QUARK MASSES

Written by A. Manohar (University of California, San Diego).

A. Introduction

This note discusses some of the theoretical issues involved in the determination of quark masses. Unlike the leptons, quarks are confined inside hadrons and are not observed as physical particles. Quark masses cannot be measured directly, but must be determined indirectly through their influence on hadron properties. As a result, the values of the quark masses depend on precisely how they are defined; there is no one definition that is the obvious choice. Though one often speaks loosely of quark masses as one would of the electron or muon mass, any careful statement of a quark mass value must make reference to a particular computational scheme that is used to extract the mass from observations. It is important to keep this scheme dependence in mind when using the quark mass values tabulated in the data listings.

The simplest way to define the mass of a quark is by making a fit of the hadron mass spectrum to a nonrelativistic quark model. The quark masses are defined as the values obtained from the fit. The resulting masses only make sense in the limited context of a particular quark model. They depend on the phenomenological potential used, and on how relativistic effects are modelled. The quark masses used in potential models also cannot be connected with the quark mass parameters in the QCD Lagrangian. Fortunately, there exist other definitions of the quark mass that have a more general significance, though they also depend on the method of calculation. The purpose of this review is to explain the most important such definitions and their interrelations.

B. Mass parameters and the QCD Lagrangian

The QCD Lagrangian for N_F quark flavors is

$$\mathcal{L} = \sum_{k=1}^{N_F} \overline{q}_k \left(i \not{D} - m_k \right) q_k - \tfrac{1}{4} G_{\mu\nu} G^{\mu\nu} \ , \qquad (1)$$

where $\not{D} = (\partial_\mu - i g A_\mu)\, \gamma^\mu$ is the gauge covariant derivative, A_μ is the gluon field, $G_{\mu\nu}$ is the gluon field strength, m_k is the mass parameter of the k^{th} quark, and q_k is the quark Dirac field. The QCD Lagrangian Eq. (1) gives finite scattering amplitudes after renormalization, a procedure that invokes a subtraction scheme to render the amplitudes finite, and requires the introduction of a dimensionful scale parameter μ. The mass parameters in the QCD Lagrangian Eq. (1) depend on the renormalization scheme used to define the theory, and also on the scale parameter μ. The most commonly used renormalization scheme for QCD perturbation theory is the $\overline{\text{MS}}$ scheme.

The QCD Lagrangian has a chiral symmetry in the limit that the quark masses vanish. This symmetry is spontaneously broken by dynamical chiral symmetry breaking, and explicitly broken by the quark masses. The nonperturbative scale of dynamical chiral symmetry breaking, Λ_χ, is around 1 GeV. It is conventional to call quarks heavy if $m > \Lambda_\chi$, so that explicit chiral symmetry breaking dominates, and light if $m < \Lambda_\chi$, so that spontaneous chiral symmetry breaking dominates. The c, b, and t quarks are heavy, and the u, d and s quarks are light. The computations for light quarks involve an expansion in m_q/Λ_χ about the limit $m_q = 0$, whereas for heavy quarks, they involve an expansion in Λ_χ/m_q about $m_q = \infty$. The corrections are largest for the s and c quarks, which are the heaviest light quark and the lightest heavy quark, respectively.

At high energies or short distances, nonperturbative effects such as chiral symmetry breaking are unimportant, and one can in principle analyze mass-dependent effects using QCD perturbation theory to extract the quark mass values. The QCD computations are conventionally performed using the $\overline{\text{MS}}$ scheme at a scale $\mu \gg \Lambda_\chi$, and give the $\overline{\text{MS}}$ "running" mass $\overline{m}(\mu)$. The μ dependence of $\overline{m}(\mu)$ at short distances can be calculated using the renormalization group equations.

For heavy quarks, one can obtain useful information on the quark masses by studying the spectrum and decays of hadrons containing heavy quarks. One method of calculation uses the heavy quark effective theory (HQET), which defines a HQET quark mass m_Q. Other commonly used definitions of heavy quark masses such as the pole mass are discussed in Sec. C. QCD perturbation theory at the heavy quark scale $\mu = m_Q$ can be used to relate the various heavy quark masses to the $\overline{\text{MS}}$ mass $\overline{m}(\mu)$, and to each other.

For light quarks, one can obtain useful information on the quark mass ratios by studying the properties of the light pseudoscalar mesons using chiral perturbation theory, which utilizes the symmetries of the QCD Lagrangian Eq. (1). The quark mass ratios determined using chiral perturbation theory are those in a subtraction scheme that is independent of the quark masses themselves, such as the $\overline{\text{MS}}$ scheme.

A more detailed discussion of the masses for heavy and light quarks is given in the next two sections. The $\overline{\text{MS}}$ scheme applies to both heavy and light quarks. It is also commonly used for predictions of quark masses in unified theories, and for computing radiative corrections in the Standard Model. For this reason, we use the $\overline{\text{MS}}$ scheme as the standard scheme in reporting quark masses. One can easily convert the $\overline{\text{MS}}$ masses into other schemes using the formulæ given in this review.

C. Heavy quarks

The commonly used definitions of the quark mass for heavy quarks are the pole mass, the $\overline{\text{MS}}$ mass, the Georgi-Politzer mass, the potential model mass used in ψ and Υ spectroscopy, and the HQET mass.

The strong interaction coupling constant at the heavy quark scale is small, and one can compute the heavy quark propagator using QCD perturbation theory. For an observable particle such as the electron, the position of the pole in the propagator is the definition of the particle mass. In QCD this definition of the quark mass is known as the pole mass m_P, and is independent of the renormalization scheme used. It is known that the on-shell quark propagator has no infrared divergences in perturbation theory [1], so this provides a perturbative definition of the quark mass. The pole mass cannot be used to arbitrarily high accuracy because of nonperturbative infrared effects in QCD. The full quark propagator has no pole because the quarks are confined, so that the pole mass cannot be defined outside of perturbation theory.

The $\overline{\text{MS}}$ running mass $\overline{m}(\mu)$ is defined by regulating the QCD theory using dimensional regularization, and subtracting the divergences using the modified minimal subtraction scheme. The $\overline{\mathrm{MS}}$ scheme is particularly convenient for Feynman diagram computations, and is the most commonly used subtraction scheme.

The Georgi-Politzer mass $\widehat{m}$ is defined using the momentum space subtraction scheme at the spacelike point $-p^2 = \widehat{m}^2$ [2]. A generalization of the Georgi-Politzer mass that is often used in computations involving QCD sum rules [3] is $\widehat{m}(\xi)$, defined at the subtraction point $p^2 = -(\xi+1)m_P^2$. QCD sum rules are discussed in more detail in the next section on light quark masses.

Lattice gauge theory calculations can be used to obtain heavy quark masses from ψ and Υ spectroscopy. The quark masses are obtained by comparing a nonperturbative computation of the meson spectrum with the experimental data. The lattice quark mass values can then be converted into quark mass values in the continuum QCD Lagrangian Eq. (1) using lattice perturbation theory at a scale given by the inverse lattice spacing. A recent computation determines the b-quark pole mass to be 5.0 ± 0.2 GeV, and the $\overline{\mathrm{MS}}$ mass to be 4.0 ± 0.1 GeV [4].

Potential model calculations of the hadron spectrum also involve the heavy quark mass. There is no way to relate the quark mass as defined in a potential model to the quark mass parameter of the QCD Lagrangian, or to the pole mass. Even in the heavy quark limit, the two masses can differ by nonperturbative effects of order Λ_{QCD}. There is also no reason why the potential model quark mass should be independent of the particular form of the potential used.

Recent work on the heavy quark effective theory [5–9] has provided a definition of the quark mass for a heavy quark that is valid when one includes nonperturbative effects and will be called the HQET mass m_Q. The HQET mass is particularly useful in the analysis of the $1/m_Q$ corrections in HQET. The HQET mass agrees with the pole mass to all orders in perturbation theory when only one quark flavor is present, but differs from the pole mass at order α_s^2 when there are additional flavors [10]. Physical quantities such as hadron masses can in principle be computed in the heavy quark effective theory in terms of the HQET mass m_Q. The computations cannot be done analytically in practice because of nonperturbative effects in QCD, which also prevent a direct extraction of the quark masses from the original QCD Lagrangian, Eq. (1). Nevertheless, for heavy quarks, it is possible to parametrize the nonperturbative effects to a given order in the $1/m_Q$ expansion in terms of a few unknown constants that can be obtained from experiment. For example, the B and D meson masses in the heavy quark effective theory are given in terms of a single nonperturbative parameter $\overline{\Lambda}$,

$$\begin{aligned} M(B) &= m_b + \overline{\Lambda} + \mathcal{O}\left(\frac{\overline{\Lambda}^2}{m_b}\right) \ , \\ M(D) &= m_c + \overline{\Lambda} + \mathcal{O}\left(\frac{\overline{\Lambda}^2}{m_c}\right) \ . \end{aligned} \tag{2}$$

This allows one to determine the mass difference $m_b - m_c = M(B) - M(D) = 3.4$ GeV up to corrections of order $\overline{\Lambda}^2/m_b - \overline{\Lambda}^2/m_c$. The extraction of the individual quark masses m_b and m_c requires some knowledge of $\overline{\Lambda}$. An estimate of $\overline{\Lambda}$ using QCD sum rules gives $\overline{\Lambda} = 0.57 \pm 0.07$ GeV [11]. The HQET masses with this value of $\overline{\Lambda}$ are $m_b = 4.74 \pm 0.14$ GeV and $m_c = 1.4 \pm 0.2$ GeV, where the spin averaged meson masses $(3M(B^*) + M(B))/4$ and $(3M(D^*) + M(D))/4$ have been used to eliminate the spin-dependent $\mathcal{O}(\overline{\Lambda}^2/m_Q)$ correction terms. The errors reflect the uncertainty in $\overline{\Lambda}$ and the unknown spin-averaged $\mathcal{O}(\overline{\Lambda}^2/m_Q)$ correction. The errors do not include any theoretical uncertainty in the QCD sum rules, which could be large. A quark model estimate suggests that $\overline{\Lambda}$ is the constituent quark mass (≈ 350 MeV), which differs significantly from the sum rule estimate. In HQET, the $1/m_Q$ corrections to heavy meson decay form-factors are also given in terms of $\overline{\Lambda}$. Thus an accurate enough measurement of these form-factors could be used to extract $\overline{\Lambda}$ directly from experiment, which then determines the quark masses up to corrections of order $1/m_Q$.

The quark mass m_Q of HQET can be related to other quark mass parameters using QCD perturbation theory at the scale

m_Q. The relation between m_Q and $\widehat{m}(\xi)$ at one loop is [12]

$$m_Q = \widehat{m}(\xi)\left[1 + \frac{\widehat{\alpha}_s(\xi)}{\pi}\frac{\xi+2}{\xi+1}\log\left(\xi+2\right)\right], \tag{3}$$

where $\widehat{\alpha}_s(\xi)$ is the strong interaction coupling constant in the momentum space subtraction scheme. The relation between m_Q and the $\overline{\text{MS}}$ mass $\overline{m}$ is known to two loops [13],

$$\begin{aligned} m_Q = \overline{m}(m_Q)\Bigg[1 &+ \frac{4\overline{\alpha}_s(m_Q)}{3\pi} \\ &+ \left(16.11 - 1.04\sum_k\left(1 - \frac{m_{Q_k}}{m_Q}\right)\right)\left(\frac{\overline{\alpha}_s(m_Q)}{\pi}\right)^2\Bigg], \end{aligned} \tag{4}$$

where $\overline{\alpha}_s(\mu)$ is the strong interaction coupling constants in the $\overline{\text{MS}}$ scheme, and the sum on k extends over all flavors Q_k lighter than Q. For the b-quark, Eq. (4) reads

$$m_b = \overline{m}_b\left(m_b\right)\left[1 + 0.09 + 0.06\right], \tag{5}$$

where the contributions from the different orders in α_s are shown explicitly. The two loop correction is comparable in size and has the same sign as the one loop term. There is presumably an error of order 0.05 in the relation between m_b and $\overline{m}_b(m_b)$ from the uncalculated higher order terms.

D. Light quarks

For light quarks, one can use the techniques of chiral perturbation theory to extract quark mass ratios. The light quark part of the QCD Lagrangian Eq. (1) has a chiral symmetry in the limit that the light quark masses are set to zero, under which left- and right-handed quarks transform independently. The mass term explicitly breaks the chiral symmetry, since it couples the left- and right-handed quarks to each other. A systematic analysis of this explicit chiral symmetry breaking provides some information on the light quark masses.

It is convenient to think of the three light quarks u, d and s as a three component column vector Ψ, and to write the mass term for the light quarks as

$$\overline{\Psi}M\Psi = \overline{\Psi}_L M\Psi_R + \overline{\Psi}_R M\Psi_L, \tag{6}$$

where M is the quark mass matrix M,

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} . \tag{7}$$

The mass term $\overline{\Psi} M \Psi$ is the only term in the QCD Lagrangian that mixes left- and right-handed quarks. In the limit that $M \to 0$, there is an independent SU(3) flavor symmetry for the left- and right-handed quarks. This $G_\chi = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ chiral symmetry of the QCD Lagrangian is spontaneously broken, which leads to eight massless Goldstone bosons, the π's, K's, and η, in the limit $M \to 0$. The symmetry G_χ is only an approximate symmetry, since it is explicitly broken by the quark mass matrix M. The Goldstone bosons acquire masses which can be computed in a systematic expansion in M in terms of certain unknown nonperturbative parameters of the theory. For example, to first order in M one finds that [14,15]

$$\begin{aligned} m^2_{\pi^0} =& B\,(m_u + m_d) \ , \\ m^2_{\pi^\pm} =& B\,(m_u + m_d) + \Delta_{em} \ , \\ m^2_{K^0} = m^2_{\overline{K}^0} =& B\,(m_d + m_s) \ , \\ m^2_{K^\pm} =& B\,(m_u + m_s) + \Delta_{em} \ , \\ m^2_\eta =& \frac{1}{3} B\,(m_u + m_d + 4m_s) \ , \end{aligned} \tag{8}$$

with two unknown parameters B and Δ_{em}, the electromagnetic mass difference. From Eq. (8), one can determine the quark mass ratios [14]

$$\begin{aligned} \frac{m_u}{m_d} =& \frac{2m^2_{\pi^0} - m^2_{\pi^+} + m^2_{K^+} - m^2_{K^0}}{m^2_{K^0} - m^2_{K^+} + m^2_{\pi^+}} = 0.56 \ , \\ \frac{m_s}{m_d} =& \frac{m^2_{K^0} + m^2_{K^+} - m^2_{\pi^+}}{m^2_{K^0} + m^2_{\pi^+} - m^2_{K^+}} = 20.1 \ , \end{aligned} \tag{9}$$

to lowest order in chiral perturbation theory. The error on these numbers is the size of the second-order corrections, which are discussed at the end of this section. Chiral perturbation theory cannot determine the overall scale of the quark masses, since it uses only the symmetry properties of M, and any multiple of M has the same G_χ transformation law as M. This can be seen from Eq. (8), where all quark masses occur only in the form Bm, so that B and m cannot be determined separately.

The mass parameters in the QCD Lagrangian have a scale dependence due to radiative corrections, and are renormalization scheme dependent. Since the mass ratios extracted using chiral perturbation theory use the symmetry transformation property of M under the chiral symmetry G_χ, it is important to use a renormalization scheme for QCD that does not change this transformation law. Any quark mass independent subtraction scheme such as $\overline{\rm MS}$ is suitable. The ratios of quark masses are scale independent in such a scheme.

The absolute normalization of the quark masses can be determined by using methods that go beyond chiral perturbation theory, such as QCD sum rules [3]. Typically, one writes a sum rule for a quantity such as B in terms of a spectral integral over all states with certain quantum numbers. This spectral integral is then evaluated by assuming it is dominated by one (or two) of the lowest resonances, and using the experimentally measured resonance parameters [16]. There are many subtleties involved, which cannot be discussed here [16].

Another method for determining the absolute normalization of the quark masses, is to assume that the strange quark mass is equal to the SU(3) mass splitting in the baryon multiplets [14,16]. There is an uncertainty in this method since in the baryon octet one can use either the Σ–N or the Λ–N mass difference, which differ by about 75 MeV, to estimate the strange quark mass. But more importantly, there is no way to relate this normalization to any more fundamental definition of quark masses.

One can extend the chiral perturbation expansion Eq. (8) to second order in the quark masses M to get a more accurate determination of the quark mass ratios. There is a subtlety that arises at second order [17], because

$$M\left(M^\dagger M\right)^{-1}\det M^\dagger \tag{10}$$

transforms in the same way under G_χ as M. One can make the replacement $M \to M(\lambda) = M + \lambda M \left(M^\dagger M\right)^{-1} \det M^\dagger$ in all formulæ,

$$M(\lambda) = \mathrm{diag}\left(m_u(\lambda)\,,\ \ m_d(\lambda)\,,\ \ m_s(\lambda)\right)$$

$$= \mathrm{diag}\left(m_u + \lambda m_d m_s\,,\ \ m_d + \lambda m_u m_s\,,\ \ m_s + \lambda m_u m_d\right)\ ,(11)$$

so it is not possible to determine λ by fitting to data. One can only determine the ratios $m_i(\lambda)/m_j(\lambda)$ using second-order chiral perturbation theory, not the desired ratios $m_i/m_j = m_i(\lambda=0)/m_j(\lambda=0)$.

Dimensional analysis can be used to estimate [18] that second-order corrections in chiral perturbation theory due to the strange quark mass are of order $\lambda m_s \sim 0.25$. The ambiguity due to the redefinition Eq. (11) (which corresponds to a second-order correction) can produce a sizeable uncertainty in the ratio m_u/m_d. The lowest-order value $m_u/m_d = 0.56$ gets corrections of order $\lambda m_s(m_d/m_u - m_u/m_d) \sim 30\%$, whereas m_s/m_d gets a smaller correction of order $\lambda m_s(m_u/m_d - m_u m_d/m_s^2) \sim 15\%$. A more quantitative discussion of second-order effects can be found in Refs. 17,19,20. Since the second-order terms have a single parameter ambiguity, the value of m_u/m_d is related to the value of m_s/m_d.

The ratio m_u/m_d is of great interest since there is no strong CP problem if $m_u = 0$. To determine m_u/m_d requires fixing λ in the mass redefinition Eq. (11). There has been considerable effort to determine the chiral Lagrangian parameters accurately enough to determine m_u/m_d, for example from the analysis of the decays $\psi' \to \psi + \pi^0, \eta$, the decay $\eta \to 3\pi$, using sum rules, and from the heavy meson mass spectrum [16,21–24]. A recent paper giving a critique of these estimates is Ref. 25.

Eventually, lattice gauge theory methods will be accurate enough to be able to compute meson masses directly from the QCD Lagrangian Eq. (1), and thus determine the light quark masses. For a reliable determination of quark masses, these computations will have to be done with dynamical fermions, and with a small enough lattice spacing that one can accurately compute the relation between lattice and continuum Lagrangians.

The quark masses for light quarks discussed so far are often referred to as current quark masses. Nonrelativistic quark models use constituent quark masses, which are of order 350 MeV for the u and d quarks. Constituent quark masses model the effects of dynamical chiral symmetry breaking, and are not related to the quark mass parameters m_k of the QCD Lagrangian Eq. (1). Constituent masses are only defined in the context of a particular hadronic model.

E. Numerical values and caveats

The quark masses in the particle data listings have been obtained by using the wide variety of theoretical methods outlined above. Each method involves its own set of approximations and errors. In most cases, the errors are a best guess at the size of neglected higher-order corrections. The expansion parameter for the approximations is not much smaller than unity (for example it is $m_K^2/\Lambda_\chi^2 \approx 0.25$ for the chiral expansion), so an unexpectedly large coefficient in a neglected higher-order term could significantly alter the results. It is also important to note that the quark mass values can be significantly different in the different schemes. For example, assuming that the b-quark pole mass is 5.0 GeV, and $\overline{\alpha}_s(m_b) \approx 0.22$ gives the $\overline{\rm MS}$ b-quark mass $\overline{m}_b(\mu = m_b) = 4.6$ GeV using the one-loop term in Eq. (4), and $\overline{m}_b(\mu = m_b) = 4.3$ GeV including the one-loop and two-loop terms. The heavy quark masses obtained using HQET, QCD sum rules, or lattice gauge theory are consistent with each other if they are all converted into the same scheme. When using the data listings, it is important to remember that the numerical value for a quark mass is meaningless without specifying the particular scheme in which it was obtained. All non-$\overline{\rm MS}$ quark masses have been converted to $\overline{\rm MS}$ values in the data listings using one-loop formulæ, unless an explicit two-loop conversion is given by the authors in the original article.

References

1. R. Tarrach, Nucl. Phys. **B183**, 384 (1981).
2. H. Georgi and H.D. Politzer, Phys. Rev. **D14**, 1829 (1976).
3. M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979).

4. C.T.H. Davies, *et al.*, Phys. Rev. Lett. **73**, 2654 (1994).
5. N. Isgur and M.B. Wise, Phys. Lett. **B232**, 113 (1989), *ibid* **B237**, 527 (1990);
M.B. Voloshin and M. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987), *ibid* **47**, 511 (1988);
S. Nussinov and W. Wetzel, Phys. Rev. **D36**, 130 (1987).
6. H. Georgi, Phys. Lett. **B240**, 447 (1990).
7. E. Eichten and B. Hill, Phys. Lett. **B234**, 511 (1990).
8. H. Georgi, in *Perspectives of the Standard Model*, ed. R.K. Ellis, C.T. Hill, and J.D. Lykken (World Scientific, Singapore, 1992);
B. Grinstein, in *High Energy Phenomenology*, ed. R. Huerta and M.A. Pérez (World Scientific, Singapore, 1992).
9. A.F. Falk, M. Neubert, and M.E. Luke, Nucl. Phys. **B388**, 363 (1992).
10. A.V. Manohar and M.B. Wise, (unpublished).
11. M. Neubert, Phys. Reports **245**, 259 (1994).
12. S. Narison, Phys. Lett. **B197**, 405 (1987).
13. N. Gray, D.J. Broadhurst, W. Grafe, and K. Schilcher, Z. Phys. **C48**, 673 (1990).
14. S. Weinberg, Trans. N.Y. Acad. Sci. **38**, 185 (1977).
15. See for example, H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, Menlo Park, 1984).
16. J. Gasser and H. Leutwyler, Phys. Reports **87**, 77 (1982).
17. D.B. Kaplan and A.V. Manohar, Phys. Rev. Lett. **56**, 2004 (1986).
18. A.V. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
19. J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
20. H. Leutwyler, Nucl. Phys. **B337**, 108 (1990).
21. P. Langacker and H. Pagels, Phys. Rev. **D19**, 2070 (1979);
H. Pagels and S. Stokar, Phys. Rev. **D22**, 2876 (1980);
H. Leutwyler, Nucl. Phys. **B337**, 108 (1990);
J. Donoghue and D. Wyler, Phys. Rev. Lett. **69**, 3444 (1992);
K. Maltman, T. Goldman and G.L. Stephenson Jr., Phys. Lett. **B234**, 158 (1990).
22. K. Choi, Nucl. Phys. **B383**, 58 (1992).
23. J. Donoghue and D. Wyler, Phys. Rev. **D45**, 892 (1992).
24. M.A. Luty and R. Sundrum, e-print hep-ph/9502398.

25. T. Banks, Y. Nir, and N. Seiberg, *Proceedings of the 2nd IFT Workshop on Yukawa Couplings and the Origins of Mass*, Gainesville, Florida (1994).