15. THE POCKET COSMOLOGY

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15.1. The Universe Observed

15.1.1. *The Hubble expansion*:

The most fundamental discovery of modern observational cosmology is the expansion of the Universe. The expansion is just a rescaling of the Universe: the proper distance between points at rest in the cosmic rest frame scales as the cosmic scale factor $R(t)$ [sometimes denoted as $a(t)$]. The expansion rate is given by

$$H(t) \equiv \dot{R}(t)/R(t) \; . \tag{15.1}$$

In general, H is a function of time. The present value of the expansion rate, the Hubble constant H_0, can be measured in a number of ways, all of which fundamentally involve dividing the recessional velocity of a distant galaxy by its distance. It is conventional to express H_0 in terms of a dimensionless constant h: $H_0 = 100\ h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. While the linear nature of the distance–redshift relation for nearby objects (Hubble's Law) is clear (see Fig. 15.1), until recently there was a systematic uncertainty of almost a factor of two in the value of h. Today, virtually all methods are now consistent with $h = 0.65 \pm 0.05$ ($H_0 = 65 \pm 5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$), with a possible systematic error of about 10% [1].

The Hubble constant sets the scale of the Universe in both time and space: the time since the scale factor $R(t)$ was zero is measured in units of the Hubble time $H_0^{-1} = 9.778\ h^{-1}\,\mathrm{Gyr}$, and the size of the observable Universe is set by the Hubble distance $cH_0^{-1} = 2998\ h^{-1}\,\mathrm{Mpc} = 9.251\ h^{-1} \times 10^{27}\,\mathrm{cm}$. The precise relationships between H_0 and the size and age depend upon the expansion history of the Universe and are discussed in Sec. 15.2.

Another fundamental parameter is the deceleration parameter, q_0, which measures the rate of change of the expansion:

$$H_0^2 q_0 \equiv -\ddot{R}(t_0)/R(t_0) \; . \tag{15.2}$$

In a universe comprised only of matter, q_0 equals $\Omega_M/2$, where Ω_M is the fraction of critical density contributed by matter. The critical density is determined by H_0 and the gravitational constant G:

$$\begin{aligned} \rho_{\mathrm{crit}} \equiv \frac{3H_0^2}{8\pi G} &= 1.879\ h^2 \times 10^{-29}\,\mathrm{g\,cm^{-3}} \\ &= 1.054\ h^2 \times 10^4\,\mathrm{eV\,cm^{-3}} \; . \end{aligned} \tag{15.3}$$

For a matter-dominated or radiation-dominated universe the density determines the fate of the universe: a sub-critical-density universe expands forever and a super-critical-density universe recollapses. A cosmological constant (or similar form of energy density) complicates the connection between destiny and energy density.

Measurements of the distances to very distant supernovae indicate that q_0 is actually negative, *i.e.,* the Universe is accelerating (see below). If correct, this illustrates dramatically that the energy density of the Universe is dominated by something other than matter or relativistic particles, since their gravity would slow (decelerate) the expansion.

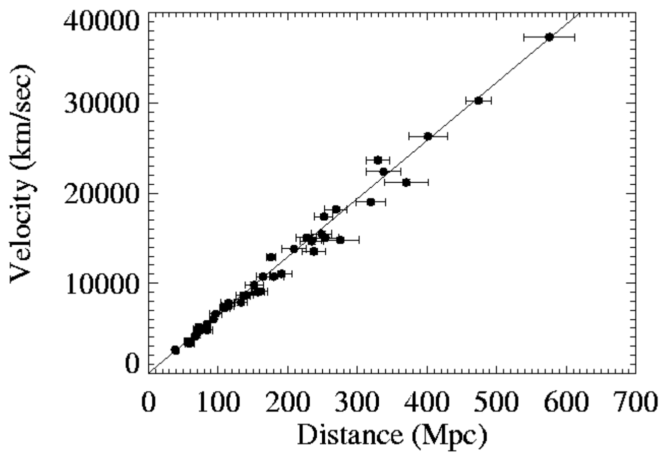

Figure 15.1: The distance–redshift diagram illustrates the expansion of the Universe. This "Hubble diagram" with linear axes is derived from a sample of type Ia supernovae and $H_0 = 65 \pm 2$ km s^{-1} Mpc^{-1} (error purely statistical; figure courtesy of Adam Riess).

15.1.2. ***The redshift***:

As the Universe expands, all distances are stretched with the cosmic scale factor, including the wavelengths of photons. (The exception to this universal stretching is the size of a bound system, *e.g.,* a galaxy, a Hydrogen atom, or a proton.) Because the Universe is expanding, photons emitted long ago are redshifted:

$$1 + z \equiv \frac{\lambda_{\rm today}}{\lambda_{\rm emission}} = \frac{R(t_0)}{R(t_{\rm emission})} \; . \tag{15.4}$$

Redshift (z) directly indicates the relative linear size of the Universe when that photon was emitted. For example, the most distant quasar has $z = 5.82$; when the light from that quasar was emitted, the Universe was a factor of $1 + z \simeq 6.82$ times smaller. The relative size of the Universe is simply $(1 + z)^{-1}$, while its age at a given redshift depends upon the expansion history.

15.1.3. *The age of the Universe*:

The relationship between the expansion age (time since zero scale factor) and the Hubble age H_0^{-1} depends upon the slowing or speeding of the expansion rate. For plausible values of the Hubble constant and expansion histories, the expansion age is between 10 and 17 Gyr. The supernova measurements that indicate the Universe is accelerating also constrain the expansion history, and imply an expansion age of $15^{+1.4}_{-1.1}$ Gyr [2].

An important cosmological consistency test is the comparison of the expansion age with independent age measurements of objects within the Universe: consistency requires the Universe to be older than any object within it. Independent age measurements include the ages of the oldest globular clusters dated by their stars, of the heavy elements as dated by radioactive decays, and of the oldest white-dwarf stars in the disk of our galaxy as measured by their cooling. The last two clocks are less easily compared to the expansion age because of the uncertainty of when the disk formed relative to the galaxy and the time history of heavy-element formation.

The reliability of the globular-cluster technique has improved in the last five years due to better stellar models, better atomic-physics data, and more accurate globular-cluster distance measurements. Current estimates for the age of the Universe based upon globular clusters are $t_0 = 14 \pm 2$ Gyr, with a possible systematic error of similar size [3]. Ages for the Universe based upon white-dwarf cooling and nuclecosmochronology are consistent with this number. While the error bars are still significant, the expansion age is comfortably consistent with the independent estimates of the age of the Universe.

15.1.4. *The composition of the Universe*:

We have taken the first steps toward a full accounting of the composition of the Universe. In units of the critical density, our assessment is

$$\begin{aligned} \text{total} : \Omega_0 &= 1 \pm 0.1 \ , \\ \text{matter} : \Omega_M &= 0.35 \pm 0.1 \ , \\ \text{energy} : \Omega_E &= 0.8 \pm 0.2 \ , \end{aligned} \tag{15.5}$$

where the errors quoted are meant to be 1σ. By matter we mean material that is nonrelativistic (*i.e.*, pressure p much smaller than its energy density). As discussed below, energy refers to components that are intrinsically relativistic; *e.g.*, photons (γ), massless neutrinos (ν), and vacuum energy (Λ).

The total of matter plus energy density has been determined by measurements of the dependence of the anisotropy of the cosmic microwave background (CMB) upon angular scale (see the review on "Cosmic background radiation" (Sec. 19) in the full *Review* and Figure 15.2).

The matter density consists of several components: optically bright baryons in stars, optically dark baryons (in hot, cold, and warm gas, neutral atomic gas, molecular clouds, and stellar remnants), neutrinos, and nonbaryonic dark matter of an unknown form, here

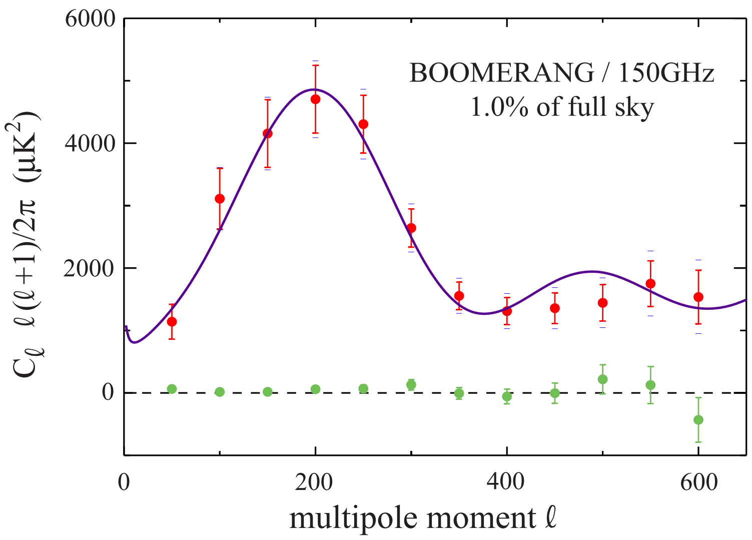

Figure 15.2: CMB angular power spectrum as determined by the Long Duration Balloon Flight of Boomerang, based on one frequency channel and 1% of the sky [4]. The curve is a flat CDM model with $\Omega_\Lambda = 0.65$. The Boomerang data by themselves imply $\Omega_0 = 1 \pm 0.06$. (The broken line and associated points show the difference of the two halves of the time stream of data; the absence of a difference indicates internal consistency.)

referred to as cold dark matter (CDM). The matter density breaks down as follows

$$\begin{aligned} \text{CDM} : \Omega_{\text{CDM}} &= 0.30 \pm 0.1 \\ \text{Baryons} : \Omega_B &= (0.019 \pm 0.001)h^{-2} \simeq 0.045 \pm 0.01 \\ &\quad \text{optically bright baryons } \Omega_* \sim 0.005 \\ &\quad \text{dark baryons } \Omega_B \sim 0.04 \\ \text{Neutrinos} : 0.10 &\gtrsim \Omega_\nu \gtrsim 0.003\ . \end{aligned} \tag{15.6}$$

The baryon density is most precisely probed by comparing the big-bang production of deuterium and the measurements of the primeval deuterium abundances in high-redshift clouds of hydrogen (see the review on "Big-bang nucleosynthesis (BBN)" (Sec. 16) in the full *Review* and Ref. 5). The total matter density is determined many ways, all of which are consistent with $\Omega_M = 0.35 \pm 0.07$. We believe that it is most cleanly determined from the baryon density and the ratio of baryonic mass to total mass in clusters of galaxies ($\equiv f_B$): $\Omega_M = \Omega_B / f_B$ [6].

The lower bound to the contribution of light neutrinos is from the SuperKamionkande evidence for neutrino oscillations involving muon neutrinos and a mass difference squared of $\mathcal{O}(10^{-2}\,\mathrm{eV}^2)$ [7]. The upper bound to Ω_ν is from the requirement that neutrinos not interfere with the formation of structure in the Universe [8]. While the neutrino contribution is small, it is comparable to that of bright stars. Finally, the CDM mass density is derived from the difference of Ω_M and Ω_B, assuming that neutrinos do not contribute significantly and that the nonbaryonic dark matter is slowly moving cold particles (the "C" in CDM).

Almost seventy years ago Zwicky pointed out that the gravity of stars in clusters of galaxies is not great enough to hold together clusters. More precise measurements today show that the total matter density is almost 100 times that of stars and that the dark matter problem is manifold. The factor of seven discrepancy between Ω_M and Ω_B is strong evidence that most of the matter is nonbaryonic. Further, the study of the formation of structure in the Universe indicates that the nonbaryonic dark matter must be slowly moving particles (cold dark matter), with the leading candidates being elementary particles left over from the earliest moments (see the review on "Dark matter" (Sec. 18) in the full *Review*). Finally, (optically) dark baryons outweigh those in stars by about a factor of ten.

Relativistic energy denoted by Ω_E appears in several forms today:

$$\begin{aligned} \text{photons} : \Omega_\gamma h^2 &= 2.471 \times 10^{-5} \\ \text{massless neutrinos} : \Omega_\nu h^2 &= 1.122 \times 10^{-5} \\ \text{dark energy} : \Omega_X &= 0.8 \pm 0.2 \ . \end{aligned} \tag{15.7}$$

The contribution of the photons in the cosmic microwave background and the (undetected) relativistic neutrino seas (two relativistic species assumed) are simple to calculate. Today, this relativistic contribution is negligible, but during the earliest moments it was the dominant component.

The mysterious entry, dark energy, is suggested by the type Ia supernovae measurements (SNeIa) that indicate the Universe is accelerating [2,9]. The supernovae data were analyzed assuming that the dark energy is a cosmological constant, and the results can be summarized by

$$\Omega_\Lambda = \frac{4}{3}\Omega_M + \frac{1}{3} \pm \frac{1}{6} \ . \tag{15.8}$$

All data are consistent with a cosmological constant of this size; however, theoretical estimates for the contribution to the cosmological constant coming from vacuum energy (zero-point energies) are at least 55 orders of magnitude larger than the critical density. This is the long-standing cosmological-constant problem.

It might well be that the resolution of the cosmological-constant puzzle is that vacuum energy does not contribute anything to the energy budget of the Universe today. If this is so, any acceleration of the expansion must be due to something else! The requirement for acceleration is an inequality involving the energy density and the pressure: $\rho + 3p < 0$. Since this component is clearly dark and relativistic ($|p| \sim \rho$ if $\rho > 0$), we have called it dark energy. Theorists have been busy, and there are already a number of interesting suggestions for the dark energy; they include a very light, slowly evolving scalar field (sometimes referred to as quintessence), vacuum energy, and a network of light, tangled topological defects. The supernova measurements, combined with other data, indicate that $-1 \leq p/\rho \leq -\frac{1}{2}$ [10].

As shown in Fig. 15.3 there is consistency between the independent determinations of the matter/energy content of the Universe. The emerging picture for the matter-energy of the Universe challenges the Standard Model of particle physics since nonbaryonic dark matter, massive neutrinos, and dark energy are not part of the Standard Model.

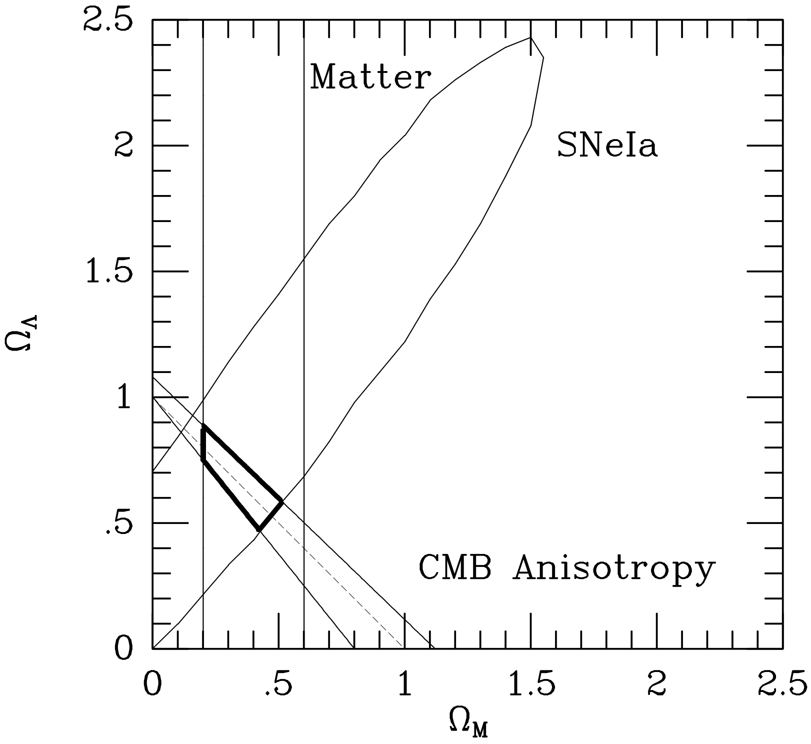

Figure 15.3: Summary of independent determinations of Ω_0, Ω_X and Ω_M, assuming the dark energy is vacuum energy (cosmological constant). Note the consistency of the three 95% confidence contours. Data are consistent with $\Omega_o = \Omega_\Lambda + \Omega_M = 1$ (dashed line).

15.1.5. *The cosmic microwave background*:

The cosmic microwave background contributes only a tiny fraction of critical density today; however, its presence means that during early history ($t < 40{,}000\,\mathrm{yrs}$) radiation dominated the energy density of the Universe. See the review on "Cosmic background radiation" (Sec. 19) in the full *Review* for a summary of the CMB with references; here we touch upon the most salient features.

The CMB is to an extraordinary precision black-body radiation (any deviation from the Planck spectrum is less than 50 parts per million and statistically insignificant). The temperature has been measured to four significant figures: $T_0 = 2.725 \pm 0.001\,\mathrm{K}$. This corresponds to a photon number density of $n_\gamma = 410.5\,\mathrm{cm}^{-3}$ and fraction of critical density $\Omega_\gamma = 2.471\ \ h^{-2} \times 10^{-5}$.

The CMB has a dipole anisotropy on the sky of amplitude $3.372 \pm 0.004\,\mathrm{mK}$, which arises from the velocity of the solar system with respect to the cosmic rest frame (defined as the CMB rest frame). This implies a solar-system velocity of $371 \pm 0.5\,\mathrm{km\,s}^{-1}$, and a velocity of the local group of $622 \pm 22\,\mathrm{km\,s}^{-1}$.

CMB anisotropy has now been detected on angular scales from 90° (multipole $l = 2$) to a fraction of a degree ($l \sim 500$), with amplitudes of tens of μK (see Figure 15.2). This anisotropy is due to inhomogeneity in the distribution of matter of the same amplitude, $\delta\rho/\rho \sim \delta T/T \sim 10^{-5}$. Since the surface of last scattering for the CMB is the Universe at about 500,000 years after the bang, this CMB anisotropy implies that the Universe at that time was very smooth, but not perfectly smooth. The level of matter inhomogeneity indicated is what is needed to explain the structure that exists today, after taking into account the growth of inhomogeneity due to the attractive force of gravity over the past 14 Gyr.

15.1.6. *Large-scale structure of the Universe*:

Einstein and others assumed isotropy and homogeneity to simplify the field equations of general relativity. While the Universe on small scales (much less than 100 Mpc) is neither isotropic or homogeneous, at early times, and on large scales today, there is ample evidence for isotropy and homogeneity. The evidence at early times is provided by the uniformity of the CMB ($\delta T/T \sim \delta\rho/\rho \sim 10^{-5}$). Redshift surveys, three-dimensional maps of the distribution of galaxies, now probe the Universe on scales as large as 300 h^{-1} Mpc. They indicate that the distribution of galaxies becomes homogeneous and isotropic on scales much greater than 100 Mpc (see Fig. 15.4). Even larger surveys to be completed over the next five years [*e.g.,* the Sloan Digital Sky Survey (SDSS) and the 2° Field project (2dF)] will probe the distribution of matter on even larger scales.

On smaller scales the Universe is highly structured: there are galaxies, small groups of galaxies, great clusters containing thousands of galaxies, superclusters, and giant sheet-like structures extending across 100 h^{-1} Mpc. This structure can be explained by the level of inhomogeneity revealed by the CMB anisotropy and the subsequent growth due to gravitational amplification, provided there is nonbaryonic dark matter.

Redshift surveys reveal the distribution of light, rather than matter itself. In principle, the two could be very different. After all, the bulk of the matter is not even baryons.

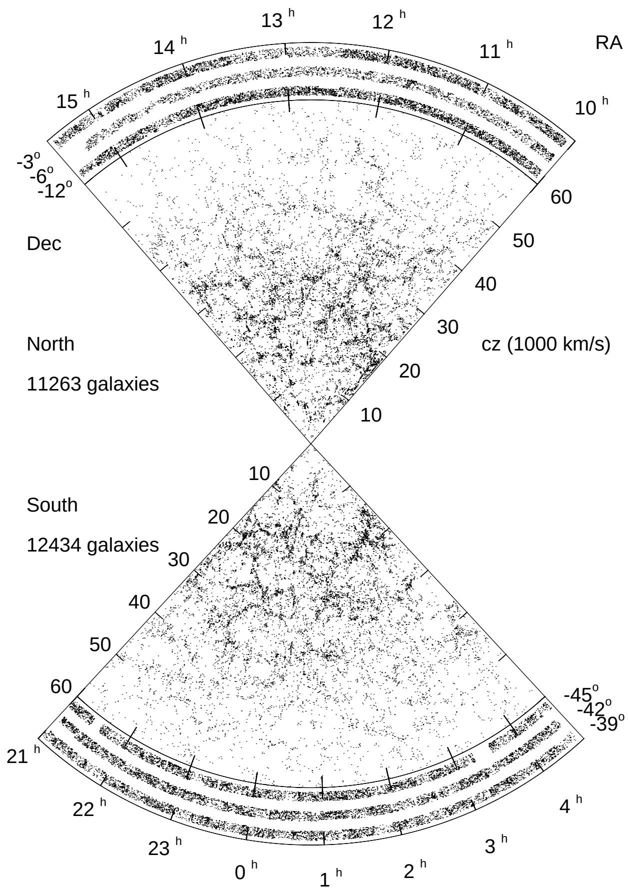

Figure 15.4: A slice from the Las Campanas Redshift Survey [11]. Each point represents a galaxy in the survey. Recessional velocity cz may be translated into distance d from us by Hubble's Law, $d = 10\ h^{-1}$ Mpc $(cz/1000$ km/s).

This problem is called biasing: light is likely to be a biased tracer of mass. The ratio of the inhomogeneity in the distribution of galaxies to that of matter is called the bias factor b. (The bias is likely to depend on scale and the type of galaxy.) A variety of studies show that biasing is important, but not overwhelming: b differs from unity by of order 50% or less. For example, the *rms* fluctuation in the number of galaxies within a sphere of radius 8 h^{-1} Mpc is unity; on the same scale the *rms* mass fluctuation has been inferred to be about 0.8, from the abundance of rich clusters and numerical simulations of structure formation.

Figure 15.5 summarizes the power spectrum $P(k) \equiv |\delta_k|^2$ of the distribution of galaxies today, where δ_k is the Fourier transform of the galaxy number density. On the very largest scales, $\lambda \gtrsim 10\ h^{-1}$ Mpc, the inhomogeneity of matter is probed by the CMB anisotropy; on small scales it is probed by the present distribution of galaxies. When the MAP and Planck CMB anisotropy maps and the 2dF and SDSS redshift surveys are complete, there will be a range of scales, from about 10 h^{-1} Mpc up to about 500 h^{-1} Mpc, where both the matter and galaxy inhomogeneity will be probed. On these scales biasing will be directly examined.

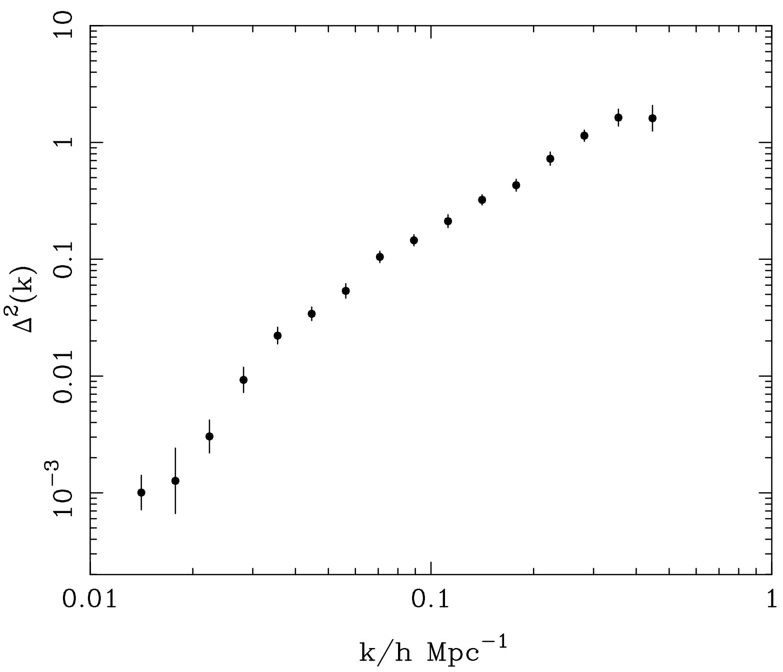

Figure 15.5: Summary of measurements of the power spectrum of the distribution of bright galaxies vs Fourier wavenumber k, shown as $\Delta(k)$ vs k [12]. $\Delta^2(k) = k^3 P(k)/2\pi^2$, which is equal to the contribution to variance of the galaxy number density divided by mean galaxy density per logarithmic interval in k $(d\sigma^2/d\ln k)$. Physically, $\Delta(k)$ roughly corresponds to the rms fluctuation in galaxy-number density in spheres of radius $\approx \pi/k$.

15.2. The Standard Cosmology

15.2.1. *Robertson–Walker line element*:

The distribution of matter in the observable Universe today is isotropic and homogeneous on the largest scales ($\gg 10\ h^{-1}\,$Mpc). The smoothness of the CMB, $\delta_T/T < 10^{-4}$ on all angular scales measured, indicates that at early times the distribution of matter and radiation were isotropic and homogeneous. Thus, for purposes of describing the present observable Universe on sufficiently large scales, as well as the Universe at early times, we may assume that the Universe is isotropic and homogeneous.

The metric for a space with homogeneous and isotropic spatial sections is the maximally symmetric Robertson-Walker (RW) metric, which can be written in the form

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\}\ , \qquad (15.9)$$

where $(t,\ r,\ \theta,\ \phi)$ are coordinates (referred to as comoving coordinates), and $R(t)$ is the cosmic scale factor. With an appropriate rescaling of the coordinates, k can be chosen to be $+1,\ \ -1$, or 0 for spaces of constant positive, negative, or zero spatial curvature, respectively. Nonetheless, there are an infinity of RW models, distinguished by their radii of spatial curvature, $R_{\rm curv} = R(t)/\sqrt{|k|}$. A convenient and widely used convention is to set the cosmic scale factor to unity today; then, the coordinate r and $1/\sqrt{|k|}$ have dimensions of length. We shall usually use this convention.

The time coordinate is just the proper (or clock) time measured by an observer at rest in the comoving frame, *i.e.*, $(r,\ \theta,\ \phi)$=const. The term *comoving* is well chosen: Observers at rest in the comoving frame remain at rest, *i.e.*, $(r,\ \theta,\ \phi)$ remain unchanged, and observers initially moving with respect to this frame will eventually come to rest in it.

15.2.2. *Particle kinematics and conservation of energy*:

If the stress-energy tensor has the form of a perfect fluid, the conservation of stress energy ($T^{\mu\nu}{}_{;\nu} = 0$) gives the first law of thermodynamics in the form

$$d(\rho R^3) = -pd(R^3)\ , \qquad (15.10)$$

or equivalently,

$$\begin{aligned} d\ln\rho &= -\,3(1+w)d\ln R \\ \Rightarrow\quad \rho &\propto \exp\left[-3\int(1+w)d\ln R\right]\ , \end{aligned} \qquad (15.11)$$

where $w \equiv p/\rho$ characterizes the equation of state of the fluid. The physical significance of the above equation is clear: The change in energy in a comoving volume element, $d(\rho R^3)$, is equal to minus the pressure times the change in volume, $-pd(R^3)$. If w is independent of time, the energy density evolves as $\rho \propto R^{-3(1+w)}$. Examples of interest include

$$\begin{aligned} \text{radiation}: \quad & p = \frac{1}{3}\rho \Longrightarrow \rho \propto R^{-4} \\ \text{matter}: \quad & p = 0 \Longrightarrow \rho \propto R^{-3} \\ \text{vacuum energy}: \quad & p = -\rho \Longrightarrow \rho \propto \text{const} \; . \end{aligned} \tag{15.12}$$

The "early" Universe was radiation dominated, and the "adolescent" Universe was matter dominated. The Universe today appears to be dominated by a form of energy similar to vacuum energy ($w \approx -1$). If the Universe underwent inflation, there was a "very early" period when the stress-energy was dominated by vacuum energy.

The equation of motion for a freely falling particle in RW space-time is very simple: the three-momentum decreases as the inverse of the cosmic scale factor:

$$|\mathbf{p}| \propto 1/R \tag{15.13}$$

For a massless particle, this is the cosmological redshift of wavelength. For a massive, nonrelativistic particle, this implies that any velocity with respect to the cosmic rest frame decreases as the inverse of the scale factor, with the particle eventually coming to rest in comoving RW coordinates.

15.2.3. *Friedmann equations*:

The dynamics of the expansion are determined from the Einstein equations. For the Robertson–Walker metric they are known as the Friedmann equations:

$$\begin{aligned} H^2 = & \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho - \frac{k}{R^2} \\ \frac{\ddot{R}}{R} = & -\frac{4\pi G}{3}(\rho + 3p) \; . \end{aligned} \tag{15.14}$$

Note that the equation for the expansion rate is the first integral of the second Friedmann equation.

These equations can be used to write the deceleration parameter as

$$q_0 = \frac{1}{2}\Omega_0 + \frac{3}{2}\sum_i w_i \Omega_i \; . \tag{15.15}$$

This formula applies to any epoch, provided the values of Ω_i corresponding to that epoch are used. For example, assuming a flat Universe and matter and cosmological-constant components,

$$q_z = \frac{1}{2} - \frac{3}{2}\left[\frac{\Omega_\Lambda}{\Omega_\Lambda + (1+z)^3(1-\Omega_\Lambda)}\right] \tag{15.16}$$

where the factor following 3/2 is $\Omega_\Lambda(z)$. It follows that the epoch of accelerated expansion ($q_z < 0$) began at redshift $z = (2\Omega_\Lambda/\Omega_M)^{1/3} - 1 \approx 0.6$ (taking $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$).

In the simple case in which the right side of the Friedmann equation is dominated by a fluid whose pressure is given by $p = w\rho$, it follows that

$$\rho \propto R^{-3(1+w)} \qquad R \propto t^{2/3(1+w)} \ , \tag{15.17}$$

This leads to the results: $R \propto t^{1/2}$ for $w = 1/3$ (radiation-dominated universe); $R \propto t^{2/3}$ for $w = 0$ (matter dominated); $R \propto \exp(H_0 t)$ for $w = -1$ (vacuum dominated); and $R \propto t$ for a curvature-dominated universe (*i.e.,* $H^2 = |k|/R^2$). Dark energy with $-1/3 > w \geq -1$ leads to the scale factor growing more rapidly than t and perhaps as rapidly as $\exp(H_0 t)$. Note that in terms of the dynamics of the expansion, curvature-domination and dark energy with $w = -1/3$ both lead to $R \propto t$.

15.2.4. *The three ages of the Universe*:

Because the energy density in relativistic particles (photons and neutrinos) evolves as R^{-4}, while that in matter evolves as R^{-3}, when $R(t) \leq R_{EQ} = 2.663 \times 10^{-4}/(\Omega_M h^2/0.156)$ the Universe was "radiation dominated." This corresponds to temperatures $T \geq T_{EQ} = 0.8819\,\mathrm{eV}(\Omega_M h^2/0.156)$. (In computing R_{EQ} we have assumed that all three neutrino species were relativistic at early times.) During the radiation era, the scale factor $R(t) \propto t^{1/2}$. Note that the 1σ uncertainty in $\Omega_M h^2$ is nearly 30%; the fiducial value $\Omega_M h^2 = 0.156$ derives from the somewhat arbitrarily selected central values, $\Omega_M = 0.35$ and $h = 2/3$;

After matter-radiation equality, the Universe begins a matter-dominated phase with scale factor $R(t) \propto t^{2/3}$. During the matter-dominated era,

$$t(z) = 16.5\,\mathrm{Gyr}/(1+z)^{3/2}(\Omega_M h^2/0.156)^{1/2} \ . \tag{15.18}$$

When the contributions to the energy density from both matter and radiation are comparable, the scale factor and age are related by

$$\frac{t}{t_{EQ}} = \frac{(R/R_{EQ} - 2)(R/R_{EQ} + 1)^{1/2} + 2}{2 - \sqrt{2}} \ , \tag{15.19}$$

This exact expression reduces to $R \propto t^{1/2}$ for $t \ll t_{EQ}$ and $R \propto t^{2/3}$ for $t \gg t_{EQ}$. The age of the Universe at matter-radiation equality is

$$\begin{aligned} t_{EQ} =& 4(\sqrt{2} - 1)H_{EQ}^{-1}/3 \\ =& 4.25 \times 10^4\,\mathrm{yrs}/(\Omega_M h^2/0.156)^2 \ . \end{aligned} \tag{15.20}$$

Last scattering of the CMB photons occurs shortly after matter-radiation equality, at a redshift $z_{LS} \simeq 1100$, when the age of the Universe was

$$t_{LS} \simeq 4.5 \times 10^5\,\mathrm{yrs}/(\Omega_M h^2/0.156)^{1/2} \ . \tag{15.21}$$

Acceleration implies that dark energy has recently began to control the behavior of the expansion. Assuming that the dark energy exists in the form of a cosmological constant, the transition to a vacuum-energy dominated age occurred at

$$R_\Lambda = (\Omega_M/\Omega_\Lambda)^{1/3} = 0.814 \tag{15.22}$$

for $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$, which corresponds to a redshift $z_\Lambda = 0.23$. Well into the Λ-dominated era, the scale factor evolves as

$$R(t) \propto \exp\left[\sqrt{\Omega_\Lambda}H_0 t\right] . \tag{15.23}$$

15.2.5. *Destiny*:

In a universe where all forms of energy density decrease more rapidly than R^{-2} ($w_i > -1/3$ for all i), there is a connection between geometry and destiny: open universes ($k \leq 0$) expand forever and closed universes ($k > 0$) recollapse. We thought until recently that we lived in this kind of universe *i.e.,* matter plus radiation. With the advent of a sizable dark energy component, all that goes out the window! For example, a closed universe with a positive cosmological constant ($w = -1$) can expand forever and an open universe with a negative cosmological constant must recollapse.

15.2.6. *Age and deceleration parameter*:

The equality $dt = RdR/H$ can be integrated to give the age of the Universe as a function of redshift:

$$t(z) = \int_z^\infty \frac{dz}{(1+z)H(z)} . \tag{15.24}$$

where the present age $t_0 = t(z=0)$ (see Fig. 15.6). An interesting example is a flat vacuum energy + matter universe:

$$t(z) = \frac{2}{3}H(z)^{-1}\Omega_\Lambda(z)^{-1/2}\ln\left[\frac{1+\Omega_\Lambda(z)^{1/2}}{\sqrt{\Omega_M(z)}}\right] . \tag{15.25}$$

where

$$\begin{aligned}
\Omega_M(z) =& \frac{\Omega_M}{\Omega_M + \Omega_\Lambda/(1+z)^3} \\
\Omega_\Lambda(z) =& 1 - \Omega_M(z) \\
H^2(z) =& H_0^2\left[\Omega_M(1+z)^3 + \Omega_\Lambda\right] .
\end{aligned} \tag{15.26}$$

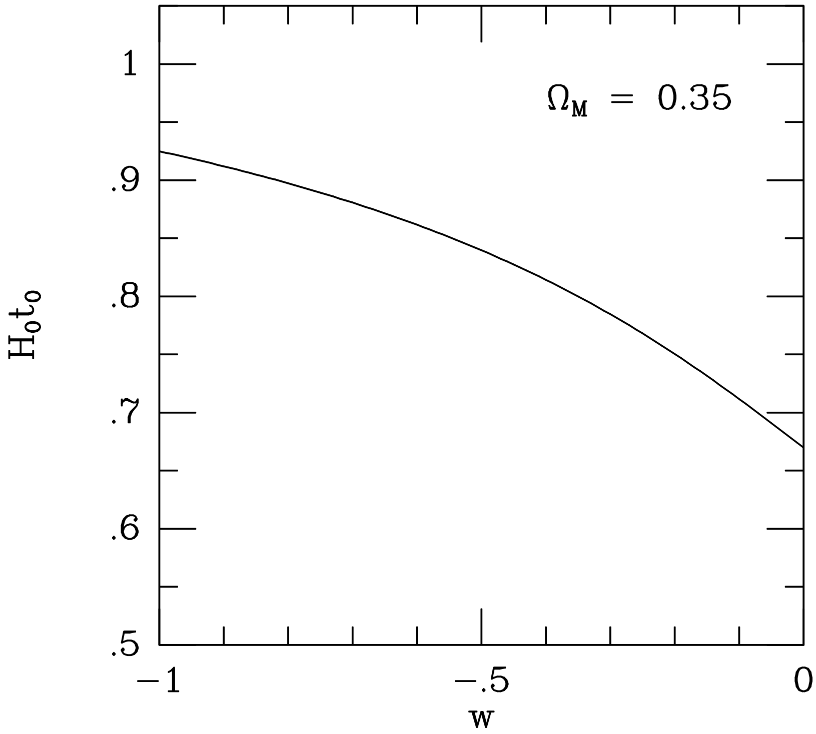

Figure 15.6: $H_0 t_0$ as a function of w, assuming a two-component universe with $\Omega_M = 0.35$ and $\Omega_X = 0.65$. $w = -1$ corresponds to a cosmological constant and $w = -1/3$ corresponds to an open universe with $\Omega_0 = \Omega_M = 0.35$. Accelerated expansion (or less decelerated expansion) leads to an older universe for a given present expansion rate; thus, $H_0 t_0$ increases with decreasing w.

15.2.7. *The classic tests*:

The behavior of the expansion, and thereby the underlying mean properties of the mass and energy in the Universe, as well as the curvature of the Universe are probed by the classical kinematic cosmological tests: the magnitude vs redshift (Hubble) diagram, the angular diameter vs redshift diagram, and the number count vs redshift test. At the heart of all three tests is the comoving distance to an object with redshift z:

$$\begin{aligned} r(z) =& \kappa^{-1/2} \sinh \left[\kappa^{-1/2} \int_0^z \frac{dx}{H(x)} \right] \\ \kappa =& (1 - \Omega_0) H_0^2 \\ H^2(z) =& H_0^2 \left[\Omega_M (1+z)^3 + \Omega_X \exp\left(3 \int (1+w) d\ln z \right) \right. \\ & \left. + (1 - \Omega_0)(1+z)^2 \right] . \end{aligned} \tag{15.27}$$

For definiteness, $k < 0$ was assumed. For $k > 0$, sinh $\rightarrow$ sin and $\kappa \rightarrow k$.

Luminosity distance as a function of redshift

$$d_L(z) = (1+z)r(z) \tag{15.28}$$

can be inferred from flux measurements of standard (or standardizable) candles such as supernovae of type Ia:

$$d_L = \sqrt{\mathcal{L}/4\pi\mathcal{F}} \tag{15.29}$$

where $\mathcal{L}$ is the luminosity of the standard candle and $\mathcal{F}$ is the measured flux. It is this technique, used with type Ia supernovae, that has revealed the acceleration of the expansion.

The angular-diameter distance

$$d_A(z) = r(z)/(1+z) \tag{15.30}$$

can be inferred through measurements of the angular size of standard rules,

$$d_A = D/\theta \tag{15.31}$$

where D is the size of the standard ruler and θ is the subtended angle. This method is central to the determination of Ω_0 from CMB anisotropy. The standard ruler is the sound horizon distance at last scattering, $D \propto v_s t_{\rm LS}$.

The comoving volume element is given by

$$\frac{dV}{d\Omega dr} = \frac{r^2}{\sqrt{1+\kappa r^2}} \qquad \Rightarrow \qquad \frac{dV}{d\Omega dz} = \frac{r^2(z)}{H(z)} \ . \tag{15.32}$$

It can be related to counts of objects of a constant (or known) comoving number density (*e.g.*, clusters or galaxies of a certain mass) vs a function of redshift,

$$\frac{dN}{dz d\Omega} = \frac{n(z)r^2(z)}{H(z)} \ . \tag{15.33}$$

Using this technique and theoretical expectations for the comoving number density of clusters in the CDM scenario, a matter density of about 0.3 has been inferred.

15.2.8. *Thermal history*:

During much of the history of the Universe, particularly the earliest history, conditions of thermal equilibrium existed. The total energy density and pressure of all species in equilibrium can be expressed in terms of the photon temperature T

$$\begin{aligned} \rho_R =& T^4 \sum_{i=\text{species}} \left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{1/2} u^2 du}{\exp(u-y_i) \pm 1} \\ p_R =& T^4 \sum_{i=\text{species}} \left(\frac{T_i}{T}\right)^4 \frac{g_i}{6\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{3/2} du}{\exp(u-y_i) \pm 1} \ , \end{aligned} \tag{15.34}$$

where $x_i \equiv m_i/T$, $y_i \equiv \mu_i/T$, the $+$ sign applies to fermions, the $-$ sign to bosons, and we have taken into account the possibility that the species i may have a thermal distribution, but with a different temperature than that of the photons. We have used natural units, where $\hbar = c = k_B = 1$.

Since the energy density and pressure of a nonrelativistic species (*i.e.*, one with mass $m \gg T$) is exponentially smaller than that of a relativistic species (*i.e.*, one with mass $m \ll T$), it is a very convenient and a good approximation to include only the relativistic species in the sums for ρ_R and p_R, in which case the above expressions greatly simplify:

$$\begin{aligned} \rho_R &= \frac{\pi^2}{30} g_* T^4 \ , \\ p_R &= \frac{\rho_R}{3} = \frac{\pi^2}{90} g_* T^4 \ , \end{aligned} \tag{15.35}$$

where g_* counts the total number of effectively massless degrees of freedom (those species with mass $m_i \ll T$),

$$g_* = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4 \ . \tag{15.36}$$

Fig. 15.7 shows $g_*(T)$ for the degrees of freedom in the Standard Model of particle physics.

During the early radiation-dominated epoch ($t \lesssim 40,000\,\text{yrs}$) $\rho \simeq \rho_R$; and further, when $g_* \simeq$ const, $p_R = \rho_R/3$ (*i.e.*, $w = 1/3$) and $R(t) \propto t^{1/2}$. From this it follows

$$\begin{aligned} H &= 1.660 \ g_*^{1/2} \frac{T^2}{m_{\text{Pl}}} \\ t &= 0.3012 \ g_*^{-1/2} \frac{m_{\text{Pl}}}{T^2} \sim \frac{\text{MeV}}{T^2} \ \text{s} \ , \end{aligned} \tag{15.37}$$

where m_{Pl} is the Planck mass 1.221×10^{19} GeV.

In the expanding Universe the entropy density s is given by

$$s \equiv \frac{\rho + p}{T} \ . \tag{15.38}$$

It is dominated by the contribution of relativistic particles, so that to a good approximation,

$$s = \frac{2\pi^2}{45} g_{*S} T^3 \ , \tag{15.39}$$

where

$$g_{*S} = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3 \ . \tag{15.40}$$

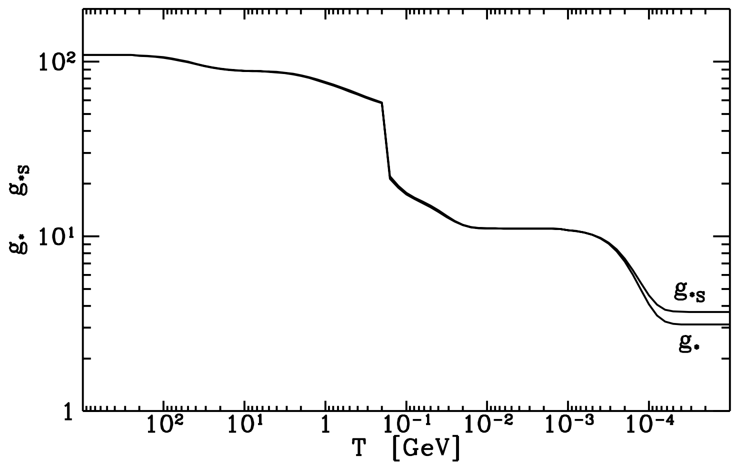

Figure 15.7: Number of relativistic degrees of freedom g_* and g_{*S} vs temperature according to the Standard Model of particle physics.

For most of the history of the Universe all particle species had a common temperature, and g_{*S} can be replaced by g_*. The annihilation of electron-positron pairs after neutrinos ceased interacting with the electromagnetic plasma ("decoupled") about 1 s after the bang leads to the slight heating of photons and $T_\gamma = (11/4)^{1/3} T_\nu$. Since then

$$\begin{aligned} g_* &= 2.0 + N_\nu \frac{7}{8} (4/11)^{4/3} = 3.363 \\ g_{*S} &= 2.0 + N_\nu \frac{7}{8} \frac{4}{11} = 43/11 = 3.91 \end{aligned} \tag{15.41}$$

where $N_\nu = 3$ has been used to obtain numerical values since much of the time since BBN all three neutrino species have been relativistic.

In the absence of an entropy producing event (*e.g.*, phase transition or particle decay), the entropy per comoving volume $S \propto R^3 s$ is conserved. The constancy of S implies that the temperature of the Universe evolves as

$$\begin{aligned} T &\propto g_{*S}^{-1/3} R^{-1} \\ \Longrightarrow R &= 3.699 \times 10^{-10}\, g_{*S}(T)^{-1/3} \frac{\text{MeV}}{T} \ . \end{aligned} \tag{15.42}$$

Whenever g_{*S} is constant, the familiar result, $T \propto R^{-1}$, obtains. The factor of $g_{*S}^{-1/3}$ enters because whenever a particle species becomes nonrelativistic and disappears, its entropy is transferred to the other relativistic particle species still present in the thermal plasma, causing T to *decrease slightly less slowly.*

Constancy of S also implies that $s \propto R^{-3}$. This means the physical size of a comoving volume element is proportional to $R^3 \propto s^{-1}$. Thus the number of some species per unit comoving volume, $N \equiv R^3 n$, is equal to the number density of that species divided by s: $N \equiv n/s$. Particle-number conservation in the expanding Universe is thus simply expressed as the constancy of n/s.

The entropy density s is proportional to the number density of relativistic particles, and therefore, to the photon number density, $s = 1.80 g_{*S} n_\gamma$. Today $s = 7.04 n_\gamma$; g_{*S} is a function of temperature, and the factor relating s and n_γ has decreased with time. However, since about 1 s that factor has been constant.

As an example of the utility of the ratio n/s, consider the baryon number. The baryon number in a comoving volume is

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} . \tag{15.43}$$

So long as baryon number nonconserving interactions are occurring very slowly, the baryon number in a comoving volume, n_B/s, is conserved. Today, there are only baryons and $s = 7.04\ n_\gamma$; thus, the baryon number of the Universe $n_B/s \simeq \eta/7$, where η is the present baryon-to-photon ratio. From BBN we know $\eta = (5.1 \pm 0.3) \times 10^{-10}$, and so we can infer that the baryon number of the Universe $n_B/s = (7.2 \pm 0.4) \times 10^{-11}$. It is believed that this tiny asymmetry between matter and antimatter arises due to B, C, and CP violating interactions that occurred out of equilibrium in the early Universe (baryogenesis).

Finally, thermal equilibrium in the expanding Universe corresponds to the limit of particle interactions occurring much more rapidly than the rate at which the temperature is dropping (set by expansion rate H). The opposite limit, a particle species that interacts slowly compared to the expansion rate (said to be decoupled), can be easily discussed. This limit applies to CMB photons after last scattering (redshift $z_{LS} \simeq 1100$) and neutrinos when the temperature of the Universe falls below about 1 MeV. The evolution of the phase-space distribution of a decoupled species is simple: particle momenta decrease as $1/R(t)$ and particle number density decrease as $1/R^3$. For a relativistic particle species that was in thermal equilibrium at decoupling (neutrinos and CMB photons), the phase-space distributions remain of the Fermi-Dirac or Bose-Einstein form, with a temperature that decreases precisely as $1/R(t)$. (Should the species eventually become nonrelativistic—for example a light neutrino species—the momentum phase-space distribution retains the FD or BE form, with $T \propto 1/R$.)

This fact explains why the CMB remains a perfect black body, and with some simple algebra and the constancy of S, how the factor of $(4/11)^{1/3}$ relating the neutrino and photon temperatures arises.

15.3. Beyond the Standard Cosmology: Inflation

Inflation is the most predictive and best developed idea about the earliest moments of the Universe. Further, its basic predictions—a flat Universe, a nearly scale-invariant spectrum of Gaussian, adiabatic density perturbations, and a nearly scale invariant spectrum of gravitational waves—are now being tested. The early results are consistent with the first two of these predictions; the third prediction will be much harder to test.

Inflation can provide insight about very fundamental issues not addressed by the standard cosmology: the origin of the large-scale isotropy and homogeneity; the origin of the small-scale inhomogeneity; the explanation for the oldness/flatness of the Universe; and in the context of simple grand unified theories, the monopole problem.

The key features of inflation are a period of accelerated expansion (typically exponential expansion), followed by an enormous release of entropy. During the period of exponential expansion a small, sub-horizon sized portion of the Universe is blown up to enormous size and made spatially flat. Quantum fluctuations in the field responsible for inflation, and in the metric of space time itself, are likewise stretched in size and eventually become density perturbations and gravitational waves. The entropy release that follows provides the heat that becomes the bath of radiation and other particles, thereby smoothly handing over the Universe to the standard hot big-bang phase. Provided that all of this occurs well before the epoch of big bang nucleosynthesis (*i.e.*, $T \gtrsim 1\,\mathrm{MeV}$ and $t \lesssim 1\,\mathrm{s}$), inflation can successfully address the fundamental questions without upsetting the success of the standard hot big-bang cosmology.

15.3.1. *Scalar-field dynamics*:

While there is no standard model of inflation, essentially all models can be described by the evolution of a scalar field ϕ initially displaced from the minimum of its potential energy curve $V(\phi)$. The evolution of the field can be described by two phases: (1) the slow roll during which nearly exponential expansion is driven by the nearly constant potential energy; (2) the coherent oscillation/reheat phase, during which the field oscillates rapidly about the minimum of its potential and eventually decays into lighter fields reheating the Universe and producing the heat of the big bang.

During the first phase, the equation of motion for the homogeneous mode scalar field in the expanding Universe is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \ , \tag{15.44}$$

which is supplemented by the Friedmann equation

$$H^2 = \frac{8\pi}{3{m_{\rm Pl}}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \ . \tag{15.45}$$

Over a small patch of the Universe, the scalar field should be smooth enough to justify the homogeneity assumption, and as inflation proceeds, the inhomogeneities in the scalar field decay away rapidly. Likewise, the energy density associated with the scalar field quickly come to dominate all other forms of energy in the Universe (*e.g.,* matter and radiation).

During the slow-roll phase the equations can be further simplified, as the $\ddot{\phi}$ term in the equation of motion and the $\dot{\phi}^2$ term in the expression for H^2 can be neglected:

$$\dot{\phi} \simeq \frac{-V'}{3H}$$
$$dN \equiv d\ln R = H dt = -\frac{8\pi}{m_{\rm Pl}{}^2} \frac{d\phi}{V'/V} , \qquad (15.46)$$

where prime denotes $d/d\phi$. These equations hold until the potential steepens and the slow-roll conditions, $m_{\rm Pl}V'/V \lesssim \sqrt{48\pi}$ and $m_{\rm Pl}{}^2 V''/V \lesssim 24\pi$, are no longer valid.

During the slow-roll phase the Universe grows in size by a factor $\exp(N)$, where N is given by the integral of dN. To solve the flatness and horizon problems, N must be greater than about 60 (the precise number depends upon when inflation takes place and the temperature to which the Universe reheats after inflation). Quantum fluctuations in ϕ, which correspond to energy density fluctuations, $\Delta\rho \sim \Delta\phi V'$, are stretched exponentially from microscopic size to astrophysical size. Likewise, quantum fluctuations in the metric undergo similar exponential stretching.

When the slow-roll phase ends, accelerated expansion ends and the scale factor grows as a power law that depends upon the shape of the potential. Ultimately, the energy in the ϕ field is transferred to other, lighter fields. These fields interact and create the thermal bath of particles that we are confident existed during the earliest moments of the Universe.

Though there are many interesting intermediate details, the reheating of the Universe involves the transition from a cold Universe dominated by the zero-momentum mode of the scalar field to a hot Universe dominated by many degrees of freedom.

While there is no standard model for inflation, typically the energy scale of inflation is $V^{1/4} \sim 10^{14}\,$GeV, though models exists with energy scales as small as 1 TeV. The potential V must be very flat, typically with a dimensionless coupling of the order of 10^{-14} (which is driven to be this small by the requirement of the density perturbation amplitude of 10^{-5}). The simplest model of inflation is a potential of the form $V(\phi) = \lambda\phi^4$; in this case, $\lambda \simeq 10^{-14}$ and the 60 or so e-folds of inflation needed to produce a large enough patch to contain our present Hubble volume occurs as ϕ rolls from $4.5 m_{\rm Pl}$ to $m_{\rm Pl}/\sqrt{2\pi}$.

15.3.2. *Predictions for observables*:

The spectrum of gravity waves (tensor perturbations) and density (or scalar) perturbations are the basis of the observables associated with inflation. They can be directly calculated from the properties of the scalar-field potential. This fact is the basis for the belief that observations may someday pin down the underlying model of inflation.

In most models both scalar and tensor perturbations have an approximately scale-invariant spectrum. In physical terms, that means that the dimensionless strain amplitude of gravity waves when they re-enter the horizon after inflation is independent of scale; in terms of the inflationary potential, that amplitude is $h_{\rm HOR} \sim H/m_{\rm Pl} \sim V^{1/2}/m_{\rm Pl}{}^2$. For density perturbations, it is the amplitude of the density perturbation at horizon crossing that is independent of scale: $(\delta\rho/\rho)_{\rm HOR} \sim H^2/\dot{\phi} \sim V^{3/2}/{m_{\rm Pl}}^3 V'$. Further, both spectra are expected to deviate from exact scale invariance by a small amount that depends upon the potential. Finally, the Fourier components of both scalar and tensor perturbations are approximately power-law in wavenumber k.

Primordial perturbations and gravity waves lead to CMB fluctuations, so measurable quantities may be expressed in terms of the inflationary potential. For instance, the scalar contribution (S) and the tensor contribution (T) to the CMB quadrupole anisotropy are

$$\begin{aligned} S \equiv & \frac{5C_2^S}{4\pi} \simeq 2.9 \, \frac{V/{m_{\rm Pl}}^4}{(m_{\rm Pl}V'/V)^2} \\ T \equiv & \frac{5C_2^T}{4\pi} \simeq 0.56(V/{m_{\rm Pl}}^4) \ , \end{aligned} \tag{15.47}$$

where C_2^S and C_2^T are the contribution of scalar and tensor perturbations to the variance of the $l = 2$ multipole amplitude $(\langle |a_{2m}|^2\rangle = C_2^S + C_2^T)$ and V is the value of the inflationary potential when the scale $k = H_0$ (present horizon scale) crossed the Hubble radius during inflation. Note, that the numerical coefficients in these expressions depend upon the composition of the Universe; the numbers shown are for $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$.

The power-law indices that characterize the scalar and gravity-wave spectra may also be expressed in terms of the inflationary potential and its derivatives:

$$\begin{aligned} n-1 = & -\frac{1}{8\pi}\left(\frac{m_{\rm Pl}V'}{V}\right)^2 + \frac{m_{\rm Pl}}{4\pi}\left(\frac{m_{\rm Pl}V'}{V}\right)' \\ n_T = & -\frac{1}{8\pi}\left(\frac{m_{\rm Pl}V'}{V}\right)^2 \ . \end{aligned} \tag{15.48}$$

Variations in the power-law indices with k may be expressed in terms of higher derivatives of V. For example,

$$\frac{dn}{d\ln k} = -\frac{m_{\rm Pl}}{8\pi}\left(\frac{m_{\rm Pl}V'}{V}\right)\frac{dn}{d\phi} \ . \tag{15.49}$$

Finally, one can in principle use these observables to solve for the inflationary potential and its first two derivatives at the value of ϕ when the scale that fixes the CMB quadrupole crossed the Hubble radius during inflation:

$$\begin{aligned} V = & 1.8T\,{m_{\rm Pl}}^4, \\ V' = & \pm\sqrt{\frac{8\pi}{7}\frac{T}{S}}\,V/m_{\rm Pl}, \\ V'' = & 4\pi\left[(n-1)+\frac{3}{7}\frac{T}{S}\right]\,V/{m_{\rm Pl}}^2, \end{aligned} \tag{15.50}$$

where the factor 1.8 depends upon the composition of the Universe and is given for $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$. The key to learning about the inflationary potential is measuring the ratio of the gravity-wave to density-perturbation contributions to the CMB quadrupole anisotropy. From that ratio, T/S, and the quadrupole anisotropy ($= T + S$), which has been measured by COBE, one can infer T. Further, if the spectrum of the inflation-produced gravity waves (n_T) can be measured, there is an important consistency test: inflation predicts $T/S = -4.9\ n_T$ (for $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$).

15.3.3. *The new standard model*:

Motivated by the predictions of inflation and the best fit for the matter/energy content of the Universe, a standard model is emerging: ΛCDM. The model is characterized by its energy/matter content; in terms of the critical density, 65% vacuum energy, 30% cold dark matter particles, and 5% baryons with a tiny bit of hot dark matter. It is a flat ($k = 0$) model with a Hubble constant of about $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and inflation-produced density perturbations that are close to being scale invariant. This model embodies all the successes of the hot big-bang cosmology, as well as the aspirations of inner space/outer space connection. Just as importantly, it is consistent with a very large (and rapidly growing) body of cosmological observations, including, the age of the Universe, the power spectrum of inhomogeneity, the CMB anisotropy measurements from $0.1°$ to $100°$, the studies of the abundance and evolution of galaxies and clusters, the mapping of dark matter in clusters and galaxies, further supernovae observations, and more.

But important questions remain: If there is dark energy, is it just a cosmological constant, and if so why is it so small? What is the nonbaryonic dark matter? What is the primeval spectrum of inhomogeneity and is it consistent with the simplest models of inflation? What are the underlying model parameters of inflation? How does baryogenesis work and can the baryon asymmetry be related to laboratory measurements of CP violation, neutrino masses or proton decay? Is there a fundamental explanation for the odd matter/energy recipe for our Universe? Will one of the seemingly minor puzzles that exist today (the sheet like structures separated by 125 h^{-1} Mpc or the disagreement between theory and observation about the structure of CDM halos) unravel the whole picture? With the flood of observations and data that are coming, we can hope to answer these questions and more in the next decade or so.

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