35. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

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35.1. Leptoproduction

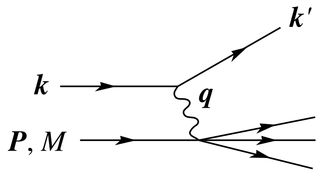

Figure 35.1: Kinematic quantities for description of lepton-nucleon scattering. k and k' are the four-momenta of incoming and outgoing leptons, P is the four-momentum of a nucleon with mass M. The exchanged particle is a γ, $W^{\pm}$, or Z^0; it transfers four-momentum $q = k - k'$ to the target.

Invariant quantities:

$\nu = \dfrac{q \cdot P}{M} = E - E'$ is the lepton's energy loss in the lab (in earlier literature sometimes $\nu = q \cdot P$). Here, E and E' are the initial and final lepton energies in the lab.

$Q^2 = -q^2 = 2(EE' - \vec{k} \cdot \vec{k}\,') - m_\ell^2 - m_{\ell'}^2$ where $m_\ell (m_{\ell'})$ is the initial (final) lepton mass. If $EE' \sin^2(\theta/2) \gg m_\ell^2,\, m_{\ell'}^2$, then

$\approx 4EE' \sin^2(\theta/2)$, where θ is the lepton's scattering angle in the lab.

$x = \dfrac{Q^2}{2M\nu}$ In the parton model, x is the fraction of the target nucleon's momentum carried by the struck quark. [See section on Quantum Chromodynamics (Sec. 9 of this *Review*.)]

$y = \dfrac{q \cdot P}{k \cdot P} = \dfrac{\nu}{E}$ is the fraction of the lepton's energy lost in the lab.

$W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$ is the mass squared of the system recoiling against the lepton.

$s = (k + P)^2 = \dfrac{Q^2}{xy} + M^2$

35.1.1. ***Leptoproduction cross sections***:

$$\frac{d^2\sigma}{dx\,dy} = \nu\,(s - M^2)\,\frac{d^2\sigma}{d\nu\,dQ^2} = \frac{2\pi\,M\nu}{E'}\frac{d^2\sigma}{d\Omega_{\rm lab}\,dE'} = x(s - M^2)\,\frac{d^2\sigma}{dx\,dQ^2}\ . \tag{35.1}$$

35.1.2. *Leptoproduction structure functions*: The neutral-current process, $eN \to eX$, at low Q^2 is just electromagnetic and parity conserving. It can be written in terms of two structure functions $F_1^{\rm em}(x,Q^2)$ and $F_2^{\rm em}(x,Q^2)$:

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\,\alpha^2(s-M^2)}{Q^4} \times \left[(1-y)\,F_2^{\rm em} + y^2\,xF_1^{\rm em} - \frac{M^2}{(s-M^2)}\,xy\,F_2^{\rm em}\right] . \tag{35.2}$$

The charged-current processes, $e^-N \to \nu X$, $\nu N \to e^-X$, and $\overline{\nu}N \to e^+X$, are parity violating and can be written in terms of three structure functions $F_1^{\rm CC}(x,Q^2)$, $F_2^{\rm CC}(x,Q^2)$, and $F_3^{\rm CC}(x,Q^2)$:

$$\frac{d^2\sigma}{dx\,dy} = \frac{G_F^2\,(s-M^2)}{2\pi}\frac{M_W^4}{(Q^2+M_W^2)^2} \tag{35.3}$$

$$\times\ \left\{\left[1-y-\frac{M^2xy}{(s-M^2)}\right]F_2^{\rm CC} + \frac{y^2}{2}\,2x\,F_1^{\rm CC} \pm (y-\frac{y^2}{2})\,x\,F_3^{\rm CC}\right\} ,$$

where the last term is positive for the e^- and ν reactions and negative for $\overline{\nu}N \to e^+X$. As explained below there are different structure functions for charge-raising and charge-lowering currents.

35.1.3. *Structure functions in the QCD parton model*: In the QCD parton model, the structure functions defined above can be expressed in terms of parton distribution functions. The quantity $f_i(x,Q^2)dx$ is the probability that a parton of type i (quark, antiquark, or gluon), carries a momentum fraction between x and $x+dx$ of the nucleon's momentum in a frame where the nucleon's momentum is large. For the cross section corresponding to the *neutral-current process* $ep \to eX$, we have for $s \gg M^2$ (in the case where the incoming electron is either left- (L) or right- (R) handed):

$$\frac{d^2\sigma}{dx\,dy} = \frac{\pi\alpha^2}{sx^2\,y^2}\left[\sum_q\left(x\,f_q\,(x,\,Q^2) + x\,f_{\overline{q}}\,(x,\,Q^2)\right)\right] \times \left[A_q + (1-y)^2\,B_q\right] . \tag{35.4}$$

Here the index q refers to a quark flavor (*i.e.*, u, d, s, c, b, or t), and

$$A_q = \left(-q_q + g_{Lq}\,g_{Le}\,\frac{Q^2}{Q^2+M_Z^2}\right)^2 + \left(-q_q + g_{Rq}\,g_{Re}\,\frac{Q^2}{Q^2+M_Z^2}\right)^2 , \tag{35.5}$$

$$B_q = \left(-q_q + g_{Rq}\,g_{Le}\,\frac{Q^2}{Q^2+M_Z^2}\right)^2 + \left(-q_q + g_{Lq}\,g_{Re}\,\frac{Q^2}{Q^2+M_Z^2}\right)^2 . \tag{35.6}$$

Here q_q is the charge of flavor q. For a left-handed electron, $g_{Re} = 0$ and $g_{Le} = (-1/2 + \sin^2\theta_W)/(\sin\theta_W\cos\theta_W)$, while for a right-handed electron, $g_{Le} = 0$ and $g_{Re} = (\sin^2\theta_W)/(\sin\theta_W\cos\theta_W)$. For the quarks, $g_{Lq} = (T_3 - q_q\sin^2\theta_W)/(\sin\theta_W\cos\theta_W)$, and $g_{Rq} = (-q_q\sin^2\theta_W)\ /\ (\sin\theta_W\cos\theta_W)$.

For neutral-current *neutrino (antineutrino) scattering*, the same formula applies with g_{Le} replaced by $g_{L\nu} = 1/(2\sin\theta_W\cos\theta_W)$ $(g_{L\overline{\nu}} = 0)$ and g_{Re} replaced by $g_{R\nu} = 0$ $[g_{R\overline{\nu}} = -1/(2\sin\theta_W\cos\theta_W)]$.

In the case of the *charged-current processes* $e^-_L p \to \nu X$ and $\overline{\nu}p \to e^+X$, Eq. (35.3) applies with

$$F_2 = 2xF_1 = 2x\Big[f_u(x,Q^2) + f_c(x,Q^2) + f_t(x,Q^2) + f_{\overline{d}}(x,Q^2) + f_{\overline{s}}(x,Q^2) + f_{\overline{b}}(x,Q^2)\Big] \ , \tag{35.7}$$

$$F_3 = 2\Big[f_u(x,Q^2) + f_c(x,Q^2) + f_t(x,Q^2) - f_{\overline{d}}(x,Q^2) - f_{\overline{s}}(x,Q^2) - f_{\overline{b}}(x,Q^2)\Big] \ . \tag{35.8}$$

For the process $\nu p \to e^- X$:

$$F_2 = 2xF_1 = 2x\Big[f_d(x,Q^2) + f_s(x,Q^2) + f_b(x,Q^2) + f_{\overline{u}}(x,Q^2) + f_{\overline{c}}(x,Q^2) + f_{\overline{t}}(x,Q^2)\Big] \ , \tag{35.9}$$

$$F_3 = 2\Big[f_d(x,Q^2) + f_s(x,Q^2) + f_b(x,Q^2) - f_{\overline{u}}(x,Q^2) - f_{\overline{c}}(x,Q^2) - f_{\overline{t}}(x,Q^2)\Big] \ . \tag{35.10}$$

35.2. e^+e^- annihilation

For pointlike, spin-1/2 fermions, the differential cross section in the c.m. for $e^+e^- \to f\overline{f}$ via single photon annihilation is (θ is the angle between the incident electron and the produced fermion; $N_c = 1$ if f is a lepton and $N_c = 3$ if f is a quark).

$$\frac{d\sigma}{d\Omega} = N_c\frac{\alpha^2}{4s}\beta\left[1 + \cos^2\theta + (1-\beta^2)\sin^2\theta\right]Q_f^2 \ , \tag{35.11}$$

where β is the velocity of the final state fermion in the c.m. and Q_f is the charge of the fermion in units of the proton charge. For $\beta \to 1$,

$$\sigma = N_c\frac{4\pi\alpha^2}{3s}Q_f^2 = N_c\frac{86.8Q_f^2\ nb}{s(\mathrm{GeV}/c^2)^2} \ . \tag{35.12}$$

At higher energies, the Z^0 (mass M_Z and width Γ_Z) must be included. If the mass of a fermion f is much less than the mass of the Z^0, then the differential cross section for $e^+e^- \to f\overline{f}$ is

$$\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4s} \Big\{ (1+\cos^2\theta) \left[Q_f^2 - 2\chi_1 v_e v_f Q_f + \chi_2 (a_e^2 + v_e^2)(a_f^2 + v_f^2) \right] + 2\cos\theta \left[-2\chi_1 a_e a_f Q_f + 4\chi_2 a_e a_f v_e v_f \right] \Big\} \tag{35.13}$$

where

$$\begin{aligned} \chi_1 &= \frac{1}{16\sin^2\theta_W \cos^2\theta_W} \frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} , \\ \chi_2 &= \frac{1}{256\sin^4\theta_W \cos^4\theta_W} \frac{s^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} , \\ a_e &= -1 , \\ v_e &= -1 + 4\sin^2\theta_W , \\ a_f &= 2T_{3f} , \\ v_f &= 2T_{3f} - 4Q_f \sin^2\theta_W , \end{aligned} \tag{35.14}$$

where $T_{3f} = 1/2$ for u, c and neutrinos, while $T_{3f} = -1/2$ for d, s, b, and negatively charged leptons.

At LEP II it may be possible to produce the orthodox Higgs boson, H, (see the mini-review on Higgs bosons) in the reaction $e^+e^- \to HZ^0$, which proceeds dominantly through a virtual Z^0. The Standard Model prediction for the cross section [3] is

$$\sigma(e^+e^- \to HZ^0) = \frac{\pi\alpha^2}{24} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_Z^2}{(s-M_Z^2)^2} \cdot \frac{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}{\sin^4\theta_W \cos^4\theta_W} . \tag{35.15}$$

where K is the c.m. momentum of the produced H or Z^0. Near the production threshold, this formula needs to be corrected for the finite width of the Z^0.

35.3. Two-photon process at e^+e^- colliders

When an e^+ and an e^- collide with energies E_1 and E_2, they emit dn_1 and dn_2 virtual photons with energies ω_1 and ω_2 and 4-momenta q_1 and q_2. In the equivalent photon approximation, the cross section for $e^+e^- \to e^+e^-X$ is related to the cross section for $\gamma\gamma \to X$ by (Ref. 1)

$$d\sigma_{e^+e^- \to e^+e^-X}(s) = dn_1\, dn_2\, d\sigma_{\gamma\gamma\to X}(W^2) \tag{35.16}$$

where $s = 4E_1E_2$, $W^2 = 4\omega_1\omega_2$ and

$$dn_i = \frac{\alpha}{\pi}\left[1 - \frac{\omega_i}{E_i} + \frac{\omega_i^2}{2E_i^2} - \frac{m_e^2\omega_i^2}{(-q_i^2)E_i^2}\right] \frac{d\omega_i}{\omega_i} \frac{d(-q_i^2)}{(-q_i^2)} . \tag{35.17}$$

After integration (including that over q_i^2 in the region $m_e^2\omega_i^2/E_i(E_i-\omega_i) \le -q_i^2 \le (-q^2)_{\rm max}$), the cross section is

$$\sigma_{e^+e^-\to e^+e^-X}(s) = \frac{\alpha^2}{\pi^2}\int_{z_{th}}^1 \frac{dz}{z}\left[f(z)\left(\ln\frac{(-q^2)_{\rm max}}{m_e^2 z}-1\right)^2 - \frac{1}{3}\left(\ln\frac{1}{z}\right)^3\right]\sigma_{\gamma\gamma\to X}(zs)\ ;$$

$$f(z) = \left(1+\tfrac{1}{2}z\right)^2\ln\frac{1}{z} - \tfrac{1}{2}(1-z)(3+z)\ ;$$

$$z = \frac{W^2}{s}\ . \tag{35.18}$$

The quantity $(-q^2)_{\rm max}$ depends on properties of the produced system X, in particular, $(-q^2)_{\rm max} \sim m_\rho^2$ for hadron production ($X = h$) and $(-q^2)_{\rm max} \sim W^2$ for lepton pair production ($X = \ell^+\ell^-$, $\ell = e,\ \mu,\ \tau$).

For production of a resonance of mass m_R and spin $J \neq 1$

$$\sigma_{e^+e^-\to e^+e^-R}(s) = (2J+1)\frac{8\alpha^2\Gamma_{R\to\gamma\gamma}}{m_R^3}$$

$$\times\left[f(m_R^2/s)\left(\ln\frac{sm_V^2}{m_e^2m_R^2}-1\right)^2 - \frac{1}{3}\left(\ln\frac{s}{m_R^2}\right)^3\right] \tag{35.19}$$

where m_V is the mass that enters into the form factor of the $\gamma\gamma \to R$ transition: $m_V \sim m_\rho$ for $R = \pi^0,\ \eta,\ f_2(1270),\ \ldots$, $m_V \sim m_R$ for $R = c\overline{c}$ or $b\overline{b}$ resonances.

35.4. Inclusive hadronic reactions

One-particle inclusive cross sections $Ed^3\sigma/d^3p$ for the production of a particle of momentum p are conveniently expressed in terms of rapidity (see above) and the momentum p_T transverse to the beam direction (defined in the center-of-mass frame)

$$E\frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi\, dy\, p_T dp_T}\ . \tag{35.20}$$

In the case of processes where p_T is large or the mass of the produced particle is large (here large means greater than 10 GeV), the parton model can be used to calculate the rate. Symbolically

$$\sigma_{\rm hadronic} = \sum_{ij}\int f_i(x_1,\ Q^2)\ f_j(x_2,\ Q^2)\ dx_1\ dx_2\ \widehat{\sigma}_{\rm partonic}\ , \tag{35.21}$$

where $f_i(x,\ Q^2)$ is the parton distribution introduced above and Q is a typical momentum transfer in the partonic process and $\widehat{\sigma}$ is the partonic cross section. Some examples will help to clarify. The production of a W^+ in pp reactions at rapidity y in the center-of-mass frame is given by

$$\frac{d\sigma}{dy} = \frac{G_F\,\pi\sqrt{2}}{3} \times \tau \Big[\cos^2\theta_c \Big(u(x_1\,,\,M_W^2)\,\overline{d}\,(x_2, M_W^2) + u(x_2\,,\,M_W^2)\,\overline{d}\,(x_1, M_W^2)\Big) + \sin^2\theta_c \Big(u(x_1\,,\,M_W^2)\,\overline{s}\,(x_2\,,\,M_W^2) + s(x_2, M_W^2)\,\overline{u}\,(x_1, M_W^2)\Big)\Big] \;, \tag{35.22}$$

where $x_1 = \sqrt{\tau}\;e^y$, $x_2 = \sqrt{\tau}\;e^{-y}$, and $\tau = M_W^2/s$. Similarly the production of a jet in pp (or $p\overline{p}$) collisions is given by

$$\frac{d^3\sigma}{d^2p_T\;dy} = \sum_{ij}\int f_i(x_1\,,\,p_T^2)\;f_j(x_2, p_T^2) \times \left[\widehat{s}\;\frac{d\widehat{\sigma}}{d\,\widehat{t}}\right]_{ij} dx_1\;dx_2\;\delta(\,\widehat{s} + \widehat{t} + \widehat{u}\,)\;, \tag{35.23}$$

where the summation is over quarks, gluons, and antiquarks. Here

$$s = (p_1 + p_2)^2\;, \tag{35.24}$$

$$t = (p_1 - p_{\rm jet})^2\;, \tag{35.25}$$

$$u = (p_2 - p_{\rm jet})^2\;, \tag{35.26}$$

p_1 and p_2 are the momenta of the incoming p and p (or $\overline{p}$) and $\widehat{s}$, $\widehat{t}$, and $\widehat{u}$ are s, t, and u with $p_1 \to x_1 p_1$ and $p_2 \to x_2\,p_2$. The partonic cross section $\widehat{s}\,[\,(d\widehat{\sigma})/(d\,\widehat{t}\,)\,]$ can be found in Ref. 2. Example: for the process $gg \to q\overline{q}$,

$$\widehat{s}\;\frac{d\sigma}{dt} = 3\alpha_s^2\;\frac{(\,\widehat{t}^{\,2} + \widehat{u}^2)}{8\widehat{s}}\;\left[\frac{4}{9\,\widehat{t}\,\widehat{u}} - \frac{1}{\widehat{s}^{\,2}}\right]\;. \tag{35.27}$$

The prediction of Eq. (35.23) is compared to data from the UA1 and UA2 collaborations in Fig. 37.8 in the Plots of Cross Sections and Related Quantities section of this *Review.*

The associated production of a Higgs boson and a gauge boson is analogous to the process $e^+e^- \to HZ^0$ in Sec. 35.2. The required parton-level cross sections [4], averaged over initial quark colors, are

$$\sigma(q_i\overline{q}_j \to W^\pm H) = \frac{\pi\alpha^2|V_{ij}|^2}{36\sin^4\theta_W}\cdot\frac{2K}{\sqrt{s}}\cdot\frac{K^2 + 3M_W^2}{(s - M_W^2)^2}$$

$$\sigma(q\overline{q} \to Z^0 H) = \frac{\pi\alpha^2(a_q^2 + v_q^2)}{144\sin^4\theta_W\cos^4\theta_W}\cdot\frac{2K}{\sqrt{s}}\cdot\frac{K^2 + 3M_Z^2}{(s - M_Z^2)^2}\;.$$

Here V_{ij} is the appropriate element of the Kobayashi-Maskawa matrix and K is the c.m. momentum of the produced H. The axial and vector couplings are defined as in Sec. 35.2.

35.5. One-particle inclusive distributions

In order to describe one-particle inclusive production in e^+e^- annihilation or deep inelastic scattering, it is convenient to introduce a fragmentation function $D_i^h\ (z, Q^2)$ where $D_i^h\ (z, Q^2)$ is the number of hadrons of type h and momentum between zp and $(z + dz)p$ produced in the fragmentation of a parton of type i. The Q^2 evolution is predicted by QCD and is similar to that of the parton distribution functions [see section on Quantum Chromodynamics (Sec. 9 of this *Review*)]. The $D_i^h(z, Q^2)$ are normalized so that

$$\sum_h \int z D_i^h\ (z, Q^2) dz = 1\ . \tag{35.28}$$

If the contributions of the Z boson and three-jet events are neglected, the cross section for producing a hadron h in e^+e^- annihilation is given by

$$\frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dz} = \frac{\sum_i e_i^2\ D_i^h\ (z, Q^2)}{\sum_i e_i^2}\ , \tag{35.29}$$

where e_i is the charge of quark-type i, σ_{had} is the total hadronic cross section, and the momentum of the hadron is $zE_{\text{cm}}/2$.

In the case of deep inelastic muon scattering, the cross section for producing a hadron of energy E_h is given by

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dz} = \frac{\sum_i e_i^2\ q_i(x, Q^2)\ D_i^h(z, Q^2)}{\sum_i e_i^2\ q_i(x, Q^2)}\ , \tag{35.30}$$

where $E_h = \nu z$. (For the kinematics of deep inelastic scattering, see Sec. 34.4.2 of the Kinematics section of this *Review.*) The fragmentation functions for light and heavy quarks have a different z dependence; the former peak near $z = 0$. They are illustrated in Figs. 36.1 and 36.2 in the section on "Heavy Quark Fragmentation in e^+e^- Annihilation" (Sec. 36 of this *Review*).

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