

12. CP VIOLATION

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The symmetries C (particle-antiparticle interchange) and P (space inversion) hold for strong and electromagnetic interactions. After the discovery of large C and P violation in the weak interactions, it appeared that the product CP was a good symmetry. In 1964 CP violation was observed in K^0 decays at a level given by the parameter $\epsilon \approx 2.3 \times 10^{-3}$. Larger CP -violation effects are anticipated in B^0 decays.

12.1. CP violation in Kaon decay

CP violation has been observed in the semi-leptonic decays $K_L^0 \rightarrow \pi^\mp \ell^\pm \nu$ and in the nonleptonic decay $K_L^0 \rightarrow 2\pi$. The experimental numbers that have been measured are

$$\delta = \frac{\Gamma(K_L^0 \rightarrow \pi^- \ell^+ \nu) - \Gamma(K_L^0 \rightarrow \pi^+ \ell^- \nu)}{\Gamma(K_L^0 \rightarrow \pi^- \ell^+ \nu) + \Gamma(K_L^0 \rightarrow \pi^+ \ell^- \nu)} \quad (12.1a)$$

$$\begin{aligned} \eta_{+-} &= A(K_L^0 \rightarrow \pi^+ \pi^-) / A(K_S^0 \rightarrow \pi^+ \pi^-) \\ &= |\eta_{+-}| e^{i\phi_{+-}} \end{aligned} \quad (12.1b)$$

$$\begin{aligned} \eta_{00} &= A(K_L^0 \rightarrow \pi^0 \pi^0) / A(K_S^0 \rightarrow \pi^0 \pi^0) \\ &= |\eta_{00}| e^{i\phi_{00}} . \end{aligned} \quad (12.1c)$$

CP violation can occur either in the $K^0 - \bar{K}^0$ mixing or in the decay amplitudes. Assuming CPT invariance, the mass eigenstates of the $K^0 - \bar{K}^0$ system can be written

$$|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle , \quad |K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle . \quad (12.2)$$

If CP invariance held, we would have $q = p$ so that K_S would be CP even and K_L CP odd. (We define $|\bar{K}^0\rangle$ as $CP |K^0\rangle$). CP violation in $K^0 - \bar{K}^0$ mixing is then given by the parameter $\tilde{\epsilon}$ where

$$\frac{p}{q} = \frac{(1 + \tilde{\epsilon})}{(1 - \tilde{\epsilon})} . \quad (12.3)$$

CP violation can also occur in the decay amplitudes

$$A(K^0 \rightarrow \pi\pi(I)) = A_I e^{i\delta_I} , \quad A(\bar{K}^0 \rightarrow \pi\pi(I)) = A_I^* e^{i\delta_I} , \quad (12.4)$$

where I is the isospin of $\pi\pi$, δ_I is the final-state phase shift, and A_I would be real if CP invariance held. The CP -violating observables are usually expressed in terms of ϵ and ϵ' defined by

$$\eta_{+-} = \epsilon + \epsilon' , \quad \eta_{00} = \epsilon - 2\epsilon' , \quad (12.5a)$$

One can then show [1]

$$\epsilon = \tilde{\epsilon} + i (\text{Im } A_0 / \text{Re } A_0) , \quad (12.5b)$$

$$\sqrt{2}\epsilon' = ie^{i(\delta_2 - \delta_0)} (\text{Re } A_2 / \text{Re } A_0) (\text{Im } A_2 / \text{Re } A_2 - \text{Im } A_0 / \text{Re } A_0) , \quad (12.5c)$$

$$\delta = 2\text{Re } \epsilon / (1 + |\epsilon|^2) \approx 2\text{Re } \epsilon . \quad (12.5d)$$

In Eq. (12.5c) small corrections of order $\epsilon' \times \text{Re } (A_2/A_0)$ are neglected and Eq. (12.5d) assumes the $\Delta S = \Delta Q$ rule.

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The quantities $\text{Im } A_0$, $\text{Im } A_2$, and $\text{Im } \epsilon$ depend on the choice of phase convention since one can change the phases of K^0 and \bar{K}^0 by a transformation of the strange quark state $|s\rangle \rightarrow |s\rangle e^{i\alpha}$; of course, observables are unchanged. It is possible by a choice of phase convention to set $\text{Im } A_0$ or $\text{Im } A_2$ or $\text{Im } \tilde{\epsilon}$ to zero, but none of these is zero with the usual phase conventions in the Standard Model. The choice $\text{Im } A_0 = 0$ is called the Wu-Yang phase convention [2] in which case $\epsilon = \tilde{\epsilon}$. The value of ϵ' is independent of phase convention and a nonzero value demonstrates *CP* violation in the decay amplitudes, referred to as direct *CP* violation. The possibility that direct *CP* violation is essentially zero and that *CP* violation occurs only in the mixing matrix was referred to as the superweak theory [3].

By applying *CPT* invariance and unitarity the phase of ϵ is given approximately by

$$\phi(\epsilon) \approx \tan^{-1} \frac{2(m_{K_L} - m_{K_S})}{\Gamma_{K_S} - \Gamma_{K_L}} = 43.49 \pm 0.08^\circ \quad (12.6a)$$

while Eq. (12.5c) gives

$$\phi(\epsilon') = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 48 \pm 4^\circ, \quad (12.6b)$$

where the numerical value is based on an analysis of $\pi\text{-}\pi$ scattering [4]. The approximation in Eq. (12.6a) depends on the assumption that direct *CP* violation is very small in all K^0 decays. This is expected to be good to a few tenths of a degree as indicated by the small value of ϵ' and of η_{+-0} , the *CP* violation parameter in the decay $K_S \rightarrow \pi^+\pi^-\pi^0$ [5], although limits on η_{000} are still poor. The relation in Eq. (12.6a) is exact in the superweak theory so this is sometimes called the superweak phase. An important point for the analysis is that $\cos[\phi(\epsilon') - \phi(\epsilon)] \simeq 1$. The consequence is that only two real quantities need be measured, the magnitude of ϵ and the value of (ϵ'/ϵ) including its sign. The measured quantity $|\eta_{00}/\eta_{+-}|^2$, which is very close to unity, is given to a good approximation by

$$|\eta_{00}/\eta_{+-}|^2 \approx 1 - 6\text{Re}(\epsilon'/\epsilon) \approx 1 - 6\epsilon'/\epsilon. \quad (12.7)$$

From the experimental measurements, one finds

$$\epsilon = (2.271 \pm 0.017) \times 10^{-3}, \quad (12.8a)$$

$$\text{Re}(\epsilon'/\epsilon) \approx \epsilon'/\epsilon = (2.1 \pm 0.5) \times 10^{-3}, \quad (12.8b)$$

$$\phi_{+-} = 43.5 \pm 0.5^\circ, \quad (12.8c)$$

$$\phi_{00} - \phi_{+-} = -0.1 \pm 0.8, \quad (12.8d)$$

$$\delta = (3.33 \pm 0.14) \times 10^{-3}. \quad (12.8e)$$

Direct *CP* violation, as indicated by ϵ'/ϵ , is expected in the Standard Model; most calculations [6] give a somewhat smaller value, but they have a large uncertainty. The value of δ agrees with Eq. (12.5d). The values of ϕ_{+-} and $\phi_{00} - \phi_{+-}$ are used to set limits on *CPT* violation. [See Tests of Conservation Laws.]

In the Standard Model, *CP* violation arises as a result of a single phase entering the CKM matrix (Sec. 11). As a result in what is now the standard phase convention, two elements have large phases, $V_{ub} \sim e^{-i\gamma}$, $V_{td} \sim e^{-i\beta}$. Because these elements have small magnitudes and involve the third generation, *CP* violation in the K^0 system is small. On the other hand, large effects are expected in the B^0 system, which is a major motivation for B factories.

12.2. *CP* violation in B decay

CP violation in the B^0 system can be observed by comparing B^0 and \bar{B}^0 decays [7]. For a final *CP* eigenstate a , the decay rate has a time dependence given by

$$\Gamma_a \sim e^{-\Gamma t} \left([1 + |\lambda_a|^2] \pm [1 - |\lambda_a|^2] \cos(\Delta Mt) \mp \text{Im } \lambda_a \sin(\Delta Mt) \right) \quad (12.9)$$

where the top sign is for B^0 and the bottom for \bar{B}^0 and

$$\lambda_a = (q_B/p_B) \bar{A}_a/A_a . \quad (12.10)$$

The quantities p_B and q_B come from the analogue for B^0 of Eq. (12.2), and $A_a(\bar{A}_a)$ is the decay amplitude to state a for $B^0(\bar{B}^0)$. However, for B^0 the eigenstates are expected to have a negligible lifetime difference and are only distinguished by the mass difference ΔM ; also as a consequence $|q_B/p_B| \approx 1$ so that $\tilde{\epsilon}_B$ is purely imaginary.

If only one quark weak transition contributes to the decay, $|\bar{A}_a/A_a| = 1$ so that $|\lambda_a| = 1$ and the $\cos(\Delta Mt)$ term vanishes. In this case, the difference between B^0 and \bar{B}^0 decays is given by the $\sin(\Delta Mt)$ term with the asymmetry coefficient

$$a_a = \frac{\Gamma_a(t) - \bar{\Gamma}_a(t)}{(\Gamma_a(t) + \bar{\Gamma}_a(t)) \sin(\Delta Mt)} = \eta_a \sin\left(2(\phi_M + \phi_D)\right) , \quad (12.11)$$

where $2\phi_M$ is the phase of the B^0 - \bar{B}^0 mixing, ϕ_D is the weak phase of the decay transition, and η_a is the *CP* eigenvalue of a .

For $B^0(\bar{B}^0) \rightarrow \psi K_S$ from the transition $b \rightarrow c\bar{c}s$, one finds in the Standard Model that the asymmetry is given directly in terms of a CKM phase with no hadronic uncertainty:

$$a_{\psi K_S} = -\sin 2\beta . \quad (12.12)$$

From the constraints on the CKM matrix (Sec. 11) $\sin 2\beta$ is predicted to be between 0.3 and 0.9. A significantly different value could be a sign of new physics.

A second decay of interest is $B^0(\bar{B}^0) \rightarrow \pi^+\pi^-$ from the transition $b \rightarrow u\bar{u}d$ with

$$a_{\pi\pi} = \sin 2(\beta + \gamma) . \quad (12.13)$$

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While either of these asymmetries could be ascribed to $B^0-\bar{B}^0$ mixing (q_B/p_B or $\tilde{\epsilon}_B$), the difference between the two asymmetries is evidence for direct *CP* violation. From Eq. (12.10) it is seen that this corresponds to a phase difference between $A_{\psi K_S}$ and $A_{\pi^+\pi^-}$. Thus this is analogous to ϵ' . In the standard phase convention, 2β in Eqs. (12.12) and (12.13) arises from $B^0-\bar{B}^0$ mixing whereas the γ in Eq. (12.13) comes from V_{ub} in the transition $b \rightarrow u\bar{u}d$. The result in Eq. (12.13) may have a sizeable correction due to what is called a penguin diagram. This is a one-loop graph producing $b \rightarrow d + \text{gluon}$ with a W and a quark, predominantly the t quark, in the loop. This leads to an amplitude proportional to $V_{tb}^*V_{td}$, which has a weak phase different from that of the original tree amplitude proportional to $V_{ub}V_{ud}^*$. There are several methods to approximately determine this correction using additional measurements [8].

CP violation in the decay amplitude is also revealed by the $\cos(\Delta Mt)$ term in Eq. (12.9) or by a difference in rates of B^+ and B^- to charge-conjugate states. These effects, however, require two contributing amplitudes to the decay (such as a tree amplitude plus a penguin) and also require final-state interaction phases. Predicted effects are very uncertain and are generally small [9].

In the case of the B_s system, the mass difference ΔM is much larger than for B^0 and has not yet been measured. As a result, it will be difficult to isolate the $\sin(\Delta Mt)$ term to measure asymmetries. Furthermore, in the Standard Model with the standard phase convention, ϕ_M is very small so that decays due to $b \rightarrow c\bar{c}s$, yielding $B_s \rightarrow \psi\eta'$, would have zero asymmetry. Decays due to $b \rightarrow u\bar{u}d$, yielding $B_s \rightarrow \rho^0 K_S$, would have an asymmetry $\sin 2\gamma$ in the tree approximation. The width difference $\Delta\Gamma$ is also expected to be much larger for B_s so that $\Delta\Gamma/\Gamma$ might be as large as 0.15. In this case, there might be a possibility of detecting *CP* violation as in the case of K^0 by observing the B_s states with different lifetimes decaying into the same *CP* eigenstate [10].

For further details, see the notes on *CP* violation in the K_L^0 , K_S^0 , and B^0 Particle Listings of this *Review*.

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