	$+$	POL
$+0.058\pm0.036$	-0.37	444	[105] CALLAHAN 66B	FBC	$+$	PI

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE	*$d\lambda_0/d\lambda_+$*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
$+0.019\pm0.005\pm0.004$		24k	[107] HORIE 01	SPEC	$+$	BR
$+0.062\pm0.024$	0.0	3000	[108] ARTEMOV 97	SPEC	$-$	DP
$+0.019\pm0.010$	$+0.03$	55k	[109] HEINTZE 77	SPEC	$+$	BR
$+0.008\pm0.097$	$+0.92$	1585	[106] BRAUN 75	HLBC	$+$	POL
-0.017 ± 0.011			[110] BRAUN 74	HLBC	$+$	$K_{\mu3}/K_{e3}$ vs. t
-0.039 ± 0.029	-1.34	3240	[105] HAIDT 71	HLBC	$+$	DP

[101] KL3FIT 02 value is from our fitted value of the $K^{\pm}_{\mu3}/K^{\pm}_{e3}$ branching ratio. Assumes $\mu - e$ universality.

[102] ARTEMOV 97B does not give $d\lambda_0/d\lambda_+$ so we take it to be zero.

[103] MERLAN 74 λ_0 and $d\lambda_0/d\lambda_+$ were calculated by us from ξ_A, λ^{μ}_{+}, and $d\xi(0)/d\lambda_+$. Their figure 6 gives $\lambda_0 = -0.025 \pm 0.012$ and no $d\lambda_0/d\lambda_+$.

[104] This value and error are taken from BRAUN 75 but correspond to the BRAUN 73C λ^{μ}_{+} result. $d\lambda_0/d\lambda_+$ is from BRAUN 73C $d\xi(0)/d\lambda_+$ in ξ_A above.

[105] λ_0 calculated by us from $\xi(0)$, λ^{μ}_{+}, and $d\xi(0)/d\lambda_+$.

[106] λ_0 value is for $\lambda_+ = 0.03$ calculated by us from $\xi(0)$ and $d\xi(0)/d\lambda_+$.

[107] HORIE 01 assumes μ-e universality in $K^{+}_{\ell3}$ decay and uses SHIMIZU 00 value $\lambda=0.0278\pm0.0040$ from $K^{\pm}_{e3}$ decay. Enters fit via $K_{\mu3}/K_{e3}$ branching ratio.

[108] ARTEMOV 97 does not give $d\lambda_0/d\lambda_+$ so we take it to be zero. Superseded by ARTEMOV 97B.

[109] HEINTZE 77 uses $\lambda_+ = 0.029 \pm 0.003$. $d\lambda_0/d\lambda_+$ estimated by us. Enters fit via $K_{\mu3}/K_{e3}$ branching ratio.

[110] BRAUN 74 is a combined $K_{\mu3}$-K_{e3} result. It is not independent of BRAUN 73C ($K_{\mu3}$) and BRAUN 73B (K_{e3}) form factor results.

$|f_S/f_+|$ FOR $K^\pm_{e3}$ DECAY

Ratio of scalar to f_+ couplings.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.045±0.033 OUR AVERAGE Error includes scale factor of 1.8.						
0.002±0.026±0.014		41k	SHIMIZU 00	SPEC	+	λ_+, f_S, f_T fit
0.070±0.016±0.016		32k	AKIMENKO 91	SPEC		λ_+, f_S, f_T, ϕ fit
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.00 ±0.10		2827	[111] BRAUN 75	HLBC	+	
<0.13	90	4017	CHIANG 72	OSPK	+	
$0.14^{+0.03}_{-0.04}$		2707	[111] STEINER 71	HLBC	+	λ_+, f_S, f_T, ϕ fit
<0.23	90		BOTTERILL 68C	ASPK		
<0.18	90		BELLOTTI 67B	HLBC		
<0.30	95		KALMUS 67	HLBC	+	

[111] Statistical errors only.

$|f_T/f_+|$ FOR $K^\pm_{e3}$ DECAY

Ratio of tensor to f_+ couplings.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.31±0.25 OUR AVERAGE Error includes scale factor of 2.4.						
0.01±0.14±0.09		41k	SHIMIZU 00	SPEC	+	λ_+, f_S, f_T fit
$0.53^{+0.09}_{-0.10}\pm0.10$		32k	AKIMENKO 91	SPEC		λ_+, f_S, f_T, ϕ fit
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.07±0.37		2827	[112] BRAUN 75	HLBC	+	
<0.75	90	4017	CHIANG 72	OSPK	+	
$0.24^{+0.16}_{-0.14}$		2707	[112] STEINER 71	HLBC	+	λ_+, f_S, f_T, ϕ fit
<0.58	90		BOTTERILL 68C	ASPK		
<0.58	90		BELLOTTI 67B	HLBC		
<1.1	95		KALMUS 67	HLBC	+	

[112] Statistical errors only.

f_T/f_+ FOR $K^\pm_{\mu3}$ DECAY

Ratio of tensor to f_+ couplings.

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*
0.02±0.12	1585	BRAUN 75	HLBC

DECAY FORM FACTORS FOR $K^\pm \to \pi^+\pi^- e^\pm \nu_e$

Given in PISLAK 01, ROSSELET 77, BEIER 73, and BASILE 71C.

DECAY FORM FACTOR FOR $K^\pm \to \pi^0\pi^0 e^\pm \nu$

Given in BOLOTOV 86B and BARMIN 88B.

$K^{\pm} \to \ell^{\pm}\nu\gamma$ FORM FACTORS

For definitions of the axial-vector F_A and vector F_V form factor, see the "Note on $\pi^{\pm} \to \ell^{\pm}\nu\gamma$ and $K^{\pm} \to \ell^{\pm}\nu\gamma$ Form Factors" in the $\pi^{\pm}$ section. In the kaon literature, often different definitions $a_K = F_A/m_K$ and $v_K = F_V/m_K$ are used.

$F_A + F_V$, SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to e\nu_e\gamma$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*
0.148±0.010 OUR AVERAGE			
0.147±0.011	51	[113] HEINTZE 79	SPEC
$0.150^{+0.018}_{-0.023}$	56	[114] HEARD 75	SPEC

[113] HEINTZE 79 quotes absolute value of $|F_A + F_V|\sin\theta_c$. We use $\sin\theta_c = V_{us} = 0.2205$.

[114] HEARD 75 quotes absolute value of $|F_A + F_V|\sin\theta_c$. We use $\sin\theta_c = V_{us} = 0.2205$.

$F_A + F_V$, SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to \mu\nu_\mu\gamma$

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
0.165±0.007±0.011		2588	[115] ADLER 00B	B787	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−1.2 to 1.1	90		DEMIDOV 90	XEBC	
< 0.23	90		[115] AKIBA 85	SPEC	

[115] Quotes absolute value. Sign not determined.

$F_A - F_V$, DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to e\nu_e\gamma$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*
<0.49	90	[116] HEINTZE 79	SPEC

[116] HEINTZE 79 quotes $|F_A - F_V| < \sqrt{11}\,|F_A + F_V|$.

$F_A - F_V$, DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \to \mu\nu_\mu\gamma$

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
−0.24 to 0.04	90	2588	ADLER 00B	B787	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−2.2 to 0.6	90		DEMIDOV 90	XEBC	
−2.5 to 0.3	90		AKIBA 85	SPEC	

$K^{\pm}$ CHARGE RADIUS

VALUE (fm)	*DOCUMENT ID*	*COMMENT*
0.560±0.031 OUR AVERAGE		
0.580±0.040	AMENDOLIA 86B	$K e \to K e$
0.530±0.050	DALLY 80	$K e \to K e$
• • • We do not use the following data for averages, fits, limits, etc. • • •		
0.620±0.037	BLATNIK 79	VMD + dispersion relations

$K^{\pm}$ REFERENCES

ADLER	02	PRL 88 041803	S. Adler *et al.*	(BNL E787 Collab.)
ADLER	02B	PR D65 052009	S. Adler *et al.*	(BNL E787 Collab.)
KL3FIT	02	RPP 2002 edition	T.G. Trippe	(PDG Collab.)
$K^{\pm}_{\mu 3}$ and $K^{0}_{\mu 3}$ Form Factors review in K^{+} Listings.				
ADLER	01	PR D63 032004	S. Adler *et al.*	(BNL E787 Collab.)
HORIE	01	PL B513 311	K. Horie *et al.*	(KEK-E426 Collab.)
PISLAK	01	PRL 87 221801	S. Pislak *et al.*	(BNL 865 Collab.)
ADLER	00	PRL 84 3768	S. Adler *et al.*	(BNL E787 Collab.)
ADLER	00B	PRL 85 2256	S. Adler *et al.*	(BNL E787 Collab.)
ADLER	00C	PRL 85 4856	S. Adler *et al.*	(BNL E787 Collab.)
APPEL	00	PRL 85 2450	R. Appel *et al.*	(BNL 865 Collab.)
Also	97	Thesis, Yale Univ.	D.R. Bergman	
Also	97	Thesis, Univ. Zurich	S. Pislak	
APPEL	00B	PRL 85 2877	R. Appel *et al.*	(BNL 865 Collab.)
MA	00	PRL 84 2580	H. Ma *et al.*	(BNL 865 Collab.)
PDG	00	EPJ C15 1	D.E. Groom *et al.*	
SHIMIZU	00	PL B495 33	S. Shimizu *et al.*	(KEK E246 Collab.)
SHIN	00	EPJ C12 627	Y.-H. Shin *et al.*	(KEK E246 Collab.)
ABE	99S	PRL 83 4253	M. Abe *et al.*	(KEK E246 Collab.)
APPEL	99	PRL 83 4482	R. Appel *et al.*	(BNL 865 Collab.)
ADLER	98	PR D58 012003	S. Adler *et al.*	(BNL E787 Collab.)
BATUSOV	98	NP B516 3	V.Y. Batusov *et al.*	
ADLER	97	PRL 79 2204	S. Adler *et al.*	(BNL E787 Collab.)
ADLER	97C	PRL 79 4756	S. Adler *et al.*	(BNL E787 Collab.)
ARTEMOV	97	PAN 60 218 Translated from YAF 60 277.	V.M. Artemov *et al.*	(JINR)
ARTEMOV	97B	PAN 60 2023 Translated from YAF 60 2205.	V.M. Artemov *et al.*	
BERGMAN	97	Thesis, Yale Univ.	D.R. Bergman	
KITCHING	97	PRL 79 4079	P. Kitching *et al.*	(BNL E787 Collab.)
PISLAK	97	Thesis, Univ. Zurich	S. Pislak	
ADLER	96	PRL 76 1421	S. Adler *et al.*	(BNL E787 Collab.)
KOPTEV	95	JETPL 61 877 Translated from ZETFP 61 865.	V.P. Koptev *et al.*	(PNPI)
AOKI	94	PR D50 69	M. Aoki *et al.*	(INUS, KEK, TOKMS)
ATIYA	93	PRL 70 2521	M.S. Atiya *et al.*	(BNL E787 Collab.)
Also	93C	PRL 71 305 (erratum)	M.S. Atiya *et al.*	(BNL E787 Collab.)
ATIYA	93B	PR D48 R1	M.S. Atiya *et al.*	(BNL E787 Collab.)
BIJNENS	93	NP B396 81	J. Bijnens, G. Ecker, J. Gasser	(CERN, BERN)
ALLIEGRO	92	PRL 68 278	C. Alliegro *et al.*	(BNL, FNAL, PSI+)
BARMIN	92	SJNP 55 547 Translated from YAF 55 976.	V.V. Barmin *et al.*	(ITEP)
IMAZATO	92	PRL 69 877	J. Imazato *et al.*	(KEK, INUS, TOKY+)
IVANOV	92	THESIS	Yu.M. Ivanov	(PNPI)
LITTENBERG	92	PRL 68 443	L.S. Littenberg, R.E. Shrock	(BNL, STON)
USHER	92	PR D45 3961	T. Usher *et al.*	(UCI)
AKIMENKO	91	PL B259 225	S.A. Akimenko *et al.*	(SERP, JINR, TBIL+)
BARMIN	91	SJNP 53 606 Translated from YAF 53 981.	V.V. Barmin *et al.*	(ITEP)
DENISOV	91	JETPL 54 558 Translated from ZETFP 54 557.	A.S. Denisov *et al.*	(PNPI)
Also	92	THESIS	Yu.M. Ivanov	(PNPI)
ATIYA	90	PRL 64 21	M.S. Atiya *et al.*	(BNL E787 Collab.)
ATIYA	90B	PRL 65 1188	M.S. Atiya *et al.*	(BNL E787 Collab.)
DEMIDOV	90	SJNP 52 1006 Translated from YAF 52 1595.	V.S. Demidov *et al.*	(ITEP)
LEE	90	PRL 64 165	A.M. Lee *et al.*	(BNL, FNAL, VILL, WASH+)
ATIYA	89	PRL 63 2177	M.S. Atiya *et al.*	(BNL E787 Collab.)
BARMIN	89	SJNP 50 421 Translated from YAF 50 679.	V.V. Barmin *et al.*	(ITEP)
BARMIN	88	SJNP 47 643 Translated from YAF 47 1011.	V.V. Barmin *et al.*	(ITEP)
BARMIN	88B	SJNP 48 1032 Translated from YAF 48 1719.	V.V. Barmin *et al.*	(ITEP)
BOLOTOV	88	JETPL 47 7 Translated from ZETFP 47 8.	V.N. Bolotov *et al.*	(ASCI)
GALL	88	PRL 60 186	K.P. Gall *et al.*	(BOST, MIT, WILL, CIT+)
BARMIN	87	SJNP 45 62 Translated from YAF 45 97.	V.V. Barmin *et al.*	(ITEP)
BOLOTOV	87	SJNP 45 1023 Translated from YAF 45 1652.	V.N. Bolotov *et al.*	(INRM)

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BOLOTOV	86	SJNP 44 73	V.N. Bolotov *et al.*	(INRM)
		Translated from YAF 44 117.		
BOLOTOV	86B	SJNP 44 68	V.N. Bolotov *et al.*	(INRM)
		Translated from YAF 44 108.		
YAMANAKA	86	PR D34 85	T. Yamanaka *et al.*	(KEK, TOKY)
Also	84	PRL 52 329	R.S. Hayano *et al.*	(TOKY, KEK)
AKIBA	85	PR D32 2911	Y. Akiba *et al.*	(TOKY, TINT, TSUK, KEK)
BOLOTOV	85	JETPL 42 481	V.N. Bolotov *et al.*	(INRM)
		Translated from ZETFP 42 390.		
ASANO	82	PL 113B 195	Y. Asano *et al.*	(KEK, TOKY, INUS, OSAK)
COOPER	82	PL 112B 97	A.M. Cooper *et al.*	(RL)
PDG	82	PL 111B	M. Roos *et al.*	(HELS, CIT, CERN)
PDG	82B	PL 111B 70	M. Roos *et al.*	(HELS, CIT, CERN)
ASANO	81B	PL 107B 159	Y. Asano *et al.*	(KEK, TOKY, INUS, OSAK)
CAMPBELL	81	PRL 47 1032	M.K. Campbell *et al.*	(YALE, BNL)
Also	83	PR D27 1056	S.R. Blatt *et al.*	(YALE, BNL)
LUM	81	PR D23 2522	G.K. Lum *et al.*	(LBL, NBS+)
LYONS	81	ZPHY C10 215	L. Lyons, C. Albajar, G. Myatt	(OXF)
DALLY	80	PRL 45 232	E.B. Dally *et al.*	(UCLA+)
WHITMAN	80	PR D21 652	R. Whitman *et al.*	(ILLC, BNL, ILL)
BARKOV	79	NP B148 53	L.M. Barkov *et al.*	(NOVO, KIAE)
BLATNIK	79	LNC 24 39	S. Blatnik, J. Stahov, C.B. Lang	(TUZL, GRAZ)
HEINTZE	79	NP B149 365	J. Heintze *et al.*	(HEIDP, CERN)
ABRAMS	77	PR D15 22	R.J. Abrams *et al.*	(BNL)
DEVAUX	77	NP B126 11	B. Devaux *et al.*	(SACL, GEVA)
HEINTZE	77	PL 70B 482	J. Heintze *et al.*	(HEIDP, CERN)
ROSSELET	77	PR D15 574	L. Rosselet *et al.*	(GEVA, SACL)
BERTRAND	76	NP B114 387	D. Bertrand *et al.*	(BRUX, KIDR, DUUC+)
BLOCH	76	PL 60B 393	P. Bloch *et al.*	(GEVA, SACL)
BRAUN	76B	LNC 17 521	H.M. Braun *et al.*	(AACH3, BARI, BELG+)
DIAMANT-...	76	PL 62B 485	A.M. Diamant-Berger *et al.*	(SACL, GEVA)
HEINTZE	76	PL 60B 302	J. Heintze *et al.*	(HEIDP)
SMITH	76	NP B109 173	K.M. Smith *et al.*	(GLAS, LIVP, OXF+)
WEISSENBE...	76	NP B115 55	A.O. Weissenberg *et al.*	(ITEP, LEBD)
BLOCH	75	PL 56B 201	P. Bloch *et al.*	(SACL, GEVA)
BRAUN	75	NP B89 210	H.M. Braun *et al.*	(AACH3, BARI, BRUX+)
CHENG	75	NP A254 381	S.C. Cheng *et al.*	(COLU, YALE)
HEARD	75	PL 55B 324	K.S. Heard *et al.*	(CERN, HEIDH)
HEARD	75B	PL 55B 327	K.S. Heard *et al.*	(CERN, HEIDH)
SHEAFF	75	PR D12 2570	M. Sheaff	(WISC)
SMITH	75	NP B91 45	K.M. Smith *et al.*	(GLAS, LIVP, OXF+)
ARNOLD	74	PR D9 1221	C.L. Arnold, B.P. Roe, D. Sinclair	(MICH)
BRAUN	74	PL 51B 393	H.M. Braun *et al.*	(AACH3, BARI, BRUX+)
MERLAN	74	PR D9 107	S. Merlan *et al.*	(YALE, BNL, LASL)
WEISSENBE...	74	PL 48B 474	A.O. Weissenberg *et al.*	(ITEP, LEBD)
ABRAMS	73B	PRL 30 500	R.J. Abrams *et al.*	(BNL)
BACKENSTO...	73	PL 43B 431	G. Backenstoss *et al.*	(CERN, KARLK, KARLE+)
BEIER	73	PRL 30 399	E.W. Beier *et al.*	(PENN)
BRAUN	73B	PL 47B 185	H.M. Braun, M. Cornelssen	(AACH3, BARI, BRUX+)
Also	75	NP B89 210	H.M. Braun *et al.*	(AACH3, BARI, BRUX+)
BRAUN	73C	PL 47B 182	H.M. Braun, M. Cornelssen	(AACH3, BARI, BRUX+)
Also	75	NP B89 210	H.M. Braun *et al.*	(AACH3, BARI, BRUX+)
LJUNG	73	PR D8 1307	D. Ljung, D. Cline	(WISC)
Also	72	PRL 28 523	D. Ljung	(WISC)
Also	72	PRL 28 1287	D. Cline, D. Ljung	(WISC)
Also	69	PRL 23 326	U. Camerini *et al.*	(WISC)
LUCAS	73	PR D8 719	P.W. Lucas, H.D. Taft, W.J. Willis	(YALE)
LUCAS	73B	PR D8 727	P.W. Lucas, H.D. Taft, W.J. Willis	(YALE)
PANG	73	PR D8 1989	C.Y. Pang *et al.*	(EFI, ARIZ, LBL)
Also	72	PL 40B 699	G.D. Cable *et al.*	(EFI, LBL)
SMITH	73	NP B60 411	K.M. Smith *et al.*	(GLAS, LIVP, OXF+)
ABRAMS	72	PRL 29 1118	R.J. Abrams *et al.*	(BNL)
ANKENBRA...	72	PRL 28 1472	C.M. Ankenbrandt *et al.*	(BNL, LASL, FNAL+)
AUBERT	72	NC 12A 509	B. Aubert *et al.*	(ORSAY, BRUX, EPOL)
CHIANG	72	PR D6 1254	I.H. Chiang *et al.*	(ROCH, WISC)
CLARK	72	PRL 29 1274	A.R. Clark *et al.*	(LBL)
EDWARDS	72	PR D5 2720	R.T. Edwards *et al.*	(ILL)
FORD	72	PL 38B 335	W.T. Ford *et al.*	(PRIN)
HOFFMASTER	72	NP B36 1	S. Hoffmaster *et al.*	(STEV, SETO, LEHI)
BASILE	71C	PL 36B 619	P. Basile *et al.*	(SACL, GEVA)
BOURQUIN	71	PL 36B 615	M.H. Bourquin *et al.*	(GEVA, SACL)

GINSBERG	71	PR D4 2893	E.S. Ginsberg	(MIT)
HAIDT	71	PR D3 10	D. Haidt	(AACH, BARI, CERN, EPOL, NIJM+)
Also	69	PL 29B 691	D. Haidt *et al.*	(AACH, BARI, CERN, EPOL+)
KLEMS	71	PR D4 66	J.H. Klems, R.H. Hildebrand, R. Stiening	(CHIC+)
Also	70	PRL 24 1086	J.H. Klems, R.H. Hildebrand, R. Stiening	(LRL+)
Also	70B	PRL 25 473	J.H. Klems, R.H. Hildebrand, R. Stiening	(LRL+)
OTT	71	PR D3 52	R.J. Ott, T.W. Pritchard	(LOQM)
ROMANO	71	PL 36B 525	F. Romano *et al.*	(BARI, CERN, ORSAY)
SCHWEINB...	71	PL 36B 246	W. Schweinberger	(AACH, BELG, CERN, NIJM+)
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BARDIN	70	PL 32B 121	D.Y. Bardin, S.N. Bilenky, B.M. Pontecorvo	(JINR)
BECHERRAWY	70	PR D1 1452	T. Becherrawy	(ROCH)
BOTTERILL	70	PL 31B 325	D.R. Botterill *et al.*	(OXF)
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MALTSEV	70	SJNP 10 678	E.I. Maltsev *et al.*	(JINR)
		Translated from YAF 10 1195.		
PANDOULAS	70	PR D2 1205	D. Pandoulas *et al.*	(STEV, SETO)
CUTTS	69	PR 184 1380	D. Cutts *et al.*	(LRL, MIT)
Also	68	PRL 20 955	D. Cutts *et al.*	(LRL, MIT)
DAVISON	69	PR 180 1333	D.C. Davison *et al.*	(UCR)
ELY	69	PR 180 1319	R.P.J. Ely *et al.*	(LOUC, WISC, LRL)
EMMERSON	69	PRL 23 393	J.M.L. Emmerson, T.W. Quirk	(OXF)
HERZO	69	PR 186 1403	D. Herzo *et al.*	(ILL)
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LOBKOWICZ	69	PR 185 1676	F. Lobkowicz *et al.*	(ROCH, BNL)
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MAST	69	PR 183 1200	T.S. Mast *et al.*	(LRL)
SELLERI	69	NC 60A 291	F. Selleri	
ZELLER	69	PR 182 1420	M.E. Zeller *et al.*	(UCLA, LRL)
BETTELS	68	NC 56A 1106	J. Bettels	(AACH, BARI, BERG, CERN, EPOL+)
Also	71	PR D3 10	D. Haidt	(AACH, BARI, CERN, EPOL, NIJM+)
BOTTERILL	68B	PRL 21 766	D.R. Botterill *et al.*	(OXF)
BOTTERILL	68C	PR 174 1661	D.R. Botterill *et al.*	(OXF)
BUTLER	68	UCRL 18420	W.D. Butler *et al.*	(LRL)
CHANG	68	PRL 20 510	C.Y. Chang *et al.*	(UMD, RUTG)
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EICHTEN	68	PL 27B 586	T. Eichten	(AACH, BARI, CERN, EPOL, ORSAY+)
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MOSCOSO	68	Thesis	L. Moscoso	(ORSAY)
AUERBACH	67	PR 155 1505	L.B. Auerbach *et al.*	(PENN, PRIN)
Also	74	PR D9 3216	L.B. Auerbach	
Erratum.				
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Also	66B	PL 20 690	E. Bellotti *et al.*	(MILA)
BISI	67	PL 25B 572	V. Bisi *et al.*	(TORI)
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CALLAHAN	66	NC 44A 90	A.C. Callahan	(WISC)
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CESTER	66	PL 21 343	R. Cester *et al.*	(PPA)
See footnote 1 in AUERBACH 67.				
Also	67	PR 155 1505	L.B. Auerbach *et al.*	(PENN, PRIN)
BIRGE	65	PR 139B 1600	R.W. Birge *et al.*	(LRL, WISC)
BISI	65	NC 35 768	V. Bisi *et al.*	(TORI)
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DEMARCO	65	PR 140B 1430	A. de Marco, C. Grosso, G. Rinaudo	(TORI, CERN)
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STAMER	65	PR 138B 440	P. Stamer *et al.*	(STEV)

YOUNG	65	Thesis UCRL 16362	P.S. Young	(LRL)
Also	67	PR 156 1464	P.S. Young, W.Z. Osborne, W.H. Barkas	(LRL)
BORREANI	64	PL 12 123	G. Borreani, G. Rinaudo, A.E. Werbrouck	(TORI)
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GIACOMELLI	64	NC 34 1134	G. Giacomelli *et al.*	(BGNA, MUNI)
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KALMUS	64	PRL 13 99	G.E. Kalmus *et al.*	(LRL, WISC)
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BOYARSKI	62	PR 128 2398	A.M. Boyarski *et al.*	(MIT)
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LITTENBERG	93	ARNPS 43 729	L.S. Littenberg, G. Valencia	(BNL, FNAL)
Rare and Radiative Kaon Decays				
RITCHIE	93	RMP 65 1149	J.L. Ritchie, S.G. Wojcicki	
"Rare *K* Decays"				
BATTISTON	92	PRPL 214 293	R. Battiston *et al.*	(PGIA, CERN, TRSTT)
Status and Perspectives of *K* Decay Physics				
BRYMAN	89	IJMP A4 79	D.A. Bryman	(TRIU)
"Rare Kaon Decays"				
CHOUNET	72	PRPL 4C 199	L.M. Chounet, J.M. Gaillard, M.K. Gaillard	(ORSAY+)
FEARING	70	PR D2 542	H.W. Fearing, E. Fischbach, J. Smith	(STON, BOHR)
HAIDT	69B	PL 29B 696	D. Haidt *et al.*	(AACH, BARI, CERN, EPOL+)
CRONIN	68B	Vienna Conf. 241	J.W. Cronin	(PRIN)
Rapporteur talk.				
WILLIS	67	Heidelberg Conf. 273	W.J. Willis	(YALE)
Rapporteur talk.				
CABIBBO	66	Berkeley Conf. 33	N. Cabibbo	(CERN)
ADAIR	64	PL 12 67	R.K. Adair, L.B. Leipuner	(YALE, BNL)
CABIBBO	64	PL 9 352	N. Cabibbo, A. Maksymowicz	(CERN)
Also	64B	PL 11 360	N. Cabibbo, A. Maksymowicz	(CERN)
Also	65	PL 14 72	N. Cabibbo, A. Maksymowicz	(CERN)
BIRGE	63	PRL 11 35	R.W. Birge *et al.*	(LRL, WISC, BARI)
BLOCK	62B	CERN Conf. 371	M.M. Block, L. Lendinara, L. Monari	(NWES, BGNA)
BRENE	61	NP 22 553	N. Brene, L. Egardt, B. Qvist	(NORD)