

36. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

Revised April 1998 by R.N. Cahn (LBNL).

36.1. Leptonproduction

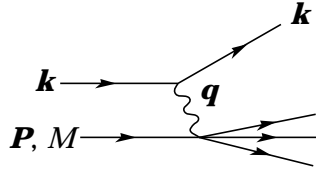


Figure 36.1: Kinematic quantities for description of lepton-nucleon scattering. k and k' are the four-momenta of incoming and outgoing leptons, P is the four-momentum of a nucleon with mass M . The exchanged particle is a γ , W^\pm , or Z^0 ; it transfers four-momentum $q = k - k'$ to the target.

Invariant quantities:

$\nu = \frac{q \cdot P}{M} = E - E'$ is the lepton's energy loss in the lab (in earlier literature sometimes $\nu = q \cdot P$). Here, E and E' are the initial and final lepton energies in the lab.

$Q^2 = -q^2 = 2(E E' - \vec{k} \cdot \vec{k}') - m_\ell^2 - m_{\ell'}^2$ where $m_\ell(m_{\ell'})$ is the initial (final) lepton mass. If $E E' \sin^2(\theta/2) \gg m_\ell^2, m_{\ell'}^2$, then

$\approx 4 E E' \sin^2(\theta/2)$, where θ is the lepton's scattering angle in the lab.

$x = \frac{Q^2}{2M\nu}$ In the parton model, x is the fraction of the target nucleon's momentum carried by the struck quark. [See section on Quantum Chromodynamics (Sec. 9 of this *Review*.)]

$y = \frac{q \cdot P}{k \cdot P} = \frac{\nu}{E}$ is the fraction of the lepton's energy lost in the lab.

$W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$ is the mass squared of the system recoiling against the lepton.

$s = (k + P)^2 = \frac{Q^2}{xy} + M^2$

36.1.1. Leptonproduction cross sections:

$$\begin{aligned} \frac{d^2\sigma}{dx dy} &= \nu (s - M^2) \frac{d^2\sigma}{d\nu dQ^2} = \frac{2\pi M\nu}{E'} \frac{d^2\sigma}{d\Omega_{\text{lab}} dE'} \\ &= x(s - M^2) \frac{d^2\sigma}{dx dQ^2}. \end{aligned} \quad (36.1)$$

2 36. Cross-section formulae for specific processes

36.1.2. Leptoproduction structure functions: The neutral-current process, $eN \rightarrow eX$, at low Q^2 is just electromagnetic and parity conserving. It can be written in terms of two structure functions $F_1^{\text{em}}(x, Q^2)$ and $F_2^{\text{em}}(x, Q^2)$:

$$\frac{d^2\sigma}{dx dy} = \frac{4\pi \alpha^2 (s - M^2)}{Q^4} \times \left[(1 - y) F_2^{\text{em}} + y^2 x F_1^{\text{em}} - \frac{M^2}{(s - M^2)} xy F_2^{\text{em}} \right]. \quad (36.2)$$

The charged-current processes, $e^-N \rightarrow \nu X$, $\nu N \rightarrow e^-X$, and $\bar{\nu}N \rightarrow e^+X$, are parity violating and can be written in terms of three structure functions $F_1^{\text{CC}}(x, Q^2)$, $F_2^{\text{CC}}(x, Q^2)$, and $F_3^{\text{CC}}(x, Q^2)$:

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi} \frac{M_W^4}{(Q^2 + M_W^2)^2} \times \left\{ \left[1 - y - \frac{M^2 xy}{(s - M^2)} \right] F_2^{\text{CC}} + \frac{y^2}{2} 2x F_1^{\text{CC}} \pm \left(y - \frac{y^2}{2} \right) x F_3^{\text{CC}} \right\}, \quad (36.3)$$

where the last term is positive for the e^- and ν reactions and negative for $\bar{\nu}N \rightarrow e^+X$. As explained below there are different structure functions for charge-raising and charge-lowering currents.

36.1.3. Structure functions in the QCD parton model: In the QCD parton model, the structure functions defined above can be expressed in terms of parton distribution functions. The quantity $f_i(x, Q^2)dx$ is the probability that a parton of type i (quark, antiquark, or gluon), carries a momentum fraction between x and $x + dx$ of the nucleon's momentum in a frame where the nucleon's momentum is large. For the cross section corresponding to the *neutral-current process* $ep \rightarrow eX$, we have for $s \gg M^2$ (in the case where the incoming electron is either left- (L) or right- (R) handed):

$$\frac{d^2\sigma}{dx dy} = \frac{\pi\alpha^2}{sx^2 y^2} \left[\sum_q \left(x f_q(x, Q^2) + x f_{\bar{q}}(x, Q^2) \right) \right] \times \left[A_q + (1 - y)^2 B_q \right]. \quad (36.4)$$

Here the index q refers to a quark flavor (*i.e.*, u, d, s, c, b , or t), and

$$A_q = \left(-q_q + g_{Lq} g_{Le} \frac{Q^2}{Q^2 + M_Z^2} \right)^2 + \left(-q_q + g_{Rq} g_{Re} \frac{Q^2}{Q^2 + M_Z^2} \right)^2, \quad (36.5)$$

$$B_q = \left(-q_q + g_{Rq} g_{Le} \frac{Q^2}{Q^2 + M_Z^2} \right)^2 + \left(-q_q + g_{Lq} g_{Re} \frac{Q^2}{Q^2 + M_Z^2} \right)^2. \quad (36.6)$$

36. Cross-section formulae for specific processes 3

Here q_q is the charge of flavor q . For a left-handed electron, $g_{Re} = 0$ and $g_{Le} = (-1/2 + \sin^2 \theta_W)/(\sin \theta_W \cos \theta_W)$, while for a right-handed electron, $g_{Le} = 0$ and $g_{Re} = (\sin^2 \theta_W)/(\sin \theta_W \cos \theta_W)$. For the quarks, $g_{Lq} = (T_3 - q_q \sin^2 \theta_W)/(\sin \theta_W \cos \theta_W)$, and $g_{Rq} = (-q_q \sin^2 \theta_W) / (\sin \theta_W \cos \theta_W)$.

For neutral-current *neutrino (antineutrino) scattering*, the same formula applies with g_{Le} replaced by $g_{L\nu} = 1/(2 \sin \theta_W \cos \theta_W)$ ($g_{L\bar{\nu}} = 0$) and g_{Re} replaced by $g_{R\nu} = 0$ [$g_{R\bar{\nu}} = -1/(2 \sin \theta_W \cos \theta_W)$].

In the case of the *charged-current processes* $e_L^- p \rightarrow \nu X$ and $\bar{\nu} p \rightarrow e^+ X$, Eq. (36.3) applies with

$$F_2 = 2xF_1 = 2x \left[f_u(x, Q^2) + f_c(x, Q^2) + f_t(x, Q^2) + f_{\bar{d}}(x, Q^2) + f_{\bar{s}}(x, Q^2) + f_{\bar{b}}(x, Q^2) \right], \quad (36.7)$$

$$F_3 = 2 \left[f_u(x, Q^2) + f_c(x, Q^2) + f_t(x, Q^2) - f_{\bar{d}}(x, Q^2) - f_{\bar{s}}(x, Q^2) - f_{\bar{b}}(x, Q^2) \right]. \quad (36.8)$$

For the process $\nu p \rightarrow e^- X$:

$$F_2 = 2xF_1 = 2x \left[f_d(x, Q^2) + f_s(x, Q^2) + f_b(x, Q^2) + f_{\bar{u}}(x, Q^2) + f_{\bar{c}}(x, Q^2) + f_{\bar{t}}(x, Q^2) \right], \quad (36.9)$$

$$F_3 = 2 \left[f_d(x, Q^2) + f_s(x, Q^2) + f_b(x, Q^2) - f_{\bar{u}}(x, Q^2) - f_{\bar{c}}(x, Q^2) - f_{\bar{t}}(x, Q^2) \right]. \quad (36.10)$$

36.2. e^+e^- annihilation

For pointlike, spin-1/2 fermions, the differential cross section in the c.m. for $e^+e^- \rightarrow f\bar{f}$ via single photon annihilation is (θ is the angle between the incident electron and the produced fermion; $N_c = 1$ if f is a lepton and $N_c = 3$ if f is a quark).

$$\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4s} \beta [1 + \cos^2 \theta + (1 - \beta^2) \sin^2 \theta] Q_f^2, \quad (36.11)$$

where β is the velocity of the final state fermion in the c.m. and Q_f is the charge of the fermion in units of the proton charge. For $\beta \rightarrow 1$,

$$\sigma = N_c \frac{4\pi\alpha^2}{3s} Q_f^2 = N_c \frac{86.8 Q_f^2 \text{ nb}}{s(\text{GeV}/c^2)^2}. \quad (36.12)$$

At higher energies, the Z^0 (mass M_Z and width Γ_Z) must be included. If the mass of a fermion f is much less than the mass of the Z^0 , then the differential cross section for $e^+e^- \rightarrow f\bar{f}$ is

4 36. Cross-section formulae for specific processes

$$\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4s} \left\{ (1 + \cos^2 \theta) \left[Q_f^2 - 2\chi_1 v_e v_f Q_f + \chi_2 (a_e^2 + v_e^2)(a_f^2 + v_f^2) \right] + 2 \cos \theta \left[-2\chi_1 a_e a_f Q_f + 4\chi_2 a_e a_f v_e v_f \right] \right\} \quad (36.13)$$

where

$$\begin{aligned} \chi_1 &= \frac{1}{16 \sin^2 \theta_W \cos^2 \theta_W} \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \\ \chi_2 &= \frac{1}{256 \sin^4 \theta_W \cos^4 \theta_W} \frac{s^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \\ a_e &= -1, \\ v_e &= -1 + 4 \sin^2 \theta_W, \\ a_f &= 2T_{3f}, \\ v_f &= 2T_{3f} - 4Q_f \sin^2 \theta_W, \end{aligned} \quad (36.14)$$

where $T_{3f} = 1/2$ for u, c and neutrinos, while $T_{3f} = -1/2$ for d, s, b , and negatively charged leptons.

At LEP II it may be possible to produce the orthodox Higgs boson, H , (see the mini-review on Higgs bosons) in the reaction $e^+e^- \rightarrow HZ^0$, which proceeds dominantly through a virtual Z^0 . The Standard Model prediction for the cross section [3] is

$$\sigma(e^+e^- \rightarrow HZ^0) = \frac{\pi\alpha^2}{24} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_Z^2}{(s - M_Z^2)^2} \cdot \frac{1 - 4\sin^2 \theta_W + 8\sin^4 \theta_W}{\sin^4 \theta_W \cos^4 \theta_W}. \quad (36.15)$$

where K is the c.m. momentum of the produced H or Z^0 . Near the production threshold, this formula needs to be corrected for the finite width of the Z^0 .

36.3. Two-photon process at e^+e^- colliders

When an e^+ and an e^- collide with energies E_1 and E_2 , they emit dn_1 and dn_2 virtual photons with energies ω_1 and ω_2 and 4-momenta q_1 and q_2 . In the equivalent photon approximation, the cross section for $e^+e^- \rightarrow e^+e^-X$ is related to the cross section for $\gamma\gamma \rightarrow X$ by (Ref. 1)

$$d\sigma_{e^+e^- \rightarrow e^+e^-X}(s) = dn_1 dn_2 d\sigma_{\gamma\gamma \rightarrow X}(W^2) \quad (36.16)$$

where $s = 4E_1E_2$, $W^2 = 4\omega_1\omega_2$ and

$$dn_i = \frac{\alpha}{\pi} \left[1 - \frac{\omega_i}{E_i} + \frac{\omega_i^2}{2E_i^2} - \frac{m_e^2 \omega_i^2}{(-q_i^2)E_i^2} \right] \frac{d\omega_i}{\omega_i} \frac{d(-q_i^2)}{(-q_i^2)}. \quad (36.17)$$

36. Cross-section formulae for specific processes 5

After integration (including that over q_i^2 in the region $m_e^2 \omega_i^2 / E_i(E_i - \omega_i) \leq -q_i^2 \leq (-q^2)_{\max}$), the cross section is

$$\begin{aligned} \sigma_{e^+e^- \rightarrow e^+e^-X}(s) &= \frac{\alpha^2}{\pi^2} \int_{z_{th}}^1 \frac{dz}{z} \left[f(z) \left(\ln \frac{(-q^2)_{\max}}{m_e^2 z} - 1 \right)^2 \right. \\ &\quad \left. - \frac{1}{3} \left(\ln \frac{1}{z} \right)^3 \right] \sigma_{\gamma\gamma \rightarrow X}(zs) ; \\ f(z) &= \left(1 + \frac{1}{2}z \right)^2 \ln \frac{1}{z} - \frac{1}{2}(1-z)(3+z) ; \\ z &= \frac{W^2}{s} . \end{aligned} \quad (36.18)$$

The quantity $(-q^2)_{\max}$ depends on properties of the produced system X , in particular, $(-q^2)_{\max} \sim m_\rho^2$ for hadron production ($X = h$) and $(-q^2)_{\max} \sim W^2$ for lepton pair production ($X = \ell^+\ell^-$, $\ell = e, \mu, \tau$).

For production of a resonance of mass m_R and spin $J \neq 1$

$$\begin{aligned} \sigma_{e^+e^- \rightarrow e^+e^-R}(s) &= (2J+1) \frac{8\alpha^2 \Gamma_{R \rightarrow \gamma\gamma}}{m_R^3} \\ &\times \left[f(m_R^2/s) \left(\ln \frac{sm_V^2}{m_e^2 m_R^2} - 1 \right)^2 - \frac{1}{3} \left(\ln \frac{s}{m_R^2} \right)^3 \right] \end{aligned} \quad (36.19)$$

where m_V is the mass that enters into the form factor of the $\gamma\gamma \rightarrow R$ transition: $m_V \sim m_\rho$ for $R = \pi^0, \eta, f_2(1270), \dots$, $m_V \sim m_R$ for $R = c\bar{c}$ or $b\bar{b}$ resonances.

36.4. Inclusive hadronic reactions

One-particle inclusive cross sections $E d^3\sigma/d^3p$ for the production of a particle of momentum p are conveniently expressed in terms of rapidity (see above) and the momentum p_T transverse to the beam direction (defined in the center-of-mass frame)

$$E \frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi dy p_T dp_T} . \quad (36.20)$$

In the case of processes where p_T is large or the mass of the produced particle is large (here large means greater than 10 GeV), the parton model can be used to calculate the rate. Symbolically

$$\sigma_{\text{hadronic}} = \sum_{ij} \int f_i(x_1, Q^2) f_j(x_2, Q^2) dx_1 dx_2 \hat{\sigma}_{\text{partonic}} , \quad (36.21)$$

where $f_i(x, Q^2)$ is the parton distribution introduced above and Q is a typical momentum transfer in the partonic process and $\hat{\sigma}$ is the partonic cross section. Some examples will

6 36. Cross-section formulae for specific processes

help to clarify. The production of a W^+ in pp reactions at rapidity y in the center-of-mass frame is given by

$$\begin{aligned} \frac{d\sigma}{dy} = & \frac{G_F \pi \sqrt{2}}{3} \\ & \times \tau \left[\cos^2 \theta_c \left(u(x_1, M_W^2) \bar{d}(x_2, M_W^2) \right. \right. \\ & \quad \left. \left. + u(x_2, M_W^2) \bar{d}(x_1, M_W^2) \right) \right. \\ & \quad \left. + \sin^2 \theta_c \left(u(x_1, M_W^2) \bar{s}(x_2, M_W^2) \right. \right. \\ & \quad \left. \left. + s(x_2, M_W^2) \bar{u}(x_1, M_W^2) \right) \right], \end{aligned} \quad (36.22)$$

where $x_1 = \sqrt{\tau} e^y$, $x_2 = \sqrt{\tau} e^{-y}$, and $\tau = M_W^2/s$. Similarly the production of a jet in pp (or $p\bar{p}$) collisions is given by

$$\begin{aligned} \frac{d^3\sigma}{d^2p_T dy} = & \sum_{ij} \int f_i(x_1, p_T^2) f_j(x_2, p_T^2) \\ & \times \left[\hat{s} \frac{d\hat{\sigma}}{d\hat{t}} \right]_{ij} dx_1 dx_2 \delta(\hat{s} + \hat{t} + \hat{u}), \end{aligned} \quad (36.23)$$

where the summation is over quarks, gluons, and antiquarks. Here

$$s = (p_1 + p_2)^2, \quad (36.24)$$

$$t = (p_1 - p_{\text{jet}})^2, \quad (36.25)$$

$$u = (p_2 - p_{\text{jet}})^2, \quad (36.26)$$

p_1 and p_2 are the momenta of the incoming p and p (or \bar{p}) and \hat{s} , \hat{t} , and \hat{u} are s , t , and u with $p_1 \rightarrow x_1 p_1$ and $p_2 \rightarrow x_2 p_2$. The partonic cross section $\hat{s}[(d\hat{\sigma})/(d\hat{t})]$ can be found in Ref. 2. Example: for the process $gg \rightarrow q\bar{q}$,

$$\hat{s} \frac{d\sigma}{dt} = 3\alpha_s^2 \frac{(\hat{t}^2 + \hat{u}^2)}{8\hat{s}} \left[\frac{4}{9\hat{t}\hat{u}} - \frac{1}{\hat{s}^2} \right]. \quad (36.27)$$

The prediction of Eq. (36.23) is compared to data from the UA1 and UA2 collaborations in Fig. 38.8 in the Plots of Cross Sections and Related Quantities section of this *Review*.

The associated production of a Higgs boson and a gauge boson is analogous to the process $e^+e^- \rightarrow HZ^0$ in Sec. 36.2. The required parton-level cross sections [4], averaged over initial quark colors, are

$$\begin{aligned} \sigma(q_i \bar{q}_j \rightarrow W^\pm H) &= \frac{\pi \alpha^2 |V_{ij}|^2}{36 \sin^4 \theta_W} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_W^2}{(s - M_W^2)^2} \\ \sigma(q\bar{q} \rightarrow Z^0 H) &= \frac{\pi \alpha^2 (a_q^2 + v_q^2)}{144 \sin^4 \theta_W \cos^4 \theta_W} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_Z^2}{(s - M_Z^2)^2}. \end{aligned}$$

Here V_{ij} is the appropriate element of the Kobayashi-Maskawa matrix and K is the c.m. momentum of the produced H . The axial and vector couplings are defined as in Sec. 36.2.

36.5. One-particle inclusive distributions

In order to describe one-particle inclusive production in e^+e^- annihilation or deep inelastic scattering, it is convenient to introduce a fragmentation function $D_i^h(z, Q^2)$ where $D_i^h(z, Q^2)$ is the number of hadrons of type h and momentum between zp and $(z + dz)p$ produced in the fragmentation of a parton of type i . The Q^2 evolution is predicted by QCD and is similar to that of the parton distribution functions [see section on Quantum Chromodynamics (Sec. 9 of this *Review*)]. The $D_i^h(z, Q^2)$ are normalized so that

$$\sum_h \int z D_i^h(z, Q^2) dz = 1. \quad (36.28)$$

If the contributions of the Z boson and three-jet events are neglected, the cross section for producing a hadron h in e^+e^- annihilation is given by

$$\frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dz} = \frac{\sum_i e_i^2 D_i^h(z, Q^2)}{\sum_i e_i^2}, \quad (36.29)$$

where e_i is the charge of quark-type i , σ_{had} is the total hadronic cross section, and the momentum of the hadron is $zE_{\text{cm}}/2$.

In the case of deep inelastic muon scattering, the cross section for producing a hadron of energy E_h is given by

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dz} = \frac{\sum_i e_i^2 q_i(x, Q^2) D_i^h(z, Q^2)}{\sum_i e_i^2 q_i(x, Q^2)}, \quad (36.30)$$

where $E_h = \nu z$. (For the kinematics of deep inelastic scattering, see Sec. 35.4.2 of the Kinematics section of this *Review*.) The fragmentation functions for light and heavy quarks have a different z dependence; the former peak near $z = 0$. They are illustrated in Figs. 37.1 and 37.2 in the section on ‘‘Heavy Quark Fragmentation in e^+e^- Annihilation’’ (Sec. 37 of this *Review*).

References:

1. V.M. Budnev, I.F. Ginzburg, G.V. Meledin, and V.G. Serbo, Phys. Reports **15C**, 181 (1975);
See also S. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. **D4**, 1532 (1971).
2. G.F. Owens, F. Reya, and M. Glück, Phys. Rev. **D18**, 1501 (1978).
3. B.W. Lee, C. Quigg, and B. Thacker, Phys. Rev. **D16**, 1519 (1977).
4. E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984).