PRODUCTION AND DECAY OF b-FLAVORED HADRONS

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This year, we opened a new chapter in B physics. A new generation of experiments, BABAR, BELLE, Hera-B, and CLEO III, saw first collisions and started to accumulate B-meson decays. The next Fermilab collider run will start soon. The long-awaited B-factory era has begun.

There is great hope these experiments will provide us with precise measurements of fundamental parameters of the Standard Model, in particular the weak-mixing angles and phase of the Cabibbo-Kobayashi-Maskawa matrix, and with it an improved understanding of CP violation and maybe even a glimpse at new physics.

While the underlying decay of the heavy quark is governed by the weak interaction, it is the strong force that is responsible for the formation of the hadrons that are observed by experimenters. Although this complicates the extraction of the the Standard Model parameters from the experimental data, it also means that decays of B mesons provide an important laboratory to test our understanding of the strong interaction.

Arguably the most exciting development since the last edition of this review is the progress in b-quark processes for which amplitudes beyond the tree level play a major role. The long sought after $B^0 \to \pi^+\pi^-$ decays have finally been observed. Many other $b \to u$ and gluonic penguin transitions have been measured. In addition to branching fractions, limits on CP asymmetries have been measured for several modes. The results on rare hadronic B decays have also been used to probe possible values of the angle γ of the CKM triangle. First attempts to measure another CKM angle, $\sin(2\beta)$, have been reported by OPAL, CDF, and ALEPH.

For $b \to c$ transitions, the CLEO Collaboration used a sample of more than 18 million B decays to update branching fractions for many exclusive hadronic decay channels. New results on semileptonic decays have been reported by CLEO

and the LEP Collaborations. Lifetime measurements improve steadily and now have reached a precision of a few percent.

Heavy-flavor physics is a very dynamic field, and in this brief review it is impossible to do justice to all recent theoretical and experimental developments. We will highlight a few new results but otherwise refer the interested reader to several excellent reviews [1–3].

Production and spectroscopy: Elementary particles are characterized by their masses, lifetimes, and internal quantum numbers. The bound states with a $\overline{b}$ quark and a u or d antiquark are referred to as the B_d ($\overline{B}^0$) and the B_u (B^+) mesons, respectively. The first excitation is called the B^* meson. B^{**} is the generic name for the four orbitally excited ($L = 1$) B-meson states that correspond to the P-wave mesons in the charm system, D^{**}. Mesons containing an s or a c quark are denoted B_s and B_c, respectively.

Experimental studies of b decay are performed at the $\Upsilon(4S)$ resonance near production threshold, as well as at higher energies in proton-antiproton collisions and Z decays. Most new results from CLEO are based on a sample of $\approx 9.7 \times 10^6$ $B\overline{B}$ events. At the Tevatron, CDF in particular has made significant contributions with 100 pb^{-1} of data. Operating at the Z resonance, each of the four LEP Collaborations recorded slightly under a million $b\overline{b}$ events, while the SLD experiment collected about 0.1 million $b\overline{b}$ events.

For quantitative studies of B decays, the initial composition of the data sample must be known. The $\Upsilon(4S)$ resonance decays only to $B^0\overline{B}^0$ and B^+B^- pairs, while at high-energy collider experiments, heavier states such as B_s or B_c mesons and b-flavored baryons are produced as well. The current experimental limit for non-$B\overline{B}$ decays of the $\Upsilon(4S)$ is less than 4% at the 95% confidence level [4]. CLEO has measured the ratio of charged to neutral $\Upsilon(4S)$ decays using exclusive $B \to \psi K^{(*)}$ decays. Assuming isospin invariance and $\tau_{B^+}/\tau_{B^0} = 1.066 \pm 0.024$ they found [5]

$$\frac{f_+}{f_0} = \frac{\mathrm{B}(\Upsilon(4S) \to B^+B^-)}{\mathrm{B}(\Upsilon(4S) \to B^0\overline{B}^0)} = 1.044 \pm 0.069^{+0.043}_{-0.045} \ . \quad (1)$$

This is consistent with equal production of B^+B^- and $B^0\overline{B}^0$ pairs, and unless explicitly stated otherwise, we will assume $f_+/f_0 = 1$. This assumption is further supported by the near equality of the B^+ and B^0 masses. Again using exclusive $B \to J/\psi K^{(*)}$ decays, CLEO determined these masses to $m(B^0) = 5.2791 \pm 0.0007 \pm 0.0003$ GeV/c^2 and $m(B^+) = 5.2791 \pm 0.0004 \pm 0.0004$ GeV/c^2, respectively [6].

At high-energy collider experiments, b quarks hadronize as $\overline{B}^0$, B^-, $\overline{B}^0_s$, and B^-_c mesons, or as baryons containing b quarks.

Over the last few years, there have been significant improvements in our understanding of the b-hadron sample composition. Table 1 summarizes the results showing the fractions f_d, f_u, f_s, and $f_{\rm baryon}$ of B^0, B^+, B^0_s, and b baryons in an unbiased sample of weakly decaying b hadrons produced at the Z resonance and in $p\overline{p}$ collisions. A detailed account can be found elsewhere in this Review [7].

Table 1: Fractions of weakly decaying b-hadron species in $Z \to b\overline{b}$ decay and in $p\overline{p}$ collisions at $\sqrt{(}s) = 1.8$ TeV.

b hadron	Fraction [%]
B^-, $\overline{B}^0$	38.9 ± 1.3
$\overline{B}^0_s$	10.7 ± 1.4
b baryons	11.6 ± 2.0

To date, the existence of the b-flavored mesons (B^-, $\overline{B}^0$, B_s, B_c, and various excitations), as well as the Λ_b baryon has been established. The current world average of the B^*–B mass difference is 45.78 ± 0.35 MeV/c^2. Using exclusive hadronic decays such as $B^0_s \to J/\psi\phi$ and $\Lambda_b \to J/\psi\Lambda$, the masses of these states are now known with the precision of a few MeV. The current world averages of the B_s and the Λ_b mass are 5.3696 ± 0.0024 GeV/c^2 and 5.624 ± 0.009 GeV/c^2, respectively. Clear evidence for the B_c, the last weakly decaying bottom meson, has been published by CDF [8]. They reconstruct the semileptonic decay $B_c \to J/\psi\ell X$, and extract a B_c mass of $6.40 \pm 0.39 \pm 0.13$ GeV/c^2.

First indications of Ξ_b production have been presented by the LEP Collaborations [9–10].

Excited B-meson states have been observed by CLEO, CUSB, LEP, and CDF. Evidence for B^{**} production has been presented by CDF and the LEP experiments [11]. Inclusively reconstructing a bottom hadron candidate combined with a charged pion from the primary vertex, they see the B^{**} as a broad resonance around 5.697 ± 0.009 GeV/c^2 in the $M(B\pi) \equiv M(B)$ mass distribution [12]. Due to the inclusive approach, the mass resolution is limited to about 40 MeV, which makes it very difficult to identify the narrow states, B_1 and B_2^*, separately. The LEP experiments have also provided evidence for excited B_s^{**} states.

Lifetimes: Precise lifetimes are key in extracting the weak parameters that are important for understanding the role of the CKM matrix in CP violation, such as the determination of V_{cb} and $B_s\overline{B}_s$ mixing measurements. In the naive spectator model, the heavy quark can decay only via the external spectator mechanism, and thus the lifetimes of all mesons and baryons containing b quarks would be equal. Nonspectator effects, such as the interference between contributing amplitudes, modify this simple picture and give rise to a lifetime hierarchy for b-flavored hadrons similar to the one in the charm sector. However, since the lifetime differences are expected to scale as $1/m_Q^2$, where m_Q is the mass of the heavy quark, the variation in the b system should be significantly smaller, of order 10% or less [14]. For the b system we expect

$$\tau(B^-) \geq \tau(\overline{B}^0) \approx \tau(B_s) > \tau(\Lambda_b^0) \gg \tau(B_c) \,. \qquad (2)$$

In the B_c, both quarks can decay weakly, resulting in its much shorter lifetime. Measurements of lifetimes for the various b-flavored hadrons thus provide a means to determine the importance of non-spectator mechanisms in the b sector.

Over the past years, the field has matured, and advanced algorithms based on impact parameter or decay length measurements exploit the potential of silicon vertex detectors. However,

in order to reach the precision necessary to test theoretical predictions, the results from different experiments need to be averaged. This is a challenging task that requires detailed knowledge of common systematic uncertainties, and correlations between the results from different experiments. The average lifetimes for b-flavored hadrons given in this edition have been determined by the LEP B Lifetimes Working Group [15]. The papers used in this calculation are listed in the appropriate sections. A detailed description of the procedures and the treatment of correlated and uncorrelated errors can be found in [16]. The new world average b-hadron lifetimes are summarized in Table 2. The first measurement of the B_c lifetime comes from the CDF Collaboration [8]. Lifetime measurements have reached a level of precision that the average b-hadron lifetime result becomes sensitive to the composition of the data sample. The result listed in Table 2 takes into account correlations between different experiments and analysis techniques, but does not correct for differences due to different admixtures of b-flavored hadrons. For inclusive lifetime measurements, the size of this effect can be estimated by dividing the available results into three sets. LEP measurements based on the identification of a lepton from the b decay yield $\tau_{b\ \mathrm{hadron}} = 1.537 \pm 0.020$ ps^{-1} [17–19]. The average b-hadron lifetime based on inclusive secondary vertex techniques is $\tau_{b\ \mathrm{hadron}} = 1.577 \pm 0.016$ ps^{-1} [18,20–24]. Finally, CDF [25] used J/ψ mesons to tag the b vertex resulting in $\tau_{b\text{-hadron}} = 1.533 \pm 0.015^{+0.035}_{-0.031}$ ps^{-1}. Contrary to what is observed, the average b lifetime determined from a sample of semileptonic decays is expected to be larger than the lifetime extracted from inclusive decays. Given the precision of the measurements, however, the discrepancy is not yet significant. The resulting average b lifetime is listed in Table 2.

For comparison with theory, lifetime ratios are preferred. Experimentally we find [15]

$$\frac{\tau_{B^+}}{\tau_{B^0}} = 1.062 \pm 0.029\,, \quad \frac{\tau_{B_s}}{\tau_{B^0}} = 0.964 \pm 0.045\,, \quad \frac{\tau_{\Lambda_b}}{\tau_{B^0}} = 0.780 \pm 0.037\,, \tag{3}$$

Table 2: Summary of inclusive and exclusive b-hadron lifetime measurements.

Particle	Lifetime [ps]
B^0	1.548 ± 0.032
B^+	1.653 ± 0.028
B_s	1.493 ± 0.062
B_c	$0.46^{+0.18}_{-0.16} \pm 0.03$
b baryon	1.208 ± 0.051
b hadron	1.564 ± 0.014

while theory makes the following predictions [26]

$$\frac{\tau_{B^+}}{\tau_{B^0}} = 1 + 0.05 \left(\frac{f_B}{200 \text{ MeV}} \right)^2 \ , \ \frac{\tau_{B_s}}{\tau_{B^0}} = 1 \pm 0.01 \ , \ \frac{\tau_{\Lambda_b}}{\tau_{B^0}} = 0.9 \ . \tag{4}$$

In conclusion, the pattern of measured B-meson lifetimes follows the theoretical expectations, and non-spectator effects are observed to be small. The short B_c lifetime has been predicted correctly. However, the Λ_b-baryon lifetime is unexpectedly short. As has been noted by several authors, the observed value of the Λ_b lifetime is quite difficult to accommodate theoretically [27–33]. This apparent breakdown of the heavy-quark expansion for inclusive, non-leptonic B decays could be caused by violations of local quark-hadron duality. Neubert, however, argues that this conclusion is premature because a reliable field-theoretical calculation is still lacking. Exploring a reasonable parameter space for the unknown hadronic matrix elements, he demonstrated that within the experimental errors, theory can accommodate the measured lifetime ratios [1]. A recent calculation based on QCD sum rules [34] arrives at a similar conclusion allowing $\tau_{\Lambda_b}/\tau_{B^0} = 0.79$–$0.87$. An initial lattice study [35], on the other hand, finds $\tau_{\Lambda_b}/\tau_{B^0} = 0.91$–$0.93$.

Similar to the kaon system, neutral B mesons contain short- and long-lived components. The lifetime difference is, of course, significantly smaller, and recent experimental limits at 95% C.L. are

$$\frac{\Delta\Gamma_d}{\Gamma_d} < 0.82 \text{ and } \frac{\Delta\Gamma_s}{\Gamma_s} < 0.65 \ . \tag{5}$$

These results are based on a comparison of direct δm measurements with χ_d measurements for B_d [36] and a combination [37] of the various B_s proper time measurements. A more restrictive limit for the B_s system can be obtained if one assumes $\Gamma_{B_s} = \Gamma_{B_d}$.

Semileptonic B decays: Measurements of semileptonic B decays are important to determine the weak couplings $|V_{cb}|$ and $|V_{ub}|$. In addition, these decays can be used to probe the dynamics of heavy quark decay. The leptonic current can be calculated exactly, while corrections due to the strong interaction are restricted to the $b \to c$ and $b \to u$ vertices, respectively.

Experimentally, semileptonic decays have the advantage of large branching ratios and the characteristic signature of the energetic charged lepton. The neutrino, however, escapes undetected so a full reconstruction of the decaying B meson is impossible. Various techniques which take advantage of production at threshold or the hermiticity of the detector have been developed by the ARGUS, CLEO, and LEP experiments to overcome this difficulty.

Several different approaches have been used to measure the inclusive semileptonic rate $B \to X\ell\nu_\ell$. These are measurements of the inclusive single lepton momentum spectrum, measurements of dilepton events using charge and angular correlations first pioneered by ARGUS [38], measurements of leptons opposite a b-tagged jet at the Z, and measurements of the separate B^- and $\overline{B}^0$ branching ratios by using events which contain a lepton and a reconstructed B meson. The double-tagged methods (lepton–lepton) have the smallest model dependence, and only the dilepton results from the the $\Upsilon(4S)$ are used. The LEP averages [39] are based primarily on single lepton measurements, which rely on modeling of the semileptonic decays. The uncertainties involved in such modeling are, by their nature, ill-defined and difficult to quantify. The average LEP [39] and the $\Upsilon(4S)$ [40] rates are listed in Table 3. Differences in B_{sl} measured at the $\Upsilon(4S)$ and the Z are expected due to the different admixture of b-flavored hadrons. Given the short Λ_b lifetime, the LEP value should be lower than the $\Upsilon(4S)$ result. Previous LEP determinations of $B \to X\ell\nu_\ell$ have been markedly

higher than the $\Upsilon(4S)$ measurements. The current LEP measurements are now in much better agreement with expectations relative to the $\Upsilon(4S)$ rate.

A few new results on the branching fractions of exclusive semileptonic B decays have been reported. The current world averages are listed in Table 3. It is interesting to compare the inclusive semileptonic branching fraction to the sum of branching fractions for exclusive modes, which agree at the 1σ level. The exclusive modes measured are consistent with saturating the inclusive rate.

The makeup of the non-D and D^* components of the B semileptonic process is a critical component in the determination of b lifetimes, B mixing, $|V_{cb}|$, and $|V_{ub}|$. It has been known for some time that the D^{**} excited states do not appear to account for the difference between the $D + D^*$ rates and the inclusive rate [41,42]. A recent inclusive $B \to D^*\pi\ell\nu_\ell X$ study by DELPHI [43] adds information regarding the breakdown into the $D^*\pi$ and $D\pi$ contributions. Unfortunately, we still lack information regarding detailed makeup of, and the hadronic mass spectrum for, this component.

Table 3: Inclusive and exclusive semileptonic branching fractions of B mesons. $\mathrm{B}(\overline{B} \to X_u\ell^-\overline{\nu}_\ell) = 0.15 \pm 0.1\%$ [44] has been included in the sum of the exclusive branching fractions.

Mode	Branching fraction [%]
$\overline{B} \to X\ell^-\overline{\nu}_\ell(\Upsilon(4S))$	$10.49 \pm 0.17 \pm 0.43$
$b \to X\ell^-\overline{\nu}_\ell(Z)$	$10.58 \pm 0.07 \pm 0.17$
$\overline{B} \to D\ell^-\overline{\nu}_\ell$	2.13 ± 0.22
$\overline{B} \to D^*\ell^-\overline{\nu}_\ell$	5.05 ± 0.25
$\overline{B} \to D^{(*)}\pi\ell^-\overline{\nu}_\ell$	2.26 ± 0.44
with $\overline{B} \to D_1^0(2420)\ell^-\overline{\nu}_\ell X$	0.74 ± 0.16
$\overline{B} \to D_2^{*0}(2460)\ell^-\overline{\nu}_\ell X$	< 0.65 90% CL
$\Sigma\mathrm{B}_{\mathrm{exclusive}}$	9.59 ± 0.56

Dynamics of semileptonic B decay and $|V_{cb}|$: Since leptons are not sensitive to the strong interaction, the amplitude for a semileptonic B decay can be factorized into two parts, a leptonic and a hadronic current. The leptonic factor can be calculated exactly, while the hadronic part is parameterized by form factors. A simple example is the transition $B \to D\ell\nu_\ell$. The differential decay rate in this case is given by

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|V_{cb}^2|P_D^3 f_+^2(q^2) \tag{6}$$

where q^2 is the mass of the virtual W $(\ell\nu_\ell)$, P_D is the D momentum and $f_+(q^2)$ is the single vector form factor which gives the probability that the final state quarks will form a D meson. Since the leptons are very light, the corresponding $f_-(q^2)$ form factor can be neglected. For $B \to D^*\ell\nu_\ell$ decays, in the limit of zero lepton mass there are three form factors which correspond to the three possible partial waves of the $B \to D^*\widehat{W}$ system (here $\widehat{W}$ is the virtual W boson, which becomes the lepton-antineutrino pair). Currently, form factors cannot be predicted by theory and need to be determined experimentally. Over the last years, however, it has been appreciated that there is a symmetry of QCD that is useful in understanding systems containing one heavy quark. This symmetry arises when the quark becomes sufficiently heavy to make its mass irrelevant to the nonperturbative dynamics of the light quarks. This allows the heavy quark degrees of freedom to be treated in isolation from the light quark degrees of freedom. This is analogous to the canonical treatment of hydrogenic atoms, in which the spin and other properties of the nucleus can be neglected. The behavior and electronic structure of the atom are determined by the light electronic degrees of freedom. Heavy quark effective theory (HQET) was created by Isgur and Wise [45], who define a single universal form factor, $\xi(v \cdot v^{'})$, known as the Isgur-Wise function. In this function, v and $v^{'}$ are the four velocities of the initial and final state heavy mesons. The Isgur-Wise function cannot be calculated from first principles, but unlike the hadronic form factors mentioned above, it is universal to leading order. In the heavy quark limit, it is the same for all

heavy meson to heavy meson transitions, and the four form factors parameterizing $B \to D^* \ell \nu_\ell$ and $B \to D \ell \nu_\ell$ decays can be related to this single function ξ.

In this framework the differential semileptonic decay rates as functions of $w = v_B \cdot v_{D^{(*)}} = (m_B^2 + m_{D^{(*)}}^2 - q^2)/2 m_B m_{D^{(*)}}$ are given by [1]

$$\begin{aligned}
\frac{d\Gamma(\overline{B} \to D^* \ell \overline{\nu}_\ell)}{dw} =& \frac{G_F^2 M_B^5}{48\pi^3} r_*^3 (1-r_*)^2 \sqrt{w^2-1}(w+1)^2 \\
& \times \left[1 + \frac{4w}{w+1}\frac{1-2wr_*+r_*^2}{(1-r_*)^2}\right] |V_{cb}|^2 \mathcal{F}^2(w) \\
\frac{d\Gamma(\overline{B} \to D \ell \overline{\nu}_\ell)}{dw} =& \frac{G_F^2 M_B^5}{48\pi^3} r^3 (1+r)^2 (w^2-1)^{3/2} |V_{cb}|^2 \mathcal{G}^2(w) \quad (7)
\end{aligned}$$

where $r_{(*)} = M_{D^{(*)}}/M_B$ and q^2 is the invariant momentum transfer. For $m_Q \to \infty$, the two form factors $\mathcal{F}(w)$ and $\mathcal{G}(w)$ coincide with the Isgur-Wise function $\xi(w)$.

Both CLEO [46] and ALEPH [47] have measured the differential decay rate distributions and extracted the ratio $\mathcal{G}(w)/\mathcal{F}(w)$ which is expected to be close to unity. The data are compatible with a universal form factor $\xi(w)$.

CLEO has also performed a direct measurement of the three form factors that are used to parameterize $B \to D^* \ell \nu_\ell$ decays [48]. These are usually expressed in terms of form factor ratios [49]. $R_1(w) = h_V(w)/h_{A_1}(w)$ and $R_2(w) = h_{A_2}(w)/h_{A_1}(w)$ where $h_V(w)$, $h_{A_1}(w)$ and $h_{A_2}(w)$ are the standard three HQET form factors in the zero lepton mass limit (see Ref. 49 and references therein). At zero recoil, *i.e.* $w = 1$, CLEO finds $R_1(1) = 1.18 \pm 0.30 \pm 0.12$ and $R_2(1) = 0.71 \pm 0.2 \pm 0.07$. While the errors are still large, this is in good agreement with a theoretical prediction of $R_1(1) = 1.3 \pm 0.1$ and $R_2(1) = 0.8 \pm 0.2$ [1].

The universal form factor $\xi(w)$ describes the overlap of wave functions of the light degrees of freedom in the initial and final heavy meson. At zero recoil, *i.e.*, when the two mesons move with the same velocity, the overlap is perfect and the form factor is absolutely normalized, $\xi(1) = 1$. In principle, all that experimentalists have to do to extract a model-independent value for $|V_{cb}|$ is to measure $d\Gamma(B \to D^{(*)} \ell \nu_\ell)/dw$ for $w \to 1$. However, in the real, world the b and c quarks are not infinitely heavy, so corrections to the limiting case have to be calculated.

The evaluation of $\mathcal{F}(1)$ and $\mathcal{G}(1)$ remains a topic of some theoretical controversy [1,50–54]. A middle ground could be characterized as

$$\begin{aligned} \mathcal{F}(1) &= 0.92 \pm 0.05 \ , \\ \mathcal{G}(1) &= 1.00 \pm 0.07 \ . \end{aligned} \tag{8}$$

The calculations of $\mathcal{F}(1)$ and $\mathcal{G}(1)$ most commonly accepted have relied upon some of the OPE techniques, and in fact these results are correlated at some level with the inclusive rate calculations. Concerns about duality violation, for example, enter these determinations as well. Other, "exclusive approaches" (see Ref. 54 and references therein) yield results similar to the values quoted and are free from duality uncertainties. However, they rely on modelling to estimate exclusive matrix elements, for which uncertainties are very difficult to quantify. Recently, there has been a prototype lattice determination that obtained an $\mathcal{F}(1)$ value only very slightly higher than the above with a preliminary uncertainty of 3.3%. These results are encouraging, and are free of the intimate correlation with the inclusive calculations. To fully understand the uncertainties, an unquenched calculation is needed.

Measurements of $\mathcal{F}(1)|V_{cb}|$ have been performed by the ALEPH, ARGUS, CLEO, DELPHI, and OPAL experiments. Because the differential decay rate actually vanishes at zero recoil, experimentally the decay rate must be measured as a function of w and extrapolated to zero. This requires a parameterization of the shape of the form factor $\mathcal{F}(w)$. Initial measurements used a linear parameterization and fit the slope and $\mathcal{F}(1)|V_{cb}|$ simultaneously. $\mathcal{F}(w)$ must have a positive curvature, so this linear parameterization results in an intercept that is biased low by about 2.6% [55]. More recent determinations [47,56–58] have used dispersion relation calculations [59,60] that relate the curvature to the slope. In either case, the slope and intercept parameters are highly correlated and require simultaneous averaging [61].

$|V_{cb}|$ from exclusive $D*\ell\nu_\ell$ determinations and from inclusive determinations (discussed below) are summarized in Table 4. The various averages are in good agreement. Because of the

correlations between slope and $\mathcal{F}(1)|V_{cb}|$, and the different meanings of the slopes in the linear and dispersion-relation-based parameterizations, the older CLEO [62] and ARGUS [63] $D^*\ell\nu$ measurements, based on the linear parameterization, have not here been averaged with the LEP results [47,57–58], based on the dispersion-relation parameterization. Determinations of $|V_{cb}|$ based on the $B \to D\ell\nu_\ell$ process [47,56] give consistent results, but with a factor of two larger uncertainty.

Table 4: Current determinations of $|V_{cb}|$. The inclusive branching fractions have been adjusted for a $1.5 \pm 1.0\%$ $b \to u$ component relative to $b \to c$ [44]. The uncertainties are experimental followed by theoretical.

Mode	$\lvert V_{cb}\rvert$
$\overline{B} \to D^*\ell^-\overline{\nu}_\ell$ [64]	$0.0367 \pm 0.0023 \pm 0.0018$
	(Dispersion relation $\mathcal{F}(w)$ parameterization)
$\overline{B} \to D^*\ell^-\overline{\nu}_\ell$ [65]	$0.0392 \pm 0.0030 \pm 0.0019$
	(Linear $\mathcal{F}(w)$ parameterization (+ bias correction))
$\Gamma(b \to c\ell\nu_\ell)$	$0.0408 \pm 0.0005 \pm 0.0025$
	(B^0, B^+, B_s, and b-baryon admixture at the Z)
$\Gamma(B \to X_c\ell\nu_\ell)$	$0.0400 \pm 0.0010 \pm 0.0024$
	(B^0, B^+ admixture at the $\Upsilon(4S)$)

Heavy Quark Symmetry (HQS) has also allowed remarkable precision in the calculation of the semileptonic width $\Gamma(B \to X_c\ell\nu_\ell)$. The operator product expansion (OPE) of the width in terms of the (inverse) heavy quark mass and in α_s appears free of $1/m_b$ corrections, and at $1/m_b^2$ is given by [66]

$$\Gamma_{SL}(B) = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3} \times$$
$$\left[z_0 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \right) - 2 \left(1 - \frac{m_c^2}{m_b^2} \right)^4 \frac{\mu_G^2}{m_b^2} - \frac{2\alpha_S}{3\pi} z_0^{(1)} + ... \right] . \quad (9)$$

At $1/m_b^2$, three nonperturbative parameters enter the expansion of the differential decay rate: μ_π^2 (or, closely related λ_1), which is related to the average kinetic energy of the b quark in the meson; μ_G^2 (or λ_2), which is related to the hyperfine splitting and can be determined from the B–B^* mass

difference; and $\overline{\Lambda}$, which relates the quark mass to the meson mass. This last enters implicitly since the b quark mass, not the B meson mass has been used. The parameters z_0 and $z_0^{(1)}$ are known phase space factors that depend on m_c^2/m_b^2. Bigi [51] suggests an uncertainty of approximately 6% on $|V_{cb}|$ from such a calculation. Various calculations [67–68] are consistent with a central value

$$|V_{cb}| = 0.0411 \sqrt{\frac{\mathcal{B}(B \to X_c \ell \nu)}{0.105}} \sqrt{\frac{1.55 \text{ ps}}{\tau_B}} \left(1 - 0.024 \frac{\mu_\pi^2 - 0.5\text{GeV}^2}{0.2\text{GeV}^2}\right). \quad (10)$$

Combined with the semileptonic branching fractions at the $\Upsilon(4S)$ and the Z quoted above, one obtains the inclusive determinations of $|V_{cb}|$ listed in Table 4. These agree with the exclusive determinations.

The validity of the OPE-based calculation rests upon the assumption of quark–hadron duality. The uncertainty induced from this assumption is unknown. While expected to be small [69–71], there has been a suggestion that the assumption could mask corrections of order $1/m_b$ [72]. A 5% effect, for example, cannot be ruled out at this time.

Moments of the inclusive lepton [73] and hadron mass [74–76] spectra can be used both to determine the nonperturbative parameters and to test the OPE/HQS framework at the $1/m_b^2$ level. A preliminary moment analysis by CLEO [77] suggests that the parameters derived from the leptonic moments may be inconsistent with those from the hadronic moments. A variety of explanations for this exist: an experimental problem, slow convergence of the $1/m$ expansion for the higher moments, or more fundamentally, duality violation. Further investigation is required.

Semileptonic $b \to u$ transitions: The simplest diagram for a rare B decay is obtained by replacing the $b \to c$ spectator diagram with a CKM suppressed $b \to u$ transition. These decays probe the small CKM matrix element V_{ub}, the magnitude of which sets bounds on the combination $\rho^2 + \eta^2$ in the Wolfenstein parameterization of the CKM matrix [78]. As with

V_{cb}, extraction of V_{ub} has been attempted using both inclusive and exclusive semileptonic B decays. An accurate method of determining V_{ub} has been somewhat elusive. With exclusive techniques, the heavy–to–light $b \to u$ transition has no theoretical analogue to the zero recoil ($w = 1$) point in the heavy–to–heavy $b \to c$ transition of $B \to D^* \ell \nu$. Rather than calculating a correction of order 10% to the unit form factor expected for a heavy–to–heavy transition at $w = 1$ (in the infinite mass limit), the absolute normalization of the form factors must be predicted. This normalization dominates the uncertainty in exclusive determinations of V_{ub}.

There have been two exclusive V_{ub} analyses by the CLEO Collaboration: a simultaneous measurement of the $B \to \pi \ell \nu_\ell$ and the $B \to \rho \ell \nu_\ell$ transitions [79], and a second measurement of the $B \to \rho \ell \nu_\ell$ rate [80]. The results of the two analyses are largely statistically independent, and their results have been combined, with correlated uncertainties accounted for, to obtain $|V_{ub}| = (3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55) \times 10^{-3}$, where the final error is the uncertainty from the form factors. New calculations based on light cone sum rules [81–83] and lattice calculations [84,85,86] promise to result in uncertainties in the 10% to 15% range soon. Uncertainties below 10% will require either unquenched lattice calculations or accurate measurements of the rate for $B \to K^* \ell^+ \ell^-$, which would allow one to extract $|V_{ub}|/|V_{cs}|$ from a double ratio of B and D decays [87].

In principle, the fully inclusive rate can be calculated reliably enough (barring an unexpectedly large violation of quark-hadron duality) to determine $|V_{ub}|$ with an accuracy under 10% [51]. Realizing this accuracy is extremely difficult in practice because the ferocious background from $b \to c \ell \nu_\ell$ decays forces experiments to limit measurement to a restricted region of the total phase space. Restriction of the theoretical rate to the restricted region can introduce large uncertainties in the calculation that can be difficult to quantify.

The published inclusive analyses at the $\Upsilon(4S)$ [88] have focused on leptons in the endpoint region of the single lepton spectrum, which are kinematically incompatible with coming from a $b \to c$ transition. Models were used to estimate the

rate into the endpoint, from which $|V_{ub}/V_{cb}| = (0.08 \pm 0.02)$ is obtained. The error is dominated by the theoretical uncertainty, which has been very difficult to quantify. Because the endpoint region extends beyond the partonic endpoint and the size of the endpoint is of order $\Lambda_{\rm QCD}$, an infinite series of terms in the OPE rate calculation become equally important [89]. While the leading singularities can be resummed into a structure function [90,91], the structure function is unknown.

Another method for extracting $|V_{ub}|$ from the endpoint has been proposed [92] based on earlier suggestions [90,91] that involve comparison of the endpoint lepton spectrum to the photon spectrum in $b \to s\gamma$. These decays share the same structure function, and the comparison results in a large cancellation of the theoretical uncertainties. In principle, this technique could lead to a determination of $|V_{ub}|$ with an uncertainty under 10%.

Over the past several years, the ALEPH [93], DELPHI [94], and L3 [95] experiments have attempted inclusive measurements of the $b \to u\ell\nu_\ell$ rate. The approaches are disparate, but tend to be sensitive to $b \to u\ell\nu$ primarily when the mass of the hadronic system (m_{X_u}) is in the region $m_{X_u} \lesssim M_D$. They are sensitive to a significantly larger portion of the phase space than the endpoint analyses, but at the cost of very large backgrounds from $b \to c\ell\nu_\ell$ decays (signal:background ratios of order 1:10). The branching fractions obtained are listed in Table 5. An average by the LEP Heavy Flavour Group [37] results in $|V_{ub}| = 4.04^{+0.41}_{-0.46}(\exp)^{+0.43}_{-0.48}(b \to c)^{+0.24}_{-0.25}(b \to u) \pm 0.02(\tau_b) \pm 0.19(\mathrm{HQS})$. A note of caution, however. While observation of these decays at LEP is an experimental tour de force, the aggressive systematic errors assigned to unknown aspects of $b \to c\ell\nu_\ell$ and $b \to u\ell\nu_\ell$ processes remain a topic of discussion in the community. Among the concerns: the large uncertainties in the makeup of the non-D and D^* components of the background and the need for modeling of the $b \to u\ell\nu_\ell$ decays to correct for the smearing and nonuniform efficiency over the phase space of the decay.

A new proposal [89] to measure $|V_{ub}|$ inclusively in a restricted region of q^2 has promise. As mentioned above, measurements in the lepton endpoint region suffer from significant theoretical uncertainties from unknown structure functions.

Analyses restricted to the hadronic mass range $m_{X_u} < \sqrt{\overline{\Lambda} m_b}$ are affected by similar uncertainties, so the level appears to be much reduced [96,97], about 10%. The proposed method offers suppression of $b \to c\ell\nu_\ell$ background without introducing such uncertainties.

So far, the various determinations of $|V_{ub}|$ have produced consistent results. However, with the many theoretical and experimental difficulties with the measurements to date, the authors agree with the conservative assessment of the current uncertainties presented in the CKM review [98].

Table 5: Inclusive semileptonic branching fractions for $b \to u\ell\nu_\ell$ measured at LEP.

Experiment	Branching Fraction $[10^{-3}]$
ALEPH [93]	$1.73 \pm 0.55 \pm 0.55$
DELPHI [94]	$1.57 \pm 0.35 \pm 0.55$
L3 [95]	$3.3 \pm 1.0 \pm 1.7$

Hadronic B decays: In hadronic decays of B mesons, the underlying weak transition of the b quark is overshadowed by strong interaction effects caused by the surrounding cloud of light quarks and gluons. While this complicates the extraction of CKM matrix elements from experimental results, it also turns the B meson into an excellent laboratory to study perturbative and non-perturbative QCD, hadronization, and Final State Interaction (FSI) effects.

The precision of the experimental data has steadily improved over the past years. In 1997 CLEO updated most branching fractions for exclusive $B \to (n\pi)^- D^{(*)}$ and $B \to J/\psi K^{(*)}$ transitions. Tighter limits on color suppressed decays such as $\overline{B} \to D^0\pi^0$ have been presented [99]. Updated measurements of the polarization in $B \to J/\psi K^*$ resolved an outstanding discrepancy between theory and experiment [100]. Angular distributions have been studied for other B decays with two vector mesons in the final state including $B \to D^*\rho$, $B \to D^*D^*$, and $B \to D^*D_s^*$. CLEO found the relative phases of the helicity amplitudes in $B \to D^*\rho^-$ decays to be non-zero [101],

implying that FSI effects may play a role in B decays after all. $B^0 \to D^{*+}D^{*-}$ decays have been observed with a branching fraction of $(9.9^{+4.2}_{-3.3} \pm 1.2) \times 10^{-4}$, providing unambiguous evidence for Cabibbo–suppressed $b \to ccd$ transitions [102,103].

Gronau and Wyler [104] first suggested that decays of the type $B \to DK$ can be used to extract the angle γ of the CKM unitarity triangle, $\gamma \approx \arg(V_{ub})$. The first example of such a Cabibbo–suppressed mode has been observed by CLEO [105]:

$$\frac{\mathrm{B}(B^- \to D^0 K^-)}{\mathrm{B}(B^- \to D^0 \pi^-)} = 0.055 \pm 0.014 \pm 0.005 \ . \qquad (11)$$

Measurements of exclusive hadronic B decays have reached sufficient precision to challenge our understanding of the dynamics of these decays. It has been suggested that in analogy to semileptonic decays, two-body hadronic decays of B mesons can be expressed as the product of two independent hadronic currents, one describing the formation of a charm meson and the other the hadronization of the remaining $\overline{u}d$ (or $\overline{c}s$) system from the virtual W^-. Qualitatively, for a B decay with a large energy release, the $\overline{u}d$ pair, which is produced as a color singlet, travels fast enough to leave the interaction region without influencing the second hadron formed from the c quark and the spectator antiquark. The assumption that the amplitude can be expressed as the product of two hadronic currents is called "factorization" in this paper. By comparing exclusive hadronic B decays to the corresponding semileptonic modes the factorization hypothesis has been experimentally confirmed for certain $b \to c$ decays with large energy release [100]. An example is given by the longitudinal polarization of ρ mesons in $B \to D^*\rho$ decays, which was recently updated by the CLEO Collaboration [101]. Their result of $\Gamma_L/\Gamma = 0.878 \pm 0.034 \pm 0.040$ agrees well with the factorization expectation, 0.85–0.88 [106–109].

For internal spectator decays, the validity of the factorization hypothesis is also questionable and requires experimental verification. The naive color transparency argument used in the previous sections is not applicable to decays such as $B \to J/\psi K$, and there is no corresponding semileptonic decay for comparison. For internal spectator decays, one can only compare experimental observables to quantities predicted by

models based on factorization. Two such quantities are the production ratio

$$\mathcal{R} = \frac{\mathrm{B}(B \to J/\psi K^*)}{\mathrm{B}(B \to J/\psi K)} \tag{12}$$

and the amount of longitudinal polarization Γ_L/Γ in $B \to J/\psi K^*$ decays. The CLEO Collaboration published new data on $B \to$ charmonium transitions [110].

$$\mathcal{R} = 1.45 \pm 0.20 \pm 0.17 \ , \ \Gamma_L/\Gamma = 0.52 \pm 0.07 \pm 0.04 \ , \tag{13}$$

are now consistent with factorization-based models.

In the decays of charm mesons, the effect of color suppression is obscured by the effects of FSI or reduced by nonfactorizable effects. Because of the larger mass of the b quark, a more consistent pattern of color-suppression is expected in the B system, and current experimental results seem to support that color-suppression is operative in hadronic decays of B mesons. Besides $B \to$ charmonium transitions, no other color-suppressed decay has been observed experimentally [99]. The current upper limit on $\mathrm{B}(\overline{B}^0 \to D^0\pi^0)$ is 0.012% at 90% C.L.

By comparing hadronic B^- and $\overline{B}^0$ decays, the relative contributions from external and internal spectator decays have been disentangled. For all decay modes studied, the B^- branching fraction was found to be larger than the corresponding $\overline{B}^0$ branching ratio, indicating constructive interference between the external and internal spectator amplitudes. In the BSW model [111], the two amplitudes are proportional to effective coefficients, a_1 and a_2, respectively. A least squares fit using experimental results and a model by Neubert *et al.* [112] gives

$$a_2/a_1 = 0.22 \pm 0.04 \pm 0.06 \ , \tag{14}$$

where we have ignored uncertainties in the theoretical predictions. The second error is due to the uncertainty in the B-meson production fractions (f_+, f_0) and lifetimes (τ_+, τ_0) that enter into the determination of a_2/a_1 in the combination $(f_+\tau_+/f_0\tau_0)$. As this ratio increases, the value of a_2/a_1 decreases. Varying $(f_+\tau_+/f_0\tau_0)$ in the allowed experimental range excludes a negative value of a_2/a_1. Other uncertainties in the

magnitude of the decay constants f_D and f_{D^*}, as well as in the hadronic form factors, can change the magnitude of a_2/a_1, but not its sign.

The magnitude of a_2 determined from this fit to the ratio of B^- and B^0 branching fractions is consistent with the value of $|a_2|$ determined from the fit to the $B \to J/\psi X$ decay modes, which only proceed via the color suppressed amplitude. The coefficient a_1 also shows little or no process dependency.

The observation that the coefficients a_1 and a_2 have the same relative sign in B^- decay came as a surprise, since destructive interference was observed in hadronic charm decay. The sign of a_2 disagrees with the theoretical extrapolation from the fit to charm meson decays using the BSW model. It also disagrees with the expectation from the $1/N_c$ rule [113]. The result may be consistent with the expectation of perturbative QCD [114]. B. Stech proposed that the observed interference pattern in charged B and D decay can be understood in terms of the running strong coupling constant α_s [115]. A solution based on PQCD factorization theorems has been suggested by B. Tseng and H.N. Li [116].

Although constructive interference has been observed in all the B^- modes studied so far, these comprise only a small fraction of the total hadronic rate. It is conceivable that higher–multiplicity B^- decays demonstrate a very different behavior.

It is intriguing that $|a_1|$ determined from the $B \to D^{(*)}\pi$, $D^{(*)}\rho$ modes agrees well with the value of a_1 extracted from $B \to DD_s$ decays. The observation of color-suppressed decays such as $\overline{B}^0 \to D^0\pi^0$ would give another measure of $|a_2|$ complementary to that obtained from $B \to$ charmonium decays.

In summary, experimental results on exclusive B decay match very nicely with theoretical expectations. Unlike charm, the b quark appears to be heavy enough so that corrections due to the strong interaction are small. Factorization and color-suppression are at work. An intriguing pattern of constructive interference in charged B decays has been observed.

Inclusive hadronic decays: Over the last years, inclusive B decays have become an area of intensive studies, experimentally as well as theoretically. Since the hadronization process

to specific final state mesons is not involved in inclusive calculations, the theoretical results and predictions are generally believed to be more reliable.

CLEO and the LEP Collaborations presented new measurements of inclusive $b \to c$ transitions that can be used to extract n_c, the number of charm quarks produced per b decay. Naively we expect $n_c = 115\%$, with the additional 15% coming from the fragmentation of the W boson to $\overline{c}s$. This expectation can be verified experimentally by adding all inclusive $b \to c$ branching fractions. Using CLEO and DELPHI results, we can perform the calculation shown in Table 6. Modes with 2 charm quarks in the final state are counted twice. For the unobserved $B \to \eta_c X$ decay, we take the experimental upper limit. B_s mesons and b baryons produced at the Z, but not at the $\Upsilon(4S)$, cause the increase in D_s and Λ_c production rates seen by LEP. To first order, however, this should not affect the charm yield, as it should be compensated by reduced branching fractions for D mesons. This reduction is not reflected in the current data, but the errors in the D branching fractions are still large. In addition, there are significant uncertainties in the D_s and Λ_c absolute branching fractions.

New measurements of the multiplicity of charm quarks per b decay have also been reported by ALEPH and OPAL [117]. Combining this with the DELPHI results yields a new correlated average of $n_c = 1.151 \pm 0.022 \pm 0.022 \pm 0.051$, where the errors are statistical, systematic and due to the uncertainties in charm branching fractions [118]. There is now good agreement between the results from the $\Upsilon(4S)$ and the Z^0.

The $b \to c\overline{c}s$ transition: It was previously assumed that the conventional $b \to c\overline{u}d \to DX$ and $b \to c\overline{c}s \to DD_sX$ mechanisms account for all D-meson production in B decay. Buchalla *et al.* [120] suggested that a significant fraction of D mesons could also arise from $b \to c\overline{c}s$ transitions with light quark pair production at the upper vertex, *i.e.* $b \to c\overline{c}s \to D\overline{D}X_s$. The two mechanisms can be distinguished by the different final states they produce. In the first case the final state includes only D mesons, whereas in the second case two D mesons can be produced, one of which has to be a $\overline{D}$.

Table 6: Charm yield per B decay.

	Channel	Branching fraction [%] $\Upsilon(4S)$ [100]	LEP (DELPHI) [119]
	$B \to D^0 X$	63.6 ± 3.0	60.05 ± 4.29
+	$B \to D^+ X$	23.5 ± 2.7	23.01 ± 2.13
+	$B \to D_s^+ X$	12.1 ± 1.7	16.65 ± 4.50
+	$B \to \Lambda_c^+ X$	2.9 ± 2.0	8.90 ± 3.0
+	$B \to \Xi_c^{+,0} X$	2.0 ± 1.0	4.00 ± 1.60
+	$2\times\ B \to J/\psi_{\rm direct} X$	0.8 ± 0.08	
+	$2\times\ B \to \psi(2S)_{\rm direct} X$	0.35 ± 0.05	
+	$2\times\ B \to \chi_{c1} X$	0.37 ± 0.07	
+	$2\times\ B \to \chi_{c2} X$	0.25 ± 0.1	
+	$2\times\ B \to \eta_c X$	< 0.9 (90%C.L.)	
+	$2\times\ b \to (c\overline{c}) X$		2.00 ± 0.65
	n_c	110 ± 5	115.1 ± 7.4

Table 7: CLEO results on $B \to DDK$ decays.

Mode	Branching fraction
$\mathrm{B}(\overline{B}^0 \to D^{*+}\overline{D}^0 K^-)$	$0.45^{+0.25}_{-0.19} \pm 0.08\%$
$\mathrm{B}(B^- \to D^{*0}\overline{D}^0 K^-)$	$0.54^{+0.33}_{-0.24} \pm 0.12\%$
$\mathrm{B}(\overline{B}^0 \to D^{*+}\overline{D}^{*0} K^-)$	$1.30^{+0.61}_{-0.47} \pm 0.27\%$
$\mathrm{B}(B^- \to D^{*0}\overline{D}^{*0} K^-)$	$1.45^{+0.78}_{-0.58} \pm 0.36\%$

Two routes to search for this addition to $\Gamma(b \to c\overline{c}s)$ have been pursued experimentally. In an exclusive search for $B \to D\overline{D}K$ decays, CLEO required the final state to include a D and a $\overline{D}$ meson. Statistically significant signals are observed for several $D^{(*)}\overline{D}^{(*)}$ combinations. The preliminary CLEO results are listed in Table 7 [121]. While the observation of these decays proves the existence of $\overline{D}$-meson production at the upper vertex, a more inclusive measurement is needed to estimate the overall magnitude of this effect. A recent CLEO analysis exploits the fact that the flavor of the final state D-meson tags the decay mechanism. High momentum leptons ($p_\ell > 1.4$ GeV/c) are used to classify the flavor of the decaying B meson. $b \to c\overline{u}d$ transitions lead to $D\ell^+$ combinations, while the observation of $\overline{D}\ell^+$ identifies the new $b \to c\overline{c}s$ mechanism.

Angular correlations are used to remove combinations with both particles coming from the same B meson. CLEO finds [122]

$$\frac{\Gamma(\overline{B} \to \overline{D}X)}{\Gamma(\overline{B} \to DX)} = 0.100 \pm 0.026 \pm 0.016 \ , \tag{15}$$

which implies

$$\mathrm{B}(\overline{B} \to \overline{D}X) = 0.079 \pm 0.022 \ . \tag{16}$$

We can now calculate $n_{cc} = \mathrm{B}(b \to c\overline{c}s)$. n_{cc} is related to n_c, the number of charm quarks produced per b decay

$$n_c = 1 + n_{cc} - n_{B \to \text{no charm}} \ . \tag{17}$$

Using the data listed in Table 6 and the above result, we find

$$n_{cc} = (23.9 \pm 3.0)\% \ . \tag{18}$$

The contribution from $B \to \Xi_c^0 X$ was reduced by 1/3 to take into account the fraction that is not produced by the $b \to c\overline{c}s$ subprocess, but by $b \to c\overline{u}d \ + \ s\overline{s}$ quark pair production.

This result is consistent with theoretical predictions, $n_{cc} = 22 \pm 6\%$ [28,123]. $b \to D\overline{D}X$ decays have also been observed at LEP and at the SLC. ALEPH [102] finds

$$\mathrm{B}(B \to D^0\overline{D}^0 X + D^0 D^{\mp} X) = 0.078^{+0.02\ +0.017+0.005}_{-0.018\ -0.015-0.004} \ , \tag{19}$$

where the last error reflects the uncertainty in D meson branching fractions. DELPHI and SLD look for double charm decays of b hadrons by selecting events that are consistent with having two decay vertices. They find $n_{2c} = (13.6 \pm 4.2)\%$ [124] and $n_{2c} = (16.2 \pm 1.9 \pm 4.2)\%$ [125], respectively. n_{2c} does not include $B \to$Charmonium production. Taking this into account we find that these results are consistent with n_{cc}. DELPHI used a b-tagging technique to measure the inclusive charmless B branching fraction to 0.033 ± 0.021. Subtracting charmonium production allows them to set an upper limit on charmless b decays of 3.7% at 95% CL [124].

Charm Counting and the Semileptonic Branching Fraction: The charm yield per B meson decay is related to an intriguing puzzle in B physics: the experimental value for the

semileptonic branching ratio of B mesons, $\mathrm{B}(B \to X\ell\nu) = 10.49 \pm 0.17 \pm 0.43\%$ ($\Upsilon(4S)$), is significantly below the theoretical lower bound $\mathrm{B} > 12.5\%$ from QCD calculations within the parton model [126]. Since the semileptonic and hadronic widths are connected via

$$1/\tau = \Gamma = \Gamma_{\mathrm{Semileptonic}} + \Gamma_{\mathrm{Hadronic}}$$

an enhanced hadronic rate is necessary to accommodate the low semileptonic branching fraction. The hadronic width, which can be expressed as

$$\Gamma_{\mathrm{Hadronic}} = \Gamma(b \to c\bar{c}s) + \Gamma(b \to c\bar{u}d) + \Gamma(b \to sg \ + \ \mathrm{no\,charm})$$

is constraint by another experimental quantity, n_c, the average number of charm quarks produced per b decay.

For years it has been difficult to accommodate the experimental results with the theoretical preference for a larger values for $\mathrm{B_{sl}}$, n_c and n_{cc}. Additional confusion has been caused by an apparent discrepancy between LEP (Z^0) and CLEO ($\Upsilon(4S)$) results. The latter issue, however, has been resolved with both the LEP average for $\mathrm{B_{sl}}$ and n_c coming down. There is now good agreement between the experiments. Several explanations of this $n_c/\mathrm{B_{sl}}$ discrepancy have been proposed:

1. enhancement of $b \to c\bar{c}s$ due to large QCD corrections or a breakdown of local duality;
2. enhancement of $b \to c\bar{u}d$ due to non-perturbative effects;
3. enhancement of $b \to sg$ and/or $b \to dg$ due to New Physics;
4. systematic problem in the experimental results;

or the problem could be caused by some combination of the above.

Arguably the most intriguing solution to this puzzle would be an enhanced $b \to sg$ rate but as we will see in the next section, new results from CLEO and LEP show no indication for New Physics and place tight limits on this process.

$\mathrm{B}(b \to c\bar{u}d)$ has been calculated to next-to-leading order. Bagan *et al.* [127] find:

$$r_{ud} = \frac{\mathrm{B}(b \to c\bar{u}d)}{\mathrm{B}(b \to c\ell\nu)} = 4.0 \pm 0.4 \to \mathrm{B}(b \to c\bar{u}d)_{\mathrm{Theory}} = 41 \pm 4\%$$

which compares well with the experimental value of $43 \pm 6\%$ [100] but the errors are still too large to completely rule out an enhanced $b \to c\bar{u}d$ rate.

The theoretically preferred solution calls for an enhancement of the $b \to c\bar{c}s$ channel [127,28]. Increasing the $b \to c\bar{c}s$ component, however, would increase the average number of c quarks produced per b quark decay as well as n_{cc}, the number of b decays with 2 charm quarks in the final state. This is not supported by the data, in particular the value of n_c appears to be too low at the few σ-level. Systematic problems with D meson branching fractions have been pointed out as potential solution [128] but new results from ALEPH [129] and CLEO [130] on $\mathrm{B}(D^0 \to K^-\pi^+)$ make this less likely.

After years of experimental and theoretical efforts the missing charm/B_{sl} problem has begun to fade away. The discrepancy between experiments at the $\Upsilon(4S)$ and the Z^0 has been resolved. More data are needed to either resolve this issue or to demonstrate that the problem persists.

Rare B decays: All B-meson decays that do not occur through the usual $b \to c$ transition are known as rare B decays. These include both tree level semileptonic and hadronic $b \to u$ decays that are suppressed by the small CKM matrix element V_{ub}, as well as higher order processes such as electromagnetic and gluonic penguin decays. Branching fractions are typically around 10^{-5}, for exclusive channels, and sophisticated background suppression techniques are essential for these analyses.

Arguably the most exciting new experimental results since the last edition of this review are in the field of rare B decays. For many charmless B-decay modes the addition of new data and the refinement of analysis techniques allowed CLEO to observe signals where previously there have been upper limits. For other channels new tighter upper limits have been published.

Hadronic $b \to u$ transitions: Using almost 20 million charged and neutral B decays, CLEO successfully reconstructed a handful of exclusive hadronic $B^0 \to \pi^+\pi^-$ decays [131]. As can be seen in Table 8, the branching fraction for this mode is about a factor of 4 smaller than the rate of $B \to K\pi$ transitions. This is not good news for CP-violation studies. Not only is the branching fraction very small, but in addition the analysis will be complicated by "penguin pollution."

A theoretically clean method to determine the sum of the angles $\beta + \gamma$ of the unitarity triangle has been proposed by Snyder and Quinn [136]. They suggest that a sample of 10^3 $B \to \rho\pi$ decays, together with a Dalitz plot analysis, allow a measurement of $\beta+\gamma$ to about 6°. CLEO has recently measured the branching fraction for these modes [132]

$$\mathrm{B}(B^+ \to \rho^0\pi^+) \;=\; (1.5 \pm 0.5 \pm 0.4) \times 10^{-5} \qquad (20)$$

$$\mathrm{B}(B^0 \to \rho^\pm\pi^\mp) \;=\; (3.5^{+1.1}_{-1.0} \pm 0.5) \times 10^{-5} \qquad (21)$$

but it will take a while before a sufficiently large data sample will be available.

Electromagnetic penguin decays: The observation of the decay $B \to K^*(892)\gamma$, reported in 1993 by the CLEO II experiment, provided first evidence for the one-loop penguin diagram [138]. Using a larger data sample, the analysis was re-done in 1999 [139] yielding a total of 125 events and

$$\mathrm{B}(B^0 \to K^{*0}\gamma) = (4.55^{+0.72}_{-0.68} \pm 0.34) \times 10^{-5}\ , \qquad (22)$$

$$\mathrm{B}(B^+ \to K^{*+}\gamma) = (3.76^{+0.89}_{-0.83} \pm 0.28) \times 10^{-5}\ . \qquad (23)$$

The decay $B \to K_2^*(1430)\gamma$ was seen with a branching fraction of $(1.66^{+0.59}_{-0.53} \pm 0.13) \times 10^{-5}$. No evidence for the decays $B \to \rho\gamma$ and $B \to \omega\gamma$ was found. The current upper limit for the ratio

Table 8: Summary of CLEO results on $B \to \pi\pi, K\pi$, and KK branching fractions. The branching fractions and the 90% C.L. upper limits are given in units of 10^{-5}. Using the notation of Gronau *et al.* [137], the third column indicates the dominant amplitudes for each decay (T, C, P, E denote tree, color suppressed, penguin, and exchange amplitudes and the unprimed (primed) amplitudes refer to $\overline{b} \to \overline{u}u\overline{d}$ ($\overline{b} \to \overline{u}u\overline{s}$) transitions, respectively.)

Mode ($B \to$)	B	Amplitude	Theoretical expectation
$\pi^+\pi^-$	$0.43^{+0.16}_{-0.14} \pm 0.05$	$-(T+P)$	0.8–2.6
$\pi^+\pi^0$	< 1.3	$-(T+C)/\sqrt{(2)}$	0.4–2.0
$\pi^0\pi^0$	< 0.93	$-(C-P)/\sqrt{(2)}$	0.006–0.1
$K^+\pi^-$	$1.72^{+0.25}_{-0.24} \pm 0.12$	$-(T'+P')$	0.7–2.4
$K^+\pi^0$	$1.16^{+0.30+0.14}_{-0.27-0.13}$	$-(T'+C'+P')/\sqrt{(2)}$	0.3–1.3
$K^0\pi^-$	$1.82^{+0.46}_{-0.40} \pm 0.16$	P'	0.8–1.5
$K^0\pi^0$	$1.46^{+0.59+0.24}_{-0.51-0.33}$	$-(C'-P')/\sqrt{(2)}$	0.3–0.8
K^+K^-	< 0.19	E	—
K^+K^0	< 0.51	P	0.07–0.13
K^0K^0	< 1.7	P	0.07–0.12

$\mathrm{B}(B \to (\rho/\omega)\gamma)/\mathrm{B}(B \to K^*\gamma)$ is 0.32 at 90% CL. The limit on the ratio of branching fractions implies that $|V_{td}/V_{ts}| < 0.75$ at 90% CL.

The observed branching fractions were used to constrain a large class of Standard Model extensions [140]. However, due to the uncertainties in the hadronization, only the inclusive $b \to s\gamma$ rate can be reliably compared with theoretical calculations. This rate can be measured from the endpoint of the inclusive photon spectrum in B decay. CLEO [141] found

$$\mathrm{B}(b \to s\gamma) = (3.15 \pm 0.35 \pm 0.41) \times 10^{-4}\,(\mathrm{CLEO})\ , \qquad (24)$$

to be compared to the Standard Model rate [142–144] of

$$\mathrm{B}(b \to s\gamma)_{SM} = (3.28 \pm 0.33) \times 10^{-4}\ . \qquad (25)$$

ALEPH used a lifetime tagged sample of $Z \to b\overline{b}$ events to search for high-energy photons in the hemisphere opposite to the tag. This allows them to measure the photon spectrum from B decays which ultimately leads to [145]

$$\mathrm{B}(b \to s\gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}\,(\mathrm{ALEPH}). \qquad (26)$$

Our theoretical understanding of inclusive $b \to s\gamma$ transitions has been significantly enhanced by two new calculations that now include all terms to next-to-leading order [142–144]. The expected Standard Model rate, while slightly larger now, is still consistent with both the CLEO and ALEPH results. The substantially reduced uncertainties result in tighter constraints on new physics such as double Higgs models [146].

Gluonic penguin decays: A larger total rate is expected for gluonic penguins, the counterpart of $b \to s\gamma$ with the photon replaced by a gluon.

Experimentally, it is a major challenge to measure the inclusive $b \to sg$ rate. The virtual gluon hadronizes as a $q\overline{q}$ pair without leaving a characteristic signature in the detector. CLEO extended D–ℓ correlation measurements described in the section on hadronic B decays to obtain the flavor specific decay rate $\Gamma(\overline{B} \to DX)_{\text{lower vertex}}/\Gamma_{\text{total}}$. This quantity should be 1 minus corrections for charmonium production, $b \to u$ transitions, $B \to$ baryons, and D_s production at the lower vertex. Most importantly, the $b \to sg$ rate must also be subtracted. To remove uncertainties due to $\mathrm{B}(D^0 \to K^-\pi^+)$, CLEO normalizes to $\Gamma(\overline{B} \to DX\ell\nu_\ell)/\Gamma(\overline{B} \to X\ell\nu_\ell)$. Their preliminary result is

$$\frac{\Gamma(\overline{B} \to DX)_{\text{lower vertex}}/\Gamma_{\text{total}}}{\Gamma(\overline{B} \to DX\ell\nu_\ell)/\Gamma(\overline{B} \to X\ell\nu_\ell)} = 0.901 \pm 0.034 \pm 0.014 \quad (27)$$

whereas $0.903 \pm 0.018 - (b \to sg)$ was expected. This corresponds to an upper limit of $\mathrm{B}(b \to sg) < 6.8\%$ at 90% CL [122]. DELPHI [147] studied the p_T spectrum of charged kaons in B decays and found a model-dependent limit $\mathrm{B}(b \to sg) < 5\%$ (95% C.L.). These results agree well with the Standard Model prediction of $\mathrm{B}(\overline{B} \to \text{no charm}) = (1.6 \pm 0.8)\%$ [148], and there is little experimental support for new physics and an enhanced $b \to sg$ rate [149]. However, experimental uncertainties are still large, and it is too early to draw final conclusions.

Exclusive decays such as $B \to K^+\pi^-$ are suppressed at tree level and are expected to proceed via loop processes. CLEO studied these decay modes, and all 4 $K\pi$ combinations have been observed [131]. The results are listed in Table 8.

B($B^+ \to K^0\pi^+$) is of particular interest since it directly measures the strength of the gluonic penguin amplitude (Table 8). The smaller rate measured for $B^0 \to K^+\pi^-$ could indicate that the two amplitudes contributing to this channel interfere destructively. This observation has been extended by Fleischer and Mannel [151] to place some constraints on γ, the phase of V_{ub}.

CLEO extended their search of charmless B decay to modes including light meson resonances such as ρ, K^*, ω, η, and η' [132,133–135]. Statistically significant signals have been seen in several channels; the results are summarized in Table 9.

Table 9: Summary of new CLEO results on rare B decays involving light meson resonances.

Mode	Branching fraction ($\times 10^{-5}$)
$B \to \omega\pi^+$	$1.13^{+0.33}_{-0.29} \pm 0.15$
$B \to \eta' K^+$	$8.0^{+1.0}_{-0.9} \pm 0.7$
$B \to \eta' K^0$	$8.9^{+1.8}_{-1.6} \pm 0.9$
$B \to \eta' X_s$	$62 \pm 16^{+13}_{-20}$ $(2.0 < p_{\eta'} < 2.7\,\mathrm{GeV/c})$
$B \to \eta K^{*+}$	$2.6^{+1.0}_{-0.8} \pm 0.3$
$B \to \eta K^{*0}$	$1.4^{+0.6}_{-0.5} \pm 0.2$

A surprisingly large signal has been observed for $B \to \eta' K$, while no evidence for ηK or $\eta' K^*$ final states has been found [152].

The interpretation of these results is subject of an ongoing discussion. It has been suggested that interference between different penguin amplitudes causes B($B \to \eta' K$) to be larger than B($B \to \eta K$) [153,154]. This hypothesis is supported by the rate seen by CLEO for $B \to K^*\eta$. Other proposals try to explain the large $\eta' K$ rate by the anomalous coupling of the η' to glue [155,156], a $c\bar{c}$ component in the η' [157], or by an enhanced $b \to sg$ rate due to some new physics [158]. Additional experimental input to this puzzle comes from a CLEO measurement of inclusive η' production. At high momenta, the η' spectrum is dominated by $B \to \eta' X_s$ decays, and a study

of the system recoiling against the η' shows that large masses $m(X_s)$ are preferred [135].

In summary, gluonic penguin decays and hadronic $b \to u$ transitions have been established. Many decay modes have been observed for the first time, and the emerging pattern is full of surprises. The observed penguin effects are large and while old favorites such as $B^0 \to \pi^+\pi^-$ might be less useful for CP-violation studies, there is hope that new opportunities will open up.

CP asymmetries and outlook: Perhaps the most exciting aspect of B physics over the past two years is that experiments have just begun to have sensitivity to CP asymmetries at a nontrivial level. CDF, for example, has contributed the measurement [159] $\sin(2\beta) = 0.79^{+0.41}_{-0.44}$ using the "golden" $B^0 \to J/\psi K_s$ mode. This measurement provides the first indication that B system favors a positive $\sin(2\beta)$, an assumption we have made based on constraints from ϵ in the K^0 system. OPAL [160] and ALEPH [161] have also attempted $\sin(2\beta)$ determinations.

The rate asymmetry for $b \to s\gamma$ versus $\overline{b} \to \overline{s}\gamma$ can constrain non-Standard Model physics, with some models resulting in asymmetries as large as 40% [162,163]. CLEO has recently measured this asymmetry [164], finding $\mathcal{A} = (0.16 \pm 0.14 \pm 0.05) \times (1.0 \pm 0.14)$ with 1/3 of their total B sample. The 90% confidence interval is $-0.09 < \mathcal{A} < 0.42$.

The CLEO II experiment has been placing limits on CP asymmetries. Nonzero results would indicate direct CP violation in the B system [165,166]. In $B^\pm \to J/\psi K^\pm$, one expects the Standard Model CP asymmetry to be very small, though asymmetries as large as 10% are possible in particular two-Higgs doublet models [167]. CLEO finds [166] $\mathcal{A} = (1.8 \pm 4.3 \pm 0.4)\%$, consistent with the Standard Model. In the case of direct CP violation in $B \to K\pi$ decays, CP asymmetries as large as 0.3–0.5 are allowed [165,168] given the expected ratio of the tree–level to the penguin amplitude of about 1/4 [169,170]. CLEO has measured the CP asymmetry in the $K\pi$ modes [165], and finds, for example, that in the $K^\pm\pi^\mp$ mode, $\mathcal{A} = -0.04 \pm 0.16$ with

the 90% confidence interval $-0.30 < \mathcal{A} < 0.22$. All measured asymmetries are consistent with zero.

Finally, various authors have been studying the rare B-decay modes observed by CLEO to probe constraints on the phase γ of V_{ub} [169–175]. Some of the most recent studies make aggressive modeling assumptions, but seem to fit the data at hand well. Such studies prefer a γ in the second quadrant, but the experimental and modeling uncertainties are too large to draw any conclusion yet.

Over the next few years, final analyses from the full CLEO II dataset will have emerged and we will see new results from the large datasets expected from the new B-factories BABAR, BELLE and CLEO III, and from the RUN II at Fermilab, and, further afield, from LHCb at the LHC. They promise a rich spectrum of rare and precision measurements that have the potential to affect fundamentally our understanding of the Standard Model and CP-violating phenomena.

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