ELECTROWEAK MODEL AND CONSTRAINTS ON NEW PHYSICS

To reduce the size of this section's PostScript file, we have divided it into three PostScript files. We present the following index:

PART 1

PART 2

PART 3

10.4. *W* and *Z* decays

The partial decay width for gauge bosons to decay into massless fermions $f_1\overline{f}_2$ is

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226.5 \pm 0.3 \quad \text{MeV} \quad , \tag{10.35a}$$

$$\Gamma(W^+ \to u_i\overline{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi} \, |V_{ij}|^2 \approx (707 \pm 1)\,|V_{ij}|^2 \quad \text{MeV} \quad , \tag{10.35b}$$

$$\Gamma(Z \to \psi_i\overline{\psi}_i) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi} \left[g_V^{i2} + g_A^{i2} \right] \tag{10.35c}$$

$$\approx \begin{cases} 167.25 \pm 0.08 \quad \text{MeV } (\nu\overline{\nu}), & 84.01 \pm 0.05 \quad \text{MeV } (e^+e^-), \\ 300.3 \pm 0.2 \quad \text{MeV } (u\overline{u}), & 383.1 \pm 0.2 \quad \text{MeV } (d\overline{d}), \\ 376.0 \mp 0.1 \quad \text{MeV } (b\overline{b}), & \end{cases}$$

where the numerical values are for $(m_t, M_H) = (175 \pm 5 \text{ GeV}, M_Z)$. For leptons $C = 1$, while for quarks $C = 3\Big(1 + \alpha_s(M_V)/\pi \; +1.409\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3\Big)$, where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections [62] for massless quarks [63]. The $Z \to f\overline{f}$ widths contain a number of additional corrections: universal (non-singlet) top-mass contributions [64]; fermion mass effects and further QCD corrections proportional to m_q^2 [65] (m_q is the running quark mass evaluated at the Z scale) which are different for vector and axial-vector partial widths; and singlet contributions starting from two-loop order which are large, strongly top-mass dependent, family universal, and flavor non-universal [66]. All QCD effects are known and included up to three loop order. The QED factor $1 + 3\alpha q_f^2/4\pi$, as well as two-loop $\alpha\alpha_s$ and α^2 corrections [67,68] are also included. Working in the on-shell scheme, *i.e.*, expressing the widths in terms of $G_F M_{W,Z}^3$, incorporates the largest radiative corrections from the running QED coupling [18,69]. Electroweak corrections to the Z widths are then incorporated by replacing $g_{V,A}^{i2}$ by $\overline{g}_{V,A}^{i2}$. Hence, in the on-shell scheme the Z widths are proportional to $\rho_i \sim 1 + \rho_t$. The $\overline{\text{MS}}$ normalization (see the end of the previous section) accounts also for the leading electroweak corrections [22]. There is additional (negative) quadratic m_t dependence in the $Z \to b\overline{b}$ vertex corrections [70] which causes $\Gamma(b\overline{b})$ to decrease with m_t. The dominant effect is to multiply $\Gamma(b\overline{b})$ by the vertex correction $1 + \delta\rho_{b\overline{b}}$, where $\delta\rho_{b\overline{b}} \sim 10^{-2}(-\frac{1}{2}\frac{m_t^2}{M_Z^2} + \frac{1}{5})$. In practice, the corrections are included in ρ_b and κ_b, as discussed before.

For 3 fermion families the total widths are predicted to be

$$\Gamma_Z \approx 2.496 \pm 0.001 \text{ GeV} \quad , \tag{10.36}$$

$$\Gamma_W \approx 2.093 \pm 0.002 \text{ GeV} \quad . \tag{10.37}$$

We have assumed $\alpha_s = 0.120$. An uncertainty in α_s of ± 0.003 introduces an additional uncertainty of 0.1% in the hadronic widths, corresponding to ± 1.6 MeV in Γ_Z. These predictions are to be compared with the experimental results $\Gamma_Z = 2.4948 \pm 0.0025$ GeV and $\Gamma_W = 2.062 \pm 0.059$ GeV.

10.5. Experimental results

Table 10.3: Principal LEP and other recent observables, compared with the Standard Model predictions for $M_Z = 91.1867 \pm 0.0020$ GeV, $M_H = M_Z$, and the global best fit values $m_t = 173 \pm 4$ GeV, $\alpha_s = 0.1214 \pm 0.0031$, and $\widehat{\alpha}(M_Z)^{-1} = 127.90 \pm 0.07$. The LEP averages of the ALEPH, DELPHI, L3, and OPAL results include common systematic errors and correlations [57]. $\bar{s}_\ell^2(A_{FB}^{(0,q)})$ is the effective angle extracted from the hadronic charge asymmetry. The values of $\Gamma(\ell^+\ell^-)$, $\Gamma(\text{had})$, and $\Gamma(\text{inv})$ are not independent of Γ_Z, R_ℓ, and σ_{had}. The first M_W value is from CDF, UA2, and DØ [71] while the second includes the measurements at LEP [57]. M_W and M_Z are correlated, but the effect is negligible due to the tiny M_Z error. The four values of A_ℓ are (i) from A_{LR} for hadronic final states [59]; (ii) the combined value from SLD including leptonic asymmetries; (iii) from the total τ polarization; and (iv) from the angular distribution of the τ polarization. The two values of s_W^2 from deep-inelastic scattering are from CCFR [36] and the global average, respectively. Similarly, the $g_{V,A}^{\nu e}$ are from CHARM II [41] and the world average. The second errors in Q_W are theoretical [48,49]. Older low-energy results are not listed but are included in the fits. In the Standard Model predictions, the uncertainty is from M_Z, m_t, $\Delta\alpha(M_Z)$ and α_s. In parentheses we show the shift in the predictions when M_H is changed to 300 GeV which is its 90% CL upper limit. The errors in Γ_Z, $\Gamma(\text{had})$, R_ℓ, and σ_{had} are completely dominated by the uncertainty in α_s.

Quantity	Value	Standard Model
m_t [GeV]	175 ± 5	$173 \pm 4\,(+5)$
M_W [GeV]	80.405 ± 0.089	$80.377 \pm 0.023\,(-0.036)$
	80.427 ± 0.075	
M_Z [GeV]	91.1867 ± 0.0020	$91.1867 \pm 0.0020\,(+0.0001)$
Γ_Z [GeV]	2.4948 ± 0.0025	$2.4968 \pm 0.0017\,(-0.0007)$
$\Gamma(\mathrm{had})$ [GeV]	1.7432 ± 0.0023	$1.7433 \pm 0.0016\,(-0.0005)$
$\Gamma(\mathrm{inv})$ [MeV]	500.1 ± 1.8	$501.7 \pm 0.2\,(-0.1)$
$\Gamma(\ell^+\ell^-)$ [MeV]	83.91 ± 0.10	$84.00 \pm 0.03\,(-0.04)$
σ_{had} [nb]	41.486 ± 0.053	$41.469 \pm 0.016\,(-0.005)$
R_ℓ	20.775 ± 0.027	$20.754 \pm 0.020\,(+0.003)$
R_b	0.2170 ± 0.0009	$0.2158 \pm 0.0001\,(-0.0002)$
R_c	0.1734 ± 0.0048	$0.1723 \pm 0.0001\,(+0.0001)$
$A_{FB}^{(0,\ell)}$	0.0171 ± 0.0010	$0.0162 \pm 0.0003\,(-0.0004)$
$A_{FB}^{(0,b)}$	0.0984 ± 0.0024	$0.1030 \pm 0.0009\,(-0.0013)$
$A_{FB}^{(0,c)}$	0.0741 ± 0.0048	$0.0736 \pm 0.0007\,(-0.0010)$
$A_{FB}^{(0,s)}$	0.118 ± 0.018	$0.1031 \pm 0.0009\,(-0.0013)$
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2322 ± 0.0010	$0.2315 \pm 0.0002\,(+0.0002)$
A_ℓ	0.1550 ± 0.0034	$0.1469 \pm 0.0013\,(-0.0018)$
	0.1547 ± 0.0032	
	0.1411 ± 0.0064	
	0.1399 ± 0.0073	
A_b	0.900 ± 0.050	$0.9347 \pm 0.0001\,(-0.0002)$
A_c	0.650 ± 0.058	$0.6678 \pm 0.0006\,(-0.0008)$
$s_W^2(\nu\mathrm{N})$	0.2236 ± 0.0041	$0.2230 \pm 0.0004\,(+0.0007)$
	0.2260 ± 0.0039	
$g_V^{\nu e}$	-0.035 ± 0.017	$-0.0395 \pm 0.0005\,(+0.0002)$
	-0.041 ± 0.015	
$g_A^{\nu e}$	-0.503 ± 0.017	$-0.5064 \pm 0.0002\,(+0.0002)$
	-0.507 ± 0.014	
$Q_W(\mathrm{Cs})$	$-72.41 \pm 0.25 \pm 0.80$	$-73.12 \pm 0.06\,(+0.01)$
$Q_W(\mathrm{Tl})$	$-114.8 \pm 1.2 \pm 3.4$	-116.7 ± 0.1

The values of the principal Z pole observables are listed in Table 10.3, along with the Standard Model predictions for $M_Z = 91.1867 \pm 0.0020$, $m_t = 173 \pm 4$ GeV, $M_H = M_Z$ and $\alpha_s = 0.1214 \pm 0.0031$. Note, that the values of the Z pole observables (as well as M_W) differ from those in the Particle Listings because they include recent preliminary results [57,58,59,71]. The values and predictions of M_W [57,71], the Q_W for cesium [44] and thallium [45], and recent results from deep inelastic [32–36] and $\nu_\mu e$ scattering [39–41] are also listed. The agreement is excellent. Even the largest discrepancies, A^0_{LR}, $A^{(0,b)}_{FB}$, and $A^{(0,\tau)}_{FB}$, deviate by only 2.4 σ, 1.9 σ and 1.7 σ, respectively.

Other observables like $R_b = \Gamma(b\bar{b})/\Gamma(\text{had})$ and $R_c = \Gamma(c\bar{c})/\Gamma(\text{had})$ which showed significant deviations in the past, are now in perfect (R_c) or at least better agreement. In particular, R_b whose measured value deviated as much as 3.7 σ from the Standard Model prediction is now only 1.3 σ high. Many types of new physics could contribute to R_b (the implications of this possibility for the value of $\alpha_s(M_Z)$ extracted from the fits are discussed below) and A_b and as a consequence to $A^{(0,b)}_{FB} = \frac{3}{4} A_e A_b$. Indeed, A_b can be extracted from $A^{(0,b)}_{FB}$ when A_e is taken from leptonic asymmetries (using lepton universality), and combined with the measurement at the SLC. The result, $A_b = 0.877 \pm 0.023$, is 2.5 σ below the Standard Model prediction. (Alternatively, one can use $A_\ell = 0.1469 \pm 0.0013$ from the global fit and obtain $A_b = 0.894 \pm 0.021$ which is 1.9 σ low.) However, this deviation of about 6% cannot arise from new physics radiative corrections since a 30% correction to $\widehat{\kappa}_b$ would be necessary to account for the central value of A_b. Only a new type of physics which couples at the tree level preferentially to the third generation, and which does not contradict R_b (including the off-peak R_b measurements by DELPHI [72]), can conceivably account for a low A_b [73].

The left-right asymmetry, $A^0_{LR} = 0.1550 \pm 0.0034$ [59], based on all hadronic data from 1992–1996 has moved closer to the Standard Model expectation of 0.1469 ± 0.0013 than previous values. However, because of the smaller error A^0_{LR} is still 2.4 σ above the Standard Model prediction. There is also an experimental difference of ~ 1.9 σ between the SLD value of $A_\ell(\text{SLD}) = 0.1547 \pm 00032$ from all A_{LR} and $A^{FB}_{LR}(\ell)$ data on one hand, and the LEP value $A_\ell(\text{LEP}) = 0.1461 \pm 0.0033$ obtained from $A^{(0,\ell)}_{FB}$, $A_e(\mathcal{P}_\tau)$, $A_\tau(\mathcal{P}_\tau)$ on the other hand, in both cases assuming lepton-family universality.

Despite these discrepancies the χ^2 value of the fit for the Standard Model is excellent. It is 25 for 30 d.o.f. when fitting to the independent observables in Table 10.3, and 181 for 209 d.o.f. when the older neutral current observables are included. The probability of a larger χ^2 is 0.73 and 0.92 for the two cases, respectively. (The low χ^2 for the older data is likely due to overly conservative estimates of systematic errors.)

With the latest value of $A^{(0,\tau)}_{FB}$ the data is now in reasonable agreement with lepton-family universality, which will be assumed. The observables in Table 10.3 (including correlations on the LEP lineshape and LEP/SLD heavy flavor observables), as well as all low-energy neutral-current data [16,17], are used in the global fits described below. The parameter $\sin^2\theta_W$ can be determined from Z pole observables, M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results [16], shown in Table 10.4, are in impressive agreement with each other, indicating the quantitative success of the Standard Model. The one discrepancy is the value $\widehat{s}^2_Z = 0.23023 \pm 0.00043$ from A_ℓ(SLD) which is 2.3 σ below the value 0.23124 ± 0.00017 from the global fit to all data and 2.6 σ below the value 0.23144 ± 0.00019 obtained from all data other than A_ℓ(SLD).

The data allow a simultaneous determination of $\sin^2\theta_W$, m_t, and the strong coupling $\alpha_s(M_Z)$. The latter is determined mainly from R_ℓ, Γ_Z, and $\sigma_{\rm had}$, and is only weakly correlated with the other variables. The global fit to all data, including the CDF/DØ value, $m_t = 175 \pm 5$ GeV, yields

$$\begin{aligned} \widehat{s}^2_Z &= 0.23124 \pm 0.00017\,(+0.00024)\ , \\ m_t &= 173 \pm 4\,(+5)\ \text{GeV}\ , \\ \alpha_s(M_Z) &= 0.1214 \pm 0.0031\,(+0.0018)\ , \\ M_H &= M_Z\ . \end{aligned} \tag{10.38}$$

In parentheses we show the effect of changing M_H to 300 GeV which is the conservative 90% CL upper limit (see below). In all fits, the errors include full statistical, systematic, and theoretical uncertainties. The $\widehat{s}^2_Z$ error reflects the error on $\overline{s}^2_f \sim \pm 0.00023$ from the Z pole asymmetries. In the on-shell scheme one has $s^2_W = 0.22304 \pm 0.00044$, the larger error due to the stronger sensitivity to m_t. The extracted value of α_s is based on a formula with negligible theoretical uncertainty (± 0.0005 in α_s) if one assumes the exact validity of the Standard Model. It is in excellent agreement with other precise values [74], such as 0.122 ± 0.005 from τ decays, 0.121 ± 0.005 from jet-event shapes in e^+e^- annihilation, and the very recent result [75], 0.119 ± 0.002 (exp) ± 0.004 (scale), from deep-inelastic scattering. It is slightly higher than the values from lattice calculations of the $b\overline{b}$ (0.1174 ± 0.0024 [76]) and $c\overline{c}$ (0.116 ± 0.003 [77]) spectra, and from decays of heavy quarkonia (0.112 ± 0.006 [74]). For more details, see our Section 9 on "Quantum Chromodynamics" in this *Review*. The average $\alpha_s(M_Z)$ obtained from Section 9 when ignoring the precision measurements discussed in this Section is 0.1178 ± 0.0023. We use this value as an external constraint for the second fit in Table 10.5. The resulting value,

$$\alpha_s = 0.1191 \pm 0.0018\,(+0.0006)\ , \tag{10.39}$$

can be regarded as the present world average.

The value of R_b is 1.3 σ above the Standard Model expectation. If this is not just a fluctuation but is due to a new physics contribution to the $Z \to b\overline{b}$ vertex (many types would couple preferentially to the third family), the value of $\alpha_s(M_Z)$ extracted from the hadronic Z width would be reduced [17]. Allowing for this possibility one obtains $\alpha_s(M_Z) = 0.1166 \pm 0.0048\,(+0.0007)$. Similar remarks apply in principle for R_c and the other quark and lepton flavors, and one should keep in mind that the Z lineshape value of α_s is very sensitive to many types of new physics.

The data indicate a preference for a small Higgs mass. There is a strong correlation between the quadratic m_t and logarithmic M_H terms in $\widehat{\rho}$ in all of the indirect data except for the $Z \to b\overline{b}$ vertex. Therefore, observables (other than R_b) which favor m_t

Table 10.4: Values obtained for s_W^2 (on-shell) and $\widehat{s}_Z^2(\overline{\text{MS}})$ from various reactions assuming the global best fit values (for $M_H = M_Z$) $m_t = 173 \pm 4$ GeV and $\alpha_s = 0.1214 \pm 0.0031$.

Reaction	s_W^2	$\widehat{s}_Z^2$
M_Z	0.2231 ± 0.0005	0.2313 ± 0.0002
M_W	0.2228 ± 0.0006	0.2310 ± 0.0005
Γ_Z/M_Z^3, R, $\sigma_{\text{had}} M_Z^2$	0.2235 ± 0.0011	0.2316 ± 0.0011
$A_{FB}^{(0,\ell)}$	0.2225 ± 0.0007	0.2307 ± 0.0006
LEP asymmetries	0.2235 ± 0.0004	0.2317 ± 0.0003
A_{LR}^0	0.2220 ± 0.0005	0.2302 ± 0.0004
$\overline{A}_b, \overline{A}_c$	0.230 ± 0.016	0.239 ± 0.016
Deep inelastic (isocalar)	0.226 ± 0.004	0.234 ± 0.004
$\nu_\mu(\overline{\nu}_\mu)p \to \nu_\mu(\overline{\nu}_\mu)p$	0.203 ± 0.032	0.211 ± 0.032
$\nu_\mu(\overline{\nu}_\mu)e \to \nu_\mu(\overline{\nu}_\mu)e$	0.221 ± 0.008	0.229 ± 0.008
atomic parity violation	0.220 ± 0.003	0.228 ± 0.003
SLAC eD	0.213 ± 0.019	0.222 ± 0.018
All data	0.2230 ± 0.0004	0.23124 ± 0.00017

values higher than the Tevatron range favor lower values of M_H. This effect is enhanced by R_b, which has little direct M_H dependence but favors the lower end of the Tevatron m_t range. M_W has additional M_H dependence through $\Delta\widehat{r}_W$ which is not coupled to m_t^2 effects. The strongest individual pulls towards smaller M_H are from M_W, A_{LR}^0, and $A_{FB}^{(0,\ell)}$ (when combined with M_Z), as well as R_b. The difference in χ^2 for the global fit is $\Delta\chi^2 = \chi^2(M_H = 1000 \text{ GeV}) - \chi^2(M_H = 77 \text{ GeV}) = 16.6$. Hence, the data favor a small value of M_H, as in supersymmetric extensions of the Standard Model, and m_t on the lower side of the Tevatron range. If one allows M_H as a free fit parameter and does not include any constraints from direct Higgs searches, one obtains $M_H = 69^{+85}_{-43}$ GeV, *i.e.*, the central value below the direct lower bound, $M_H \geq 77$ GeV (95% CL) [78]. Including the results of the direct searches as an extra contribution to the likelihood function drives the best fit value to the present kinematic reach ($M_H \sim 83$ GeV), and we obtain the upper limit $M_H < 236$ (287) GeV at 90 (95)% CL. The extraction of M_H from the precision data depends strongly on the value used for $\alpha(M_Z)$. The value derived by Martin and Zeppenfeld [11] relying on the predictions of perturbative QCD down to smaller values of $\sqrt{s}$ is higher and has a smaller stated error. Using this value would give a best fit at $M_H = 140$ GeV, and an upper limit $M_H < 300$ (361) GeV at 90 (95)% CL. Clearly, a consensus on the applicability of perturbative QCD in e^+e^- annihilation is highly desirable.

The most deviating observable, A_{LR}, has a strong impact on the Higgs mass limits as well [17,79]. The Introduction to this *Review* suggests an unbiased treatment of deviating observables r through the introduction of scale factors S_r. It is instructive to study the impact of this more conservative procedure on M_H. For the case of a fit to the Standard Model, we define

$$S_r = \max(\sqrt{\chi_r^2}, 1) \ , \tag{10.40}$$

where χ_r^2 is the χ^2 contribution of observable r to a global fit in which M_H is allowed as a free fit parameter (with no direct constraints included). We then repeat the fit with all errors multiplied by S_r, and proceed iteratively until the procedure has converged. This way we obtain

$$S_{A^0_{LR}} = 2.76, \qquad S_{A^{(0,b)}_{FB}} = 2.05, \qquad S_{A^{(0,\tau)}_{FB}} = 1.83,$$
$$S_{A^{FB}_{LR}(\tau)} = 1.45, \qquad S_{A^{FB}_{LR}(\mu)} = 1.34, \qquad S_{R_b} = 1.33,$$

as well as $S_{A_e(\mathcal{P}_\tau)} = 1.02$, and $S_r = 1$ for all other observables. The result of the global fit is

$$\begin{aligned} \widehat{s}^{\,2}_Z &= 0.23141 \pm 0.00031 \ , \\ m_t &= 174 \pm 5 \text{ GeV} \ , \\ \alpha_s(M_Z) &= 0.1222 \pm 0.0034 \ , \\ M_H &= 122^{+134}_{-77} \text{ GeV} \ , \end{aligned} \tag{10.41}$$

where the larger errors compared to Eq. (10.38) are from M_H rather than the S_r. Since the central value of M_H is much larger than the present direct lower bound, and $\log(M_H)$ is approximately normal distributed, it is justified to include the error due to M_H (with all correlations properly taken into account) in a Gaussian way in the uncertainties of the other parameters. For comparison with other fits we also list the results for fixed M_H in Table 10.5. Including the direct constraint we obtain an upper limit $M_H < 329$ (408) GeV at 90 (95)% CL, which is higher by $\mathcal{O}(100$ GeV$)$ than the one without scale factors. It is in good agreement with the bound we obtained above by switching to the higher $\alpha(M_Z)$. Indeed, both analyses decrease the impact of A_{LR} on the Higgs mass limit.

A few comments are in order: (i) The procedure used here is not unambiguous. It depends on whether results from different experiments (*e.g.*, the various experimental groups at LEP or the Tevatron) are combined or used as individual pieces of input. We use combined result, primarily in order to avoid insurmountable complications with cross correlations between different experimental groups on top of the correlations between the observables. Even the result on a single observable quoted by an individual group, is in general a combination of various channels, with different types of systematic errors (which are the prime reason for the introduction of scale factors in the first place). Thus, ideally, one would prefer to define the S_r at this level. In practice, however, this is virtually impossible to achieve. In the case of M_W we use the individual determinations, since they are uncorrelated and are based on entirely different processes. (ii) None of the definitions of scale factors in the Introduction to this *Review* is directly applicable to our case. However, we have tried to work as closely as possible in spirit to the definitions given there. One major difference is that central values of fit parameters (in particular of M_H) change upon introducing S_r; on the other hand, central values of measurements remain unchanged. (iii) The procedure used here relies on the validity of the Standard Model, since in the presence of new physics, some discrepancies will be shifted into new physics parameters. When fits to new types of physics are to be compared to Standard Model fits as is done in Section 10.5 one has to refrain from using scale factors.

One can also carry out a fit to the indirect data alone, *i.e.*, without including the value $m_t = 175 \pm 5$ GeV observed directly by CDF and DØ. (The indirect prediction is for the $\overline{\text{MS}}$ mass which is in the end converted to the pole mass using an BLM optimized [80] version of the two-loop perturbative QCD formula [81]; this should correspond approximately to the kinematic mass extracted from the collider events.) One obtains $m_t = 170 \pm 7\ (+14)$ GeV, with little change in the $\sin^2\theta_W$ and α_s values, in remarkable agreement with the direct CDF/DØ value. The results of fits to various combinations of the data are shown in Table 10.5 and the relation between $\widehat{s}^2_Z$ and m_t for various observables in Fig. 10.1.

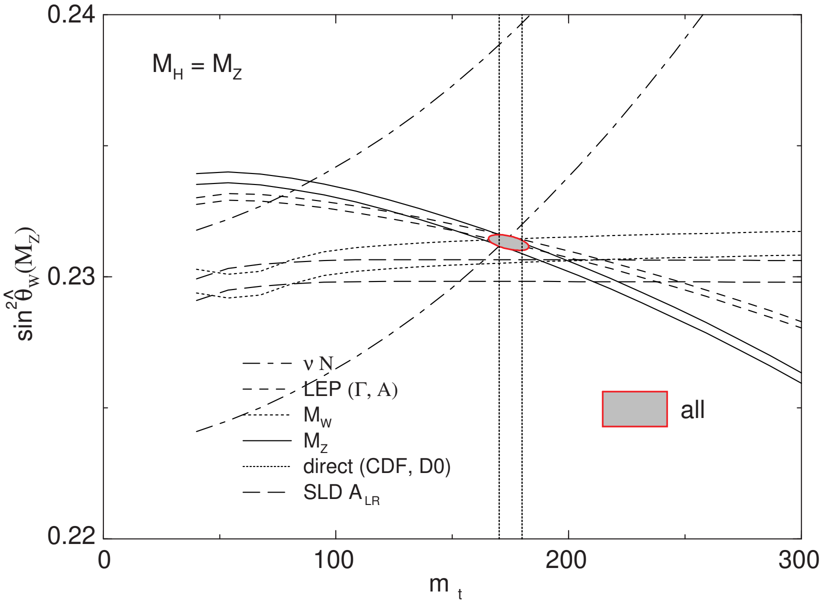

Figure 10.1: One-standard-deviation uncertainties in $\sin^2\widehat{\theta}_W$ as a function of m_t, the direct CDF and DØ range 175 ± 5 GeV, and the 90% CL region in $\sin^2\widehat{\theta}_W - m_t$ allowed by all data, assuming $M_H = M_Z$.

Table 10.5: Values of $\widehat{s}^2_Z$ and s^2_W (in parentheses), α_s, and m_t for various combinations of observables. The central values and uncertainties are for $M_H = M_Z$ while the third numbers show the shift (positive unless specified) from changing M_H to 300 GeV.

Data	$\widehat{s}^2_Z$ (s^2_W)	α_s (M_Z)	m_t [GeV]
All indirect + m_t	0.23124(17)(24) (0.2230±0.0004 (+0.0007))	0.1214(31)(18)	173(4)(5)
All indirect + $m_t + \alpha_s$	0.23121(17)(22) (0.2230±0.0004 (+0.0007))	0.1191(18)(6)	173(4)(5)
All indirect + $m_t + S_r$	0.23133(20)(32) (0.2232±0.0005 (+0.0008))	0.1218(31)(21)	173(4)(5)
All indirect	0.23129(19)(11) (0.2234±0.0007 (−0.0002))	0.1216(31)(14)	170(7)(14)
Z pole	0.23135(21)(10) (0.2236±0.0008 (−0.0003))	0.1218(31)(13)	168(8)(14)
LEP 1	0.23170(24)(13) (0.2247±0.0009 (−0.0002))	0.1232(31)(14)	160(8)(14)
SLD + M_Z	0.23023(43) (0.2192±0.0017 (−0.0008))	0.1200 (fixed)	203(13)(17)
$A^{(0,b)}_{FB}$ + M_Z	0.23209(45) (0.2261±0.0018 (−0.0009))	0.1200 (fixed)	147(17)(21)
M_W + M_Z	0.23101(43)(22) (0.2221±0.0015)	0.1200 (fixed)	181(12)(12)

Using $\alpha(M_Z)$ and $\widehat{s}^2_Z$ as inputs, one can predict $\alpha_s(M_Z)$ assuming grand unification. One predicts [82] $\alpha_s(M_Z) = 0.130 \pm 0.001 \pm 0.01$ for the simplest theories based on the minimal supersymmetric extension of the Standard Model, where the first (second) uncertainty is from the inputs (thresholds). This is consistent with the experimental $\alpha_s(M_Z) = 0.1216 \pm 0.0031 \pm 0.0003$ from the Z lineshape (with the second error corresponding to $M_H < 150$ GeV, as is appropriate to the lower M_H range appropriate for supersymmetry) and with the world average 0.119 ± 0.002. Nonsupersymmetric unified theories predict the low value $\alpha_s(M_Z) = 0.073 \pm 0.001 \pm 0.001$. See also the note on "Low-Energy Supersymmetry" in the Particle Listings.

One can also determine the radiative correction parameters Δr: including the CDF and DØ data, one obtains $\Delta r = 0.0355 \pm 0.0014$ $(+0.0021)$ and $\Delta\widehat{r}_W =$

0.0697 ± 0.0005 ($+0.0001$), in excellent agreement with the predictions 0.0349 ± 0.0020 and 0.0698 ± 0.0007. M_W measurements [57,71] (when combined with M_Z) are equivalent to measurements of $\Delta r = 0.0325 \pm 0.0045$.

Table 10.6: Values of the model-independent neutral-current parameters, compared with the Standard Model predictions for $M_Z = 91.1867 \pm 0.0020$ GeV, $M_H = M_Z$, and the global best fit values $m_t = 173 \pm 4$ GeV, $\alpha_s = 0.1214 \pm 0.0031$, and $\widehat{\alpha}(M_Z)^{-1} = 127.90 \pm 0.07$. There is a second $g_{V,A}^{\nu e}$ solution, given approximately by $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$, which is eliminated by e^+e^- data under the assumption that the neutral current is dominated by the exchange of a single Z. θ_i, $i = L$ or R, is defined as $\tan^{-1}[\epsilon_i(u)/\epsilon_i(d)]$.

Quantity	Experimental Value	Standard Model Prediction	Correlation	
$\epsilon_L(u)$	0.328 ± 0.016	0.3461 ± 0.0002		
$\epsilon_L(d)$	-0.440 ± 0.011	-0.4292 ± 0.0002		non-
$\epsilon_R(u)$	-0.179 ± 0.013	-0.1548 ± 0.0001		Gaussian
$\epsilon_R(d)$	$-0.027\ ^{+0.077}_{-0.048}$	0.0775 ± 0.0001		
g_L^2	0.3009 ± 0.0028	0.3040 ± 0.0003		
g_R^2	0.0328 ± 0.0030	0.0300		small
θ_L	2.50 ± 0.035	2.4629 ± 0.0001		
θ_R	$4.56\ ^{+0.42}_{-0.27}$	5.1765		
$g_V^{\nu e}$	-0.041 ± 0.015	-0.0395 ± 0.0005		-0.04
$g_A^{\nu e}$	-0.507 ± 0.014	-0.5064 ± 0.0002		
C_{1u}	-0.216 ± 0.046	-0.1885 ± 0.0003	-0.997	-0.78
C_{1d}	0.361 ± 0.041	0.3412 ± 0.0002		0.78
$C_{2u} - \frac{1}{2}C_{2d}$	-0.03 ± 0.12	-0.0488 ± 0.0008		

Most of the parameters relevant to ν-hadron, νe, e-hadron, and e^+e^- processes are determined uniquely and precisely from the data in "model independent" fits (*i.e.*, fits which allow for an arbitrary electroweak gauge theory). The values for the parameters defined in Eqs. (10.12)–(10.14) are given in Table 10.6 along with the predictions of the Standard Model. The agreement is excellent. The low-energy e^+e^- results are difficult to present in a model-independent way because Z propagator effects are non-negligible at TRISTAN, PETRA, and PEP energies. However, assuming e-μ-τ universality, the lepton asymmetries imply [55] $4(g_A^e)^2 = 0.99 \pm 0.05$, in good agreement with the Standard Model prediction $\simeq 1$.

The results presented here are generally in reasonable agreement with the ones obtained by the LEP Electroweak Working Group [57]. We obtain slightly higher values for α_s and significantly lower best fit values for M_H. We could trace the differences to be due to (i) the inclusion of recent higher order radiative corrections, in particular, $\mathcal{O}(\alpha^2 m_t^2)$ [26] and $\mathcal{O}(\alpha\alpha_s)$ vertex [68] corrections, as well as the leading $\mathcal{O}(\alpha_s^4)$ contribution to hadronic Z decays; (ii) the use of a slightly higher value of $\alpha(M_Z)$ [9]; (iii) a more complete set of low energy data (which is not very important for Standard Model fits, but is for physics beyond the Standard Model); and (iv) scheme dependences. Taking into account these differences, the agreement is excellent.