NEUTRINO MASS

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While there is no unequivocal evidence for neutrino mass, it is natural to suspect that the neutrinos, like the charged leptons and the quarks, have nonzero masses. Evidence of these masses is being sought through experiments on neutrinos created astrophysically, in the earth's atmosphere, by accelerators, by reactors, and by nuclear decays, and in studies of reactions where neutrinos appear only as virtual particles.

In the decay

$$W^+ \to \ell^+ \nu_\ell \tag{1}$$

of a W boson into a charged lepton of "flavor" $\ell(e, \mu$, or $\tau)$, the accompanying neutrino is referred to as ν_ℓ, the neutrino of flavor ℓ. Neutrinos of different flavor are different objects. When an energetic ν_ℓ undergoes a charged-current weak interaction, it produces a charged lepton ℓ of the same flavor as the neutrino [1].

If neutrinos have masses, then a neutrino of definite flavor, ν_ℓ, need not be a mass eigenstate. Indeed, if leptons behave like quarks, the ν_ℓ is a coherent linear superposition of mass eigenstates, given by

$$|\nu_\ell\rangle = \sum_m U_{\ell m} |\nu_m\rangle \ . \tag{2}$$

Here, the ν_m are the mass eigenstates, and the coefficients $U_{\ell m}$ form a matrix U known as the leptonic mixing matrix. There are at least three ν_m, and perhaps more. However, it is usually assumed that no more than three ν_m make significant contributions to Eq. (2). Then U is a 3×3 matrix, and according to the electroweak Standard Model (SM), extended to include neutrino masses, it is unitary.

The relation (2) means that when, for example, a W^+ decays to an e^+ and a neutrino, the neutrino with probability $|U_{e1}|^2$ is a ν_1, with probability $|U_{e2}|^2$ is a ν_2, and so on. This behavior is an exact leptonic analogue of what is known to occur when a W^+ decays to quarks.

If each neutrino of definite flavor is a coherent superposition of mass eigenstates, then we will have *neutrino oscillation* [2].

This is the spontaneous metamorphosis of a neutrino of one flavor into one of another flavor as the neutrino propagates.

To understand neutrino oscillation, let us consider how a neutrino born as the ν_ℓ of Eq. (2) evolves in time. First, we apply Schrödinger's equation to the ν_m component of ν_ℓ in the rest frame of that component. This tells us that [3]

$$|\nu_m(\tau_m)\rangle = e^{-iM_m\tau_m}|\nu_m(0)\rangle \ , \tag{3}$$

where M_m is the mass of ν_m, and τ_m is time in the ν_m frame. In terms of the time t and position L in the laboratory frame, the Lorentz-invariant phase factor in Eq. (3) may be written

$$e^{-iM_m\tau_m} = e^{-i(E_m t - p_m L)} \ . \tag{4}$$

Here, E_m and p_m are respectively the energy and momentum of ν_m in the laboratory frame. In practice, our neutrino will be extremely relativistic, so we will be interested in evaluating the phase factor of Eq. (4) where $t \approx L$, where it becomes $\exp[-i(E_m - p_m)L]$.

Imagine now that our ν_ℓ has been produced with a definite momentum p, so that all of its mass-eigenstate components have this common momentum. Then the ν_m component has $E_m = \sqrt{p^2 + M_m^2} \approx p + M_m^2/2p$, assuming that all neutrino masses M_m are small compared to the neutrino momentum. The phase factor of Eq. (4) is then approximately

$$e^{-i(M_m^2/2p)L} \ . \tag{5}$$

Alternatively, suppose that our ν_ℓ has been produced with a definite energy E, so that all of its mass-eigenstate components have this common energy [4]. Then the ν_m component has $p_m = \sqrt{E^2 - M_m^2} \approx E - M_m^2/2E$. The phase factor of Eq. (4) is then approximately

$$e^{-i(M_m^2/2E)L} \ . \tag{6}$$

Since highly relativistic neutrinos have $E \approx p$, the phase factors (5) and (6) are approximately equal. Thus, it doesn't matter whether our ν_ℓ is created with definite momentum or definite energy.

From Eq. (2) and either Eq. (5) or Eq. (6), it follows that after a neutrino born as a ν_ℓ has propagated a distance L, its state vector has become

$$|\nu_\ell(L)\rangle \approx \sum_m U_{\ell m} e^{-i(M_m^2/2E)L} |\nu_m\rangle \ . \tag{7}$$

Using the unitarity of U to invert Eq. (2), and inserting the result in Eq. (7), we find that

$$|\nu_\ell(L)\rangle \approx \sum_{\ell'} \left[\sum_m U_{\ell m} e^{-i(M_m^2/2E)L} U^*_{\ell' m} \right] |\nu_{\ell'}\rangle \ . \tag{8}$$

We see that our ν_ℓ, in traveling the distance L, has turned into a superposition of all the flavors. The probability that it has flavor ℓ', $P(\nu_\ell \to \nu_{\ell'}; L)$, is obviously given by

$$P(\nu_\ell \to \nu_{\ell'};\ L) = |\langle \nu_{\ell'} | \nu_\ell(L) \rangle|^2 = \left| \sum_m U_{\ell m} e^{-i(M_m^2/2E)L} U^*_{\ell' m} \right|^2 \ . \tag{9}$$

The quantum mechanics of neutrino oscillation leading to the result Eq. (9) is somewhat subtle. It has been analyzed using wave packets [5], treating a propagating neutrino as a virtual particle [6], evaluating the phase acquired by a propagating mass eigenstate in terms of the proper time of propagation [3], requiring that a neutrino's flavor cannot change unless the neutrino travels [4], and taking different neutrino mass eigenstates to have both different momenta and different energies [7]. The subtleties of oscillation are still being explored and discussed.

Frequently, a neutrino oscillation experiment is analyzed assuming that only two neutrino flavors, ν_e and ν_μ for example, mix appreciably. Then the mixing matrix U takes the form

$$U = \begin{pmatrix} \cos\theta_{e\mu} & \sin\theta_{e\mu} \\ -\sin\theta_{e\mu} & \cos\theta_{e\mu} \end{pmatrix} \ , \tag{10}$$

where $\theta_{e\mu}$ is the ν_e–ν_μ mixing angle. Inserting this matrix into Eq. (9), we find that

$$P(\nu_e \to \nu_\mu;\ L) = \sin^2 2\theta_{e\mu} \sin^2 \left(\Delta M_{12}^2 L/4E \right) \ . \tag{11}$$

Here, $\Delta M_{12}^2 \equiv M_1^2 - M_2^2$, where ν_1 and ν_2 are the mass eigenstates which make up ν_e and ν_μ. If the omitted factors of $\hbar$ and

c are inserted into the argument $\Delta M_{12}^2 L/4E$ of the oscillatory sine function, it becomes 1.27 ΔM_{12}^2 (eV^2)L (km)/E (GeV). The probability that a ν_e will retain its original flavor during propagation over a distance L is simply

$$P(\nu_e \to \nu_e;\ L) = 1 - P(\nu_e \to \nu_\mu;\ L)\ . \tag{12}$$

Under some important circumstances, a "two-neutrino" formula virtually identical to that of Eq. (11) accurately describes neutrino oscillation even when all three neutrino flavors mix. One of these circumstances is when all mixing angles are small. That is, each neutrino of definite flavor is dominantly one mass eigenstate, plus only small amounts of the other two. In this circumstance, let us refer to the dominant mass eigenstate component of ν_e as ν_1, that of ν_μ as ν_2, and that of ν_τ as ν_3. Then the mixing matrix U is approximately

$$U \approx \begin{pmatrix} 1 & \theta_{e\mu} & \theta_{e\tau} \\ -\theta_{e\mu} & 1 & \theta_{\mu\tau} \\ -\theta_{e\tau} & -\theta_{\mu\tau} & 1 \end{pmatrix}\ , \tag{13}$$

where θ_{ab} is the (small) ν_{ℓ_a}–ν_{ℓ_b} mixing angle. Inserting this mixing matrix in Eq. (9), we find that through second order in the mixing angles,

$$P(\nu_{\ell_a} \to \nu_{\ell_b \neq \ell_a};\ L) \approx (2\theta_{ab})^2 \sin^2\left(\Delta M_{ij}^2 L/4E\right)\ . \tag{14}$$

Here, $\Delta M_{ij}^2 \equiv M_i^2 - M_j^2$, where ν_i and ν_j are, respectively, the dominant mass eigenstate components of ν_{ℓ_a} and ν_{ℓ_b}. We see that when all mixing angles are small, the oscillation between any pair of neutrino flavors is indeed described by a two-neutrino formula just like Eq. (11), but for each pair of flavors, there is a different mixing angle and a different ΔM^2. In addition, in contrast to Eq. (12), the probability that a neutrino (say, a ν_e) retains its original flavor is now given by

$$P(\nu_e \to \nu_e;\ L) = 1 - P(\nu_e \to \nu_\mu;\ L) - P(\nu_e \to \nu_\tau;\ L)\ . \tag{15}$$

Another interesting situation occurs when there is a neutrino mass hierarchy, $M_3 \gg M_2 \gg M_1$, so that $\Delta M_{32}^2 \approx \Delta M_{31}^2 \gg \Delta M_{21}^2$. Then there is a region of L/E in which $\Delta M_{21}^2 L/E$ is negligible compared to unity, but $\Delta M^2_{32}\ L/E$ is not. For L/E in this region, it follows from Eq. (9) and the unitarity of U that [8]

$$P(\nu_{\ell_a} \to \nu_{\ell_b \neq \ell_a};\ L) \approx |2U_{a3}U_{b3}|^2 \sin^2\left(\Delta M^2_{32} L/4E\right) \quad . \qquad (16)$$

Once again, the oscillation probability has the same form as when just two neutrinos mix. Furthermore, Eq. (16) holds whether the mixing angles are large or small. However, the parameters in Eq. (16) have a different meaning from those in the true two-neutrino formula, Eq. (11). In Eq. (16), the coefficient $|2U_{a3}U_{b3}|^2$ is, in general, *not* $\sin^2 2\theta_{ab}$, as it would be in the two-neutrino case. (To be sure, $|2U_{a3}U_{b3}|^2$ never exceeds unity, anymore than $\sin^2 2\theta_{ab}$ does.) In addition, in Eq. (16), the mass splitting which appears is always the same one—ΔM^2_{32}—regardless of which neutrino flavors are being considered.

In a beam of neutrinos born with flavor ℓ_a, neutrino oscillation can be sought in two ways: First, one may seek the *appearance* in the beam of neutrinos of a different flavor, ℓ_b. Secondly, one may seek a *disappearance* of some of the original ν_{ℓ_a} flux, or an L-dependence of this flux.

Clearly, no oscillation is expected unless L/E of the experiment is sufficiently large that the phase factors $\exp(-iM^2_m\ L/2E)$ in Eq. (9) differ appreciably from one another. Otherwise, $\mathrm{P}(\nu_\ell \to \nu_{\ell'}; L) = |\sum_m U_{\ell m}U^*_{\ell' m}|^2 = \delta_{\ell\ell'}$. Now, with omitted factors of $\hbar$ and c inserted, the relative phase of $\exp(-iM^2_i L/2E)$ and $\exp(-iM^2_j L/2E)$ is $2.54\ \Delta M^2_{ij}(\mathrm{eV}^2)\ L(\mathrm{km})/E(\mathrm{GeV})$. Thus, for example, an experiment in which neutrinos with $E \approx 1$ GeV travel 1 km between production and detection will be sensitive to $\Delta M^2 \gtrsim 1\ \mathrm{eV}^2$.

A more direct way than neutrino oscillation experiments to search for neutrino mass is to look for its kinematical effects in decays which produce a neutrino. In the decay $X \to Y\ell^+\nu_\ell$, where X is a hadron and Y is zero or more hadrons, the momenta of ℓ^+ and the particles in Y will obviously be modified if ν_ℓ has a mass. If ν_ℓ is a superposition of mass eigenstates ν_m, then $X \to Y\ell^+\nu_\ell$ is actually the sum of the decays $X \to Y\ell^+\nu_m$ yielding every ν_m light enough to be emitted. Thus, if, for example, one ν_m is much heavier than the others, the energy spectrum of ℓ^+ may show a threshold rise where the ℓ^+ energy becomes low enough for the heavy ν_m to be emitted [9]. However, if neutrino mixing is small, then the decays $X \to Y\ell^+\nu_m$ yield almost always the neutrino mass eigenstate which is the dominant component of ν_ℓ. The kinematics of ℓ^+ and Y then reflect the mass of this mass eigenstate.

From kinematical studies of the particles produced in $^3\mathrm{H} \to {}^3\mathrm{He}\ e^-\,\overline{\nu}_e, \pi \to \mu\nu_\mu$, and $\tau \to n\pi\nu_\tau$, upper limits have been derived for M_1, M_2, and M_3, respectively. Here, we assume mixing to be small, and, as before, call the dominant mass-eigenstate components of ν_e, ν_μ, and ν_τ, respectively, ν_1, ν_2, and ν_3. In the case of the decay $^3\mathrm{H} \to {}^3\mathrm{He}\ e^-\,\overline{\nu}_e$, the upper bound on the neutrino mass is derived from study of the e^- energy spectrum. It should be noted that in several experiments, the theoretical expression used to describe this spectrum does not produce a good fit, either for $M_1 = 0$ or for $M_1 > 0$ [10]. Indeed, the best fit is achieved for an unphysical, *negative* value of M_1^2. Thus, the quoted limits on M_1 must be interpreted with caution.

Neutrinos carry neither electric charge nor, as far as we know, any other charge-like quantum numbers. To be sure, it may be that the reason an interacting "neutrino" creates an ℓ^-, while an "antineutrino" creates an ℓ^+, is that neutrinos and antineutrinos carry opposite values of a conserved "lepton number." However, there may be no lepton number. Even then, the fact that "neutrinos" and "antineutrinos" interact differently can be easily understood. One need only note that, in practice, the particles we call "neutrinos" are always left-handed, while the ones we call "antineutrinos" are right-handed. Since the weak interactions are not invariant under parity, it is then possible to attribute the difference between the interactions of "neutrinos" and "antineutrinos" to the fact that these particles are oppositely polarized.

If the neutrino mass eigenstates do not carry any charge-like attributes, they may be their own antiparticles. A neutrino which is its own antiparticle is called a Majorana neutrino, while one which is not is called a Dirac neutrino.

If neutrinos are of Majorana character, we can have neutrinoless double beta-decay ($\beta\beta_{0\nu}$), in which one nucleus decays to another by emitting two electrons and nothing else. This process can be initiated through the emission of two virtual W bosons by the parent nucleus. One of these W bosons then emits an electron and an accompanying virtual "antineutrino." In the Majorana case, this "antineutrino" is no different from a "neutrino," except for its right-handed helicity. If the virtual neutrino has a mass, then (like the e^+ in nuclear β-decay), it is not fully right-handed, but has a small amplitude, proportional to its mass, for being left-handed. Its left-handed component is precisely what we call a "neutrino," and can be absorbed by the second virtual W boson to create the second outgoing electron. This mechanism yields for $\beta\beta_{0\nu}$ an amplitude proportional to an effective neutrino mass $\langle M\rangle$, given in a common phase convention by [11]

$$\langle M\rangle = \sum_m U_{em}^2 M_m \ . \tag{17}$$

Experimental upper bounds on the $\beta\beta_{0\nu}$ rate are used to derive upper bounds on $\langle M\rangle$. Note that, owing to possible phases in the mixing matrix elements U_{em}, the relation between $\langle M\rangle$ and the actual masses M_m of the neutrino mass eigenstates can be somewhat complicated. The process $\beta\beta_{0\nu}$ is discussed further by P. Vogel in this *Review*.

If neutrinos are their own antiparticles, then their magnetic and electric dipole moments must vanish. To see why, recall that CPT invariance requires that the dipole moments of the electron and its antiparticle be equal and opposite. Similarly, CPT invariance would require that the dipole moments of a neutrino and its antiparticle be equal and opposite. But, if the antiparticle of the neutrino is the neutrino itself, this means that the dipole moments must vanish [12].

If neutrinos are not their own antiparticles, then they can have dipole moments. However, for a Dirac neutrino mass eigenstate ν_m, the magnetic dipole moment μ_m predicted by the Standard Model (extended to include neutrino masses) is only [13]

$$\mu_m = 3.2 \times 10^{-19} M_m(\text{eV})\mu_B \ , \tag{18}$$

where μ_B is the Bohr magneton.

Whether neutrinos are their own antiparticles or not, there may be *transition* magnetic and electric dipole moments. These induce the transitions $\nu_m \rightarrow \nu_{m' \neq m}\gamma$.

A Majorana neutrino, being its own antiparticle, obviously consists of just two states: spin up and spin down. In contrast, a Dirac neutrino, together with its antiparticle, consists of four states: the spin-up and spin-down neutrino states, plus the spin-up and spin-down antineutrino states. A four-state Dirac neutrino may be pictured as comprised of two degenerate two-state Majorana neutrinos. Conversely, in the field-theory description of neutrinos, by introducing so-called Majorana mass terms, one can split a Dirac neutrino, D, into two nondegenerate Majorana neutrinos, ν and N. In some extensions of the SM, it is natural for the D, ν, and N masses, M_D, M_ν, and M_N, to be related by

$$M_\nu M_N \approx M_D^2 \ . \tag{19}$$

In these extensions, it is also natural for M_D to be of the order of $M_{\ell \text{ or } q}$, the mass of a typical charged lepton or quark. Then we have [14]

$$M_\nu M_N \sim M_{\ell \text{ or } q}^2 \ . \tag{20}$$

Suppose now that $M_N \gg M_{\ell \text{ or } q}$, so that N is a very heavy neutrino which has not yet been observed. Then relation (20), known as the seesaw relation, implies that $M_\nu \ll M_{\ell \text{ or } q}$. Thus, ν is a candidate for one of the light neutrino mass eigenstates which make up ν_e, ν_μ, and ν_τ. So long as N is heavy, the seesaw relation explains, without fine tuning, why a mass eigenstate component of ν_e, ν_μ, or ν_τ will be light. Interestingly, the picture from which the seesaw relation arises predicts that the mass eigenstate components of ν_e, ν_μ, and ν_τ are Majorana neutrinos.

In early 1998, there are three observed hints of neutrino oscillation, and thus of neutrino mass. These hints are the behavior of solar neutrinos, the behavior of atmospheric neutrinos, and the results of the LSND experiment.

The flux of solar neutrinos has been detected on earth by several experiments [15] with different neutrino energy thresholds. In every experiment, the flux is found to be below the corresponding prediction of the Standard Solar Model (SSM) [16]. The discrepancies between the observed fluxes and the SSM predictions have proven very difficult to explain by simply modifying the SSM, without invoking neutrino mass [17]. Indeed, we know of no attempt which has succeeded. By contrast, all the existing observations can successfully and elegantly be explained if one does invoke neutrino mass. The most popular explanation of this type is based on the Mikheyev-Smirnov-Wolfenstein (MSW) effect—a matter-enhanced neutrino oscillation [18].

The neutrinos produced by the nuclear processes that power the sun are electron neutrinos ν_e. With some probability, the MSW effect converts a ν_e into a neutrino ν_x of another flavor. Depending on the specific version of the effect, ν_x is a ν_μ, a ν_τ, a ν_μ–ν_τ mixture, or perhaps a sterile neutrino ν_s. Since present solar neutrino detectors are sensitive to a ν_e, but wholly, or at least largely, insensitive to a ν_μ, ν_τ, or ν_s, the flavor conversion accounts for the low observed fluxes.

The MSW $\nu_e \to \nu_x$ conversion results from interaction between neutrinos and solar electrons as the neutrinos travel outward from the solar core, where they were produced. The conversion requires that, somewhere in the sun, the total energy of a ν_e of given momentum, including the energy of its interaction with the solar electrons, equal the total energy of the ν_x of the same momentum, so that we have an energy level crossing. Given the typical density of solar electrons, and the typical momenta of solar neutrinos, the condition that there be a level crossing requires that

$$M_{\nu_x}^2 - M_{\nu_e}^2 \equiv \Delta M_{\nu_x \nu_e}^2 \sim 10^{-5}\mathrm{eV}^2 \ , \tag{21}$$

where M_{ν_e} is the mass of the dominant mass eigenstate component of ν_e, and M_{ν_x} is the mass of ν_x.

The solar neutrino observations can also be explained by supposing that on their way from the sun to the earth, the electron neutrinos produced in the solar core undergo vacuum oscillation into neutrinos of another flavor [19]. Assuming that only two neutrino flavors are important to this oscillation, the oscillation probability is described by an expression of the form given by Eq. (11). To explain the observed suppression of the solar ν_e flux to less than half the predicted value at some energies, and to accommodate the observation that the suppression is energy-dependent, the argument $[1.27\Delta M^2(\text{eV}^2)L(\text{km})/E(\text{GeV})]$ of the oscillatory factor in Eq. (11) must be of order unity when L is the distance from the sun to the earth, and $E \simeq 1$ MeV is the typical energy of a solar neutrino. Perhaps this apparent coincidence makes the vacuum oscillation explanation of the solar neutrino observations less likely than the MSW explanation. To have $[1.27\Delta M^2(\text{eV}^2)L(\text{km})/E(\text{GeV})]\sim 1$, we require that $\Delta M^2 \sim 10^{-10}$ eV^2.

The solar neutrino experiments, and the comparison between their results and theoretical predictions, are discussed in some detail by K. Nakamura in this *Review*.

Neutrinos created in the earth's atmosphere by cosmic rays result largely from the cosmic-ray-induced production of pions, which then decay via the chain $\pi \to \mu\nu_\mu,\ \mu \to e\nu_e\nu_\mu$. As we see, this chain produces neutrinos in the ratio $\nu_\mu : \nu_e = 2:1$. Since the various neutrinos from the chain have different energy spectra, this $2:1$ ratio does not hold at a given neutrino energy, but it is believed that the actual $\nu_\mu : \nu_e$ ratio is known to 5% [20]. However, measurements of this ratio in underground detectors yield [21]

$$R \equiv \frac{(\nu_\mu : \nu_e)_{\text{Data}}}{(\nu_\mu : \nu_e)_{\text{MC}}} \approx 0.6 \pm 0.1 \ , \qquad (22)$$

where $(\nu_\mu : \nu_e)_{\text{MC}}$ is the $\nu_\mu : \nu_e$ ratio expected on the basis of a Monte Carlo simulation. In addition, it is found that the quantity R depends on the direction from which the neutrinos are coming: For upward-going neutrinos, which must have been produced in the atmosphere on the side of the earth opposite to where the detector is located, and then traveled $\sim 10^4$ km, the diameter of the earth, to reach the detector, R has an anomalously low value. But for downward-going neutrinos, which must have been produced in the atmosphere just above the detector and traveled only ~ 10 km to reach it, R is consistent with unity [22].

The atmospheric neutrino results have been interpreted as $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_e$ oscillation, described by an expression like that of Eq. (11). To accommodate the fact that the upward-going neutrinos oscillate, making R anomalously low, we must have $[1.27\Delta M^2(\text{eV}^2)L(\text{km})/E(\text{GeV})] \gtrsim 1$ when $L \sim 10^4$ km and $E \sim 1$ GeV, a typical energy for the neutrinos studied. This requires $\Delta M^2 \gtrsim 10^{-4}$ eV^2. To accommodate the fact that the downward-going neutrinos do not oscillate (since for them R is not anomalous), we must have $[1.27\Delta M^2(\text{eV}^2)L(\text{km})/E(\text{GeV})] \ll 1$ when $L \sim 10$ km and $E \sim 1$ GeV. This requires $\Delta M^2 \lesssim 10^{-2}$ eV^2. Thus, the favored ΔM^2 range is

$$10^{-4} \lesssim \Delta M^2 \lesssim 10^{-2} \text{ eV}^2 \ . \tag{23}$$

The size of the observed effect implies that the mixing angle is near maximal:

$$\sin^2 2\theta \approx 1 \ . \tag{24}$$

In view of a recent bound on $\nu_e \leftrightarrow \nu_\mu$ oscillation from the CHOOZ reactor experiment [23], the $\nu_\mu \to \nu_\tau$ interpretation of the atmospheric neutrino data is more likely than the $\nu_\mu \to \nu_e$ interpretation.

The LSND experiment [24] has studied neutrinos from stopped positively-charged pions, which decay via the chain

$$\begin{array}{l} \pi^+ \to \mu^+\nu_\mu \\ \qquad\quad \hookrightarrow e^+\nu_e\overline{\nu}_\mu \end{array} \tag{25}$$

We note that this chain does not produce $\overline{\nu}_e$, but an excess of $\overline{\nu}_e$ over expected background is reported by the experiment. This excess is interpreted as arising from oscillation of the $\overline{\nu}_\mu$ which the chain does produce into $\overline{\nu}_e$. Since the experiment has $L(\text{km})/E(\text{GeV}) \sim 1$, the implied mass splitting is $\Delta M^2 \gtrsim 1$ eV^2.

More recently, the same experiment has studied the neutrinos from the decay

$$\pi^+ \to \mu^+\nu_\mu \tag{26}$$

of positively-charged pions in flight. This decay does not produce ν_e, but the experiment reports a ν_e signal above background [25]. This signal is interpreted as coming from $\nu_\mu \to \nu_e$ oscillation. The regions of ΔM^2 and $\sin^2 2\theta$ favored by the stopped pion and decay-in-flight data are consistent [25,26].

Suppose we assume that the behavior of the solar, atmospheric, and LSND neutrinos are all to be understood in terms of neutrino oscillation. What neutrino masses are then suggested?

If there are only three neutrinos of definite flavor, ν_e, ν_μ, and ν_τ, made up out of just three neutrinos of definite mass, ν_1, ν_2, and ν_3, then there are only three mass splittings ΔM^2_{ij}, and they obviously satisfy

$$\Delta M^2_{12} + \Delta M^2_{23} + \Delta M^2_{31} =$$
$$(M_1^2 - M_2^2) + (M_2^2 - M_3^2) + (M_3^2 - M_1^2) = 0 \ . \qquad (27)$$

Now, the ΔM^2 required by the MSW explanation of the solar neutrino data is $\sim 10^{-5}$ eV^2, Eq. (21), and that required by the vacuum oscillation explanation is only 10^{-10} eV^2. The ΔM^2 required by the vacuum oscillation interpretation of the atmospheric neutrino anomaly is $\sim 10^{-(2-4)}$eV^2, Eq. (23). Finally, the ΔM^2 favored by the vacuum oscillation explanation of the LSND data is $\gtrsim 1$ eV^2. Since the ΔM^2 values required to explain the solar, atmospheric, and LSND effects are of three different orders of magnitude, there is no way these three ΔM^2 values can add up to zero, as demanded by Eq. (27). Thus, it appears that one cannot explain all three of the existing hints of neutrino oscillation without introducing a fourth neutrino. Since this neutrino is known to make no contribution to the width of the Z^0 [27], it must be a neutrino which does not participate in the normal weak interactions—a "sterile" neutrino.

Despite this argument, interesting attempts have been made to make do with just three neutrinos. In one of these [28], there is a neutrino mass hierarchy of the sort described before Eq. (16), with $\Delta M^2_{32} \approx \Delta M^2_{31} \gg \Delta M^2_{21}$. The large mass splitting, ΔM^2_{32}, is taken to be ~ 0.4 eV^2, and the small one, ΔM^2_{21}, to be $\sim$ (3–10) $\times$ 10^{-5}eV^2. The LSND results are interpreted as $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ oscillation governed by the large mass splitting. The solar neutrino observations are explained in terms of an MSW $\nu_e \to \nu_\mu$ conversion governed by the small mass splitting. The atmospheric neutrino anomaly, which appears naively to require an intermediate ΔM^2, is explained as a combination of oscillation effects involving both the large ΔM^2_{32} and the small ΔM^2_{21}. This scheme does not quite fit all the data, but it is intriguingly close.

If one assumes that a sterile neutrino cannot be avoided, then all three hints of neutrino oscillation can be accommodated, for example, with the following four neutrinos: A nearly degenerate pair, ν_3, ν_2, with $M_3 \approx M_2 \sim 1$ eV, and a much lighter pair, ν_1, ν_s, in which ν_s is the sterile neutrino [29]. We take the mass M_s of ν_s to be $\sim 3 \times 10^{-3}$ eV, and $M_1 \ll M_s$. The flavor neutrinos ν_τ and ν_μ are each 50–50 mixtures of ν_3 and ν_2, in accord with the suggestion from the atmospheric neutrino data that ν_τ and ν_μ are maximally mixed. The ν_e is dominantly ν_1. The mass splitting $M_3^2 - M_2^2$ is chosen to be $\lesssim 10^{-2}$eV^2 to facilitate the $\nu_\mu \to \nu_\tau$ oscillation interpretation of the atmospheric anomaly. The splitting $M_s^2 - M_1^2 \approx M_s^2 \sim 10^{-5}$ eV^2 allows us to interpret the solar neutrino observations as reflecting MSW conversion of ν_e to the sterile ν_s. Finally, the mass-squared splitting of ~ 1 eV^2 between the heavier pair and the lighter one enables us to explain the LSND data in terms of $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ oscillation.

The existing hints of neutrino oscillation, and the possible neutrino-mass scenarios which they suggest, will be probed in future neutrino experiments.

In addition to the ν_e, ν_μ, and ν_τ sections, the *Review of Particle Physics* includes sections on "Number of Light Neutrino Types," "Heavy Lepton Searches," and "Searches for Massive Neutrinos and Lepton Mixing."

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