## $K^{\pm}_{\ell 3}$ AND $K^{0}_{\ell 3}$ FORM FACTORS

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Assuming that only the vector current contributes to $K \rightarrow \pi\ell\nu$ decays, we write the matrix element as

$$M \propto f_+(t)\left[(P_K + P_\pi)_\mu \overline{\ell}\gamma_\mu(1+\gamma_5)\nu\right] + f_-(t)\left[m_\ell\overline{\ell}(1+\gamma_5)\nu\right] \ , \quad (1)$$

where $P_K$ and $P_\pi$ are the four-momenta of the $K$ and $\pi$ mesons, $m_\ell$ is the lepton mass, and $f_+$ and $f_-$ are dimensionless form factors which can depend only on $t = (P_K - P_\pi)^2$, the square of the four-momentum transfer to the leptons. If time-reversal invariance holds, $f_+$ and $f_-$ are relatively real. $K_{\mu 3}$ experiments measure $f_+$ and $f_-$, while $K_{e3}$ experiments are sensitive only to $f_+$ because the small electron mass makes the $f_-$ term negligible.

*(a) $K_{\mu 3}$ experiments*. Analyses of $K_{\mu 3}$ data frequently assume a linear dependence of $f_+$ and $f_-$ on $t$, *i.e.*,

$$f_\pm(t) = f_\pm(0)\left[1 + \lambda_\pm(t/m_\pi^2)\right] \quad (2)$$

Most $K_{\mu 3}$ data are adequately described by Eq. (2) for $f_+$ and a constant $f_-$ (*i.e.*, $\lambda_- = 0$). There are two equivalent parametrizations commonly used in these analyses:

*(1) $\lambda_+, \xi(0)$ parametrization*. Analyses of $K_{\mu 3}$ data often introduce the ratio of the two form factors

$$\xi(t) = f_-(t)/f_+(t) \ .$$

The $K_{\mu 3}$ decay distribution is then described by the two parameters $\lambda_+$ and $\xi(0)$ (assuming time reversal invariance and $\lambda_- = 0$). These parameters can be determined by three different methods:

*Method A*. By studying the Dalitz plot or the pion spectrum of $K_{\mu 3}$ decay. The Dalitz plot density is (see, *e.g.*, Chounet *et al.* [1]):

$$\rho(E_\pi, E_\mu) \propto f_+^2(t)\left[A + B\xi(t) + C\xi(t)^2\right] \ ,$$

where

$$A = m_K \left(2E_\mu E_\nu - m_K E'_\pi\right) + m_\mu^2 \left(\frac{1}{4}E'_\pi - E_\nu\right) \ ,$$

$$B = m_\mu^2 \left(E_\nu - \frac{1}{2}E'_\pi\right) \ ,$$

$$C = \frac{1}{4} m_\mu^2 E'_\pi \ ,$$

$$E'_\pi = E_\pi^{\max} - E_\pi = \left(m_K^2 + m_\pi^2 - m_\mu^2\right)/2m_K - E_\pi \ .$$

Here $E_\pi$, $E_\mu$, and $E_\nu$ are, respectively, the pion, muon, and neutrino energies in the kaon center of mass. The density $\rho$ is fit to the data to determine the values of $\lambda_+, \xi(0)$, and their correlation.

*Method B.* By measuring the $K_{\mu 3}/K_{e3}$ branching ratio and comparing it with the theoretical ratio (see, *e.g.*, Fearing *et al.* [2]) as given in terms of $\lambda_+$ and $\xi(0)$, assuming $\mu$-$e$ universality:

$$\Gamma(K^\pm_{\mu 3})/\Gamma(K^\pm_{e3}) = 0.6457 + 1.4115\lambda_+ + 0.1264\xi(0) + 0.0192\xi(0)^2 + 0.0080\lambda_+\xi(0) \ ,$$

$$\Gamma(K^0_{\mu 3})/\Gamma(K^0_{e3}) = 0.6452 + 1.3162\lambda_+ + 0.1264\xi(0) + 0.0186\xi(0)^2 + 0.0064\lambda_+\xi(0) \ .$$

This cannot determine $\lambda_+$ and $\xi(0)$ simultaneously but simply fixes a relationship between them.

*Method C.* By measuring the muon polarization in $K_{\mu 3}$ decay. In the rest frame of the $K$, the $\mu$ is expected to be polarized in the direction $\mathbf{A}$ with $\mathbf{P} = \mathbf{A}/\left|\mathbf{A}\right|$, where $\mathbf{A}$ is given (Cabibbo and Maksymowicz [3]) by

$$\mathbf{A} = a_1(\xi)\mathbf{p}_\mu - a_2(\xi)\left[\frac{\mathbf{p}_\mu}{m_\mu}\left(m_K - E_\pi + \frac{\mathbf{p}_\pi \cdot \mathbf{p}_\mu}{\left|\mathbf{p}_\mu\right|^2}(E_\mu - m_\mu)\right) + \mathbf{p}_\pi\right] + m_K \mathrm{Im}\xi(t)(\mathbf{p}_\pi \times \mathbf{p}_\mu) \ .$$

If time-reversal invariance holds, $\xi$ is real, and thus there is no polarization perpendicular to the $K$-decay plane. Polarization experiments measure the weighted average of $\xi(t)$ over the $t$ range of the experiment, where the weighting accounts for the variation with $t$ of the sensitivity to $\xi(t)$.

*(2)* $\lambda_+, \lambda_0$ *parametrization.* Most of the more recent $K_{\mu3}$ analyses have parameterized in terms of the form factors $f_+$ and $f_0$ which are associated with vector and scalar exchange, respectively, to the lepton pair. $f_0$ is related to $f_+$ and $f_-$ by

$$f_0(t) = f_+(t) + \left[t/(m_K^2 - m_\pi^2)\right] f_-(t) \ .$$

Here $f_0(0)$ must equal $f_+(0)$ unless $f_-(t)$ diverges at $t = 0$. The earlier assumption that $f_+$ is linear in $t$ and $f_-$ is constant leads to $f_0$ linear in $t$:

$$f_0(t) = f_0(0)\left[1 + \lambda_0(t/m_\pi^2)\right] \ .$$

With the assumption that $f_0(0) = f_+(0)$, the two parametrizations, $(\lambda_+, \xi(0))$ and $(\lambda_+, \lambda_0)$ are equivalent as long as correlation information is retained. $(\lambda_+, \lambda_0)$ correlations tend to be less strong than $(\lambda_+, \xi(0))$ correlations.

The experimental results for $\xi(0)$ and its correlation with $\lambda_+$ are listed in the $K^\pm$ and $K_L^0$ sections of the Particle Listings in section $\xi_{\rm A}$, $\xi_{\rm B}$, or $\xi_{\rm C}$ depending on whether method A, B, or C discussed above was used. The corresponding values of $\lambda_+$ are also listed.

Because recent experiments tend to use the $(\lambda_+, \lambda_0)$ parametrization, we include a subsection for $\lambda_0$ results. Wherever possible we have converted $\xi(0)$ results into $\lambda_0$ results and vice versa.

See the 1982 version of this note [4] for additional discussion of the $K_{\mu3}^0$ parameters, correlations, and conversion between parametrizations, and also for a comparison of the experimental results.

*(b)* $K_{e3}$ *experiments.* Analysis of $K_{e3}$ data is simpler than that of $K_{\mu3}$ because the second term of the matrix element assuming a pure vector current [Eq. (1) above] can be neglected. Here $f_+$ is usually assumed to be linear in $t$, and the linear coefficient $\lambda_+$ of Eq. (2) is determined.

If we remove the assumption of a pure vector current, then the matrix element for the decay, in addition to the terms in Eq. (2), would contain

$$+2m_K\, f_S\, \overline{\ell}(1+\gamma_5)\nu$$

$$+(2f_T/m_K)(P_K)_\lambda(P_\pi)_\mu\, \overline{\ell}\, \sigma_{\lambda\mu}(1+\gamma_5)\nu \ ,$$

where $f_S$ is the scalar form factor, and $f_T$ is the tensor form factor. In the case of the $K_{e3}$ decays where the $f_-$ term can be neglected, experiments have yielded limits on $|f_S/f_+|$ and $|f_T/f_+|$.

**References**

1. L.M. Chounet, J.M. Gaillard, and M.K. Gaillard, Phys. Reports **4C**, 199 (1972).
2. H.W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. **D2**, 542 (1970).
3. N. Cabibbo and A. Maksymowicz, Phys. Lett. **9**, 352 (1964).
4. Particle Data Group, Phys. Lett. **111B**, 73 (1982).