10. ELECTROWEAK MODEL AND CONSTRAINTS ON NEW PHYSICS

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10.1. Introduction

The standard electroweak model is based on the gauge group [1] SU(2) $\times$ U(1), with gauge bosons W_μ^i, $i = 1, 2, 3$, and B_μ for the SU(2) and U(1) factors, respectively, and the corresponding gauge coupling constants g and g'. The left-handed fermion fields $\psi_i = \begin{pmatrix} \nu_i \\ \ell_i^- \end{pmatrix}$ and $\begin{pmatrix} u_i \\ d_i' \end{pmatrix}$ of the i^{th} fermion family transform as doublets under SU(2), where $d_i' \equiv \sum_j V_{ij}\, d_j$, and V is the Cabibbo-Kobayashi-Maskawa mixing matrix. (Constraints on V are discussed in the section on the Cabibbo-Kobayashi-Maskawa mixing matrix.) The right-handed fields are SU(2) singlets. In the minimal model there are three fermion families and a single complex Higgs doublet $\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.

After spontaneous symmetry breaking the Lagrangian for the fermion fields is

$$\begin{aligned} \mathscr{L}_F = & \sum_i \overline{\psi}_i \left(i\, \partial\!\!\!/ - m_i - \frac{g m_i H}{2 M_W} \right) \psi_i \\ & - \frac{g}{2\sqrt{2}} \sum_i \overline{\psi}_i\, \gamma^\mu\, (1 - \gamma^5)(T^+\, W_\mu^+ + T^-\, W_\mu^-)\, \psi_i \\ & - e \sum_i q_i\, \overline{\psi}_i\, \gamma^\mu\, \psi_i\, A_\mu \\ & - \frac{g}{2\cos\theta_W} \sum_i \overline{\psi}_i\, \gamma^\mu (g_V^i - g_A^i \gamma^5)\, \psi_i\, Z_\mu\ . \end{aligned} \qquad (10.1)$$

$\theta_W \equiv \tan^{-1}(g'/g)$ is the weak angle; $e = g \sin\theta_W$ is the positron electric charge; and $A \equiv B\cos\theta_W + W^3 \sin\theta_W$ is the (massless) photon field. $W^\pm \equiv (W^1 \mp i W^2)/\sqrt{2}$ and $Z \equiv -B\sin\theta_W + W^3\cos\theta_W$ are the massive charged and neutral weak boson fields, respectively. T^+ and T^- are the weak isospin raising and lowering operators. The vector and axial vector couplings are

$$g_V^i \equiv t_{3L}(i) - 2 q_i \sin^2\theta_W\ , \qquad (10.2a)$$

$$g_A^i \equiv t_{3L}(i)\ , \qquad (10.2b)$$

where $t_{3L}(i)$ is the weak isospin of fermion i ($+1/2$ for u_i and ν_i; $-1/2$ for d_i and e_i) and q_i is the charge of ψ_i in units of e.

The second term in $\mathscr{L}_F$ represents the charged-current weak interaction [2]. For example, the coupling of a W to an electron and a neutrino is

$$-\frac{e}{2\sqrt{2}\sin\theta_W} \left[W_\mu^-\ \overline{e}\ \gamma^\mu (1 - \gamma^5)\nu + W_\mu^+\ \overline{\nu}\ \gamma^\mu\ (1 - \gamma^5) e \right]\ . \qquad (10.3)$$

For momenta small compared to M_W, this term gives rise to the effective four-fermion interaction with the Fermi constant given (at tree level, *i.e.*, lowest order in perturbation theory) by $G_F/\sqrt{2} = g^2/8M_W^2$. CP violation is incorporated in the Standard Model by a single observable phase in V_{ij}. The third term in $\mathscr{L}_F$ describes electromagnetic interactions (QED), and the last is the weak neutral-current interaction.

In Eq. (10.1), m_i is the mass of the i^{th} fermion ψ_i. For the quarks these are the current masses. For the light quarks, as described in the Particle Listings, $\widehat{m}_u \approx$ 1–5 MeV, $\widehat{m}_d \approx$ 3–9 MeV, and $\widehat{m}_s \approx$ 75–170 MeV. These are running $\overline{\rm MS}$ masses evaluated at the scale $\mu = 2$ GeV. (In this section we denote quantities defined in the $\overline{\rm MS}$ scheme by a caret; the exception is the strong coupling constant, α_s, which will always correspond to the $\overline{\rm MS}$ definition and where the caret will be dropped.) For the heavier quarks, $\widehat{m}_c(\mu = \widehat{m}_c) \approx$ 1.15–1.35 GeV and $\widehat{m}_b(\mu = \widehat{m}_b) \approx$ 4.0–4.4 GeV. The average of the recent CDF [4] and DØ [5] values for the top quark "pole" mass is $m_t = 174.3 \pm 5.1$ GeV. We will use this value for m_t (together with $M_H = 100$ GeV) for the numerical values quoted in Sec. 10.2–Sec. 10.4. See "The Note on Quark Masses" in the Particle Listings for more information.

H is the physical neutral Higgs scalar which is the only remaining part of ϕ after spontaneous symmetry breaking. The Yukawa coupling of H to ψ_i, which is flavor diagonal in the minimal model is $g m_i/2M_W$. In nonminimal models there are additional charged and neutral scalar Higgs particles [6].

10.2. Renormalization and radiative corrections

The Standard Model has three parameters (not counting the Higgs boson mass, M_H, and the fermion masses and mixings). A particularly useful set is:

(*a*) The fine structure constant $\alpha = 1/137.0359895(61)$, determined from the quantum Hall effect. In most electroweak-renormalization schemes, it is convenient to define a running α dependent on the energy scale of the process, with $\alpha^{-1} \sim 137$ appropriate at very low energy. (The running has also been observed directly. [7]) For scales above a few hundred MeV this introduces an uncertainty due to the low-energy hadronic contribution to vacuum polarization. In the modified minimal subtraction ($\overline{\rm MS}$) scheme [8] (used for this *Review*), and with $\alpha_s(M_Z) = 0.120$ for the QCD coupling at M_Z, one has $\widehat{\alpha}(m_\tau)^{-1} = 133.513 \pm 0.026$ and $\widehat{\alpha}(M_Z)^{-1} = 127.934 \pm 0.027$ [9]. The non-linear α_s dependence of $\widehat{\alpha}(M_Z)$ and the resulting correlation with the input variable α_s, is fully taken into account in the fits. The uncertainty is from e^+e^- annihilation data below 1.8 GeV [10], from uncalculated higher order perturbative and non-perturbative QCD corrections, and from the $\overline{\rm MS}$ quark masses, $\widehat{m}_c(\widehat{m}_c) = 1.31 \pm 0.07$ and $\widehat{m}_b(\widehat{m}_b) = 4.24 \pm 0.11$ [9]. Such a short distance mass definition (unlike the pole mass) is free from non-perturbative and renormalon uncertainties. Various recent evaluations of the contributions of the five light quark flavors, $\Delta\alpha_{\rm had}^{(5)}$, to the conventional (on-shell) QED coupling, $\alpha(M_Z) = \dfrac{\alpha}{1 - \Delta\alpha}$, are summarized in Table 10.1. Most of the older results relied on $e^+e^- \to$ hadrons cross-section measurements up to energies of 40 GeV which were somewhat higher than the QCD prediction, suggested stronger running, and were less precise. The most recent results assume the validity of perturbative QCD (PQCD) at scales of 1.8 GeV and above (outside of resonance regions), and are in very good agreement with each other. They imply higher central values for the extracted M_H by $\mathcal{O}(20$ GeV). On the other hand, the upper limits for M_H are all similar due to a compensation of the latter effect and the higher precision. Further improvement of this dominant theoretical uncertainty in the interpretation of precision data will require better measurements of the cross-section for $e^+e^- \to$ hadrons at low energy.

(*b*) The Fermi constant, $G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2}, determined from the muon lifetime formula [22,23],

$$\begin{aligned} \tau_\mu^{-1} = & \frac{G_F^2 m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right) \\ & \times \left[1 + \left(\frac{25}{8} - \frac{\pi^2}{2}\right)\frac{\alpha(m_\mu)}{\pi} + C_2 \frac{\alpha^2(m_\mu)}{\pi^2}\right]\ , \end{aligned} \qquad (10.4a)$$

where

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x\ , \qquad (10.4b)$$

$$C_2 = \frac{156815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\zeta(3) + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2\ln(2)\ , \qquad (10.4c)$$

and

$$\alpha(m_\mu)^{-1} = \alpha^{-1} - \frac{2}{3\pi}\ln\left(\frac{m_\mu}{m_e}\right) + \frac{1}{6\pi} \approx 136\ . \qquad (10.4d)$$

Table 10.1: Recent evaluations of the on-shell $\Delta\alpha^{(5)}_{\rm had}(M_Z)$. For better comparison we adjusted central values and errors to correspond to a common and fixed value of $\alpha_s(M_Z) = 0.120$. References quoting results without the top quark decoupled are converted to the five flavor definition. Ref. [20] uses $\Lambda_{\rm QCD} = 380 \pm 60$ MeV; for the conversion we assumed $\alpha_s(M_Z) = 0.118 \pm 0.003$.

Reference	Result	Comment
Martin&Zeppenfeld [11]	0.02744 ± 0.00036	PQCD for $\sqrt{s} > 3$ GeV
Eidelman&Jegerlehner [12]	0.02803 ± 0.00065	PQCD for $\sqrt{s} > 40$ GeV
Geshkenbein&Morgunov [13]	0.02780 ± 0.00006	$\mathcal{O}(\alpha_s)$ resonance model
Burkhardt&Pietrzyk [14]	0.0280 ± 0.0007	PQCD for $\sqrt{s} > 40$ GeV
Swartz [15]	0.02754 ± 0.00046	use of fitting function
Alemany, Davier, Höcker [16]	0.02816 ± 0.00062	includes τ decay data
Krasnikov&Rodenberg [17]	0.02737 ± 0.00039	PQCD for $\sqrt{s} > 2.3$ GeV
Davier&Höcker [10]	0.02784 ± 0.00022	PQCD for $\sqrt{s} > 1.8$ GeV
Kühn&Steinhauser [18]	0.02778 ± 0.00016	complete $\mathcal{O}(\alpha_s^2)$
Erler [9]	0.02779 ± 0.00020	converted from $\overline{\rm MS}$ scheme
Davier&Höcker [19]	0.02770 ± 0.00015	use of QCD sum rules
Groote *et al.* [20]	0.02787 ± 0.00032	use of QCD sum rules
Jegerlehner [21]	0.02778 ± 0.00024	converted from MOM scheme

The $\mathcal{O}(\alpha^2)$ corrections to μ decay have been completed recently [23]. The remaining uncertainty in G_F is from the experimental input.

(*c*) The Z boson mass, $M_Z = 91.1872 \pm 0.0021$ GeV, determined from the Z lineshape scan at LEP 1 [24].

With these inputs, $\sin^2\theta_W$ can be calculated when values for m_t and M_H are given; conversely (as is done at present), M_H can be constrained by $\sin^2\theta_W$. The value of $\sin^2\theta_W$ is extracted from Z pole observables, the W mass, and neutral-current processes [25], and depends on the renormalization prescription. There are a number of popular schemes [27–32] leading to values which differ by small factors depending on m_t and M_H. The notation for these schemes is shown in Table 10.2. Discussion of the schemes follows the table.

Table 10.2: Notations used to indicate the various schemes discussed in the text. Each definition of $\sin\theta_W$ leads to values that differ by small factors depending on m_t and M_H.

Scheme	Notation
On-shell	$s_W = \sin\theta_W$
NOV	$s_{M_Z} = \sin\theta_W$
$\overline{\rm MS}$	$\widehat{s}_Z = \sin\theta_W$
$\overline{\rm MS}$ *ND*	$\widehat{s}_{ND} = \sin\theta_W$
Effective angle	$\overline{s}_f = \sin\theta_W$

(*i*) The on-shell scheme [27] promotes the tree-level formula $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ to a definition of the renormalized $\sin^2\theta_W$ to all orders in perturbation theory, *i.e.*, $\sin^2\theta_W \to s_W^2 \equiv 1 - M_W^2/M_Z^2$. This scheme is simple conceptually. However, M_W is known much less precisely than M_Z and in practice one extracts s_W^2 from M_Z alone using

$$M_W = \frac{A_0}{s_W(1-\Delta r)^{1/2}} \ , \qquad (10.5a)$$

$$M_Z = \frac{M_W}{c_W} \ , \qquad (10.5b)$$

where $c_W \equiv \cos\theta_W$, $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.2805(2)$ GeV, and Δr includes the radiative corrections relating α, $\alpha(M_Z)$, G_F, M_W, and M_Z. One finds $\Delta r \sim \Delta r_0 - \rho_t/\tan^2\theta_W$, where $\Delta r_0 = 1 - \alpha/\widehat{\alpha}(M_Z) = 0.0664(2)$ is due to the running of α and $\rho_t = 3G_F m_t^2/8\sqrt{2}\pi^2 = 0.00952(m_t/174.3 \text{ GeV})^2$ represents the dominant (quadratic) m_t dependence. There are additional contributions to Δr from bosonic loops, including those which depend logarithmically on M_H. One has $\Delta r = 0.0350 \mp 0.0019 \pm 0.0002$, where the second uncertainty is from $\alpha(M_Z)$. Thus the value of s_W^2 extracted from M_Z includes an uncertainty (∓ 0.0006) from the currently allowed range of m_t.

(*ii*) A more precisely determined quantity $s^2_{M_Z}$ can be obtained from M_Z by removing the (m_t, M_H) dependent term from Δr [28], *i.e.*,

$$s^2_{M_Z} c^2_{M_Z} \equiv \frac{\pi\alpha(M_Z)}{\sqrt{2}\, G_F\, M_Z^2} \ . \qquad (10.6)$$

Using $\alpha(M_Z)^{-1} = 128.92 \pm 0.03$ yields $s^2_{M_Z} = 0.23105 \mp 0.00008$. The small uncertainty in $s^2_{M_Z}$ compared to other schemes is because most of the m_t dependence has been removed by definition. However, the m_t uncertainty reemerges when other quantities (*e.g.*, M_W or other Z pole observables) are predicted in terms of M_Z.

Both s_W^2 and $s^2_{M_Z}$ depend not only on the gauge couplings but also on the spontaneous-symmetry breaking, and both definitions are awkward in the presence of any extension of the Standard Model which perturbs the value of M_Z (or M_W). Other definitions are motivated by the tree-level coupling constant definition $\theta_W = \tan^{-1}(g'/g)$.

(*iii*) In particular, the modified minimal subtraction ($\overline{\rm MS}$) scheme introduces the quantity $\sin^2\widehat{\theta}_W(\mu) \equiv \widehat{g}'^2(\mu)/\left[\widehat{g}^2(\mu) + \widehat{g}'^2(\mu)\right]$, where the couplings $\widehat{g}$ and $\widehat{g}'$ are defined by modified minimal subtraction and the scale μ is conveniently chosen to be M_Z for electroweak processes. The value of $\widehat{s}_Z^2 = \sin^2\widehat{\theta}_W(M_Z)$ extracted from M_Z is less sensitive than s_W^2 to m_t (by a factor of $\tan^2\theta_W$), and is less sensitive to most types of new physics than s_W^2 or $s^2_{M_Z}$. It is also very useful for comparing with the predictions of grand unification. There are actually several variant definitions of $\sin^2\widehat{\theta}_W(M_Z)$, differing according to whether or how finite $\alpha\ln(m_t/M_Z)$ terms are decoupled (subtracted from the couplings). One cannot entirely decouple the $\alpha\ln(m_t/M_Z)$ terms from all electroweak quantities because $m_t \gg m_b$ breaks SU(2) symmetry. The scheme that will be adopted here decouples the $\alpha\ln(m_t/M_Z)$ terms from the $\gamma - Z$ mixing [8,29], essentially eliminating any $\ln(m_t/M_Z)$ dependence in the formulae for asymmetries at the Z pole when written in terms of $\widehat{s}_Z^2$. (A similar definition is used for $\widehat{\alpha}$.) The various definitions are related by

$$\widehat{s}_Z^2 = c\,(m_t, M_H) s_W^2 = \overline{c}\,(m_t, M_H)\, s^2_{M_Z} \ , \qquad (10.7)$$

where $c = 1.0371 \pm 0.0021$ and $\overline{c} = 1.0004 \mp 0.0007$. The quadratic m_t dependence is given by $c \sim 1 + \rho_t/\tan^2\theta_W$ and $\overline{c} \sim 1 - \rho_t/(1 - \tan^2\theta_W)$, respectively. The expressions for M_W and M_Z in the $\overline{\rm MS}$ scheme are

$$M_W = \frac{A_0}{\widehat{s}_Z(1 - \Delta\widehat{r}_W)^{1/2}} \ , \qquad (10.8a)$$

$$M_Z = \frac{M_W}{\widehat{\rho}^{1/2}\widehat{c}_Z} \ , \qquad (10.8b)$$

and one predicts $\Delta\widehat{r}_W = 0.0695 \pm 0.0001 \pm 0.0002$. $\Delta\widehat{r}_W$ has no quadratic m_t dependence, because shifts in M_W are absorbed into the observed G_F, so that the error in $\Delta\widehat{r}_W$ is dominated by $\Delta r_0 = 1 - \alpha/\widehat{\alpha}(M_Z)$, which induces the second quoted uncertainty. The quadratic m_t dependence has been shifted into $\widehat{\rho} \sim 1 + \rho_t$, where including bosonic loops, $\widehat{\rho} = 1.0107 \pm 0.0006$.

(*iv*) A variant $\overline{\rm MS}$ quantity $\widehat{s}^2_{\rm ND}$ (used in the 1992 edition of this *Review*) does not decouple the $\alpha\ln(m_t/M_Z)$ terms [30]. It is related to $\widehat{s}^2_Z$ by

$$\widehat{s}^2_Z = \widehat{s}^2_{\rm ND}/\left(1+\frac{\widehat{\alpha}}{\pi}d\right) , \qquad (10.9a)$$

$$d = \frac{1}{3}\left(\frac{1}{\widehat{s}^2}-\frac{8}{3}\right)\left[(1+\frac{\alpha_s}{\pi})\ln\frac{m_t}{M_Z}-\frac{15\alpha_s}{8\pi}\right], \qquad (10.9b)$$

Thus, $\widehat{s}^2_Z - \widehat{s}^2_{\rm ND} \sim -0.0002$ for $m_t = 174.3$ GeV.

(*v*) Yet another definition, the effective angle [31,32] $\overline{s}^2_f$ for Z coupling to fermion f, is described in Sec. 10.3.

Experiments are now at such a level of precision that complete $\mathcal{O}(\alpha)$ radiative corrections must be applied. For neutral-current and Z pole processes, these corrections are conveniently divided into two classes:

1. QED diagrams involving the emission of real photons or the exchange of virtual photons in loops, but not including vacuum polarization diagrams. These graphs often yield finite and gauge-invariant contributions to observable processes. However, they are dependent on energies, experimental cuts, *etc.*, and must be calculated individually for each experiment.
2. Electroweak corrections, including $\gamma\gamma$, γZ, ZZ, and WW vacuum polarization diagrams, as well as vertex corrections, box graphs, *etc.*, involving virtual W's and Z's. Many of these corrections are absorbed into the renormalized Fermi constant defined in Eq. (10.4). Others modify the tree-level expressions for Z pole observables and neutral-current amplitudes in several ways [25]. One-loop corrections are included for all processes. In addition, certain two-loop corrections are also important. In particular, two-loop corrections involving the top-quark modify ρ_t in $\widehat{\rho}$, Δr, and elsewhere by

$$\rho_t \to \rho_t[1+R(M_H,m_t)\rho_t/3] . \qquad (10.10)$$

$R(M_H,m_t)$ is best described as an expansion in M_Z^2/m_t^2. The unsuppressed terms were first obtained in Ref. 33, and are known analytically [34]. Contributions suppressed by M_Z^2/m_t^2 were studied in Ref. 35 with the help of small and large Higgs mass expansions, which can be interpolated. These contributions are about as large as the leading ones in Refs. 33 and 34. A subset of the relevant two-loop diagrams has also been calculated numerically without any heavy mass expansion [36]. This serves as a valuable check on the M_H dependence of the leading terms obtained in Refs. 33–35. The difference turned out to be small. For M_H above its lower direct limit, $-17 < R < -12$. Mixed QCD-electroweak loops of order $\alpha\alpha_s m_t^2$ [37] and $\alpha\alpha_s^2 m_t^2$ [38] increase the predicted value of m_t by 6%. This is, however, almost entirely an artifact of using the pole mass definition for m_t. The equivalent corrections when using the $\overline{\rm MS}$ definition $\widehat{m}_t(\widehat{m}_t)$ increase m_t by less than 0.5%. The leading electroweak [33,34] and mixed [39] two-loop terms are also known for the $Z\to b\bar{b}$ vertex, but not the respective subleading ones. $\mathcal{O}(\alpha\alpha_s)$-vertex corrections involving massless quarks have been obtained in Ref. [40]. Since they add coherently, the resulting effect is sizable, and shifts the extracted $\alpha_s(M_Z)$ by $\approx +0.0007$. Corrections of the same order to $Z\to b\bar{b}$ decays have also been completed [41].

Throughout this Review we utilize electroweak radiative corrections from the program GAPP, which works entirely in the $\overline{\rm MS}$ scheme, and which is independent of the package ZFITTER.

10.3. Cross-section and asymmetry formulas

It is convenient to write the four-fermion interactions relevant to ν-hadron, ν-e, and parity violating e-hadron neutral-current processes in a form that is valid in an arbitrary gauge theory (assuming massless left-handed neutrinos). One has

$$-\mathscr{L}^{\nu{\rm Hadron}} = \frac{G_F}{\sqrt{2}}\,\overline{\nu}\,\gamma^\mu\,(1-\gamma^5)\nu$$
$$\times\sum_i\left[\epsilon_L(i)\,\overline{q}_i\,\gamma_\mu(1-\gamma^5)q_i+\epsilon_R(i)\,\overline{q}_i\,\gamma_\mu(1+\gamma^5)q_i\right] , \qquad (10.11)$$

$$-\mathscr{L}^{\nu e} = \frac{G_F}{\sqrt{2}}\,\overline{\nu}_\mu\,\gamma^\mu(1-\gamma^5)\nu_\mu\,\overline{e}\,\gamma_\mu(g_V^{\nu e}-g_A^{\nu e}\gamma^5)e \qquad (10.12)$$

(for ν_e-e or $\overline{\nu}_e$-e, the charged-current contribution must be included), and

$$-\mathscr{L}^{e{\rm Hadron}} = -\frac{G_F}{\sqrt{2}}$$
$$\times\sum_i\left[C_{1i}\,\overline{e}\,\gamma_\mu\,\gamma^5\,e\,\overline{q}_i\,\gamma^\mu\,q_i+C_{2i}\,\overline{e}\,\gamma_\mu\,e\,\overline{q}_i\,\gamma^\mu\,\gamma^5\,q_i\right] . \qquad (10.13)$$

(One must add the parity-conserving QED contribution.)

The Standard Model expressions for $\epsilon_{L,R}(i)$, $g^{\nu e}_{V,A}$, and C_{ij} are given in Table 10.3. Note, that $g^{\nu e}_{V,A}$ and the other quantities are coefficients of effective four-fermi operators, which differ from the quantities defined in Eq. (10.2) in the radiative corrections and in the presence of possible physics beyond the Standard Model.

A precise determination of the on-shell s^2_W, which depends only very weakly on m_t and M_H, is obtained from deep inelastic neutrino scattering from (approximately) isoscalar targets [42]. The ratio $R_\nu \equiv \sigma^{NC}_{\nu N}/\sigma^{CC}_{\nu N}$ of neutral- to charged-current cross-sections has been measured to 1% accuracy by the CDHS [43] and CHARM [44] collaborations at CERN, and the CCFR [45] collaboration at Fermilab has obtained an even more precise result, so it is important to obtain theoretical expressions for R_ν and $R_{\overline{\nu}} \equiv \sigma^{NC}_{\overline{\nu}N}/\sigma^{CC}_{\overline{\nu}N}$ to comparable accuracy. Fortunately, most of the uncertainties from the strong interactions and neutrino spectra cancel in the ratio. The largest theoretical uncertainty is associated with the c-threshold, which mainly affects σ^{CC}. Using the slow rescaling prescription [46] the central value of $\sin^2\theta_W$ from CCFR varies as $0.0111(m_c\,[{\rm GeV}]-1.31)$, where m_c is the effective mass which is numerically close to the $\overline{\rm MS}$ mass $\widehat{m}_c(\widehat{m}_c)$, but their exact relation is unknown at higher orders. For $m_c = 1.31\pm0.24$ GeV (determined from ν-induced dimuon production [47]) this contributes ±0.003 to the total uncertainty $\Delta\sin^2\theta_W \sim \pm0.004$. (The experimental uncertainty is also ±0.003.) This uncertainty largely cancels, however, in the Paschos-Wolfenstein ratio [48],

$$R^- = \frac{\sigma^{NC}_{\nu N}-\sigma^{NC}_{\overline{\nu}N}}{\sigma^{CC}_{\nu N}-\sigma^{CC}_{\overline{\nu}N}} . \qquad (10.14)$$

It was measured recently by the NuTeV collaboration [49] for the first time, and required a high-intensity and high-energy anti-neutrino beam.

A simple zeroth-order approximation is

$$R_\nu = g_L^2 + g_R^2 r , \qquad (10.15a)$$

$$R_{\overline{\nu}} = g_L^2 + \frac{g_R^2}{r} , \qquad (10.15b)$$

$$R^- = g_L^2 - g_R^2 , \qquad (10.15c)$$

where

$$g_L^2 \equiv \epsilon_L(u)^2+\epsilon_L(d)^2 \approx \frac{1}{2}-\sin^2\theta_W+\frac{5}{9}\sin^4\theta_W , \qquad (10.16a)$$

$$g_R^2 \equiv \epsilon_R(u)^2+\epsilon_R(d)^2 \approx \frac{5}{9}\sin^4\theta_W , \qquad (10.16b)$$

and $r \equiv \sigma^{CC}_{\overline{\nu}N}/\sigma^{CC}_{\nu N}$ is the ratio of $\overline{\nu}$ and ν charged-current cross-sections, which can be measured directly. (In the simple parton model, ignoring hadron energy cuts, $r \approx (\frac{1}{3}+\epsilon)/(1+\frac{1}{3}\epsilon)$, where $\epsilon \sim 0.125$ is the ratio of the fraction of the nucleon's momentum carried by antiquarks to that carried by quarks.) In practice, Eq. (10.15) must be corrected for quark mixing, quark sea effects, c-quark threshold effects, nonisoscalarity, $W-Z$ propagator differences, the finite muon mass, QED and electroweak radiative corrections. Details of the neutrino spectra, experimental cuts, x and Q^2 dependence of structure functions, and longitudinal structure functions enter only at the level of these corrections and therefore lead to very small uncertainties. The CCFR group quotes $s^2_W = 0.2236\pm0.0041$ for $(m_t,M_H) = (175,150)$ GeV with very little sensitivity to (m_t,M_H). The NuTeV collaboration finds $s^2_W = 0.2253\pm0.0022$ using the

Table 10.3: Standard Model expressions for the neutral-current parameters for ν-hadron, ν-e, and e-hadron processes. At tree level, $\rho = \kappa = 1$, $\lambda = 0$. If radiative corrections are included, $\rho^{NC}_{\nu N} = 1.0083$, $\widehat{\kappa}_{\nu N}(\langle Q^2\rangle = -10 \text{ GeV}^2) = 0.9980$, $\widehat{\kappa}_{\nu N}(\langle Q^2\rangle = -35 \text{ GeV}^2) = 0.9965$, $\lambda_{uL} = -0.0031$, $\lambda_{dL} = -0.0025$, and $\lambda_{dR} = 2\,\lambda_{uR} = 7.5 \times 10^{-5}$. For ν-e scattering, $\rho_{\nu e} = 1.0129$ and $\widehat{\kappa}_{\nu e} = 0.9967$ (at $\langle Q^2\rangle = 0.$). For atomic parity violation and the SLAC polarized electron experiment, $\rho'_{eq} = 0.9878$, $\rho_{eq} = 1.0008$, $\widehat{\kappa}'_{eq} = 1.0026$, $\widehat{\kappa}_{eq} = 1.0300$, $\lambda_{1d} = -2\,\lambda_{1u} = 3.7 \times 10^{-5}$, $\lambda_{2u} = -0.0121$ and $\lambda_{2d} = 0.0026$. The dominant m_t dependence is given by $\rho \sim 1 + \rho_t$, while $\widehat{\kappa} \sim 1$ ($\overline{\text{MS}}$) or $\kappa \sim 1 + \rho_t/\tan^2\theta_W$ (on-shell).

Quantity	Standard Model Expression
$\epsilon_L(u)$	$\rho^{NC}_{\nu N}\left(\frac{1}{2} - \frac{2}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}^2_Z\right) + \lambda_{uL}$
$\epsilon_L(d)$	$\rho^{NC}_{\nu N}\left(-\frac{1}{2} + \frac{1}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}^2_Z\right) + \lambda_{dL}$
$\epsilon_R(u)$	$\rho^{NC}_{\nu N}\left(-\frac{2}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}^2_Z\right) + \lambda_{uR}$
$\epsilon_R(d)$	$\rho^{NC}_{\nu N}\left(\frac{1}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}^2_Z\right) + \lambda_{dR}$
$g^{\nu e}_V$	$\rho_{\nu e}\left(-\frac{1}{2} + 2\widehat{\kappa}_{\nu e}\,\widehat{s}^2_Z\right)$
$g^{\nu e}_A$	$\rho_{\nu e}\left(-\frac{1}{2}\right)$
C_{1u}	$\rho'_{eq}\left(-\frac{1}{2} + \frac{4}{3}\widehat{\kappa}'_{eq}\,\widehat{s}^2_Z\right) + \lambda_{1u}$
C_{1d}	$\rho'_{eq}\left(\frac{1}{2} - \frac{2}{3}\widehat{\kappa}'_{eq}\,\widehat{s}^2_Z\right) + \lambda_{1d}$
C_{2u}	$\rho_{eq}\left(-\frac{1}{2} + 2\widehat{\kappa}_{eq}\,\widehat{s}^2_Z\right) + \lambda_{2u}$
C_{2d}	$\rho_{eq}\left(\frac{1}{2} - 2\widehat{\kappa}_{eq}\,\widehat{s}^2_Z\right) + \lambda_{2d}$

same reference values. Combining all of the precise deep-inelastic measurements, one obtains $s^2_W = 0.2253 \pm 0.0021$.

The laboratory cross-section for $\nu_\mu e \to \nu_\mu e$ or $\overline{\nu}_\mu e \to \overline{\nu}_\mu e$ elastic scattering is

$$\frac{d\sigma_{\nu_\mu,\overline{\nu}_\mu}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi} \times\left[(g^{\nu e}_V \pm g^{\nu e}_A)^2 + (g^{\nu e}_V \mp g^{\nu e}_A)^2(1-y)^2 - (g^{\nu e 2}_V - g^{\nu e 2}_A)\frac{y\, m_e}{E_\nu}\right], \tag{10.17}$$

where the upper (lower) sign refers to $\nu_\mu(\overline{\nu}_\mu)$, and $y \equiv E_e/E_\nu$ (which runs from 0 to $(1 + m_e/2E_\nu)^{-1}$) is the ratio of the kinetic energy of the recoil electron to the incident ν or $\overline{\nu}$ energy. For $E_\nu \gg m_e$ this yields a total cross-section

$$\sigma = \frac{G_F^2\, m_e\, E_\nu}{2\pi}\left[(g^{\nu e}_V \pm g^{\nu e}_A)^2 + \frac{1}{3}(g^{\nu e}_V \mp g^{\nu e}_A)^2\right]. \tag{10.18}$$

The most accurate leptonic measurements [50–52] of $\sin^2\theta_W$ are from the ratio $R \equiv \sigma_{\nu_\mu e}/\sigma_{\overline{\nu}_\mu e}$ in which many of the systematic uncertainties cancel. Radiative corrections (other than m_t effects) are small compared to the precision of present experiments and have negligible effect on the extracted $\sin^2\theta_W$. The most precise experiment (CHARM II) [52] determined not only $\sin^2\theta_W$ but $g^{\nu e}_{V,A}$ as well. The cross-sections for ν_e-e and $\overline{\nu}_e$-e may be obtained from Eq. (10.17) by replacing $g^{\nu e}_{V,A}$ by $g^{\nu e}_{V,A} + 1$, where the 1 is due to the charged-current contribution.

The SLAC polarized-electron experiment [53] measured the parity-violating asymmetry

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \tag{10.19}$$

where $\sigma_{R,L}$ is the cross-section for the deep-inelastic scattering of a right- or left-handed electron: $e_{R,L}N \to e$X. In the quark parton model

$$\frac{A}{Q^2} = a_1 + a_2\,\frac{1-(1-y)^2}{1+(1-y)^2}, \tag{10.20}$$

where $Q^2 > 0$ is the momentum transfer and y is the fractional energy transfer from the electron to the hadrons. For the deuteron or other isoscalar targets, one has, neglecting the s-quark and antiquarks,

$$a_1 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{1u} - \frac{1}{2}C_{1d}\right) \approx \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(-\frac{3}{4} + \frac{5}{3}\sin^2\theta_W\right), \tag{10.21a}$$

$$a_2 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{2u} - \frac{1}{2}C_{2d}\right) \approx \frac{9G_F}{5\sqrt{2}\pi\alpha}\left(\sin^2\theta_W - \frac{1}{4}\right). \tag{10.21b}$$

There are now precise experiments measuring atomic parity violation [54] in cesium (at the 0.4% level) [55], thallium [56], lead [57], and bismuth [58]. The uncertainties associated with atomic wave functions are quite small for cesium [59], and have been reduced recently to about 0.4% [60]. In the past, the semi-empirical value of the tensor polarizability added another source of theoretical uncertainty [61]. The ratio of the off-diagonal hyperfine amplitude to the polarizability has now been measured directly by the Boulder group [60]. Combined with the precisely known hyperfine amplitude [62] one finds excellent agreement with the earlier results, reducing the overall theory uncertainty to only 0.5% (while slightly increasing the experimental error). The theoretical uncertainties are 3% for thallium [63] but larger for the other atoms. For heavy atoms one determines the "weak charge"

$$Q_W = -2\left[C_{1u}\,(2Z+N) + C_{1d}(Z+2N)\right] \approx Z(1 - 4\sin^2\theta_W) - N. \tag{10.22}$$

The recent Boulder experiment in cesium also observed the parity-violating weak corrections to the nuclear electromagnetic vertex (the anapole moment [64]).

In the future it should be possible to reduce the theoretical wave function uncertainties by taking the ratios of parity violation in different isotopes [54,65]. There would still be some residual uncertainties from differences in the neutron charge radii, however [66].

The forward-backward asymmetry for $e^+e^- \to \ell^+\ell^-$, $\ell = \mu$ or τ, is defined as

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \tag{10.23}$$

where $\sigma_F(\sigma_B)$ is the cross-section for ℓ^- to travel forward (backward) with respect to the e^- direction. A_{FB} and R, the total cross-section relative to pure QED, are given by

$$R = F_1, \tag{10.24}$$

$$A_{FB} = 3F_2/4F_1, \tag{10.25}$$

where

$$F_1 = 1 - 2\chi_0\, g^e_V\, g^\ell_V\, \cos\delta_R + \chi_0^2\left(g^{e2}_V + g^{e2}_A\right)\left(g^{\ell 2}_V + g^{\ell 2}_A\right), \tag{10.26a}$$

$$F_2 = -2\chi_0\, g^e_A\, g^\ell_A\, \cos\delta_R + 4\chi_0^2\, g^e_A\, g^\ell_A\, g^e_V\, g^\ell_V, \tag{10.26b}$$

$$\tan\delta_R = \frac{M_Z\Gamma_Z}{M_Z^2 - s}, \tag{10.27}$$

$$\chi_0 = \frac{G_F}{2\sqrt{2}\pi\alpha}\,\frac{sM_Z^2}{\left[(M_Z^2 - s)^2 + M_Z^2\Gamma_Z^2\right]^{1/2}}, \tag{10.28}$$

and $\sqrt{s}$ is the CM energy. Eq. (10.26) is valid at tree level. If the data is radiatively corrected for QED effects (as described above), then the remaining electroweak corrections can be incorporated [67,68] (in an approximation adequate for existing PEP, PETRA, and TRISTAN data, which are well below the Z pole) by replacing χ_0 by $\chi(s) \equiv (1 + \rho_t)\chi_0(s)\alpha/\alpha(s)$, where $\alpha(s)$ is the running QED coupling, and evaluating g_V in the $\overline{\text{MS}}$ scheme. Formulas for $e^+e^- \to$ hadrons may be found in Ref. 69.

At LEP and SLC, there are high-precision measurements of various Z pole observables [70–78]. These include the Z mass and total width, Γ_Z, and partial widths $\Gamma(f\overline{f})$ for $Z \to f\overline{f}$ where fermion $f = e$, μ, τ, hadrons, b, or c. It is convenient to use the variables M_Z, Γ_Z, $R_\ell \equiv \Gamma(\text{had})/\Gamma(\ell^+\ell^-)$, $\sigma_{\text{had}} \equiv 12\pi\Gamma(e^+e^-)\Gamma(\text{had})/M_Z^2\,\Gamma_Z^2$, $R_b \equiv \Gamma(b\overline{b})/\Gamma(\text{had})$, and $R_c \equiv \Gamma(c\overline{c})/\Gamma(\text{had})$, most of which are weakly correlated experimentally. ($\Gamma(\text{had})$ is the partial width into hadrons.) $\mathcal{O}(\alpha^3)$ QED corrections introduce a large anticorrelation (-28%) between Γ_Z and σ_{had}, while the anticorrelation between R_b and R_c (-14%) is smaller than previously. R_ℓ is insensitive to m_t except for the $Z \to b\overline{b}$ vertex and final state corrections and the implicit dependence through $\sin^2\theta_W$. Thus it is especially useful for constraining α_s. The width for invisible decays [24], $\Gamma(\text{inv}) = \Gamma_Z - 3\Gamma(\ell^+\ell^-) - \Gamma(\text{had}) = 498.8 \pm 1.5$ MeV, can be used to determine the number of neutrino flavors much lighter than $M_Z/2$, $N_\nu = \Gamma(\text{inv})/\Gamma^{\text{theory}}(\nu\overline{\nu}) = 2.983 \pm 0.009$ for $(m_t, M_H) = (174.3, 100)$ GeV.

There are also measurements of various Z pole asymmetries. These include the polarization or left-right asymmetry

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \; , \tag{10.29}$$

where $\sigma_L(\sigma_R)$ is the cross-section for a left-(right)-handed incident electron. A_{LR} has been measured precisely by the SLD collaboration at the SLC [71,72], and has the advantages of being extremely sensitive to $\sin^2\theta_W$ and that systematic uncertainties largely cancel. In addition, the SLD collaboration has extracted the final-state couplings A_b, A_c [24,73], A_s [74], A_τ, and A_μ [72,75] from left-right forward-backward asymmetries, using

$$A_{LR}^{FB}(f) = \frac{\sigma_{LF}^f - \sigma_{LB}^f - \sigma_{RF}^f + \sigma_{RB}^f}{\sigma_{LF}^f + \sigma_{LB}^f + \sigma_{RF}^f + \sigma_{RB}^f} = \frac{3}{4}A_f \; , \tag{10.30}$$

where, for example, σ_{LF} is the cross-section for a left-handed incident electron to produce a fermion f traveling in the forward hemisphere. Similarly, A_τ is measured at LEP [24,76] through the negative total τ polarization, $\mathcal{P}_\tau$, and A_e is extracted from the angular distribution of $\mathcal{P}_\tau$. An equation such as (10.30) assumes that initial state QED corrections, photon exchange, $\gamma - Z$ interference, the tiny electroweak boxes, and corrections for $\sqrt{s} \neq M_Z$ are removed from the data, leaving the pure electroweak asymmetries. This allows the use of effective tree-level expressions,

$$A_{LR} = A_e P_e \; , \tag{10.31}$$

$$A_{FB} = \frac{3}{4}A_f\,\frac{A_e + P_e}{1 + P_e A_e} \; , \tag{10.32}$$

where

$$A_f \equiv \frac{2\overline{g}_V^f\,\overline{g}_A^f}{\overline{g}_V^{f2} + \overline{g}_A^{f2}} \; , \tag{10.33}$$

and

$$\overline{g}_V^f = \sqrt{\rho_f}\,(t_{3L}^{(f)} - 2q_f\kappa_f\sin^2\theta_W) \; , \tag{10.33b}$$

$$\overline{g}_A^f = \sqrt{\rho_f}\,t_{3L}^{(f)} \; . \tag{10.33c}$$

P_e is the initial e^- polarization, so that the second equality in Eq. (10.30) is reproduced for $P_e = 1$, and the Z pole forward-backward asymmetries at LEP ($P_e = 0$) are given by $A_{FB}^{(0,f)} = \frac{3}{4}A_e A_f$ where $f = e$, μ, τ, b, c, s [77], and q, and where $A_{FB}^{(0,q)}$ refers to the hadronic charge asymmetry. Corrections for t-channel exchange and s/t-channel interference cause $A_{FB}^{(0,e)}$ to be strongly anticorrelated with R_e (-36%). The initial state coupling, A_e, is also determined through the left-right charge asymmetry [78] and in polarized Bhabba scattering at the SLC [72,75].

The electroweak-radiative corrections have been absorbed into corrections $\rho_f - 1$ and $\kappa_f - 1$, which depend on the fermion f and on the renormalization scheme. In the on-shell scheme, the quadratic m_t dependence is given by $\rho_f \sim 1 + \rho_t$, $\kappa_f \sim 1 + \rho_t/\tan^2\theta_W$, while in $\overline{\text{MS}}$, $\widehat{\rho}_f \sim \widehat{\kappa}_f \sim 1$, for $f \neq b$ ($\widehat{\rho}_b \sim 1 - \frac{4}{3}\rho_t$, $\widehat{\kappa}_b \sim 1 + \frac{2}{3}\rho_t$). In the $\overline{\text{MS}}$ scheme the normalization is changed according to $G_F M_Z^2/2\sqrt{2}\pi \to \widehat{\alpha}/4\widehat{s}_Z^2\widehat{c}_Z^2$. (If one continues to normalize amplitudes by $G_F M_Z^2/2\sqrt{2}\pi$, as in the 1996 edition of this *Review*, then $\widehat{\rho}_f$ contains an additional factor of $\widehat{\rho}$.) In practice, additional bosonic and fermionic loops, vertex corrections, leading higher order contributions, *etc.*, must be included. For example, in the $\overline{\text{MS}}$ scheme one has $\widehat{\rho}_\ell = 0.9979$, $\widehat{\kappa}_\ell = 1.0013$, $\widehat{\rho}_b = 0.9866$ and $\widehat{\kappa}_b = 1.0068$. It is convenient to define an effective angle $\overline{s}_f^2 \equiv \sin^2\overline{\theta}_{Wf} \equiv \widehat{\kappa}_f\widehat{s}_Z^2 = \kappa_f s_W^2$, in terms of which $\overline{g}_V^f$ and $\overline{g}_A^f$ are given by $\sqrt{\rho_f}$ times their tree-level formulae. Because $\overline{g}_V^\ell$ is very small, not only $A_{LR}^0 = A_e$, $A_{FB}^{(0,\ell)}$, and $\mathcal{P}_\tau$, but also $A_{FB}^{(0,b)}$, $A_{FB}^{(0,c)}$, $A_{FB}^{(0,s)}$, and the hadronic asymmetries are mainly sensitive to $\overline{s}_\ell^2$. One finds that $\widehat{\kappa}_f$ ($f \neq b$) is almost independent of (m_t, M_H), so that one can write

$$\overline{s}_\ell^2 \sim \widehat{s}_Z^{\,2} + 0.00029 \; . \tag{10.34}$$

Thus, the asymmetries determine values of $\overline{s}_\ell^2$ and $\widehat{s}_Z^{\,2}$ almost independent of m_t, while the κ's for the other schemes are m_t dependent.

The Z boson properties are extracted assuming the Standard Model expressions for the $\gamma - Z$ interference terms. These have also been tested experimentally by performing more general fits [79] to the LEP data obtained at CM energies of about 91, 130, and 172 GeV. Assuming family universality this approach introduces three additional parameters relative to the standard fit [76],

$$j_{\text{had}}^{\text{tot}} \sim g_V^\ell g_V^{\text{had}} = 0.14 \pm 0.14 \; , \tag{10.35a}$$

$$j_\ell^{\text{tot}} \sim g_V^\ell g_V^\ell = 0.004 \pm 0.012 \; , \tag{10.35b}$$

$$j_\ell^{\text{fb}} \sim g_A^\ell g_A^\ell = 0.780 \pm 0.013 \; , \tag{10.35c}$$

where the first two parameters describe the $\gamma - Z$ interference contribution to the total hadronic and leptonic cross-sections, and the third to the leptonic forward-backward asymmetries. The results in Eq. (10.35) are in good agreement with the Standard Model expectations [76], 0.22, 0.004, and 0.799, respectively. This is a valuable test of the Standard Model; but it should be cautioned that new physics is not expected to be described by this set of parameters, since (i) they do not account for extra interactions beyond the standard weak neutral current, and (ii) the photonic amplitude remains fixed to its Standard Model value.

As another test, strong constraints on anomalous triple gauge couplings were obtained at LEP 2 above the W^+W^- threshold and by DØ at the Tevatron. While there are a total of 14 independent couplings, one can use SU(2) × U(1) gauge invariance, discrete symmetries, and LEP 1 constraints to reduce the number of triple gauge couplings to three. Each coupling is extracted from the data by setting the other two to zero (the SM value). Including the run at CM energy of 189 GeV, LEP 2 quotes the results [24],

$$\Delta\kappa_\gamma = 0.038^{+0.079}_{-0.075} \; , \tag{10.36a}$$

$$\Delta g_1^Z = -0.010 \pm 0.033 \; , \tag{10.36b}$$

$$\lambda_\gamma = -0.037^{+0.035}_{-0.036} \; , \tag{10.36c}$$

in excellent agreement with Standard Model expectations. Eq. (10.36a) can be used to rule out Kaluza-Klein theories which predict $\Delta\kappa_\gamma = -3$ [80]. In addition, the first direct limits on anomalous quartic gauge couplings were obtained by OPAL [81] through measurements of the $W^+W^-\gamma$ cross-section and of acoplanar photon pair events.

The CLEO collaboration [82] reported a precise measurement of the flavor changing transition $b \to s\gamma$. The result for the branching fraction is

$$\mathcal{B}(b \to s\gamma) = (3.37 \pm 0.37 \pm 0.34 \pm 0.24^{+0.35}_{-0.16} \pm 0.38) \times 10^{-4} \; , \tag{10.37}$$

where the first three errors are the quoted statistical, systematical, and model uncertainties, respectively. The fourth uncertainty accounts for the extrapolation from the finite photon energy cutoff (2.1 GeV) to the full theoretical branching ratio [83], and the last one is our estimate of the theory uncertainty (excluding parametric errors such as from α_s). It is advantageous to normalize the result with respect to the semi-leptonic branching fraction [84,85], $\mathcal{B}(b \to ce\nu) = 0.1034 \pm 0.0046$, yielding

$$R = \frac{\mathcal{B}(b \to s\gamma)}{\mathcal{B}(b \to ce\nu)} = (3.26^{+0.75}_{-0.68}) \times 10^{-3} \; , \qquad (10.38)$$

and to use the variable $\ln R = -5.73 \pm 0.22$ in electroweak fits to assure an approximately Gaussian error [86]. This measurement is to be compared to the next-to-leading order calculations of Refs. 85,87.

The present world average of the muon anomalous magnetic moment is

$$a_\mu^{\rm exp} = \frac{g_\mu - 2}{2} = (116592300 \pm 840) \times 10^{-11} \; , \qquad (10.39)$$

while the estimated SM electroweak contribution [88], $a_\mu^{\rm EW} = (151 \pm 4) \times 10^{-11}$, is much smaller than the uncertainty. However, a new experiment at BNL is expected to reduce the experimental error to $\pm 40 \times 10^{-11}$ or better. The limiting factor will then be the uncertainty from the hadronic contribution [19], $a_\mu^{\rm had} = (6924 \pm 62) \times 10^{-11}$, which has recently been estimated with the help of τ decay data and finite-energy QCD sum rule techniques. This result constitutes a major improvement over previous ones which had more than twice the uncertainty [12]. It would be important to verify it, and reduce the error even further to meet the experimental precision. Additional hadronic uncertainties are induced by the light-by-light scattering contribution [89], $a_\mu^{\rm LBLS} = (-92 \pm 32) \times 10^{-11}$, and other subleading hadronic contributions [90], $a_\mu^{\rm had}\left[\left(\frac{\alpha}{\pi}\right)^3\right] = (-100 \pm 6) \times 10^{-11}$. The SM prediction is

$$a_\mu^{\rm theory} = (116591596 \pm 67) \times 10^{-11} \; . \qquad (10.40)$$

With the anticipated accuracy at BNL it will be possible to explore new physics (specifically supersymmetry in the large $\tan\beta$ region [91]) up to energies of 5 TeV and more. If greater precision is achieved, it will be important to properly correlate the theoretical error on $a_\mu^{\rm had}$ with the one in $\Delta\alpha_{\rm had}^{(5)}$.

10.4. *W* and *Z* decays

The partial decay width for gauge bosons to decay into massless fermions $f_1\overline{f}_2$ is

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226.5 \pm 0.3 \text{ MeV} \; , \qquad (10.41a)$$

$$\Gamma(W^+ \to u_i \overline{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi} \, |V_{ij}|^2 \approx (707 \pm 1) \, |V_{ij}|^2 \text{ MeV} \; , \qquad (10.41b)$$

$$\Gamma(Z \to \psi_i \overline{\psi}_i) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi} \left[g_V^{i2} + g_A^{i2} \right] \qquad (10.41c)$$

$$\approx \begin{cases} 300.3 \pm 0.2 \text{ MeV } (u\overline{u}), & 167.24 \pm 0.08 \text{ MeV } (\nu\overline{\nu}), \\ 383.1 \pm 0.2 \text{ MeV } (d\overline{d}), & 84.01 \pm 0.05 \text{ MeV } (e^+e^-), \\ 375.9 \mp 0.1 \text{ MeV } (b\overline{b}). & \end{cases}$$

For leptons $C = 1$, while for quarks $C = 3\Big(1 + \alpha_s(M_V)/\pi + 1.409\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3\Big)$, where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections [92] for massless quarks [93]. The $Z \to f\overline{f}$ widths contain a number of additional corrections: universal (non-singlet) top-mass contributions [94]; fermion mass effects and further QCD corrections proportional to $\widehat{m}_q^2(M_Z^2)$ [95] which are different for vector and axial-vector partial widths; and singlet contributions starting from two-loop order which are large, strongly top-mass dependent, family universal, and flavor non-universal [96]. All QCD effects are known and included up to three loop order. The QED factor $1 + 3\alpha q_f^2/4\pi$, as well as two-loop $\alpha\alpha_s$ and α^2 self-energy corrections [97] are also included. Working in the on-shell scheme, *i.e.*, expressing the widths in terms of $G_F M_{W,Z}^3$, incorporates the largest radiative corrections from the running QED coupling [27,98]. Electroweak corrections to the Z widths are then incorporated by replacing $g_{V,A}^{i2}$ by $\overline{g}_{V,A}^{i2}$. Hence, in the on-shell scheme the Z widths are proportional to $\rho_i \sim 1 + \rho_t$. The $\overline{\rm MS}$ normalization accounts also for the leading electroweak corrections [31]. There is additional (negative) quadratic m_t dependence in the $Z \to b\overline{b}$ vertex corrections [99] which causes $\Gamma(b\overline{b})$ to decrease with m_t. The dominant effect is to multiply $\Gamma(b\overline{b})$ by the vertex correction $1 + \delta\rho_{b\overline{b}}$, where $\delta\rho_{b\overline{b}} \sim 10^{-2}(-\frac{1}{2}\frac{m_t^2}{M_Z^2} + \frac{1}{5})$. In practice, the corrections are included in ρ_b and κ_b, as discussed before.

For 3 fermion families the total widths are predicted to be

$$\Gamma_Z \approx 2.4963 \pm 0.0012 \text{ GeV} \; , \qquad (10.42)$$

$$\Gamma_W \approx 2.0927 \pm 0.0025 \text{ GeV} \; . \qquad (10.43)$$

We have assumed $\alpha_s(M_Z) = 0.1200$. An uncertainty in α_s of ± 0.0028 introduces an additional uncertainty of 0.1% in the hadronic widths, corresponding to ± 1.4 MeV in Γ_Z. These predictions are to be compared with the experimental results $\Gamma_Z = 2.4944 \pm 0.0024$ GeV [24] and $\Gamma_W = 2.06 \pm 0.05$ GeV [100].

10.5. Experimental results

The values of the principal Z pole observables are listed in Table 10.4, along with the Standard Model predictions for $M_Z = 91.1870 \pm 0.0021$ GeV, $M_H = 98^{+57}_{-38}$ GeV, $m_t = 172.9 \pm 4.6$ GeV, $\alpha_s(M_Z) = 0.1192 \pm 0.0028$, and $\widehat{\alpha}(M_Z)^{-1} = 127.938 \pm 0.027$ ($\Delta\alpha_{\rm had}^{(5)} \approx 0.02776 \pm 0.00020$). Note, that the values of the Z pole observables (as well as M_W) differ from those in the Particle Listings because they include recent preliminary results [24,72]. The values and predictions of M_W [24,101]; the Q_W for cesium [55,60] and thallium [56]; deep inelastic [43–45,49] and ν_μ-e scattering [50–52]; and the $b \to s\gamma$ observable [82] are also listed. The agreement is very good. Even the largest discrepancies, $A_{FB}^{(0,b)}$ and Q_W(Cs), deviate by only 2.3 σ. The hadronic peak cross-section, $\sigma_{\rm had}$, the A_{LR}^0 from hadronic final states, and the R^ν result by the CHARM collaboration deviate by 1.7 σ; all the other observables agree with the Standard Model prediction at the 1.5 σ level or better. Other observables like $R_b = \Gamma(b\overline{b})/\Gamma(\text{had})$ and $R_c = \Gamma(c\overline{c})/\Gamma(\text{had})$ which showed significant deviations in the past, are now in reasonable agreement. In particular, R_b whose measured value deviated as much as 3.7 σ from the Standard Model prediction is now only 0.9 σ (0.3%) high.

A_b can be extracted from $A_{FB}^{(0,b)}$ when $A_e = 0.1497 \pm 0.0016$ is taken from a fit to leptonic asymmetries (using lepton universality), and combined with the measurement at the SLC. The result, $A_b = 0.892 \pm 0.016$, is 2.7 σ below the Standard Model prediction.† However, it would be extremely difficult to account for this nearly 5% deviation by new physics radiative corrections since a 25% correction to $\widehat{\kappa}_b$ would be necessary to account for the central value of A_b. If this deviation is due to new physics, it is most likely of tree-level type affecting preferentially the third generation. It seems difficult, however, to simultaneously account for R_b, which has been measured on the Z peak, off-peak [103], and recently at LEP 2 [24]). $A_{FB}^{(b)} = 0.44 \pm 0.12$ has also been measured at LEP 2 [24], and found to be 1.2 σ below the Standard Model prediction (0.58).

The left-right asymmetry, $A_{LR}^0 = 0.15108 \pm 0.00218$ [72], based on all hadronic data from 1992–1998 has moved closer to the Standard Model expectation of 0.1475 ± 0.0013 than previous values. The combined value of $A_\ell = 0.1512 \pm 0.0020$ from SLD (using lepton-family universality) is still 1.8 σ above the Standard Model prediction; but there is now only a minor experimental difference of ~ 1.2 σ between this SLD value and the LEP value, $A_\ell = 0.1471 \pm 0.0026$, obtained from a fit to $A_{FB}^{(0,\ell)}$, $A_e(\mathcal{P}_\tau)$, and $A_\tau(\mathcal{P}_\tau)$, again assuming universality.

† Alternatively, one can use $A_\ell = 0.1471 \pm 0.0026$, which is from LEP alone and in excellent agreement with the Standard Model, and obtain $A_b = 0.904 \pm 0.018$ which is 1.7 σ low. This illustrates that some of the discrepancy is related to the one in A_{LR}.

Table 10.4: Principal Z-pole and other recent observables, compared with the Standard Model predictions for the global best fit values $M_Z = 91.1870 \pm 0.0021$ GeV, $M_H = 98^{+57}_{-38}$ GeV, $m_t = 172.9 \pm 4.6$ GeV, $\alpha_s(M_Z) = 0.1192 \pm 0.0028$, and $\widehat{\alpha}(M_Z)^{-1} = 127.938 \pm 0.027$. The LEP averages of the ALEPH, DELPHI, L3, and OPAL results include common systematic errors and correlations [24,76]. The heavy flavour results of LEP and SLD are based on common inputs and correlated, as well [73]. $\overline{s}^2_\ell(A^{(0,q)}_{FB})$ is the effective angle extracted from the hadronic charge asymmetry. The values of $\Gamma(\ell^+\ell^-)$, $\Gamma(\text{had})$, and $\Gamma(\text{inv})$ are not independent of Γ_Z, the R_ℓ, and σ_{had}. The first M_W value is from CDF, UA2, and DØ [101] while the second one is from LEP 2 [24]. The first M_W and M_Z are correlated, but the effect is negligible due to the tiny M_Z error. The three values of A_e are (i) from A_{LR} for hadronic final states [71]; (ii) from A_{LR} for leptonic final states and from polarized Bhabba scattering [75]; and (iii) from the angular distribution of the τ polarization. The two A_τ values are from SLD and the total τ polarization, respectively. The two values of R^ν from deep-inelastic scattering (DIS) are from CDHS [43] and CHARM [44], respectively; similarly, κ^ν (proportional to R^ν) is from CCFR [45]. The two values for $g^{\nu e}_{V,A}$ are from CHARM II [52] and the world average. The second errors in Q_W and DIS are theoretical. In the Standard Model predictions, the uncertainty is from M_Z, M_H, m_t, $\widehat{\alpha}(M_Z)^{-1}$, and α_s, and their correlations have been accounted for. The errors in Γ_Z, $\Gamma(\text{had})$, R_ℓ, and σ_{had} are largely dominated by the uncertainty in α_s.

Quantity	Value	Standard Model	Pull
m_t [GeV]	174.3 ± 5.1	172.9 ± 4.6	0.3
M_W [GeV]	80.448 ± 0.062	80.378 ± 0.020	1.1
	80.350 ± 0.056		−0.5
M_Z [GeV]	91.1872 ± 0.0021	91.1870 ± 0.0021	0.1
Γ_Z [GeV]	2.4944 ± 0.0024	2.4956 ± 0.0016	−0.5
$\Gamma(\text{had})$ [GeV]	1.7439 ± 0.0020	1.7422 ± 0.0015	—
$\Gamma(\text{inv})$ [MeV]	498.8 ± 1.5	501.65 ± 0.15	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.96 ± 0.09	84.00 ± 0.03	—
σ_{had} [nb]	41.544 ± 0.037	41.480 ± 0.014	1.7
R_e	20.803 ± 0.049	20.740 ± 0.018	1.3
R_μ	20.786 ± 0.033	20.741 ± 0.018	1.4
R_τ	20.764 ± 0.045	20.786 ± 0.018	−0.5
R_b	0.21642 ± 0.00073	0.2158 ± 0.0002	0.9
R_c	0.1674 ± 0.0038	0.1723 ± 0.0001	−1.3
$A^{(0,e)}_{FB}$	0.0145 ± 0.0024	0.0163 ± 0.0003	−0.8
$A^{(0,\mu)}_{FB}$	0.0167 ± 0.0013		0.3
$A^{(0,\tau)}_{FB}$	0.0188 ± 0.0017		1.5
$A^{(0,b)}_{FB}$	0.0988 ± 0.0020	0.1034 ± 0.0009	−2.3
$A^{(0,c)}_{FB}$	0.0692 ± 0.0037	0.0739 ± 0.0007	−1.3
$A^{(0,s)}_{FB}$	0.0976 ± 0.0114	0.1035 ± 0.0009	−0.5
$\overline{s}^2_\ell(A^{(0,q)}_{FB})$	0.2321 ± 0.0010	0.2315 ± 0.0002	0.6

Table 10.4: (continued)

Quantity	Value	Standard Model	Pull
A_e	0.15108 ± 0.00218	0.1475 ± 0.0013	1.7
	0.1558 ± 0.0064		1.3
	0.1483 ± 0.0051		0.2
A_μ	0.137 ± 0.016		−0.7
A_τ	0.142 ± 0.016		−0.3
	0.1425 ± 0.0044		−1.1
A_b	0.911 ± 0.025	0.9348 ± 0.0001	−1.0
A_c	0.630 ± 0.026	0.6679 ± 0.0006	−1.5
A_s	0.85 ± 0.09	0.9357 ± 0.0001	−1.0
R^-	$0.2277 \pm 0.0021 \pm 0.0007$	0.2299 ± 0.0002	−1.0
κ^ν	$0.5820 \pm 0.0027 \pm 0.0031$	0.5831 ± 0.0004	−0.3
R^ν	$0.3096 \pm 0.0033 \pm 0.0028$	0.3091 ± 0.0002	0.1
	$0.3021 \pm 0.0031 \pm 0.0026$		−1.7
$g^{\nu e}_V$	-0.035 ± 0.017	-0.0397 ± 0.0003	—
	-0.041 ± 0.015		−0.1
$g^{\nu e}_A$	-0.503 ± 0.017	-0.5064 ± 0.0001	—
	-0.507 ± 0.014		0.0
$Q_W(\text{Cs})$	$-72.06 \pm 0.28 \pm 0.34$	-73.09 ± 0.03	2.3
$Q_W(\text{Tl})$	$-114.8 \pm 1.2 \pm 3.4$	-116.7 ± 0.1	0.5
$\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\nu)}$	$3.26^{+0.75}_{-0.68} \times 10^{-3}$	$3.15^{+0.21}_{-0.20} \times 10^{-3}$	0.1

Despite these discrepancies the goodness of the fit to all data is reasonable with a $\chi^2/\text{d.o.f.} = 42/37$. The probability of a larger χ^2 is 27%. The observables in Table 10.4, as well as some other less precise observables, are used in the global fits described below. The correlations on the LEP lineshape, the LEP/SLD heavy flavor, and the deep inelastic scattering observables, are included. There are also small correlations between some of the SLD measurements, and between the two observables from the τ polarization at LEP, which have not been fully investigated, yet.

The data allow a simultaneous determination of M_H, m_t, $\sin^2\theta_W$, and the strong coupling $\alpha_s(M_Z)$. ($\Delta\alpha^{(5)}_{\text{had}}$ is also allowed to float in the fits, subject to the theoretical constraints [9] described in Sec. 10.2.) α_s is determined mainly from R_ℓ, Γ_Z, and σ_{had}, and is only weakly correlated with the other variables. The global fit to all data, including the CDF/DØ value, $m_t = 174.3 \pm 5.1$ GeV, yields

$$\begin{aligned} M_H &= 98^{+57}_{-38} \text{ GeV} , \\ m_t &= 172.9 \pm 4.6 \text{ GeV} , \\ \widehat{s}^2_Z &= 0.23117 \pm 0.00016 , \\ \alpha_s(M_Z) &= 0.1192 \pm 0.0028 . \end{aligned} \tag{10.44}$$

In the on-shell scheme one has $s^2_W = 0.22302 \pm 0.00040$, the larger error due to the stronger sensitivity to m_t, while the corresponding

Table 10.5: Values of $\widehat{s}^2_Z$, s^2_W, α_s, and M_H [in GeV] for various (combinations of) observables. Unless indicated otherwise, the top quark mass, $m_t = 174.3 \pm 5.1$ GeV, is used as an additional constraint in the fits. The (†) symbol indicates a fixed parameter.

Data	$\widehat{s}^2_Z$	s^2_W	$\alpha_s(M_Z)$	M_H
All data	0.23117(16)	0.2230(4)	0.1192(28)	98^{+57}_{-38}
All data (incl. α_s)	0.23116(16)	0.2230(4)	0.1184(12)	97^{+56}_{-37}
All indirect (no m_t)	0.23114(17)	0.2232(5)	0.1190(28)	69^{+80}_{-33}
Z pole (no m_t)	0.23120(18)	0.2233(6)	0.1191(28)	77^{+102}_{-38}
LEP 1 (no m_t)	0.23156(23)	0.2240(7)	0.1208(30)	166^{+270}_{-95}
SLD + M_Z	0.23070(28)	0.2220(6)	0.1200 (†)	40^{+38}_{-22}
$A^{(b,c)}_{FB}$ + M_Z	0.23204(34)	0.2251(9)	0.1200 (†)	516^{+521}_{-258}
M_W + M_Z	0.23107(42)	0.2227(9)	0.1200 (†)	85^{+112}_{-60}
M_Z	0.23115(18)	0.2229(6)	0.1200 (†)	100 (†)
Q_W	0.2269(18)	0.2186(19)	0.1200 (†)	100 (†)
DIS (isoscalar)	0.2335(22)	0.2252(21)	0.1200 (†)	100 (†)
SLAC eD	0.222(18)	0.213(19)	0.1200 (†)	100 (†)
elastic $\nu_\mu(\overline{\nu_\mu})e$	0.229(8)	0.221(8)	0.1200 (†)	100 (†)
elastic $\nu_\mu(\overline{\nu_\mu})p$	0.211(32)	0.203(32)	0.1200 (†)	100 (†)

effective angle is related by Eq. (10.34), *i.e.*, $\overline{s}_\ell^2 = 0.23147 \pm 0.00016$. In all fits, the errors include full statistical, systematic, and theoretical uncertainties. The $\widehat{s}_Z^2$ ($\overline{s}_\ell^2$) error reflects the error on $\overline{s}_f^2 = 0.23151 \pm 0.00017$ from a fit to the Z pole asymmetries.

The weak mixing angle can be determined from Z pole observables, M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results (for the older low-energy neutral-current data see [25,26]) shown in Table 10.5, are in reasonable agreement with each other, indicating the quantitative success of the Standard Model. The largest discrepancy is the value $\widehat{s}_Z^2 = 0.23204 \pm 0.00034$ from the forward-backward asymmetries into bottom and charm quarks combined with M_Z, which is 2.6 σ above the value 0.23117 ± 0.00016 from the global fit to all data. Similarly, the SLD asymmetries, when combined with M_Z, yield $\widehat{s}_Z^2 = 0.23070 \pm 0.00028$, which is 1.7 σ low. The new value of Q_W from atomic parity violation corresponds (for $M_H = 100$ GeV) to $\widehat{s}_Z^2 = 0.2269 \pm 0.0018$, which is 2.4 σ low.

The extracted value of $\alpha_s(M_Z)$ is based on a formula with negligible theoretical uncertainty (± 0.0005 in $\alpha_s(M_Z)$) if one assumes the exact validity of the Standard Model. It is in excellent agreement with other precise values, such as 0.1202 ± 0.0027 (ALEPH) and 0.1219 ± 0.0020 (OPAL) from τ decays [104], 0.120 ± 0.005 from jet-event shapes in e^+e^- annihilation, 0.119 ± 0.002 (exp) ± 0.004 (scale) from deep-inelastic scattering [105], and 0.1174 ± 0.0024 ($b\overline{b}$) [106] and 0.116 ± 0.003 ($c\overline{c}$) [107] from lattice calculations of quarkonium spectra. The results from the τ lifetime have been converted from the 3-flavor definition, $\alpha_s^{(3)}(m_\tau) = 0.334 \pm 0.022$ (ALEPH) and $\alpha_s^{(3)}(m_\tau) = 0.348 \pm 0.021$ (OPAL), to the 5-flavor definition at the Z scale using the four-loop QCD β-function [108] with three-loop matching [109]. We note, that this introduces an asymmetric error (the lower error bar being larger), and that the quoted OPAL error for $\alpha_s^{(5)}(M_Z)$ is slightly underestimated given their result for $\alpha_s^{(3)}(m_\tau)$. For more details, see our Section 9 on "Quantum Chromodynamics" in this *Review*. The average $\alpha_s(M_Z)$ obtained from Section 9 when ignoring the precision measurements discussed in this Section is 0.1182 ± 0.0013. We use this value as an external constraint for the second fit in Table 10.5. The resulting value, $\alpha_s(M_Z) = 0.1184 \pm 0.0012$, can be regarded as the present world average. One should keep in mind, however, that the Z lineshape value of α_s is very sensitive to many types of new physics.

The data indicate a preference for a small Higgs mass. There is a strong correlation between the quadratic m_t and logarithmic M_H terms in $\widehat{\rho}$ in all of the indirect data except for the $Z \to b\overline{b}$ vertex. Therefore, observables (other than R_b) which favor m_t values higher than the Tevatron range favor lower values of M_H. This effect is enhanced by R_b, which has little direct M_H dependence but favors the lower end of the Tevatron m_t range. M_W has additional M_H dependence through $\Delta\widehat{r}_W$ which is not coupled to m_t^2 effects. The strongest individual pulls towards smaller M_H are from M_W and A_{LR}^0. The difference in χ^2 for the global fit is $\Delta\chi^2 = \chi^2(M_H = 1000 \text{ GeV}) - \chi^2_{\text{min}} = 30.4$. Hence, the data favor a small value of M_H, as in supersymmetric extensions of the Standard Model, and m_t on the lower side of the Tevatron range. The central value of the global fit result, $M_H = 98^{+57}_{-38}$ GeV, is close to the present kinematic reach at LEP 2, and slightly above the direct lower bound, $M_H \geq 95.2$ GeV (95% CL) [110].

The 90% central confidence range from all precision data is

$$42 \text{ GeV} \leq M_H \leq 201 \text{ GeV} .$$

Including the results of the direct searches as an extra contribution to the likelihood function drives the 95% upper limit to $M_H \leq 231$ GeV. As two further refinements, we account for (i) theoretical uncertainties from uncalculated higher order contributions by allowing the T parameter (see next subsection) subject to the constraint $T = 0 \pm 0.02$, (ii) the M_H dependence of the correlation matrix which gives slightly more weight to lower Higgs masses [112]. The resulting limits at 95 (90, 99)% CL are

$$M_H \leq 235 \text{ (205, 306) GeV} ,$$

respectively. The extraction of M_H from the precision data depends strongly on the value used for $\alpha(M_Z)$. Upper limits, however, are more robust due to two compensating effects: the older results indicated more QED running and were less precise, yielding M_H distributions which were broader with centers shifted to smaller values.

One can also carry out a fit to the indirect data alone, *i.e.*, without including the value, $m_t = 174.3 \pm 5.1$ GeV, observed directly by CDF and DØ. (The indirect prediction is for the $\overline{\text{MS}}$ mass, $\widehat{m}_t(\widehat{m}_t) = 158.7^{+9.1}_{-7.0}$ GeV, which is in the end converted to the pole mass using a BLM optimized [113] version of the two-loop perturbative QCD formula [114]; this should correspond approximately to the kinematic mass extracted from the collider events.) One obtains $m_t = 168.2^{+9.6}_{-7.4}$ GeV, with little change in the $\sin^2\theta_W$ and α_s values, in remarkable agreement with the direct CDF/DØ value. The central M_H value of this fit (see the third line of Table 10.5) is below the direct lower bound; keeping $M_H = 100$ GeV fixed results in $m_t = 172.2 \pm 4.0$ GeV in even better agreement. The relations between M_H and m_t for various observables are shown in Fig. 10.1.

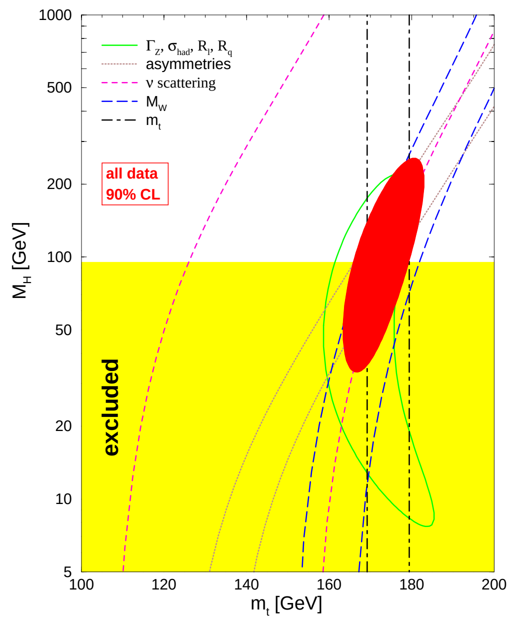

Figure 10.1: One-standard-deviation (39.35%) uncertainties in M_H as a function of m_t for various inputs, and the 90% CL region ($\Delta\chi^2 = 4.605$) allowed by all data. $\alpha_s(M_Z) = 0.120$ is assumed except for the fit to all data. The 95% direct lower limit from LEP 2 is also shown.

Using $\alpha(M_Z)$ and $\widehat{s}_Z^2$ as inputs, one can predict $\alpha_s(M_Z)$ assuming grand unification. One predicts [115] $\alpha_s(M_Z) = 0.130 \pm 0.001 \pm 0.01$ for the simplest theories based on the minimal supersymmetric extension of the Standard Model, where the first (second) uncertainty is from the inputs (thresholds). This is slightly larger, but consistent with the experimental $\alpha_s(M_Z) = 0.1192 \pm 0.0028$ from the Z lineshape, and with the world average 0.1184 ± 0.0012. Nonsupersymmetric unified theories predict the low value $\alpha_s(M_Z) = 0.073 \pm 0.001 \pm 0.001$. See also the note on "Low-Energy Supersymmetry" in the Particle Listings.

One can also determine the radiative correction parameters Δr: from the global fit one obtains $\Delta r = 0.0354 \pm 0.0012$ and $\Delta\widehat{r}_W = 0.0694 \pm 0.0004$. M_W measurements [24,101] (when combined with M_Z) are equivalent to measurements of $\Delta r = 0.0345 \pm 0.0025$, in excellent agreement with the result from all indirect data, $\Delta r = 0.0357 \pm 0.0014$. Fig. 10.2 shows the 1 σ contours in the $M_W - m_t$ plane from the direct and indirect determinations, as well as the combined 90% CL region. The indirect determination uses M_Z from LEP 1 as input, which is defined assuming an s-dependent decay width. M_W then corresponds to the s-dependent width definition, as well, and can be directly compared with the results from the Tevatron and LEP 2 which have been obtained using the same definition. The difference to a constant width definition is formally only of $\mathcal{O}(\alpha^2)$, but is strongly enhanced since the decay channels add up coherently. It is about 34 MeV for M_Z and 27 MeV for M_W. The residual difference between working consistently with one or the other definition is about 3 MeV, *i.e.*, of typical size for non-enhanced (and generally uncalculated) $\mathcal{O}(\alpha^2)$ corrections.

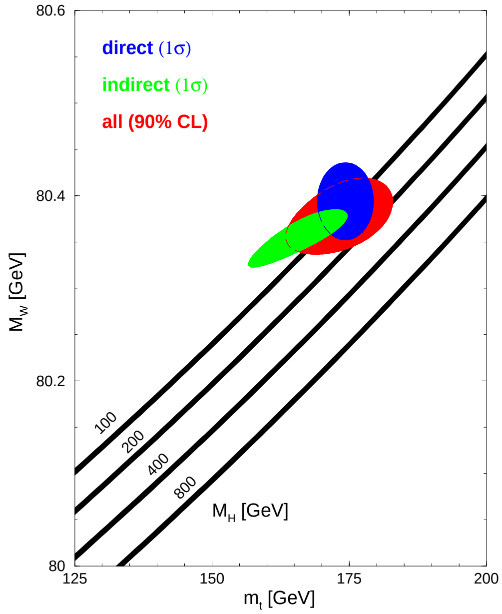

Figure 10.2: One-standard-deviation (39.35%) region in M_W as a function of m_t for the direct and indirect data, and the 90% CL region ($\Delta\chi^2 = 4.605$) allowed by all data. The Standard Model prediction as a function of M_H is also indicated. The widths of the M_H bands reflect the theoretical uncertainty from $\alpha(M_Z)$ for $\alpha_s(M_Z) = 0.120$.

Most of the parameters relevant to ν-hadron, ν-e, e-hadron, and e^+e^- processes are determined uniquely and precisely from the data in "model independent" fits (*i.e.*, fits which allow for an arbitrary electroweak gauge theory). The values for the parameters defined in Eqs. (10.11)–(10.13) are given in Table 10.6 along with the predictions of the Standard Model. The agreement is reasonable. The low-energy e^+e^- results are difficult to present in a model-independent way because Z propagator effects are non-negligible at TRISTAN, PETRA, and PEP energies. However, assuming e-μ-τ universality, the lepton asymmetries imply [69] $4(g_A^e)^2 = 0.99 \pm 0.05$, in good agreement with the Standard Model prediction $\simeq 1$.

The results presented here are generally in reasonable agreement with the ones obtained by the LEP Electroweak Working Group [24,76]. We obtain slightly higher best fit values for α_s and M_H. We could trace most of the differences to be due to (i) the inclusion of recent higher order radiative corrections, in particular, the leading $\mathcal{O}(\alpha_s^4)$ contribution to hadronic Z decays [111]; (ii) a different evaluation of $\alpha(M_Z)$ [9]; (iii) slightly different data sets; and (iv) scheme dependences. Taking into account these differences, the agreement is excellent.

10.6. Constraints on new physics

The Z pole, W mass, and neutral-current data can be used to search for and set limits on deviations from the Standard Model. In particular, the combination of these indirect data with the direct CDF and DØ value for m_t allows to set stringent limits on new physics. We will mainly discuss the effects of exotic particles (with heavy masses $M_{\rm new} \gg M_Z$ in an expansion in $M_Z/M_{\rm new}$) on the gauge boson self-energies. (Brief remarks are made on new physics which is not of this type.) Most of the effects on precision measurements can be described by three gauge self-energy parameters S, T, and U. We will define these, as well as related parameters, such as ρ_0, ϵ_i, and $\widehat{\epsilon}_i$, to arise from new physics only. *I.e.*, they are equal to zero ($\rho_0 = 1$) exactly in the Standard Model, and do not include any contributions

Table 10.6: Values of the model-independent neutral-current parameters, compared with the Standard Model predictions for the global best fit values $M_Z = 91.1870 \pm 0.0021$ GeV, $M_H = 98^{+57}_{-38}$ GeV, $m_t = 172.9 \pm 4.6$ GeV, $\alpha_s(M_Z) = 0.1192 \pm 0.0028$, and $\widehat{\alpha}(M_Z)^{-1} = 127.938 \pm 0.027$. There is a second $g_{V,A}^{\nu e}$ solution, given approximately by $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$, which is eliminated by e^+e^- data under the assumption that the neutral current is dominated by the exchange of a single Z. The ϵ_L, as well as the ϵ_R, are strongly correlated and non-Gaussian, so that for implementations we recommend the parametrization using g_i and $\theta_i = \tan^{-1}[\epsilon_i(u)/\epsilon_i(d)]$, $i = L$ or R. θ_R is only weakly correlated with the g_i, while the correlation coefficient between θ_R and θ_L is 0.27.

Quantity	Experimental Value	Standard Model Prediction	Correlation	
$\epsilon_L(u)$	0.330 ± 0.016	0.3459 ± 0.0002		
$\epsilon_L(d)$	-0.439 ± 0.011	-0.4291 ± 0.0002		non-
$\epsilon_R(u)$	$-0.176\ ^{+0.014}_{-0.006}$	-0.1550 ± 0.0001		Gaussian
$\epsilon_R(d)$	$-0.023\ ^{+0.070}_{-0.047}$	0.0776		
g_L^2	0.3020 ± 0.0019	0.3038 ± 0.0003	0.32	−0.39
g_R^2	0.0315 ± 0.0016	0.0301		−0.10
θ_L	2.50 ± 0.034	2.4631 ± 0.0001		
θ_R	$4.58\ ^{+0.40}_{-0.27}$	5.1765		
$g_V^{\nu e}$	-0.041 ± 0.015	-0.0397 ± 0.0003		−0.04
$g_A^{\nu e}$	-0.507 ± 0.014	-0.5064 ± 0.0001		
C_{1u}	-0.211 ± 0.041	-0.1886 ± 0.0002	−0.9996	−0.78
C_{1d}	0.359 ± 0.037	0.3413 ± 0.0002		0.78
$C_{2u} - \frac{1}{2}C_{2d}$	-0.04 ± 0.12	-0.0491 ± 0.0005		

from m_t or M_H, which are treated separately. Our treatment differs from most of the original papers.

Many extensions of the Standard Model are described by the ρ_0 parameter,

$$\rho_0 \equiv M_W^2/(M_Z^2\, \widehat{c}_Z^{\,2}\, \widehat{\rho}) \ , \tag{10.45}$$

which describes new sources of SU(2) breaking that cannot be accounted for by the Standard Model Higgs doublet or m_t effects. In the presence of $\rho_0 \neq 1$, Eq. (10.45) generalizes Eq. (10.8*b*), while Eq. (10.8*a*) remains unchanged. Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect the radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (10.11)–(10.13), (10.28), and Γ_Z in Eq. (10.41). There is enough data to determine ρ_0, M_H, m_t, and α_s, simultaneously. From the global fit,

$$\rho_0 = 0.9998^{+0.0011}_{-0.0006} \ , \tag{10.46}$$

$$95 \text{ GeV} < M_H < 211 \text{ GeV} \ , \tag{10.47}$$

$$m_t = 173.6 \pm 4.9 \ \text{GeV} \ , \tag{10.48}$$

$$\alpha_s(M_Z) = 0.1194 \pm 0.0028 \ , \tag{10.49}$$

where the lower limit on M_H is the direct search bound. (If the direct limit is ignored one obtains $M_H = 72^{+125}_{-36}$ and $\rho_0 = 0.9995^{+0.0013}_{-0.0009}$). The error bar in Eq. (10.46) is highly asymmetric: at the 2 σ level one has $\rho_0 = 0.9998^{+0.0034}_{-0.0012}$ and $M_H < 1002$ GeV. Clearly, in the presence of ρ_0 upper limits on M_H become very weak.

The result in Eq. (10.46) is in remarkable agreement with the Standard Model expectation, $\rho_0 = 1$. It can be used to constrain higher-dimensional Higgs representations to have vacuum expectation values of less than a few percent of those of the doublets. Indeed, the relation between M_W and M_Z is modified if there are Higgs multiplets with weak isospin $> 1/2$ with significant vacuum expectation values. In order to calculate to higher orders in such theories one must define a set of four fundamental renormalized parameters which one may conveniently choose to be α, G_F, M_Z, and M_W, since M_W and M_Z are directly measurable. Then $\widehat{s}^2_Z$ and ρ_0 can be considered dependent parameters.

Eq. (10.46) can also be used to constrain other types of new physics. For example, nondegenerate multiplets of heavy fermions or scalars break the vector part of weak SU(2) and lead to a decrease in the value of M_Z/M_W. A nondegenerate SU(2) doublet $\binom{f_1}{f_2}$ yields a positive contribution to ρ_0 [116] of

$$\frac{CG_F}{8\sqrt{2}\pi^2}\Delta m^2 \ , \tag{10.50}$$

where

$$\Delta m^2 \equiv m_1^2 + m_2^2 - \frac{4m_1^2 m_2^2}{m_1^2 - m_2^2} \ln\frac{m_1}{m_2} \geq (m_1 - m_2)^2 \ , \tag{10.51}$$

and $C = 1$ (3) for color singlets (triplets). Thus, in the presence of such multiplets, one has

$$\frac{3G_F}{8\sqrt{2}\pi^2} \sum_i \frac{C_i}{3} \Delta m_i^2 = \rho_0 - 1 \ , \tag{10.52}$$

where the sum includes fourth-family quark or lepton doublets, $\binom{t'}{b'}$ or $\binom{E^0}{E^-}$, and scalar doublets such as $\binom{\tilde{t}}{\tilde{b}}$ in supersymmetry (in the absence of $L-R$ mixing). This implies

$$\sum_i \frac{C_i}{3} \Delta m_i^2 \leq (100 \ \text{GeV})^2 \tag{10.53}$$

at 95% CL. The corresponding constraints on nondegenerate squark and slepton doublets are even stronger, $\Delta m_i^2 \leq (69 \ \text{GeV})^2$. This is due to the MSSM Higgs mass bound, $m_{h^0} < 150$ GeV, and the strong correlation between m_{h^0} and ρ_0 (81%).

Nondegenerate multiplets usually imply $\rho_0 > 1$. Similarly, heavy Z' bosons decrease the prediction for M_Z due to mixing and generally lead to $\rho_0 > 1$ [117]. On the other hand, additional Higgs doublets which participate in spontaneous symmetry breaking [118], heavy lepton doublets involving Majorana neutrinos [119], and the vacuum expectation values of Higgs triplets or higher-dimensional representations can contribute to ρ_0 with either sign. Allowing for the presence of heavy degenerate chiral multiplets (the S parameter, to be discussed below) affects the determination of ρ_0 from the data, at present leading to a smaller value (for fixed M_H).

A number of authors [120–125] have considered the general effects on neutral current and Z and W boson observables of various types of heavy (*i.e.*, $M_{\text{new}} \gg M_Z$) physics which contribute to the W and Z self-energies but which do not have any direct coupling to the ordinary fermions. In addition to nondegenerate multiplets, which break the vector part of weak SU(2), these include heavy degenerate multiplets of chiral fermions which break the axial generators. The effects of one degenerate chiral doublet are small, but in technicolor theories there may be many chiral doublets and therefore significant effects [120].

Such effects can be described by just three parameters, S, T, and U at the (electroweak) one loop level. (Three additional parameters are needed if the new physics scale is comparable to M_Z [126].) T is proportional to the difference between the W and Z self-energies at $Q^2 = 0$ (*i.e.*, vector SU(2)-breaking), while S ($S+U$) is associated with the difference between the Z (W) self-energy at $Q^2 = M^2_{Z,W}$ and $Q^2 = 0$ (axial SU(2)-breaking). Denoting the contributions of new physics to the various self-energies by Π^{new}_{ij}, we have

$$\widehat{\alpha}(M_Z)T \equiv \frac{\Pi^{\text{new}}_{WW}(0)}{M_W^2} - \frac{\Pi^{\text{new}}_{ZZ}(0)}{M_Z^2} \ , \tag{10.54a}$$

$$\begin{aligned}\frac{\widehat{\alpha}(M_Z)}{4\widehat{s}^2_Z\widehat{c}^2_Z}S \equiv &\frac{\Pi^{\text{new}}_{ZZ}(M_Z^2) - \Pi^{\text{new}}_{ZZ}(0)}{M_Z^2} \\ &- \frac{\widehat{c}^2_Z - \widehat{s}^2_Z}{\widehat{c}_Z\widehat{s}_Z}\frac{\Pi^{\text{new}}_{Z\gamma}(M_Z^2)}{M_Z^2} - \frac{\Pi^{\text{new}}_{\gamma\gamma}(M_Z^2)}{M_Z^2} \ ,\end{aligned} \tag{10.54b}$$

$$\begin{aligned}\frac{\widehat{\alpha}(M_Z)}{4\widehat{s}^2_Z}(S+U) \equiv &\frac{\Pi^{\text{new}}_{WW}(M_W^2) - \Pi^{\text{new}}_{WW}(0)}{M_W^2} \\ &- \frac{\widehat{c}_Z}{\widehat{s}_Z}\frac{\Pi^{\text{new}}_{Z\gamma}(M_Z^2)}{M_Z^2} - \frac{\Pi^{\text{new}}_{\gamma\gamma}(M_Z^2)}{M_Z^2} \ .\end{aligned} \tag{10.54c}$$

S, T, and U are defined with a factor proportional to $\widehat{\alpha}$ removed, so that they are expected to be of order unity in the presence of new physics. In the $\overline{\text{MS}}$ scheme as defined in Ref. [29], the last two terms in Eq. (10.54b) and Eq. (10.54c) can be omitted (as was done in earlier editions of this *Review*). They are related to other parameters (S_i, h_i, $\widehat{\epsilon}_i$) defined in [29,121,122] by

$$\begin{aligned}&T = h_V = \widehat{\epsilon}_1/\alpha \ , \\ &S = h_{AZ} = S_Z = 4\widehat{s}^2_Z\widehat{\epsilon}_3/\alpha \ , \\ &U = h_{AW} - h_{AZ} = S_W - S_Z = -4\widehat{s}^2_Z\widehat{\epsilon}_2/\alpha \ .\end{aligned} \tag{10.55}$$

A heavy nondegenerate multiplet of fermions or scalars contributes positively to T as

$$\rho_0 - 1 = \frac{1}{1-\alpha T} - 1 \simeq \alpha T \ , \tag{10.56}$$

where ρ_0 is given in Eq. (10.52). The effects of nonstandard Higgs representations cannot be separated from heavy nondegenerate multiplets unless the new physics has other consequences, such as vertex corrections. Most of the original papers defined T to include the effects of loops only. However, we will redefine T to include all new sources of SU(2) breaking, including nonstandard Higgs, so that T and ρ_0 are equivalent by Eq. (10.56).

A multiplet of heavy degenerate chiral fermions yields

$$S = C\sum_i \Big(t_{3L}(i) - t_{3R}(i)\Big)^2/3\pi \ , \tag{10.57}$$

where $t_{3L,R}(i)$ is the third component of weak isospin of the left- (right-) handed component of fermion i and C is the number of colors. For example, a heavy degenerate ordinary or mirror family would contribute $2/3\pi$ to S. In technicolor models with QCD-like dynamics, one expects [120] $S \sim 0.45$ for an isodoublet of technifermions, assuming $N_{TC} = 4$ technicolors, while $S \sim 1.62$ for a full technigeneration with $N_{TC} = 4$; T is harder to estimate because it is model dependent. In these examples one has $S \geq 0$. However, the QCD-like models are excluded on other grounds (flavor-changing neutral currents, and too-light quarks and pseudo-Goldstone bosons [127]). In particular, these estimates do not apply to models of walking technicolor [127], for which S can be smaller or even negative [128]. Other situations in which $S < 0$, such as loops involving scalars or Majorana particles, are also possible [129]. Supersymmetric extensions of the Standard Model generally give very small effects [130]. Most simple types of new physics yield $U = 0$, although there are counter-examples, such as the effects of anomalous triple-gauge vertices [122].

The Standard Model expressions for observables are replaced by

$$\begin{aligned}M_Z^2 &= M_{Z0}^2 \ \frac{1-\alpha T}{1 - G_F M_{Z0}^2 S/2\sqrt{2}\pi} \ , \\ M_W^2 &= M_{W0}^2 \ \frac{1}{1 - G_F M_{W0}^2 (S+U)/2\sqrt{2}\pi} \ ,\end{aligned} \tag{10.58}$$

where M_{Z0} and M_{W0} are the Standard Model expressions (as functions of m_t and M_H) in the $\overline{\text{MS}}$ scheme. Furthermore,

$$\begin{aligned}\Gamma_Z &= \frac{1}{1-\alpha T} M_Z^3 \beta_Z \ , \\ \Gamma_W &= M_W^3 \beta_W \ , \\ A_i &= \frac{1}{1-\alpha T} A_{i0} \ ,\end{aligned} \tag{10.59}$$

where β_Z and β_W are the Standard Model expressions for the reduced widths Γ_{Z0}/M_{Z0}^3 and Γ_{W0}/M_{W0}^3, M_Z and M_W are the physical masses, and A_i (A_{i0}) is a neutral current amplitude (in the Standard Model).

The data allow a simultaneous determination of $\widehat{s}^2_Z$ (from the Z pole asymmetries), S (from M_Z), U (from M_W), T (mainly from

Γ_Z), α_s (from R_ℓ and $\sigma_{\rm had}$), and m_t (from CDF and DØ), with little correlation among the Standard Model parameters:

$$\begin{aligned} S &= -0.07 \pm 0.11\ (-0.09)\ , \\ T &= -0.10 \pm 0.14\ (+0.09)\ , \\ U &= \ \ 0.11 \pm 0.15\ (+0.01)\ , \end{aligned} \tag{10.60}$$

and $\widehat{s}^2_Z = 0.23117 \pm 0.00017$, $\alpha_s(M_Z) = 0.1203 \pm 0.0031$, $m_t = 173.4 \pm 4.9$ GeV, where the uncertainties are from the inputs. The central values assume $M_H = 100$ GeV, and in parentheses we show the change for $M_H = 300$ GeV. As can be seen, the Standard Model parameters (U) can be determined with no (little) M_H dependence. On the other hand, S, T, and M_H cannot be obtained simultaneously, because the Higgs boson loops themselves are resembled approximately by oblique effects. The first Eq. (10.60) shows that negative contributions to the S parameter can weaken or entirely remove the strong constraints on M_H from the Standard Model fits. The parameters in Eqs. (10.60) which by definition are due to new physics only, are all consistent with the Standard Model values of zero. Using Eq. (10.56) the value of ρ_0 corresponding to T is 0.9992 ± 0.0011 $(+0.0007)$. The values of the $\widehat{\epsilon}$ parameters defined in Eq. (10.55) are

$$\begin{aligned} \widehat{\epsilon}_3 &= -0.0006 \pm 0.0009\ (-0.0008)\ , \\ \widehat{\epsilon}_1 &= -0.0008 \pm 0.0011\ (+0.0007)\ , \\ \widehat{\epsilon}_2 &= -0.0009 \pm 0.0013\ (-0.0001)\ . \end{aligned} \tag{10.61}$$

Unlike the original definition, we defined the quantities in Eqs. (10.61) to vanish identically in the absence of new physics and to correspond directly to the parameters S, T, and U in Eqs. (10.60). There is a strong correlation (81%) between the S and T parameters. The allowed region in $S - T$ is shown in Fig. 10.3. From Eqs. (10.60) one obtains $S \leq 0.11\,(0.01)$ and $T \leq 0.13\,(0.22)$ at 95% CL for $M_H = 100$ GeV (300 GeV). If one fixes $M_H = 600$ GeV and requires the constraint $S \geq 0$ (as is appropriate in QCD-like technicolor models) then $S \leq 0.09$. This rules out simple technicolor models with many techni-doublets and QCD-like dynamics.

An extra generation of ordinary fermions is excluded at the 99.6% CL on the basis of the S parameter alone. This result assumes that there are no new contributions to T or U. Allowing a contribution of 0.18 ± 0.08 to T reduces the CL to 97%. This is in agreement with a fit to the number of light neutrinos, $N_\nu = 2.985 \pm 0.008$ (which favors a larger value for $\alpha_s(M_Z) = 0.1229 \pm 0.0034$ mainly from R_ℓ). However, the S parameter fit is valid even for a very heavy fourth family neutrino.

Although S is consistent with zero, the electroweak asymmetries, especially the SLD left-right asymmetry and Q_W, favor $S < 0$. The simplest origin of $S < 0$ would probably be an additional heavy Z' boson [117], which could mimic $S < 0$. Similarly, there is a slight indication of negative T, while, as discussed above, nondegenerate scalar or fermion multiplets generally predict $T > 0$.

There is no simple parametrization that is powerful enough to describe the effects of every type of new physics on every possible observable. The S, T, and U formalism describes many types of heavy physics which affect only the gauge self-energies, and it can be applied to all precision observables. However, new physics which couples directly to ordinary fermions, such as heavy Z' bosons [117] or mixing with exotic fermions [131] cannot be fully parametrized in the S, T, and U framework. It is convenient to treat these types of new physics by parametrizations that are specialized to that particular class of theories (*e.g.*, extra Z' bosons), or to consider specific models (which might contain, *e.g.*, Z' bosons and exotic fermions with correlated parameters). Constraints on various types of new physics are reviewed in [26,132,133]. Fits to models with technicolor, extended technicolor, and supersymmetry are described, respectively, in [134], [135], and [86,136]. In a new development, the effects of compactified extra spatial dimensions at the TeV scale have been considered in Ref. 137.

An alternate formalism [138] defines parameters, ϵ_1, ϵ_2, ϵ_3, ϵ_b in terms of the specific observables M_W/M_Z, $\Gamma_{\ell\ell}$, $A_{FB}^{(0,\ell)}$, and R_b.

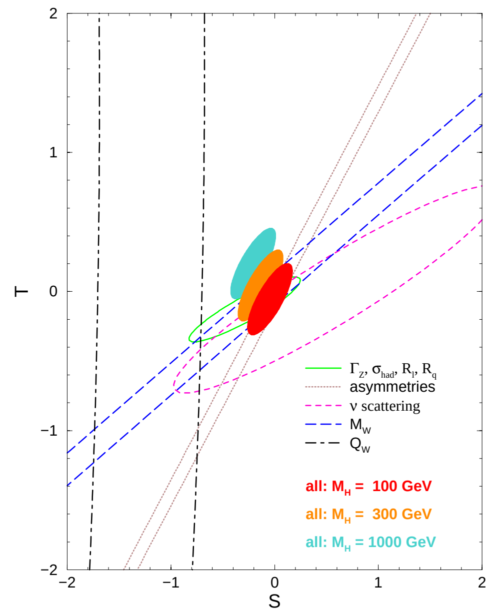

Figure 10.3: 1 σ constraints (39.35%) on S and T from various inputs. S and T represent the contributions of new physics only. (Uncertainties from m_t are included in the errors.) The contours assume $M_H = 100$ GeV except for the central and upper 90% CL contours allowed by all data, which are for $M_H = 300$ GeV and 1000 GeV, respectively. Data sets not involving M_W are insensitive to U. Due to higher order effects, however, $U = 0$ has to be assumed in all fits. $\alpha_s(M_Z) = 0.120$ is assumed for the 1 σ constraints, while in the fits to all data α_s is allowed to float.

The definitions coincide with those for $\widehat{\epsilon}_i$ in Eqs. (10.54) and (10.55) for physics which affects gauge self-energies only, but the ϵ's now parametrize arbitrary types of new physics. However, the ϵ's are not related to other observables unless additional model-dependent assumptions are made. Another approach [139–141] parametrizes new physics in terms of gauge-invariant sets of operators. It is especially powerful in studying the effects of new physics on non-Abelian gauge vertices. The most general approach introduces deviation vectors [132]. Each type of new physics defines a deviation vector, the components of which are the deviations of each observable from its Standard Model prediction, normalized to the experimental uncertainty. The length (direction) of the vector represents the strength (type) of new physics.

Table 10.7: 95% CL lower mass limits (in GeV) on various extra Z' bosons, appearing in models of unification and string theory. ρ_0 free indicates a completely arbitrary Higgs sector, while $\rho_0 = 1$ restricts to Higgs doublets and singlets with still unspecified charges.

Z'	ρ_0 free	$\rho_0 = 1$
Z_χ	551	545
Z_ψ	151	146
Z_η	379	365
Z_{LR}	570	564
Z_{SM}	822	809
$Z_{\rm string}$	582	578

One of the best motivated kinds of physics beyond the Standard Model besides supersymmetry are extra Z' bosons. They do not spoil the observed approximate gauge coupling unification, and appear copiously in many Grand Unified Theories (GUTs) and most superstring models. For example, the $SO(10)$ GUT contains an extra $U(1)$ as can be seen from its maximal subgroup,

$SU(5) \times U(1)_\chi$. Similarly, the E_6 GUT contains the subgroup $SO(10) \times U(1)_\psi$. It possesses only axial-vector couplings to the ordinary fermions, and its mass is generally less constrained. The Z_η boson is the linear combination $\sqrt{3/8}\, Z_\chi - \sqrt{5/8}\, Z_\psi$. The Z_{LR} boson occurs in left-right models with gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \subset SO(10)$. The sequential Z_{SM} boson is defined to have the same couplings to fermions as the SM Z boson. Such a boson is not expected in the context of gauge theories unless it has different couplings to exotic fermions than the ordinary Z. However, it serves as a useful reference case when comparing constraints from various sources. It could also play the role of an excited state of the ordinary Z in models with extra dimensions at the weak scale. Finally, we consider a superstring motivated Z_{string} boson appearing in a specific model [142]. The potential Z' boson is in general a superposition of the SM Z and the new boson associated with the extra $U(1)$. The mixing angle θ satisfies,

$$\tan^2\theta = \frac{M^2_{Z_1^0} - M^2_Z}{M^2_{Z'} - M^2_{Z_1^0}},$$

where $M_{Z_1^0}$ is the SM value for M_Z in the absence of mixing. Note, that $M_Z < M_{Z_1^0}$, and that the SM Z couplings are changed by the mixing. If the Higgs $U(1)'$ quantum numbers are known, there will be an extra constraint,

$$\theta = C \frac{g_2}{g_1} \frac{M_Z^2}{M_{Z'}^2} \,, \tag{10.62}$$

where $g_{1,2}$ are the $U(1)$ and $U(1)'$ gauge couplings with $g_2 = \sqrt{\frac{5}{3}}\, \sin\theta_W \sqrt{\lambda}\, g_1$. $\lambda = 1$ (which we assume) if the GUT group breaks directly to $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)} \times \mathrm{U(1)}'$. C is a function of vacuum expectation values. For minimal Higgs sectors it can be found in reference [117]. Table 10.7 shows the 95% CL lower mass limits obtained from a somewhat earlier data set [143] for ρ_0 free and $\rho_0 = 1$, respectively. In cases of specific minimal Higgs sectors where C is known, the Z' mass limits are generally pushed into the TeV region. For more details see Ref. 143 and the Section on "The Z' Searches" in this *Review*. The more recent values for $Q_W(\mathrm{Cs})$ and σ_{had} used in this Review modify the results and even suggest the possible existence of Z' [144].

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