FITS FOR K_L^0 CP-VIOLATION PARAMETERS

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In recent years, K_L^0 CP-violation experiments have improved our knowledge of CP-violation parameters and their consistency with the expectations of CPT invariance and unitarity. For definitions of K_L^0 CP-violation parameters and a brief discussion of the theory, see the article "CP Violation" by L. Wolfenstein in Section 12 of this *Review*.

This note describes our two fits for the CP-violation parameters in $K_L^0 \to \pi^+\pi^-$ and $\pi^0\pi^0$ decay, one for the phases ϕ_{+-} and ϕ_{00}, and another for the amplitudes $|\eta_{+-}|$ and $|\eta_{00}|$.

Fit to ϕ_{+-}, ϕ_{00}, $\Delta\phi$, Δm, and τ_S data: We perform a joint fit to the data on ϕ_{+-}, ϕ_{00}, the phase difference $\Delta\phi = \phi_{00} - \phi_{+-}$, the $K_L^0 - K_S^0$ mass difference Δm, and the K_S^0 mean life τ_S, including the effects of correlations. Measurements of ϕ_{+-} and ϕ_{00} are highly correlated with Δm and τ_S. Some measurements of τ_S are correlated with Δm. The correlations are given in the footnotes of the ϕ_{+-} and ϕ_{00} sections of the K_L^0 Particle Listings and the τ_S section of the K_S^0 Particle listings. In editions of the Review prior to 1996, we adjusted the experimental values of ϕ_{+-} and ϕ_{00} to account for correlations with Δm and τ_S but did not include the effects of these correlations when evaluating Δm and τ_S. In 1996, we introduced a joint fit including these correlations. In this fit, the ϕ_{+-} measurements have a strong influence on the fitted value of Δm. This is because the CERN NA31 vacuum regeneration experiments (CAROSI 90 [1] and GEWENIGER 74B [2]), the Fermilab E773/E731 regenerator experiments (SCHWINGENHEUER 95 [3] and GIBBONS 93 [4]), and the CPLEAR $K^0 - \overline{K}^0$ asymmetry experiment (APOSTOLAKIS 99C [5]) have very different dependences of ϕ_{+-} on Δm, as can be seen from their diagonal bands in Fig. 1.

The region where the ϕ_{+-} bands from these experiments cross gives a powerful measurement of Δm which decreases the fitted Δm value relative to our pre-1996 average Δm and earlier measurements such as CULLEN 70 [6], GEWENIGER 74C [7], and GJESDAL 74 [8]. This decrease brings the Δm-dependent ϕ_{+-} measurements into good agreement with each other and with ϕ(superweak), where

$$\phi(\text{superweak}) = \tan^{-1}\left(\frac{2\Delta m}{\Delta\Gamma}\right) = \tan^{-1}\left(\frac{2\Delta m \tau_S \tau_L}{\hbar(\tau_L - \tau_S)}\right) . \quad (1)$$

The (ϕ_{+-}, τ_S) correlations influence the τ_S fit result in a similar manner, as can be seen in Fig. 2. The influence of the ϕ_{+-} experiments is not as great on τ_S as it is on Δm because the indirect measurements of τ_S derived from the diagonal crossing bands in Fig. 2 are not as precise as the direct measurements of τ_S from E773 (SCHWINGENHEUER 95 [3]), E731 (GIBBONS 93 [4]), and NA31 (BERTANZA 97 [9]).

In Fig. 1 [Fig. 2] the slope of the diagonal ϕ_{+-} bands shows the Δm [τ_S] dependence; the unseen τ_S [Δm] dependent term is evaluated using the fitted τ_S [Δm]. The vertical half-width σ_ϕ of each band is the ϕ_{+-} error for fixed Δm [τ_S] and includes the systematic error due to the error in the fitted τ_S [Δm].

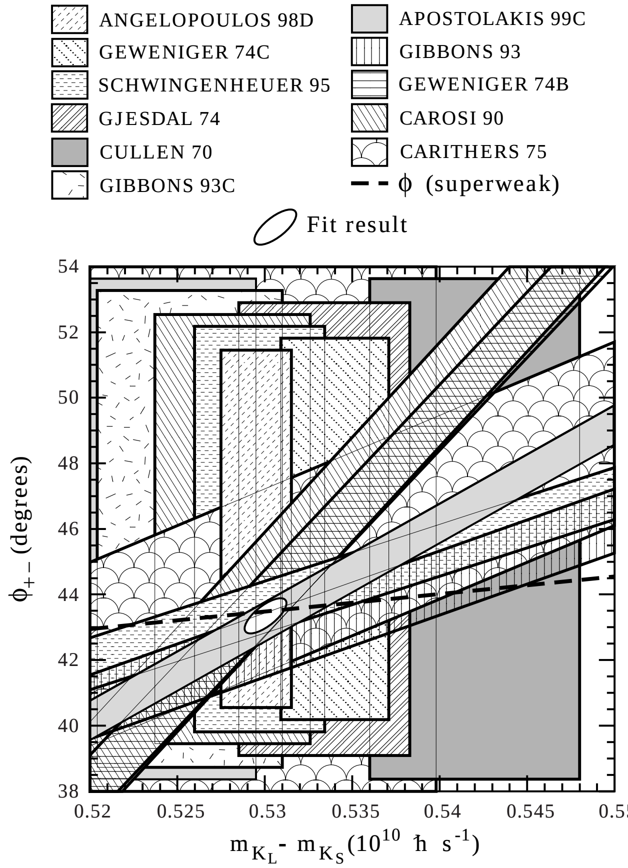

Figure 1: ϕ_{+-} vs Δm. Δm measurements appear as vertical bands spanning $\Delta m \pm 1\sigma$, some of which are cut near the top to aid the eye. The ϕ_{+-} measurements appear as diagonal bands spanning $\phi_{+-} \pm \sigma_\phi$. The dashed line shows ϕ(superweak). The ellipse shows the 1σ contour of the fit result. See Table 1 for data references.

Table 2 gives the resulting fit values for the parameters and Table 3 gives the correlation matrix. The resulting ϕ_{+-} is in good agreement with $\phi(\text{superweak}) = 43.49 \pm 0.07^\circ$ obtained from Eq. (1) using Δm and τ_S from Table 2.

The χ^2 is 16.0 for 20 degrees of freedom, indicating good agreement of the input data. Nevertheless, there has been criticism that Fermilab E773 (SCHWINGENHEUER 95 [3]) and E731 (GIBBONS 93 [4]) measure $\phi_{+-} - \phi_f$ and calculate the regeneration phase ϕ_f from the power law momentum dependence of the regeneration amplitude using analyticity and dispersion relations. In the E731 result, a systematic error of ± 0.5 degrees for departures from a pure power-law is included. For the E773 result, they modeled a variety of effects that do distort the amplitude from a pure power law and ascribed a $\pm 0.35^\circ$ systematic error from uncertainties in these effects. Even so, the E731 result remains valid within its quoted errors. KLEINKNECHT 94 [16] and KLEINKNECHT 95 [17] argue that these systematic errors should be around 3°, primarily because of the absence of data on the momentum dependence of the regeneration amplitude above 160 GeV/c. BRIERE 95 [18] and BRIERE 95C [19] reply that the current understanding

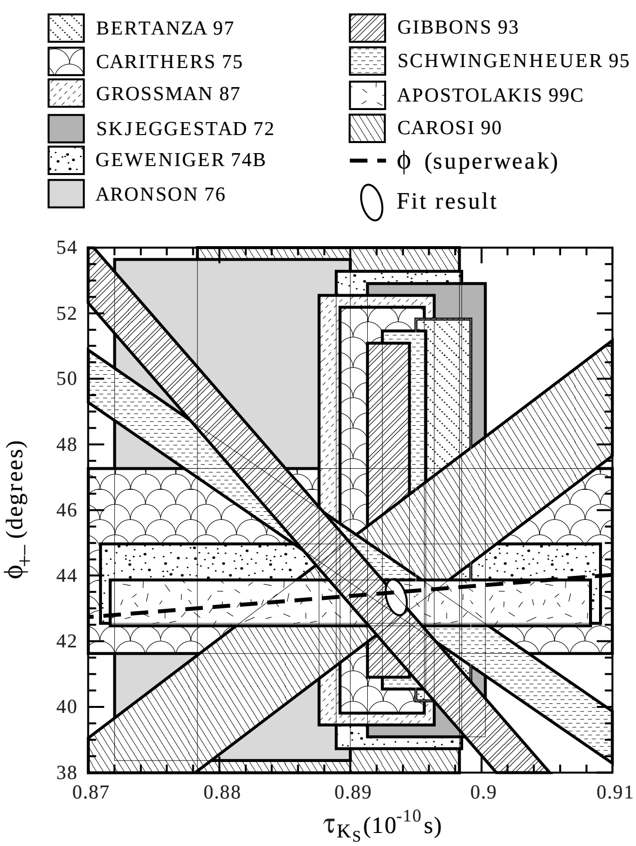

Figure 2: ϕ_{+-} vs τ_S. τ_S measurements appear as vertical bands spanning $\tau_S \pm 1\sigma$, some of which are cut near the top to aid the eye. The ϕ_{+-} measurements appear as diagonal bands spanning $\phi_{+-} \pm \sigma_\phi$. The dashed line shows ϕ(superweak). The ellipse shows the fit result's 1σ contour. See Table 1 for data references.

Table 1: References and location of input data for Fig. 1 and Fig. 2. Unless otherwise indicated by a footnote, a check (✓) indicates that the data can be found in the ϕ_{+-} or Δm sections of the K_L Particle Listings, or the τ_S section of the K_S Particle Listings, according to the column headers.

Location of input data					
Fig. 1		Fig. 2			
ϕ_{+-}	Δm	ϕ_{+-}	τ_S	PDG Document ID	Ref.
✓	✓	✓		APOSTOLAKIS 99C	[5]
✓		✓	✓	GIBBONS 93	[4]
✓	✓	✓	✓	SCHWINGENHEUER 95	[3]
✓		✓	✓	GEWENIGER 74B	[2]
✓	✓*	✓	✓*	CAROSI 90	[1]
✓	✓†	✓	✓	CARITHERS 75	[10]
	✓			ANGELOPOULOS 98D	[11]
	✓			GEWENIGER 74C	[7]
	✓			GJESDAL 74	[8]
	✓			CULLEN 70	[6]
	✓			GIBBONS 93C	[12]
			✓	BERTANZA 97	[9]
			✓	GROSSMAN 87	[13]
			✓	SKJEGGESTAD 72	[14]
			✓	ARONSON 76	[15]

* from $\phi_{00}(\Delta m, \tau_S)$ in ϕ_{00} Particle Listings.
† from $\tau_S(\Delta m)$ in τ_S Particle Listings.

Table 2: Results of the fit for ϕ_{+-}, ϕ_{00}, $\phi_{00} - \phi_{+-}$, Δm, and τ_S. The fit has $\chi^2 = 16.0$ for 20 degrees of freedom (24 measurements -5 parameters $+1$ constraint).

Quantity	Fit Result
ϕ_{+-}	$43.3 \pm 0.5°$
Δm	$(0.5300 \pm 0.0012) \times 10^{10}\hbar$ s^{-1}
τ_S	$(0.8935 \pm 0.0008) \times 10^{-10}$s
ϕ_{00}	$43.2 \pm 1.0°$
$\Delta\phi$	$-0.1 \pm 0.8°$

Table 3: Correlation matrix for the fitted parameters.

	ϕ_{+-}	Δm	τ_S	ϕ_{00}	$\Delta\phi$
ϕ_{+-}	1.00	0.71	−0.30	0.54	−0.02
Δm	0.71	1.00	−0.19	0.43	0.04
τ_S	−0.30	−0.19	1.00	−0.14	0.04
ϕ_{00}	0.54	0.43	−0.14	1.00	0.83
$\Delta\phi$	−0.02	0.04	0.04	0.83	1.00

of regeneration is sufficient to allow a precise and reliable correction for the region above 160 GeV/c. The question is one of judgement about the reliability of the assumptions used. In the absence of any contradictory evidence, we choose to accept the judgement of the E731/E773 experimenters in setting their systematic errors.

Fit for ϵ'/ϵ, $|\eta_{+-}|$, $|\eta_{00}|$, and B($K_L \to \pi\pi$)

We list measurements of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$ and ϵ'/ϵ. Independent information on $|\eta_{+-}|$ and $|\eta_{00}|$ can be obtained from measurements of the K_L^0 and K_S^0 lifetimes (τ_L, τ_S) and branching ratios (B) to $\pi\pi$, using the relations

$$|\eta_{+-}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^+\pi^-)}{\tau_L} \; \frac{\tau_S}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}\right]^{1/2} , \qquad (2a)$$

$$|\eta_{00}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\tau_L} \; \frac{\tau_S}{\mathrm{B}(K_S^0 \to \pi^0\pi^0)}\right]^{1/2} . \qquad (2b)$$

For historical reasons the branching ratio fits and the CP-violation fits are done separately, but we want to include the influence of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$, and ϵ'/ϵ measurements on B($K_L^0 \to \pi^+\pi^-$) and B($K_L^0 \to \pi^0\pi^0$) and vice versa. We approximate a global fit to all of these measurements by first performing two independent fits: 1) BRFIT, a fit to the K_L^0 branching ratios, rates, and mean life, and 2) ETAFIT, a fit to the $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{+-}/\eta_{00}|$, and ϵ'/ϵ measurements. The results from fit 1, along with the K_S^0 values from this edition are used to compute values of $|\eta_{+-}|$ and $|\eta_{00}|$ which are included as measurements in the $|\eta_{00}|$ and $|\eta_{+-}|$ sections with a document ID of BRFIT 00. Thus the fit values of $|\eta_{+-}|$ and $|\eta_{00}|$ given in this edition include both the direct measurements and the results from the branching ratio fit.

The process is reversed in order to include the direct $|\eta|$ measurements in the branching ratio fit. The results from fit 2 above (before including BRFIT 00 values) are used along with the K_L^0 and K_S^0 mean lives and the $K_S^0 \to \pi\pi$ branching fractions to compute the K_L^0 branching ratios $\Gamma(K_L^0 \to \pi^+\pi^-)/\Gamma(\text{total})$ and $\Gamma(K_L^0 \to \pi^0\pi^0)/\Gamma(K_L^0 \to \pi^+\pi^-)$. These branching ratio values are included as measurements in the branching ratio section with a document ID of ETAFIT 00. Thus the K_L^0 branching ratio fit values in this edition include the results of direct measurements of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$, and ϵ'/ϵ. A more detailed discussion of these fits is given in the 1990 edition of this *Review* [20].

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