NEUTRINO PHYSICS AS EXPLORED BY FLAVOR CHANGE

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I. The physics of flavor change: The rather convincing evidence that atmospheric neutrinos change from one flavor to another has now been joined by new, very strong evidence that the solar neutrinos do this as well. Neutrino flavor change implies that neutrinos have nonzero masses. That is, there is a spectrum of three or more neutrino mass eigenstates, $\nu_1, \nu_2, \nu_3, \ldots$, that are the analogues of the charged-lepton mass eigenstates, e, μ, and τ. Neutrino flavor change also implies leptonic mixing. That is, the weak interaction coupling the W boson to a charged lepton and a neutrino can couple any charged-lepton mass eigenstate ℓ_α to any neutrino mass eigenstate ν_i. Here, $\alpha = e, \mu$, or τ, and ℓ_e is the electron, etc. Leptonic W^+ decay can yield a particular ℓ_α^+ in association with any ν_i. The amplitude for this decay to produce the specific combination $\ell_\alpha^+ + \nu_i$ is $U_{\alpha i}^*$, where U is the unitary leptonic mixing matrix [1]. Thus, the neutrino state created in the decay $W^+ \to \ell_\alpha^+ + \nu$ is the state

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle \ . \tag{1}$$

This superposition of neutrino mass eigenstates, produced in association with the charged lepton of "flavor" α, is the state we refer to as the neutrino of flavor α.

While there are only three (known) charged lepton mass eigenstates, the experimental results suggest that perhaps there are more than three neutrino mass eigenstates. If, for example, there are four ν_i, then one linear combination of them,

$$|\nu_s\rangle = \sum_i U_{si}^* |\nu_i\rangle \ , \tag{2}$$

does not have a charged-lepton partner, and consequently does not couple to the Standard Model W boson. Indeed, since the decays $Z \to \nu_\alpha \overline{\nu}_\alpha$ of the Standard Model Z boson have been found to yield only three distinct neutrinos ν_α of definite flavor [2], ν_s does not couple to the Z boson either. Such a neutrino, which does not have any Standard Model weak couplings, is referred to as a "sterile" neutrino. Despite its name, this neutrino might participate in feeble interactions that lie beyond the Standard Model.

To understand neutrino flavor change, or "oscillation," in vacuum, let us consider how a neutrino born as the ν_α of Eq. (1) evolves in time. First, we apply Schrödinger's equation to the ν_i component of ν_α in the rest frame of that component. This tells us that [3]

$$|\nu_i(\tau_i)\rangle = e^{-im_i\tau_i}|\nu_i(0)\rangle \ , \tag{3}$$

where m_i is the mass of ν_i, and τ_i is time in the ν_i frame. In terms of the time t and position L in the laboratory frame, the Lorentz-invariant phase factor in Eq. (3) may be written

$$e^{-im_i\tau_i} = e^{-i(E_it-p_iL)} \ . \tag{4}$$

Here, E_i and p_i are respectively the energy and momentum of ν_i in the laboratory frame. In practice, our neutrino will be extremely relativistic, so we will be interested in evaluating the phase factor of Eq. (4) where $t \approx L$, and where it becomes $\exp[-i(E_i - p_i)L]$.

Imagine now that our ν_α has been produced with a definite momentum p, so that all of its mass-eigenstate components have this common momentum. Then the ν_i component has $E_i = \sqrt{p^2 + m_i^2} \approx p + m_i^2/2p$, assuming that all neutrino masses m_i are small compared to the neutrino momentum. The phase factor of Eq. (4) is then approximately

$$e^{-i(m_i^2/2p)L} \ . \tag{5}$$

From this expression and Eq. (1), it follows that after a neutrino born as a ν_α has propagated a distance L, its state vector has become

$$|\nu_\alpha(L)\rangle \approx \sum_i U^*_{\alpha i} e^{-i(m_i^2/2E)L}|\nu_i\rangle \ . \tag{6}$$

Here, $E \simeq p$ is the average energy of the various mass eigenstate components of the neutrino. Using the unitarity of U to invert Eq. (1), and inserting the result in Eq. (6), we find that

$$|\nu_\alpha(L)\rangle \approx \sum_\beta \left[\sum_i U^*_{\alpha i} e^{-i(m_i^2/2E)L} U_{\beta i}\right] |\nu_\beta\rangle \ . \tag{7}$$

We see that our ν_α, in traveling the distance L, has turned into a superposition of all the flavors. The probability that it has flavor β, $P(\nu_\alpha \to \nu_\beta)$, is obviously $|\langle \nu_\beta | \nu_\alpha(L)\rangle|^2$. From Eq. (7) and the unitarity of U, we easily find that

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = {} & \delta_{\alpha\beta} \\ & -4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2[1.27\, \Delta m^2_{ij}(L/E)] \\ & +2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin[2.54\, \Delta m^2_{ij}(L/E)] \ . \end{aligned} \tag{8}$$

Here, $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ is in eV^2, L is in km, and E is in GeV. We have used the fact that when the previously omitted factors of $\hbar$ and c are included,

$$\Delta m^2_{ij}(L/4E) \cong 1.27\, \Delta m^2_{ij}(\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})} \ . \tag{9}$$

Assuming that CPT invariance holds,

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\beta \to \nu_\alpha) \ . \tag{10}$$

But, from Eq. (8) we see that

$$P(\nu_\beta \to \nu_\alpha; U) = P(\nu_\alpha \to \nu_\beta; U^*) \ . \tag{11}$$

Thus, when CPT holds,

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta; U) = P(\nu_\alpha \to \nu_\beta; U^*) \ . \tag{12}$$

That is, the probability for oscillation of an antineutrino is the same as that for a neutrino, except that the mixing matrix U is replaced by its complex conjugate. Thus, if U is not real, the neutrino and antineutrino oscillation probabilities can differ by having opposite values of the last term in Eq. (8). When CPT holds, any difference between these probabilities indicates a violation of CP invariance.

The quantum mechanics of neutrino oscillation leading to the result Eq. (8) is somewhat subtle. To do justice to the physics requires a more refined treatment [4] than the one we have given. Sophisticated treatments continue to yield new insights [5]. However, they lead to the same oscillation probability as we have found here.

As we shall see, the (Mass)^2 splittings Δm^2_{ij} called for by the various reported signals of oscillation are quite different from one another. It may be that one splitting, ΔM^2, is much bigger than all the others. If that is the case, then for an oscillation experiment with L/E such that $\Delta M^2 L/E = \mathcal{O}(1)$, Eq. (8) simplifies considerably, becoming

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_{\beta\neq\alpha}) \cong S_{\alpha\beta}\,\sin^2[1.27\,\Delta M^2(L/E)]\ , \qquad (13)$$

and

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) \cong 1 - 4\,T_\alpha(1-T_\alpha)\sin^2[1.27\,\Delta M^2(L/E)]\ . \qquad (14)$$

Here,

$$S_{\alpha\beta} \equiv 4\left|\sum_{i\ \mathrm{Up}} U^*_{\alpha i}U_{\beta i}\right|^2 \qquad (15)$$

and

$$T_\alpha = \sum_{i\ \mathrm{Up}} |U_{\alpha i}|^2\ , \qquad (16)$$

where "i Up" denotes a sum over only those neutrino mass eigenstates that lie *above* ΔM^2 or, alternatively, only those that lie *below* it. The unitarity of U guarantees that summing over either of these two clusters will yield the same results for $S_{\alpha\beta}$ and for $T_\alpha(1-T_\alpha)$.

The situation described by Eqs. (13–16) may be called "quasi-two-neutrino oscillation." It has also been called "one mass scale dominance" [6]. It corresponds to an experiment whose L/E is such that the experiment can "see" only the big splitting ΔM^2. To this experiment, all the neutrinos above ΔM^2 appear to be a single neutrino, as do all those below ΔM^2.

The relations of Eqs. (13–16) also apply to the special case where, to a good approximation, only two mass eigenstates, and two corresponding flavors, are relevant. One encounters this case when, for example, only two mass eigenstates couple significantly to the charged lepton with which the neutrino being studied is produced. When only two mass eigenstates count, there is only a single splitting, Δm^2, and, omitting irrelevant phase factors, the unitary mixing matrix U takes the form

$$U = \begin{array}{cc} & \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} & \left[\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right] \end{array} \quad . \tag{17}$$

Here the symbols above and to the left of the matrix label the columns and rows, and θ is referred to as the mixing angle. From Eqs. (15) -(16), we now have $S_{\alpha\beta} = \sin^2 2\theta$ and $4\,T_\alpha(1 - T_\alpha) = \sin^2 2\theta$, so that Eqs. (13)-(14) become

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_{\beta\neq\alpha}) = \sin^2 2\theta\ \sin^2[1.27\,\Delta m^2(L/E)] \ , \tag{18}$$

and

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta\ \sin^2[1.27\,\Delta m^2(L/E)] \ . \tag{19}$$

Many experiments have been analyzed using these two expressions, which are quite accurate even when it is actually quasi-two-neutrino oscillation, rather than a genuine two-neutrino situation, that is being studied. For quasi-two-neutrino oscillation, "Δm^2" is really ΔM^2, of course, while "$\sin^2 2\theta$" is actually $S_{\alpha\beta}$ in the case of an appearance experiment, and $4\,T_\alpha(1 - T_\alpha)$ in the case of a disappearance experiment.

When neutrinos travel through matter (e.g. in the Sun, Earth, or a supernova), their coherent forward scattering from particles they encounter along the way can significantly modify their propagation [7]. As a result, the probability for changing flavor can be rather different than it is in vacuum [8]. Flavor change that occurs in matter, and that grows out of the interplay between flavor-nonchanging neutrino-matter interactions and neutrino mass and mixing, is known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

One can describe neutrino propagation through matter via a Schrödinger equation. This equation governs the evolution of a neutrino state vector with several components, one for each flavor. The effective Hamiltonian in the equation, a matrix $\mathcal{H}$ in neutrino flavor space, differs from its vacuum counterpart by the addition of interaction energies arising from the coherent forward neutrino scattering. For example, the ν_e–ν_e element of $\mathcal{H}$ includes the interaction energy

$$V = \sqrt{2}\, G_F N_e \ , \tag{20}$$

arising from charged-current-induced ν_e forward scattering from ambient electrons. Here, G_F is the Fermi constant, and N_e is the number of electrons per unit volume. In addition, the ν_e–ν_e, ν_μ–ν_μ, and ν_τ–ν_τ elements of $\mathcal{H}$ all contain a common interaction energy growing out of neutral-current-induced forward scattering. However, when one is not considering the possibility of transitions to sterile neutrino flavors, this common interaction energy merely adds to $\mathcal{H}$ a multiple of the identity matrix, and such an addition has no effect on flavor transitions.

The effect of matter is illustrated by the propagation of solar neutrinos through solar matter. Experimental bounds on the oscillation of reactor $\overline{\nu}_e$ into other flavors [9] tell us that, very likely, only two neutrino mass eigenstates, ν_1 and ν_2, are significantly involved in the evolution of the solar neutrinos. Correspondingly, only two flavors are involved: the ν_e flavor with which every solar neutrino is born, and the effective flavor ν_x—some linear combination of ν_μ and ν_τ—which it may become. The Hamiltonian $\mathcal{H}$ is then a 2×2 matrix in ν_e–ν_x space. Apart from an irrelevant multiple of the identity, for a distance r from the center of the Sun, $\mathcal{H}$ is given by

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_V + \mathcal{H}_M(r) \\ &= \frac{\Delta m^2_\odot}{4E} \begin{bmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{bmatrix} + \begin{bmatrix} V(r) & 0 \\ 0 & 0 \end{bmatrix} \ . \end{aligned} \tag{21}$$

Here, the first matrix $\mathcal{H}_V$ is the Hamiltonian in vacuum, and the second matrix $\mathcal{H}_M(r)$ is the modification due to matter. In $\mathcal{H}_V$, $\theta_\odot$ is the solar mixing angle defined by the two-neutrino mixing matrix of Eq. (17) with $\theta = \theta_\odot, \nu_\alpha = \nu_e$, and $\nu_\beta = \nu_x$. The splitting $\Delta m^2_\odot$ is $m^2_2 - m^2_1$, and for the present purpose we *define* ν_2 to be the heavier of the two mass eigenstates, so that $\Delta m^2_\odot$ is positive. In $\mathcal{H}_M(r)$, $V(r)$ is the interaction energy of Eq. (20) with the electron density $N_e(r)$ evaluated at distance r from the Sun's center.

From Eqs. (18–19) (with $\theta = \theta_\odot$), we see that two-neutrino oscillation in vacuum cannot distinguish between a mixing angle $\theta_\odot$ and an angle $\theta'_\odot = \pi/2 - \theta_\odot$. But these two mixing angles represent physically different situations. Suppose, for example, that $\theta_\odot < \pi/4$. Then, from Eq. (17) we see that if the mixing angle is $\theta_\odot$, the lighter mass eigenstate (defined to be ν_1) is more ν_e than ν_x, while if it is $\theta'_\odot$, then this mass eigenstate is more ν_x than ν_e. While oscillation in vacuum cannot discriminate between these two possibilities, neutrino propagation through solar matter can do so. The neutrino interaction energy V of Eq. (20) is of definite, positive sign [10]. Thus, the ν_e–ν_e element of the solar $\mathcal{H}$, $-(\Delta m^2_\odot/4E)\cos 2\theta_\odot + V(r)$, has a different size when the mixing angle is $\theta'_\odot = \pi/2 - \theta_\odot$ than it does when this angle is $\theta_\odot$. As a result, the flavor content of the neutrinos coming from the Sun can be different in the two cases [11].

Analyses of all the solar neutrino data, including very recent results on the neutral-current interactions of solar neutrinos in the Sudbury Neutrino Observatory (SNO), suggest that the most likely explanation of the behavior of the solar neutrinos is the Large-Mixing-Angle (LMA) variant of the MSW effect. A preference for this explanation has been found in several analyses that take into account the latest results [12–14]. The preference is illustrated by Fig. 1, obtained in an analysis by the SNO collaboration [15].

Let us estimate the probability $P(\nu_e \to \nu_e)$ that a solar neutrino which undergoes the LMA MSW effect in the Sun still has its original ν_e flavor when it arrives at the Earth. We focus on the neutrinos produced by ^{8}B decay, which are at the high-energy end of the solar neutrino spectrum. At $r \simeq 0$, where the solar neutrinos are created, the electron density $N_e \simeq 6 \times 10^{25}$ / cm^3 [16] yields for the interaction energy V of Eq. (20) the value 0.75×10^{-5} eV^2 / MeV. Thus, for $\Delta m^2_\odot$ in the favored region (cf. Fig. 1) and E a typical ^{8}B neutrino energy ($\sim$6-7 MeV), $\mathcal{H}_M$ dominates over $\mathcal{H}_V$. This means that, in first approximation, $\mathcal{H}(r \simeq 0)$ is diagonal. Thus, a ^{8}B neutrino is born not only in a ν_e flavor eigenstate, but also, again in first approximation, in an eigenstate of the Hamiltonian $\mathcal{H}(r \simeq 0)$. Since $V > 0$, the neutrino will be in the heavier of the two eigenstates. Now,

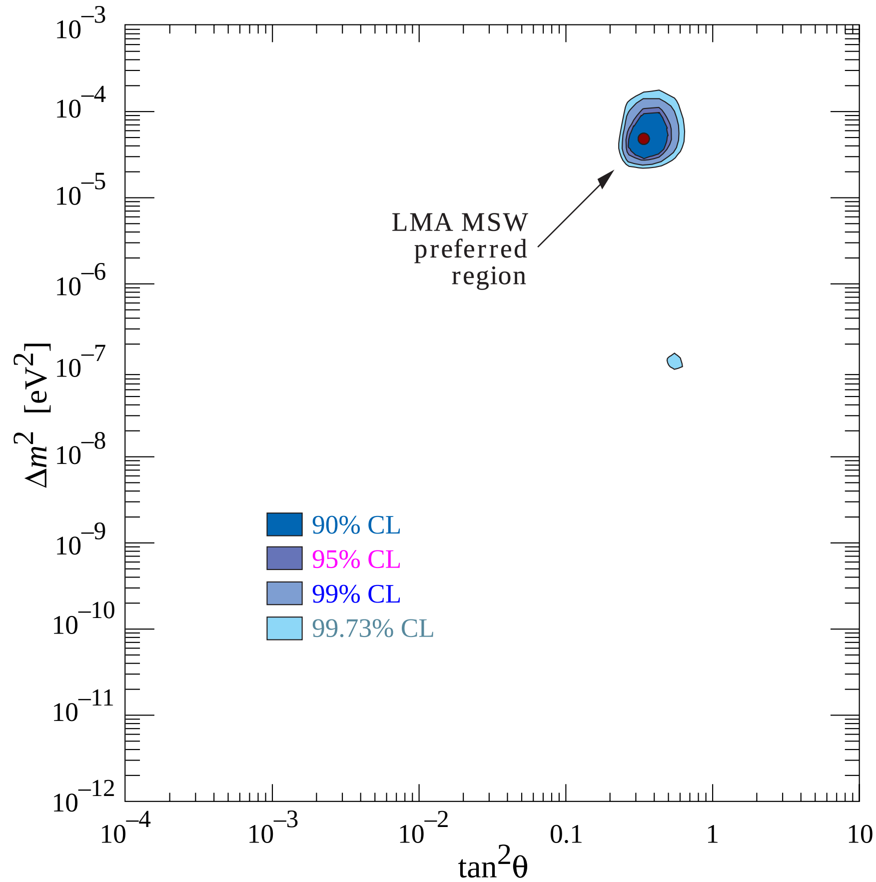

Figure 1: Allowed regions of the solar neutrino parameter space [15]. The splitting Δm^2 is $\Delta m^2_\odot$, and the angle θ is $\theta_\odot$.

under the conditions where the LMA MSW effect occurs, the propagation of a neutrino from $r \simeq 0$ to the outer edge of the Sun is adiabatic. That is, $N_e(r)$ changes sufficiently slowly that we may solve Schrödinger's equation for one r at a time, and then patch together the solutions. This means that our neutrino propagates outward through the Sun as one of the r-dependent eigenstates of the r-dependent $\mathcal{H}(r)$. Since the eigenvalues of $\mathcal{H}(r)$ do not cross at any r, and our neutrino is born in the heavier of the two $r = 0$ eigenstates, it emerges from the Sun in the heavier of the two $\mathcal{H}_V$ eigenstates. The latter is the mass eigenstate we have called ν_2, given according to Eq. (17) by

$$\nu_2 = \nu_e \sin\theta_\odot + \nu_x \cos\theta_\odot \quad . \tag{22}$$

Since this is an eigenstate of the vacuum Hamiltonian, the neutrino remains in it all the way to the surface of the Earth. The probability of observing the neutrino as a ν_e on Earth is then just the probability that ν_2 is a ν_e. That is [cf. Eq. (22)],

$$P(\nu_e \to \nu_e) = \sin^2 \theta_\odot \ . \tag{23}$$

Interestingly, if $\theta_\odot < 45°$, then this ν_e survival probability will be less than 1/2, as observed [17,18].

II. The evidence for flavor metamorphosis: There are three reported indications that neutrinos actually do change flavor in nature: the quite-convincing evidence that the atmospheric neutrinos do so, the now-compelling evidence that the solar neutrinos do, and the so-far-unconfirmed evidence that the neutrinos studied by the Liquid Scintillator Neutrino Detector (LSND) experiment do so also.

The atmospheric neutrinos are produced in the Earth's atmosphere by cosmic rays, and then detected in an underground detector. Incident on this detector are neutrinos coming from all directions, created all around the Earth in the atmosphere. The most compelling evidence that something very interesting happens to these atmospheric neutrinos en route to the detector is the fact that the detected upward-going atmospheric ν_μ flux ϕ_U (coming from all directions below the horizontal at the detector) differs from the corresponding downward-going flux ϕ_D. Suppose that neither neutrino oscillation, nor any other mechanism, decreases or increases the ν_μ flux as the neutrinos travel from their points of origin to the detector. Then, as illustrated in Fig. 2, any ν_μ that enters the sphere S defined in the figure caption will later exit this sphere. Thus, since we are dealing with a steady-state situation, the total ν_μ fluxes entering and exiting S per unit time must be equal. Now, for neutrino energies above a few GeV, the flux of cosmic rays which produce the atmospheric neutrinos is isotropic. Consequently, these neutrinos are being created at the same rate all around the Earth. Owing to this spherical symmetry, the equality between the ν_μ fluxes entering and exiting S must hold at any point of S, such as the location of the detector. Now, as shown in Fig. 2, a ν_μ *entering* S through the detector must be part of the downward-going flux ϕ_D. One *exiting* S through the detector must be part of the upward-going flux ϕ_U. Thus, the equality of the ν_μ fluxes entering and exiting S at the detector implies that $\phi_D = \phi_U$. (It is easily shown that this equality must hold not only for the integrated downward and upward fluxes, but angle by angle. That is, the flux coming down from zenith angle θ_Z must equal that coming up from angle $\pi - \theta_Z$ [19].)

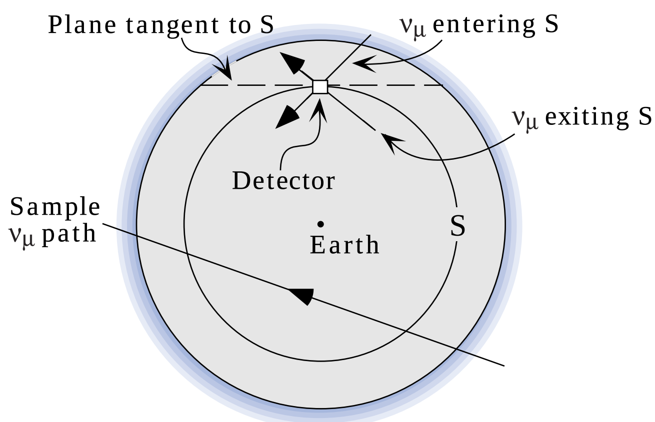

Figure 2: Atmospheric muon neutrino fluxes at an underground detector. S is a sphere centered at the center of the Earth and passing through the detector.

The underground Super-Kamiokande (SK) detector finds that for multi-GeV atmospheric muon neutrinos [20],

$$\frac{\text{Flux Up}(-1.0 < \cos\theta_Z < -0.2)}{\text{Flux Down}(+0.2 < \cos\theta_Z < +1.0)} = 0.54 \pm 0.04 \ , \qquad (24)$$

in strong disagreement with the requirement that the upward and downward fluxes be equal. Thus, some mechanism must be changing the ν_μ flux as the neutrinos travel to the detector. The most attractive candidate for this mechanism is the oscillation $\nu_\mu \to \nu_?$ of the muon neutrinos into neutrinos $\nu_?$ of another flavor. Since the upward-going muon neutrinos come from the atmosphere on the opposite side of the Earth from the detector, they travel much farther than the downward-going ones to reach the detector. Thus, they have more time to oscillate away into the other flavor, which explains why Flux Up < Flux Down. From reactor experimental limits on $P(\overline{\nu}_e \to \overline{\nu}_\mu)$ [9], which, assuming CPT invariance, are also limits on $P(\nu_\mu \to \nu_e)$, we know that $\nu_?$ is not a ν_e, except possibly a small fraction of the time [21]. Thus, $\nu_?$ is a ν_τ, a sterile neutrino ν_s, or sometimes one and sometimes the other. All of the detailed SK atmospheric neutrino data are well-described by the hypothesis that the oscillation is purely $\nu_\mu \to \nu_\tau$, and that it is a quasi-two-neutrino oscillation with a splitting $\Delta m^2_{\rm atm}$ in the 90% CL range [22].

$$1.6 \times 10^{-3}\,\mathrm{eV}^2 \lesssim \Delta m^2_{\rm atm} \lesssim 3.9 \times 10^{-3}\,\mathrm{eV}^2 \ , \qquad (25)$$

and a mixing angle $\theta_{\rm atm}$ with

$$\sin^2 2\theta_{\rm atm} > 0.92 \ . \qquad (26)$$

Other experiments favor roughly similar regions of parameter space [23,24]. We note that the constraint (25) implies that at least one mass eigenstate ν_i has a mass exceeding 0.03 eV. From several pieces of evidence, the 90% CL upper limit on the fraction of $\nu_?$ that is sterile is 19% [22].

In principle, upward-going (long-distance-traveling) muon neutrinos could be disappearing, not as a result of oscillation, but through decay into invisible daughters. This possibility is theoretically less likely than oscillation. However, there is a model of this kind [25] that proved totally compatible with all the atmospheric neutrino data for a long time. Only very recently was it finally possible for SK to show, using a neutral-current-enriched event sample, that this model is disfavored at 99% CL [26]. With neutrino decay, the expected neutral-current event rate for upward-going neutrinos is only 70% of that for downward-going ones. In contrast, with oscillation, the two rates should be equal. SK has been able to discriminate between these two possibilities.

The oscillation interpretation of the atmospheric neutrino data has received support from the KEK to Kamioka (K2K) long-baseline experiment. This experiment produces a ν_μ beam using an accelerator, measures the beam intensity with a complex of near detectors, and then measures the ν_μ flux still in the beam 250 km away using the SK detector. The L/E of this experiment is such that one expects to see an oscillation dominated by the atmospheric (mass)^2 splitting $\Delta m^2_{\rm atm}$. Whereas 80 ν_μ events would be expected in SK if there were no oscillation, and 52 events would be expected if oscillation were occurring with the parameters that fit the atmospheric data, 56 events are seen [26]. This is very consistent with oscillation, but obviously the statistics are still low.

The K2K evidence for oscillation is strengthened by this experiment's recent analysis of the shape of the ν_μ energy spectrum at both the near and SK (far) detectors [27]. When the spectral information is taken into account, it is found that for maximal mixing, the 90% CL allowed range for the (mass)^2 splitting $\Delta m^2_{\rm atm}$ is

$$1.5 \times 10^{-3}\,{\rm eV}^2 \lesssim \Delta m^2_{\rm atm} \lesssim 3.9 \times 10^{-3}\,{\rm eV}^2 \ . \qquad (27)$$

This is very consistent with the range (25) found from the atmospheric data.

The neutrinos created in the Sun have been detected on Earth by several experiments, as discussed by K. Nakamura in this *Review.* The nuclear processes that power the Sun make only ν_e, not ν_μ or ν_τ, and until recently all the solar neutrino measurements have been sensitive exclusively, or at least mostly, to ν_e. For years, the solar neutrino experiments have been reporting that the solar ν_e flux arriving at the Earth is below the one expected from neutrino production calculations. It has been hypothesized that this ν_e deficit is due to the metamorphosis, through a mechanism such as the LMA MSW effect discussed in Sec. I, of some of the electron neutrinos into neutrinos of another flavor. In that case, we should see a flux of muon and/or tau neutrinos coming from the Sun, despite the fact that the nuclear reactions in the Sun do not make neutrinos of these flavors. Thanks to SNO, we now have extremely strong evidence for these muon and/or tau neutrinos. This evidence is strengthened further by data from SK.

SNO has studied the flux of high-energy solar neutrinos from ^{8}B decay. This experiment detects these neutrinos via the reactions

$$\nu + d \rightarrow e^- + p + p \ , \tag{28}$$

$$\nu + d \rightarrow \nu + p + n \ , \tag{29}$$

and

$$\nu + e \rightarrow \nu + e \ . \tag{30}$$

The first of these reactions, charged-current deuteron breakup, can be initiated only by a ν_e. Thus, it measures the flux $\phi(\nu_e)$ of ν_e from ^{8}B decay in the Sun. The second reaction, neutral-current deuteron breakup, can be initiated with equal cross sections by neutrinos of all active flavors. Thus, it measures $\phi(\nu_e) + \phi(\nu_{\mu,\tau})$, where $\phi(\nu_{\mu,\tau})$ is the flux of ν_μ and/or ν_τ from the Sun. Finally, the third reaction, neutrino electron elastic scattering, can be triggered by a neutrino of any active flavor, but $\sigma(\nu_{\mu,\tau}\, e \rightarrow \nu_{\mu,\tau}\, e) \simeq \sigma(\nu_e\, e \rightarrow \nu_e\, e)/6.5$. Thus, this reaction measures $\phi(\nu_e) + \phi(\nu_{\mu,\tau})/6.5$.

From the observed rates for the reactions (28)–(30), SNO finds that

$$\phi(\nu_e) = (1.76 \pm 0.10) \times 10^6 \text{ cm}^{-2}\text{s}^{-1} \ , \tag{31}$$

and

$$\phi(\nu_{\mu,\tau}) = (3.41 \ ^{+0.66}_{-0.64}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1} \ , \tag{32}$$

assuming the standard shape of the ^{8}B neutrino energy spectrum [17]. We see that the flux of ν_μ and/or ν_τ from the Sun, $\phi(\nu_{\mu,\tau})$, is $5.3\,\sigma$ from zero. If the SK data on solar-neutrino electron scattering, reaction (30), are included as a further constraint, it is found that $\phi(\nu_{\mu,\tau}) = (3.45 \ ^{+0.65}_{-0.62}) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$, $5.5\,\sigma$ from zero [17]. This convincingly nonvanishing ν_μ/ν_τ flux from the Sun is "smoking-gun" evidence that the electron neutrinos produced in the solar core do indeed change flavor.

Change of neutrino flavor, whether in matter or vacuum, does not change the total neutrino flux. Thus, unless some of the solar ν_e are changing into sterile neutrinos, the total active flux measured by the neutral-current reaction (29) should agree with the predicted total solar neutrino flux based on calculations of neutrino production in the Sun. This predicted total is $(5.05\ ^{+1.01}_{-0.81}) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ [28]. By comparison, the total active flux measured by reaction (29) is $(5.09\ ^{+0.64}_{-0.61}) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$, in very good agreement. This agreement provides evidence that neutrino production in the Sun is correctly understood, and further strengthens the evidence that neutrinos really do change flavor.

Nevertheless, given the uncertainties in the calculated total neutrino production and in the measurements, it is still possible that a significant fraction of the solar ν_e that change their flavor becomes sterile. It has been found that at the $1\,\sigma$ level this fraction can be as large as 1/4 [13,29].

Assuming that no solar neutrinos become sterile, and including data from other experiments, and solar-model predictions for neutrino fluxes below the ^{8}B energy range, the SNO collaboration arrived at the favored regions of $\Delta m^2_\odot$ and $\theta_\odot$ shown in Fig. 1 [12]. The best-fit point has $\sin^2\theta_\odot = 0.25$. From Eq. (23), we then expect that the ^{8}B ν_e survival probability is approximately 0.25 as well. From Eqs. (31) and (32), the central value of this probability is $1.76/(1.76 + 3.41) = 0.34$. This number is larger than 0.25 for at least two reasons [30]. First, the neutrinos arriving at the Earth's surface are not entirely in the heavier mass eigenstate ν_2. With small probability, they are in the eigenstate ν_1, and ν_1 has higher ν_e content than does ν_2. Secondly, at night, the solar neutrinos pass through the Earth before reaching the detector. An MSW effect within the Earth is expected to increase slightly the ν_e content of these nighttime neutrinos. Indeed, SNO finds that, with $\phi_N(\phi_D)$ the nighttime (daytime) ν_e flux, $2(\phi_N - \phi_D)/(\phi_N + \phi_D) = (7.0 \pm 4.9\ ^{+1.3}_{-1.2})\%$ [12].

The neutrinos studied by the LSND experiment [31] come from the decay $\mu^+ \to e^+\nu_e\overline{\nu}_\mu$ of muons at rest. While this decay does not produce $\overline{\nu}_e$, an excess of $\overline{\nu}_e$ over expected background is reported by the experiment. This excess is interpreted as due to oscillation of some of the $\overline{\nu}_\mu$ produced by μ^+ decay into $\overline{\nu}_e$. The related KArlsruhe Rutherford Medium Energy Neutrino (KARMEN) experiment [32] sees no indication for such an oscillation. However, the LSND and KARMEN experiments are not identical; at LSND the neutrino travels a distance $L \approx 30\,\mathrm{m}$ before detection, while at KARMEN it travels $L \approx 18\,\mathrm{m}$. The KARMEN results exclude a portion of the neutrino parameter region favored by LSND, but not all of it. A joint analysis [33] of the results of both experiments finds that a splitting $0.2 \lesssim \Delta m^2_{\mathrm{LSND}} \lesssim 1\,\mathrm{eV}^2$ and mixing $0.003 \lesssim \sin^2 2\theta_{\mathrm{LSND}} \lesssim 0.03$, or a splitting $\Delta m^2_{\mathrm{LSND}} \simeq 7\,\mathrm{eV}^2$ and mixing $\sin^2 2\theta_{\mathrm{LSND}} \simeq 0.004$, might explain both experiments.

III. Neutrino spectra and mixings: If there are only three neutrino mass eigenstates, ν_1, ν_2 and ν_3, then there are only three mass splittings Δm^2_{ij}, and they obviously satisfy

$$\Delta m^2_{32} + \Delta m^2_{21} + \Delta m^2_{13} = 0 \ . \tag{33}$$

Now, as we have seen, the Δm^2 values required to explain the flavor changes of the atmospheric, solar, and LSND neutrinos are of three different orders of magnitude. Thus, they cannot possibly obey the constraint of Eq. (33). If all three of the reported changes of flavor are genuine, then nature must contain at least four neutrino mass eigenstates [34]. As explained in Section I, one linear combination of these mass eigenstates would have to be sterile.

If only the atmospheric and solar flavor changes prove to be genuine, then nature may well contain only three neutrino mass eigenstates. The neutrino spectrum then contains two mass eigenstates separated by the splitting $\Delta m^2_{\odot}$ needed to explain the solar neutrino data, and a third eigenstate separated from the first two by the larger splitting $\Delta m^2_{\mathrm{atm}}$ called for by the atmospheric neutrino data. The solar pair—the two eigenstates separated by $\Delta m^2_{\odot}$—might be at the bottom or the top of the spectrum. The study of flavor changes of accelerator-generated neutrinos and antineutrinosthat pass through matter can discriminate between these two possibilities. If the solar pair is at the bottom, then the spectrum is of the form shown in Fig. 3. There we include the approximate flavor content of each mass eigenstate, the flavor-α fraction of eigenstate ν_i being simply $|\langle \nu_\alpha | \nu_i \rangle|^2 = |U_{\alpha i}|^2$. The flavor content shown assumes that the LMA MSW effect is the explanation of the behavior of

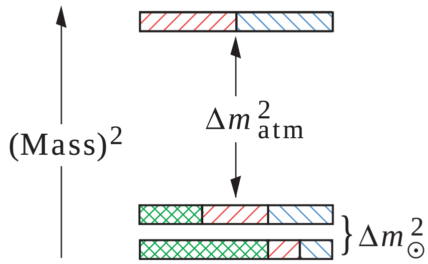

Figure 3: A three-neutrino (Mass)^2 spectrum that accounts for the flavor changes of the solar and atmospheric neutrinos. The ν_e fraction of each mass eigenstate is crosshatched, the ν_μ fraction is indicated by right-leaning hatching, and the ν_τ fraction by left-leaning hatching.

the solar neutrinos. Other explanations can give different flavor content (and yield a different $\Delta m^2_\odot$).

When there are only three neutrino mass eigenstates, and the corresponding three familiar neutrinos of definite flavor, the leptonic mixing matrix U can be written as

$$
U =
$$

$$
\begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array}
\begin{array}{c}
\begin{array}{ccc} \qquad\qquad \nu_1 \qquad\qquad & \qquad\qquad \nu_2 \qquad\qquad & \nu_3 \end{array} \\
\left[\begin{array}{ccc}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{array}\right]
\end{array}
$$

$$
\times\, \mathrm{diag}(e^{i\alpha_1/2},\ e^{i\,\alpha_2/2},\ 1)\ . \tag{34}
$$

Here, ν_1 and ν_2 are the members of the solar pair, with $m_2 > m_1$, and ν_3 is the isolated neutrino, which may be heavier or lighter than the solar pair. Inside the matrix, $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, where the three θ_{ij}'s are mixing angles. The quantities δ, α_1, and α_2 are CP-violating phases. From Eq. (8), we see that α_1 and α_2 do not affect neutrino oscillation, but these phases do affect the rate for neutrinoless double-beta decay. Apart from the phases α_1, α_2, the parametrization of the leptonic mixing matrix in Eq. (34) is identical to that [35] advocated for the quark mixing matrix by Gilman, Kleinknecht, and Renk in their article in this *Review*.

From bounds on the oscillation of reactor $\overline{\nu}_e$, $s_{13}^2 \lesssim 0.03$ at 90% CL [9]. Taking this into account, and assuming that the atmospheric neutrino mixing angle of Eq. (26) is maximal (which gives the best fit to the atmospheric data [22]), and that the solar neutrinos are governed by the LMA MSW effect, the U of Eq. (34) simplifies to

$$U = \begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{array}{c} \begin{array}{ccc} \quad\nu_1 \quad\quad & \quad\quad \nu_2 \quad\quad & \quad \nu_3 \end{array} \\ \left[\begin{array}{ccc} c\, e^{i\alpha_1/2} & s\, e^{i\alpha_2/2} & s_{13}\, e^{-i\delta} \\ -s\, e^{i\alpha_1/2}/\sqrt{2} & c\, e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \\ s\, e^{i\alpha_1/2}/\sqrt{2} & -c\, e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \end{array} \right] \end{array} . \quad (35)$$

Here, $c \equiv \cos\theta_\odot$ and $s \equiv \sin\theta_\odot$, where $\theta_\odot$ is the solar mixing angle defined in Section I and constrained by Fig. 1. With θ_{13} small, $\theta_\odot \simeq \theta_{12}$. The illustrative flavor content shown in Fig. 3 is obtained from the U of Eq. (35) taking $s_{13}^2 \simeq 0$, $s^2 \simeq 1/4$.

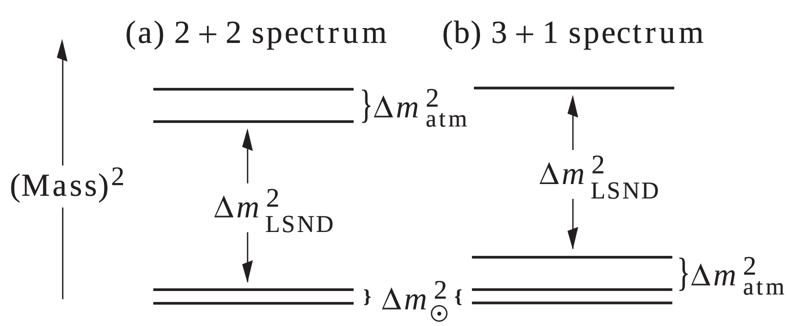

Figure 4: Possible four-neutrino (Mass)^2 spectra.

If the atmospheric, solar, and LSND neutrinos all prove to genuinely change flavor, then, as already noted, there must be at least four mass eigenstates. If there are exactly four, then the spectrum is either of the kind depicted in Fig. 4a, or of the kind shown in Fig. 4b.

In Fig. 4a, we have a “2+2” spectrum. This consists of a “solar pair” of eigenstates that are separated by the solar splitting $\Delta m_\odot^2$ and are the main contributors to the behavior of solar neutrinos, plus an "atmospheric pair" that are separated by the atmospheric splitting $\Delta m^2_{\rm atm}$ and are the main contributors to the atmospheric $\nu_\mu \rightarrow \nu_\tau$ oscillation. From bounds on reactor $\overline{\nu}_e$ oscillation [9], we know that the ν_e fraction of the atmospheric pair is less than 3%. From bounds on accelerator ν_μ oscillation [36], we know that the ν_μ fraction of the solar pair is similarly limited. Thus, the atmospheric (solar) pair of eigenstates plays only a small role in the behavior of the solar ν_e (atmospheric ν_μ). The solar and atmospheric pairs are separated from each other by the large LSND splitting $\Delta m^2_{\rm LSND}$, making possible the LSND oscillation. The solar pair may lie below the atmospheric pair, as shown in Fig. 4a, or above it.

In Fig. 4b, we have a "3+1" spectrum. This includes a trio, consisting of a solar pair separated by $\Delta m^2_\odot$, plus a third neutrino separated from the solar pair by $\Delta m^2_{\rm atm}$, and a fourth neutrino separated from the trio by $\Delta m^2_{\rm LSND}$. In the trio, the solar pair may lie below the third neutrino, as shown, or above it [37]. In addition, the fourth, isolated neutrino may lie above the other three, as shown, or below them. In the case of a 3+1 spectrum, the reactor $\overline{\nu}_e$ and accelerator ν_μ oscillation bounds mentioned previously imply that the isolated neutrino has very little ν_e or ν_μ flavor content. It is interesting to consider the possibility that it has very little ν_τ content as well, and consequently is largely sterile. Then, by unitarity, the other three neutrinos—the "3"—can have only very little sterile content. Those three neutrinos dominate the solar and atmospheric fluxes, so neither of these fluxes will contain sterile neutrinos to any significant degree. In contrast, it is characteristic of the 2+2 spectra that either the solar or atmospheric neutrino fluxes, or both, do include a substantial component of sterile neutrinos [38]. Thus, further information on the sterile neutrino content of these two fluxes can potentially discriminate between the 2+2 and 3+1 spectra.

Neither a 2+2 nor a 3+1 spectrum gives an excellent fit to all the data. In the framework of the 2+2 spectra, there is tension between the solar and atmospheric data. In that of the 3+1 spectra, there is tension between the bounds on oscillation at large L/E and LSND [39].

IV. Questions to be answered: The strong evidence for neutrino flavor metamorphosis—hence neutrino mass—opens many questions about the neutrinos. These questions, which hopefully will be answered by future experiments, include the following:

i) Do neutrinos truly change flavor?

The evidence that they do is already very strong, but further confirmation is desirable. In particular, one would like to observe the undulatory $\sin^2[1.27\,\Delta m^2\,(L/E)]$ dependence of vacuum oscillation probabilities on L/E. This so-far-unobserved characteristic signature of oscillation could in principle be seen in reactor neutrino experiments hoping to confirm the flavor changes attributed to the solar neutrinos, long base-line (LBL) accelerator experiments aimed at confirming the oscillation of atmospheric neutrinos, and short base-line accelerator experiments hoping to confirm the oscillation of the LSND neutrinos.

ii) How many neutrino species are there? Do sterile neutrinos exist?

This question may be addressed by carrying out an experiment that can confirm or refute LSND.

iii) What are the neutrino (Mass)^2 splittings? What are the mixing angles in the leptonic mixing matrix?

If $\Delta m^2_\odot$ is in the LMA-MSW-favored range, and not too small or large, it can be determined through reactor neutrino experiments. The corresponding mixing angle $\theta_\odot$ can be determined from a combination of solar, and especially reactor, neutrino data.

The allowed range for $\Delta m^2_{\rm atm}$ can be narrowed by LBL experiments. The value of $\theta_{\rm atm}$, and in particular its deviation from maximality [cf. inequality (26)], can perhaps be found by placing a detector at a suitable angle off the axis of an LBL beam. There, the neutrino flux can be highly monoenergetic, and one can run at the extremum of the oscillatory factor $\sin^2[1.27\,\Delta m^2_{\rm atm}(L/E)]$, thereby increasing one's sensitivity to the coefficient of this factor, $\sin^2 2\theta_{\rm atm}$ [40].

The splitting $\Delta m^2_{\rm LSND}$ and corresponding mixing can be ascertained through short base-line accelerator experiments.

If the leptonic mixing matrix U does turn out to be a 3×3 matrix of the form shown in Eq. (35), with $\theta_\odot$ a large angle, then U is rather different from its quark counterpart. In the latter, the diagonal elements are essentially unity, and all the off-diagonal elements are small. But in the U of Eq. (35), *all* the elements are fairly large, except for U_{e3}. This difference between the leptonic and quark mixing matrices may contain a clue about the origin of mixing.

iv) What are the masses m_i of the individual mass eigenstates ν_i?

Flavor-change experiments can determine a spectral pattern such as the one in Fig. 3, but not the distance of the entire pattern from the zero of $(\text{Mass})^2$. One might discover that distance via study of the β energy spectrum in tritium β decay, if the mass of some ν_i with appreciable coupling to an electron is large enough to be within reach of a feasible experiment.

It will be interesting to find out whether or not neutrinos are appreciably heavier than they need to be to account for the splittings required by oscillation data.

v) Is each ν_i identical to its antiparticle?

If the answer to this question is "yes," we shall refer to the neutrinos as Majorana particles, and if it is "no," as Dirac particles. Observation of neutrinoless double-beta decay would establish that the answer is "yes" [41]. Measurement of the rate for this process would also provide information concerning the scale of neutrino mass [42,43].

vi) Does the behavior of neutrinos violate CP?

From Eqs. (8), (12), and (35), we see that if the CP-violating phase δ and the small mixing angle θ_{13} are both nonvanishing, there will be CP-violating differences between neutrino and antineutrino oscillation probabilities. Observation of these differences would establish that CP violation is not a peculiarity of quarks. Since from Eq. (35) the size of all CP-violating effects depends on that of θ_{13}, it is clearly important not only to establish that $\theta_{13} \neq 0$, so that CP violation in oscillation can be nonvanishing, but also to measure θ_{13}, so that one will know how big the CP violation can be. Depending on the size of θ_{13}, measuring this angle may be possible in an

LBL accelerator experiment presently under construction, or may require a more intense neutrino beam and a more massive detector, or even a nonconventional neutrino beam produced by the decay of muons in a storage ring functioning as a "neutrino factory."

From Eq. (7) or (8), it follows that the phases $\alpha_{1,2}$ in Eq. (34) or (35) do not affect neutrino oscillation. However, these phases can lead to CP-violating values of the rate for neutrinoless double beta decay [42,43].

These and other questions about the world of neutrinos will be the focus of a major experimental program in the years to come.

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