PSEUDOSCALAR-MESON DECAY CONSTANTS

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Charged mesons

The decay constant f_P for a charged pseudoscalar meson P is defined by

$$\langle 0|A_\mu(0)|P(\mathbf{q})\rangle = i f_P\, q_\mu \ , \tag{1}$$

where A_μ is the axial-vector part of the charged weak current after a Cabibbo-Kobayashi-Maskawa mixing-matrix element $V_{qq'}$ has been removed. The state vector is normalized by $\langle P(\mathbf{q})|P(\mathbf{q}')\rangle = (2\pi)^3\ 2E_q\,\delta(\mathbf{q}-\mathbf{q}')$, and its phase is chosen to make f_P real and positive. Note, however, that in many theoretical papers our $f_P/\sqrt{2}$ is denoted by f_P.

In determining f_P experimentally, radiative corrections must be taken into account. Since the photon-loop correction introduces an infrared divergence that is canceled by soft-photon emission, we can determine f_P only from the combined rate for $P^\pm \to \ell^\pm \nu_\ell$ and $P^\pm \to \ell^\pm \nu_\ell \gamma$. This rate is given by

$$\Gamma\,(P \to \ell\nu_\ell + \ell\nu_\ell\gamma) = \frac{G_F^2 |{V_{qq'}}^2|}{8\pi} f_P^2\, m_\ell^2\, m_P \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 [1 + \mathscr{O}(\alpha)] \ . \tag{2}$$

Here m_ℓ and m_P are the masses of the lepton and meson. Radiative corrections include inner bremsstrahlung, which is independent of the structure of the meson [1–3], and also a structure-dependent term [4,5]. After radiative corrections are made, there are ambiguities in extracting f_P from experimental measurements. In fact, the definition of f_P is no longer unique.

It is desirable to define f_P such that it depends only on the properties of the pseudoscalar meson, not on the final decay products. The short-distance corrections to the fundamental electroweak constants like $G_F|V_{qq'}|$ should be separated out. Following Marciano and Sirlin [6], we define f_P with the following form for the $\mathcal{O}(\alpha)$ corrections:

$$1+\mathcal{O}(\alpha)=\left[1+\frac{2\alpha}{\pi}\ln\Big(\frac{m_Z}{m_\rho}\Big)\right]\left[1+\frac{\alpha}{\pi}F(x)\right]$$
$$\times\left\{1-\frac{\alpha}{\pi}\left[\frac{3}{2}\ln\Big(\frac{m_\rho}{m_P}\Big)+C_1+C_2\frac{m_\ell^2}{m_\rho^2}\ln\left(\frac{m_\rho^2}{m_\ell^2}\right)+C_3\frac{m_\ell^2}{m_\rho^2}+\ldots\right]\right\}\ , \tag{3}$$

where m_ρ and m_Z are the masses of the ρ meson and Z boson. Here

$$F(x)=3\ln x+\frac{13-19x^2}{8(1-x^2)}-\frac{8-5x^2}{2(1-x^2)^2}\,x^2\ln x$$
$$-2\Big(\frac{1+x^2}{1-x^2}\ln x+1\Big)\ln(1-x^2)+2\Big(\frac{1+x^2}{1-x^2}\Big)L(1-x^2)\ ,$$

with

$$x\equiv m_\ell/m_P\ ,\qquad L(z)\equiv\int_0^z\frac{\ln(1-t)}{t}\,dt\ . \tag{4}$$

The first bracket in the expression for $1+\mathcal{O}(\alpha)$ is the short-distance electroweak correction. A quarter of $(2\alpha/\pi)\ln(m_Z/m_\rho)$ is subject to the QCD correction $(1-\alpha_s/\pi)$, which leads to a reduction of the total short-distance correction of 0.00033 from the electroweak contribution alone [6]. The second bracket together with the term $-(3\alpha/2\pi)\ln(m_\rho/m_P)$ in the third bracket corresponds to the radiative corrections to the point-like pion decay ($\Lambda_{\rm cutoff}\approx m_\rho$) [2]. The rest of the corrections in the third bracket are expanded in powers of m_ℓ/m_ρ. The expansion coefficients C_1, C_2, and C_3 depend on the hadronic structure of the pseudoscalar meson and in most cases cannot be computed accurately. In particular, C_1 absorbs the uncertainty in the matching energy scale between short- and long-distance strong interactions and thus is the main source of uncertainty in determining f_{π^+} accurately.

With the experimental value for the decay $\pi^+ \to \mu^+\nu_\mu + \mu^+\nu_\mu\gamma$, one obtains

$$f_{\pi^+} = 130.7 \pm 0.1 \pm 0.36 \text{ MeV} , \tag{5}$$

where the first error comes from the experimental uncertainty on $|V_{ud}|$ and the second comes from the uncertainty on C_1 ($= 0 \pm 0.24$) [6]. Similarly, one obtains from the decay $K^+ \to \mu^+\nu_\mu + \mu^+\nu_\mu\gamma$ the decay constant

$$f_{K^+} = 159.8 \pm 1.4 \pm 0.44 \text{ MeV} , \tag{6}$$

where the first error is due to the uncertainty on $|V_{us}|$.

For the heavy pseudoscalar mesons, uncertainties in the experimental values for the decay rates are much larger than the radiative corrections. For the D^+, only an upper bound can be obtained from the published data:

$$f_{D^+} < 310 \text{ MeV (CL} = 90\%) . \tag{7}$$

For the D_s^+, the decay constant has been extracted from both the $D_s^+ \to \mu^+\nu_\mu$ and the $D_s^+ \to \tau^+\nu_\tau$ branching fractions. Two values have been reported since the last edition [7,8]:

$f_{D_s^+} = 194 \pm 35 \pm 20 \pm 14$ MeV from $D_s^+ \to \mu^+\nu_\mu$,

$f_{D_s^+} = 309 \pm 58 \pm 33 \pm 38$ MeV from $D_s^+ \to \tau^+\nu_\tau$.

There are now altogether five reported values for $f_{D_s^+}$ spread over a wide range,

$$f_{D_s^+} = 194 \text{ MeV } \sim 430 \text{ MeV} \tag{8}$$

with large uncertainties attached. We must wait for better data before giving a meaningful value for $f_{D_s^+}$. (See the measurements of the $D_s^+ \to \ell^+\nu_\ell$ modes in the Particle Listings for the numbers quoted by individual experiments.)

There have been many attempts to extract f_P from spectroscopy and nonleptonic decays using theoretical models. Since it is difficult to estimate uncertainties for them, we have listed here only values of decay constants that are obtained directly from the observation of $P^\pm \to \ell^\pm\nu_\ell$.

Light neutral mesons

The decay constants for the light neutral pseudoscalar mesons π^0, η, and η' are defined by

$$\langle 0|A_\mu(0)|P^0(\mathbf{q})\rangle = i(f_P/\sqrt{2})q_\mu \ , \tag{9}$$

where A_μ is a neutral axial-vector current of octet or singlet. However f_p for the neutral mesons cannot be extracted directly from the data.

In the limit of $m_P \to 0$, the Adler-Bell-Jackiw anomaly determines f_P through the matrix element of the two-photon decay $P^0 \to \gamma\gamma$ [9,10]. The extrapolation to the mass shell is needed to extract the physical value of f_P. In the case of f_{π^0}, the extrapolation is small and the experimental uncertainty in the π^0 lifetime dominates in the uncertainty of f_{π^0}:

$$f_{\pi^0} = 130 \pm 5 \text{ MeV} \ , \tag{10}$$

which is consistent with isospin symmetry.

For the η and η', the extrapolation to the mass shell is larger and therefore the dominance of the anomaly on the mass shell is questionable, particularly for the η'; and η–η' mixing adds to the uncertainty. If the corrections are computed for the octet with the chiral Lagrangian [11], one obtains $f_8 \approx 1.3 f_\pi$ for the decay constant of the $I = 0$ octet state. For the singlet state, *if* the $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ decay rates are fitted with the same form as the anomaly indicates, $f_1 \approx f_\pi$ would give a viable fit for $f_8 \approx 1.3 f_\pi$ and the η–η' mixing angle of $\theta_P \approx -20°$. However, because of the arbitrariness even in defining the decay constants, we do not quote numbers for f_η or $f_{\eta'}$ here.

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