11. THE CABIBBO-KOBAYASHI-MASKAWA QUARK-MIXING MATRIX

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In the Standard Model with SU(2) × U(1) as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix relating these bases was defined for six quarks and given an explicit parametrization by Kobayashi and Maskawa [1] in 1973. It generalizes the four-quark case, where the matrix is parametrized by a single angle, the Cabibbo angle [2].

By convention, the mixing is often expressed in terms of a 3×3 unitary matrix V operating on the charge $-e/3$ quark mass eigenstates (d, s, and b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{11.1}$$

The values of individual matrix elements can in principle all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Using the constraints discussed below together with unitarity, and assuming only three generations, the 90% confidence limits on the magnitude of the elements of the complete matrix are

$$\begin{pmatrix} 0.9742 \text{ to } 0.9757 & 0.219 \text{ to } 0.226 & 0.002 \text{ to } 0.005 \\ 0.219 \text{ to } 0.225 & 0.9734 \text{ to } 0.9749 & 0.037 \text{ to } 0.043 \\ 0.004 \text{ to } 0.014 & 0.035 \text{ to } 0.043 & 0.9990 \text{ to } 0.9993 \end{pmatrix} . \tag{11.2}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of others.

There are several parametrizations of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. We advocate a "standard" parametrization [3] of V that utilizes angles θ_{12}, θ_{23}, θ_{13}, and a phase, δ_{13}

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} , \tag{11.3}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for the "generation" labels $i, j = 1, 2, 3$. This has distinct advantages of interpretation, for the rotation angles are defined and labelled in a way that relates to the mixing of two specific generations and if one of these angles vanishes, so does the mixing between those two generations; in the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified with the Cabibbo angle [2]. The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases.

The matrix elements in the first row and third column, which can be directly measured in decay processes, are all of a simple form, and, as c_{13} is known to deviate from unity only in the sixth decimal place, $V_{ud} = c_{12}$, $V_{us} = s_{12}$, $V_{ub} = s_{13}\ e^{-i\delta_{13}}$, $V_{cb} = s_{23}$, and $V_{tb} = c_{23}$ to an excellent approximation. The phase δ_{13} lies in the range $0 \le \delta_{13} < 2\pi$, with nonzero values generally breaking CP invariance for the weak interactions. The generalization to the n generation case contains $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases. The range of matrix elements in Eq. (11.2) corresponds to 90% CL limits on the sines of the angles of $s_{12} = 0.219$ to 0.226, $s_{23} = 0.037$ to 0.043, and $s_{13} = 0.002$ to 0.005.

Kobayashi and Maskawa [1] originally chose a parametrization involving the four angles θ_1, θ_2, θ_3, and δ:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} , \tag{11.4}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1, 2, 3$. In the limit $\theta_2 = \theta_3 = 0$, this reduces to the usual Cabibbo mixing with θ_1 identified (up to a sign) with the Cabibbo angle [2]. Several different forms of the Kobayashi-Maskawa parametrization are found in the literature. Since all these parametrizations are referred to as "the" Kobayashi-Maskawa form, some care about which one is being used is needed when the quadrant in which δ lies is under discussion.

A popular approximation that emphasizes the hierarchy in the size of the angles, $s_{12} \gg s_{23} \gg s_{13}$, is due to Wolfenstein [4], where one sets $\lambda \equiv s_{12}$, the sine of the Cabibbo angle, and then writes the other elements in terms of powers of λ:

$$V \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \tag{11.5}$$

with A, ρ, and η real numbers that were intended to be of order unity.

More recently, another parametrization has been advocated [5]. It arises in many theories of quark masses and is particularly useful where one builds models in which initially $m_u = m_d = 0$ and there is no nontrivial phase in the CKM matrix. In this parametrization [5] no phases occur in the third row or third column of the CKM matrix, so that the CP-violating phase only occurs in the CKM matrix elements connecting first and second generation quarks. Consequently, the connection between measurements of CP-violating effects for B mesons and single CKM parameters is less obvious than in the standard parametrization.

No physics can depend on which of the above parametrizations (or any other) is used, as long as a single one is used consistently and care is taken to be sure that no other choice of phases is in conflict.

Our present knowledge of the matrix elements comes from the following sources:

(1)$|V_{ud}|$: Analyses have been performed comparing nuclear beta decays that proceed through a vector current to muon decay. Radiative corrections are essential to extracting the value of the matrix element. They already include [6] effects of order $Z\alpha^2$, and most of the theoretical argument centers on the nuclear mismatch and structure-dependent radiative corrections [7,8]. New data have been obtained on superallowed $0^+ \rightarrow 0^+$ beta decays [9].

Taking the complete data set, a value of $|V_{ud}| = 0.9740 \pm 0.0005$ has been obtained [10]. It has been argued [11] that the change in charge-symmetry-violation for quarks inside nucleons that are in nuclear matter results in an additional change in the predicted decay rate by 0.075 to 0.2%, leading to a systematic underestimate of $|V_{ud}|$. This reasoning has been used [12] to explain quantitatively the binding energy differences of the valence protons and neutrons of mirror nuclei. While it can be argued [10] that there may be double-counting of corrections, until this is settled, we take this correction as an additional uncertainty to obtain a value of $|V_{ud}| = 0.9740 \pm 0.0010$.

The theoretical uncertainties in extracting a value of $|V_{ud}|$ from neutron decays are significantly smaller than for decays of mirror nuclei, but the value depends on both the value of g_A/g_V and the neutron lifetime. Experimental progress has been made on the former quantity using very highly polarized cold neutrons together with improved detectors. Averaging over recent experiments [13] gives $g_A/g_V = -1.2715 \pm 0.0021$ and results in $|V_{ud}| = 0.9728 \pm 0.0012$ from neutron decay. Since most of the contributions to the errors in these two determinations of $|V_{ud}|$ are independent, we average them to obtain

$$|V_{ud}| = 0.9735 \pm 0.0008 \ . \tag{11.6}$$

(2)$|V_{us}|$: Analysis of K_{e3} decays yields [14]

$$|V_{us}| = 0.2196 \pm 0.0023 \ . \tag{11.7}$$

With isospin violation taken into account in K^+ and K^0 decays, the extracted values of $|V_{us}|$ are in agreement at the 1% level. A reanalysis [8] obtains essentially the same value, but quotes a somewhat smaller error, which is only statistical. The analysis [15] of hyperon decay data has larger theoretical uncertainties because of first order SU(3) symmetry breaking effects in the axial-vector couplings. This has been redone incorporating second order SU(3) symmetry breaking corrections in models [16] applied to the WA2 data [17] to give a value of $|V_{us}| = 0.2176 \pm 0.0026$, which is consistent with Eq. (11.7) using the "best-fit" model. Since the values obtained in the models differ outside the errors and generally do not give good fits, we retain the value in Eq. (11.7) for $|V_{us}|$.

(3)$|V_{cd}|$: The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. The dimuon production cross sections of the CDHS group [18] yield $\overline{B}_c\, |V_{cd}|^2 = 0.41 \pm 0.07 \times 10^{-2}$, where $\overline{B}_c$ is the semileptonic branching fraction of the charmed hadrons produced. The corresponding value from the more recent CCFR Tevatron experiment [19], where a next-to-leading-order QCD analysis has been carried out, is $0.534 \pm 0.021^{+0.025}_{-0.051} \times 10^{-2}$, where the last error is from the scale uncertainty. Assuming a similar scale error for the CDHS result and averaging these two results gives $0.49 \pm 0.05 \times 10^{-2}$. Supplementing this with data [20] on the mix of charmed particle species produced by neutrinos and PDG values for their semileptonic branching fractions (to give [19] $\overline{B}_c = 0.099 \pm 0.012$) then yields

$$|V_{cd}| = 0.224 \pm 0.016 \ . \tag{11.8}$$

(4)$|V_{cs}|$: Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange-quark density in the parton sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an SU(3)-symmetric sea, leads to a lower bound [18], $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of D_{e3} decay with the expression [21] that follows from the standard weak interaction amplitude:

$$\Gamma(D \to \overline{K} e^+ \nu_e) = |f_+^D(0)|^2 \ |V_{cs}|^2 \ (1.54 \times 10^{11} \ \mathrm{s}^{-1}) \ . \tag{11.9}$$

Here $f_+^D(q^2)$, with $q = p_D - p_K$, is the form factor relevant to D_{e3} decay; its variation has been taken into account with the parametrization $f_+^D(t)/f_+^D(0) = M^2/(M^2 - t)$ and $M = 2.1$ GeV/c^2, a form and mass consistent with direct measurements [22]. Combining data on branching fractions for D_{e3} decays with accurate values for the D lifetimes [22] yields a value of $(0.818 \pm 0.041) \times 10^{11}$ s^{-1} for $\Gamma(D \to \overline{K} e^+ \nu_e)$. Therefore

$$|f_+^D(0)|^2 \ |V_{cs}|^2 = 0.531 \pm 0.027 \ . \tag{11.10}$$

A very conservative assumption is that $|f_+^D(0)| < 1$, from which it follows that $|V_{cs}| > 0.62$. Calculations of the form factor either performed [23,24] directly at $q^2 = 0$, or done [25] at the maximum value of $q^2 = (m_D - m_K)^2$ and interpreted at $q^2 = 0$ using the measured q^2 dependence, give the value $f_+^D(0) = 0.7 \pm 0.1$. It follows that

$$|V_{cs}| = 1.04 \pm 0.16 \ . \tag{11.11}$$

Recent measurements [26] of $|V_{cs}|$ in charmed-tagged W decays give a consistent result of $|V_{cs}| = 0.97 \pm 0.09$ (stat.) $\pm\, 0.07$ (syst.). The constraint of unitarity when there are only three generations gives a much tighter bound (see below).

(5)$|V_{cb}|$: The heavy quark effective theory [27] (HQET) provides a nearly model-independent treatment of B semileptonic decays to charmed mesons, assuming that both the b and c quarks are heavy enough for the theory to apply. Measurements of the exclusive decay $B \to \overline{D}^* \ell^+ \nu_\ell$ have been used primarily to extract a value of $|V_{cb}|$ using corrections based on the HQET. Exclusive $B \to \overline{D} \ell^+ \nu_\ell$ decays give a consistent but less precise result. Analysis of inclusive decays, where the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual V–A interaction, depends on going from the quark to the hadron level. This is also understood within the context of the HQET [28], and the results for $|V_{cb}|$ are again consistent with those from exclusive decays. Combining all the LEP data on both exclusive and inclusive decays gives [29]

$$|V_{cb}| = 0.0402 \pm 0.0019 \ , \tag{11.12}$$

which is consistent with the latest CLEO result [29] from exclusive and inclusive decays, $|V_{cb}| = 0.0404 \pm 0.0034$. The combination of large data samples and the HQET make this the third most accurately measured CKM matrix element, after $|V_{ud}|$ and $|V_{us}|$.

(6)$|V_{ub}|$: The decay $b \to u\ell\overline{\nu}$ and its charge conjugate can be observed from the semileptonic decay of B mesons produced on the $\Upsilon(4S)$ $(b\overline{b})$ resonance by measuring the lepton energy spectrum above the endpoint of the $b \to c\ell\overline{\nu}_\ell$ spectrum. There the $b \to u\ell\overline{\nu}_\ell$ decay rate can be obtained by subtracting the background from nonresonant e^+e^- reactions. This continuum background is determined from auxiliary measurements off the $\Upsilon(4S)$. The interpretation of the result in terms of $|V_{ub}/V_{cb}|$ depends fairly strongly on the theoretical model used to generate the lepton energy spectrum, especially for $b \to u$ transitions [24,25,30].

The LEP experiments ALEPH [31], L3 [32], and DELPHI [33] have presented new analyses that measure the $b \to u\ell\nu_\ell$ component in b decays at the Z^0. Discrimination between u-like and c-like decays is based on up to 20 different event parameters which are sensitive to the mass of the quark of the final state. Using an extended range of the spectrum compared to the end-point analysis, this extraction of $|V_{ub}|$ is less sensitive to theoretical assumptions, but requires a detailed understanding of the decay $b \to c\ell\nu_\ell$.

The value of $|V_{ub}|$ can also be extracted from exclusive decays, such as $B \to \pi\ell\nu_\ell$ and $B \to \rho\ell\nu_\ell$, but there is an associated theoretical model dependence in the values of the matrix elements of the weak current between exclusive states. There has been a substantial increase in the data from CLEO for these exclusive decays [29], and the error on $|V_{ub}|$, arising primarily from the theoretical model dependence, is comparable to that obtained from inclusive decays. Enhanced awareness of the theoretical uncertainties and the difference between the results obtained from inclusive and exclusive analyses leads us to be even more conservative in setting the error bar than in previous reviews and we quote [34]

$$|V_{ub}/V_{cb}| = 0.090 \pm 0.025 \ . \tag{11.13}$$

(7)V_{tb}: The discovery of the top quark by the CDF and DØ collaborations utilized in part the semileptonic decays of t to b. One can set a (still rather crude) limit on the fraction of decays of the form $t \to b \ \ell^+ \ \nu_\ell$, as opposed to semileptonic t decays that involve s or d quarks, of [35]

$$\frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.99 \pm 0.29 \ . \tag{11.14}$$

(8)*Hadronic W decays*: The ratio of hadronic W decays to leptonic decays has been measured at LEP, with the result [36] that $\Sigma_{i,j}|V_{ij}|^2 = 2.032 \pm 0.032$, where the sum extends over $i = u, c$ and $j = d, s, b$. With a three-generation CKM matrix, from unitarity this sum would be expected to have the value 2. Since five of the CKM matrix elements are well measured or contribute negligibly to the sum of the squares, this measurement can also be used as a precision measurement of $|V_{cs}| = 0.9891 \pm 0.016$.

For most of these CKM matrix elements the principal error is no longer experimental, but rather theoretical. This arises from explicit model dependence in interpreting data or in the use of specific hadronic matrix elements to relate experimental measurements to weak transitions of quarks. This type of uncertainty arises even more strongly in extracting CKM matrix elements from loop diagrams, as discussed below. Such errors are not distributed in a Gaussian manner. We have taken the interpretation that a "1 σ" range in a theoretical error corresponds to a 68% likelihood that the true value lies within "± 1 σ" of the central value. While we do use the central values with the quoted errors to make a best overall fit to the CKM matrix, the result should be taken with appropriate care, and we regard extending this to multi-standard-deviation determinations of allowed regions for CKM matrix elements as unfounded.

The results for three generations of quarks, from Eqs. (11.6)–(11.8) and Eqs. (11.11)–(11.14), plus unitarity, are summarized in the matrix in Eq. (11.2). The ranges given there are different from those given in Eqs. (11.6)–(11.14) because of the inclusion of unitarity, but are consistent with the one-standard-deviation errors on the input matrix elements. Note in particular that the unitarity constraint has pushed $|V_{ud}|$ about one standard deviation higher than given in Eq. (11.6). If we had kept the error on $|V_{ud}|$ quoted by Hardy and Towner [10], we would have a violation of unitarity in the first row of the CKM matrix by about two standard deviations. While this bears watching and encourages another more accurate measurement of $|V_{us}|$, we do not see this presently as a major challenge to the validity of the three-generation Standard Model.

The data do not preclude there being more than three generations. Moreover, the entries deduced from unitarity might be altered when the CKM matrix is expanded to accommodate more generations. Conversely, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that any additional element in the first row have a magnitude $|V_{ub'}| < 0.10$. When there are more than three generations, the allowed ranges (at 90% CL) of the matrix elements connecting the first three generations are

$$\begin{pmatrix} 0.9722 \text{ to } 0.9748 & 0.216 \text{ to } 0.223 & 0.002 \text{ to } 0.005 & \dots \\ 0.209 \text{ to } 0.228 & 0.959 \text{ to } 0.976 & 0.037 \text{ to } 0.043 & \dots \\ 0 \text{ to } 0.09 & 0 \text{ to } 0.16 & 0.07 \text{ to } 0.993 & \dots \\ \vdots & \vdots & \vdots & \end{pmatrix}, \tag{11.15}$$

where we have used unitarity (or the expanded matrix) and the measurements of the magnitudes of the CKM matrix elements (including the constraint from hadronic W decays), resulting in the weak bound $|V_{tb}| > 0.07$.

Further information, particularly on CKM matrix elements involving the top quark, can be obtained from flavor-changing processes that occur at the one-loop level. We have not used this information in the discussion above since the derivation of values for V_{td} and V_{ts} in this manner from, for example, B mixing or $b \to s\gamma$, require an additional assumption that the top-quark loop, rather than new physics, gives the dominant contribution to the process in question. Conversely, the agreement of CKM matrix elements extracted from loop diagrams with the values based on direct measurements and three generations can be used to place restrictions on new physics.

The measured value [37] of $\Delta M_{B_d} = 0.473 \pm 0.016$ ps^{-1} from $B_d^0 - \overline{B}_d^0$ mixing can be turned in this way into information on $|V_{tb}{}^* V_{td}|$, assuming that the dominant contribution to the mass difference arises from the matrix element between a B_d and a $\overline{B}_d$ of an operator that corresponds to a box diagram with W bosons and top quarks as sides. Using the characteristic hadronic matrix element that then occurs, $\widehat{B}_{B_d} f_{B_d}{}^2 = (210 \pm 40 \text{ MeV})^2$ from lattice QCD calculations [38], which we regard as having become the most reliable source of such matrix elements, next-to-leading-order QCD corrections ($\eta_{\text{QCD}} = 0.55$) [39], and the running top-quark mass, $\overline{m}_t(m_t) = 166 \pm 5$ GeV, as input,

$$|V_{tb}{}^* \cdot V_{td}| = 0.0083 \pm 0.0016 \ , \tag{11.16}$$

where the uncertainty comes primarily from that in the hadronic matrix elements, whose estimated errors are combined linearly.

In the ratio of B_s to B_d mass differences, many common factors (such as the QCD correction and dependence on the top-quark mass) cancel, and we have

$$\frac{\Delta M_{B_s}}{\Delta M_{B_d}} = \frac{M_{B_s}}{M_{B_d}} \frac{\widehat{B}_{B_s} f_{B_s}^2}{\widehat{B}_{B_d} f_{B_d}^2} \frac{|V_{tb}^* \cdot V_{ts}|^2}{|V_{tb}^* \cdot V_{td}|^2} \ . \tag{11.17}$$

With the experimentally measured masses [22], $\widehat{B}_{B_s}/\widehat{B}_{B_d} = (1.14 \pm 0.13)^2$ with quite conservative error bars from lattice QCD [38], and the improved experimental lower limit [37] at 95% CL of $\Delta M_{B_s} > 14.3$ ps^{-1},

$$|V_{td}|/|V_{ts}| < 0.24 \tag{11.18}$$

Since with three generations, $|V_{ts}| \approx |V_{cb}|$, this result converts to $|V_{td}| < 0.010$, which is a significant constraint by itself (see Fig. 11.2).

The CLEO observation [40] of $b \to s\gamma$ can be translated [41] similarly into $|V_{ts}|/|V_{cb}| = 1.1 \pm 0.43$, where the large uncertainty is again dominantly theoretical. In $K^+ \to \pi^+ \nu\overline{\nu}$ there are significant contributions from loop diagrams involving both charm and top quarks. Experiment is just beginning to probe the level predicted in the Standard Model [42].

All these additional indirect constraints are consistent with the matrix elements obtained from the direct measurements plus unitarity, assuming three generations; with the recent results on B mixing and theoretical improvements in lattice calculations, adding the indirect constraints to the fit reduces the range allowed for $|V_{td}|$.

Direct and indirect information on the CKM matrix is neatly summarized in terms of "the unitarity triangle," one of six such triangles that correspond to the unitarity condition applied to two different rows or columns of the CKM matrix. Unitarity of the 3×3 CKM matrix applied to the first and third columns yields

$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td} V_{tb}^* = 0 \ . \tag{11.19}$$

The unitarity triangle is just a geometrical presentation of this equation in the complex plane [43]. We can always choose to orient the triangle so that $V_{cd}\ \ V_{cb}{}^*$ lies along the horizontal; in the parametrization we have chosen, V_{cb} is real, and V_{cd} is real to a very good approximation in any case. Setting cosines of small angles to unity, Eq. (11.19) becomes

$$V_{ub}{}^* + V_{td} = s_{12}\ \ V_{cb}{}^* \ , \tag{11.20}$$

which is shown as the unitarity triangle in Fig. 11.1(a). Rescaling the triangle by a factor $[1/|s_{12}\ \ V_{cb}|]$ so that the base is of unit length, the coordinates of the vertices become

$$A\big(\mathrm{Re}(V_{ub})/|s_{12}\ \ V_{cb}|\ ,\quad -\mathrm{Im}(V_{ub})/|s_{12}\, V_{cb}|\big)\ ,\ \ B(1,0)\ ,\ \ C(0,0)\ . \tag{11.21}$$

In the Wolfenstein parametrization [4], the coordinates of the vertex A of the unitarity triangle are simply (ρ, η), as shown in Fig. 11.1(b). The angle $\gamma = \delta_{13}$.

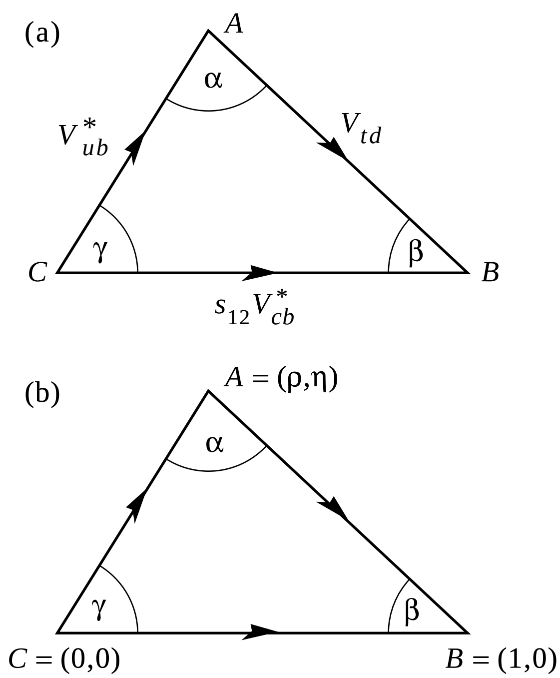

Figure 11.1: (a) Representation in the complex plane of the triangle formed by the CKM matrix elements $V_{ub}{}^*$, V_{td}, and $s_{12}\ \ V_{cb}{}^*$. (b) Rescaled triangle with vertices $A(\rho, \eta)$, $B(1,0)$, and $C(0,0)$.

CP-violating processes will involve the phase in the CKM matrix, assuming that the observed CP violation is solely related to a nonzero value of this phase. This allows additional constraints to be brought to bear. More specifically, a necessary and sufficient condition for CP violation with three generations can be formulated in a parametrization-independent manner in terms of the nonvanishing of the determinant of the commutator of the mass matrices for the charge $2e/3$ and charge $-e/3$ quarks [44]. CP-violating amplitudes or differences of rates are all proportional to the CKM factor in this quantity. This is the product of factors $s_{12}s_{13}s_{23}c_{12}c_{13}^2c_{23}s_{\delta_{13}}$ in the parametrization adopted above, and is $s_1^2s_2s_3c_1c_2c_3s_\delta$ in that of Ref. 1. With the approximation of setting cosines to unity, this is just twice the area of the unitarity triangle.

While hadronic matrix elements whose values are imprecisely known generally enter the calculations, the constraints from CP violation in the neutral kaon system, taken together with the restrictions on the magnitudes of the CKM matrix elements shown above, are tight enough to restrict considerably the range of angles and the phase of the CKM matrix. For example, the constraint obtained from the CP-violating parameter ϵ in the neutral K system corresponds to the vertex A of the unitarity triangle lying on a hyperbola for fixed values of the hadronic matrix elements [45,46]. In addition, following the initial evidence [47], it is now established that direct CP violation in the weak transition from a neutral K to two pions exists, *i.e.*, that the parameter ϵ' is nonzero [48]. However, theoretical uncertainties in hadronic matrix elements of cancelling amplitudes presently preclude this measurement from giving a significant constraint on the unitarity triangle.

The constraints on the vertex of the unitarity triangle that follow from $|V_{ub}|$, B mixing, and ϵ are shown in Fig. 11.2. The improved limit in Eq. (11.18) that arises from the ratio of B_s to B_d mixing eliminates a significant region for the vertex A of the unitarity triangle, a region otherwise allowed by direct measurements of the CKM matrix elements, essentially limiting the vertex A to be in the first quadrant (ρ positive). The limit is not far from the value we would expect from the other information on the unitarity triangle. Thus a significant increase in experimental sensitivity to B_s mixing will lead either to an observation of mixing or an indication of physics beyond the Standard Model. This limit is more robust theoretically since it depends on ratios (rather than absolute values) of hadronic matrix elements and is independent of the top mass or QCD corrections (which cancel in the ratio).

Ultimately in the Standard Model, the CP-violating process $K_L \to \pi^0\nu\overline{\nu}$ offers high precision in measuring the imaginary part of $V_{td} \cdot V_{ts}{}^*$, which, given V_{ts}, will yield the altitude of the unitarity triangle. However, the experimental upper limit is presently many orders of magnitude away from the requisite sensitivity.

For CP-violating asymmetries of neutral B mesons decaying to CP eigenstates, for certain final states arising from a single weak decay amplitude there is a direct relationship between the magnitude of the asymmetry in a given decay and $\sin 2\phi$, where $\phi = \alpha$, β, γ is an appropriate angle of the unitarity triangle [43]. The CDF Collaboration has used the decay $B_d(\overline{B}_d) \to \psi K_S$ to obtain a first indication [49] of a nonvanishing asymmetry,

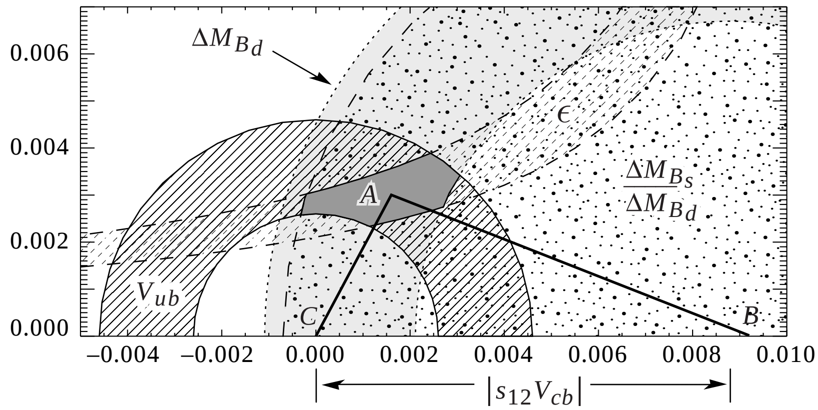

Figure 11.2: Constraints on the position of the vertex, A, of the unitarity triangle following from $|V_{ub}|$, B mixing, and the CP-violating parameter ϵ. A possible unitarity triangle is shown with A in the preferred region.

corresponding to a value of $\sin 2\beta$:

$$\sin 2\beta = 0.79^{+0.41}_{-0.44} \ . \tag{11.22}$$

This is consistent with the other information in Fig. 11.2 including having the correct sign, which is positive at the 93% CL. It presages the data that will be obtained in the next several years on both the magnitudes and relative phases of the CKM matrix elements, permitting incisive tests of this part of the Standard Model. (See Sec. 12 on CP Violation and the review on "CP Violation in B decay—Standard Model Predictions" in the B Listings.)

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