CP VIOLATION IN B DECAY – STANDARD MODEL PREDICTIONS

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The study of CP violation in B decays [1] offers an opportunity to test whether the Standard Model mechanism for CP violation, due to the phase structure of the CKM matrix, is the only source of such effects [2]. The known CP-violation effects in K decays can be accommodated by this mechanism, but do not provide a critical test of it.

The Unitarity conditions (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix")

$$V_{uq}V_{ub}^* + V_{cq}V_{cb}^* + V_{tq}V_{tb}^* = 0 \ , \tag{1}$$

with $q = s$ or $q = d$ where V_{ij} is an element of the CKM matrix can be represented as triangles in the complex plane. The three interior angles of the $q = d$ triangle are labeled

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \ , \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \ ,$$
$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \ . \tag{2}$$

In terms of the Wolfenstein parameters [3] we can also write

$$\tan\alpha = \frac{\eta}{\eta^2 - \rho(1-\rho)} \ , \quad \tan\beta = \frac{\eta}{1-\rho} \ ,$$
$$\tan\gamma = \frac{\eta}{\rho} \ . \tag{3}$$

Notice that the sign as well the magnitude of these angles is meaningful and can be measured.

A major aim of CP-violation studies of B decays is to make enough independent measurements of the sides and angles that the Unitarity triangle is overdetermined and thereby to check the validity of the Standard Model predictions that relate various measurements to aspects of this triangle. Constraints can be made on the basis of present data on the B-meson masses and lifetimes, on the ratio of charmless decays to decays with charm (V_{ub}/V_{cb}), and on ϵ [4] in K decays. These constraints have been discussed in many places in the literature; for a recent summary see Ref. 5. The range of allowed values depends on matrix element estimates, these are difficult to calculate hadronic physics effects. Improved methods to calculate such quantities, and understand the uncertainties in them, are needed to further sharpen tests of the Standard Model. Because of the uncertainties in these quantities, any given "Standard Model allowed range," for example for (ρ, η), cannot be interpreted as a statistically-based error range.

The phases in decay amplitudes which arise because of the phase in the CKM matrix, are called weak phases; the phases which arise from final state rescattering effects are referred to as strong phases. When one compares the amplitude for decay to a CP eigenstate to that for the related CP-conjugate process, the weak phase ϕ_i of each contribution changes sign, while the strong phase δ_i is unchanged:

$$\mathcal{A} = \Sigma_i \mathcal{A}_i e^{i(\delta_i+\phi_i)} \;,\; \overline{\mathcal{A}} = \Sigma_i \mathcal{A}_i e^{i(\delta_i-\phi_i)} \;. \tag{4}$$

Direct CP violation is a difference in the direct decay rate between $B \rightarrow f$ and $\overline{B} \rightarrow \overline{f}$ without any contribution from mixing effects. This requires $|\mathcal{A}| \neq |\overline{\mathcal{A}}|$, which occurs only if there is more than one term in the sum Eq. (4), and then only if the two terms have both different weak phases and different strong phases. A nonzero result for $\mathrm{Re}(\epsilon'/\epsilon)$ in K decay is a direct CP-violation effect. Direct CP violation can occur both in charged channels and in neutral channels in B decays [4].

In the Standard Model direct CP violation occurs because there are two major classes of diagrams that contribute to weak decays, tree diagrams, and penguin diagrams, examples of which are shown in Fig. 1. Tree diagrams are those in which the W does not reconnect to the quark line from which it was emitted. Penguin diagrams are loop diagrams in which the W is re absorbed on the same quark line, producing a net change of flavor, and a gluon (for a strong penguin) or a photon or Z (for an electroweak penguin) is emitted from the loop. There may be several different tree diagrams for a given process, namely W emission and decay, W decay, W exchange between the initial valence quarks, and/or valence quark-antiquark annihilation to produce the W. However all such contributions which enter a given transition do so with the same CKM (weak) phase. Direct CP violation occurs because of interference between tree diagrams and those penguin diagrams which have different weak phases than the trees. In channels where there are no tree contributions, direct CP violation can arise because of interference between different penguin contributions.

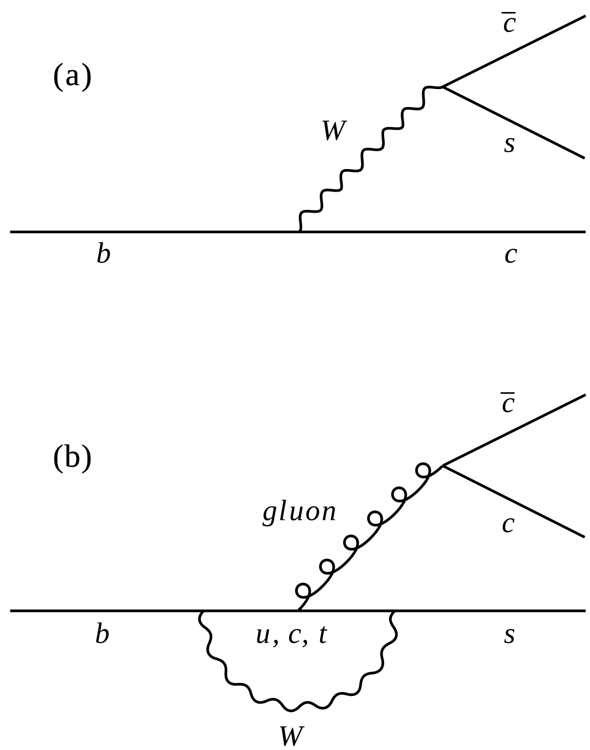

Figure 1: Quark level processes for $b \to c\bar{c}s$: (a) Tree diagram; (b) Penguin diagram. In the case of electroweak penguin contributions, the *gluon* is replaced by a Z or a γ.

To calculate the size of expected CP-violation effects one begins from the relevant quark decay diagrams. We divide the amplitudes into two factors: a CKM factor given by the CKM-matrix elements that enter at each W vertex, and a Feynman amplitude from evaluating the remainder of the diagram. The Feynman amplitude of the penguin diagram is suppressed relative to tree diagrams by a factor of order $\alpha_s(m_b)/4\pi$. Firm predictions based on this argument for the strength of the CP-violating effects in particular exclusive charged B-decay channels are not possible because the relationship between the free-quark decay diagrams and the exclusive meson-decay amplitudes depends on operator matrix elements and thus estimates are model dependent. Furthermore one cannot reliably predict the strong phases that contribute to the asymmetry.

There is one interesting exception to this last statement that gives a possible way to find large direct CP-violation effects with known strong phase differences. This is any situation where two or more resonance channels contribute to the same final state set of particles in overlapping kinematic regions. The dominant contributions to the strong phases are then the resonant decay phases, which are known from measurements that determine the resonance mass and width. These give a known strong phase contribution which varies with the kinematics of the final particles and overlays the fixed strong phase of the resonance-production process. If two such resonant channels interfere, then there is a large and kinematically-varying known contribution to the strong phase difference between the contributions of the two channels. Examples include the interference of the different ρ-π charge combinations in the three pion final states [6] or interference between different $K^*\pi$ combinations in $K\pi\pi$ states. Detailed exploration of possible applications of these ideas can be found in Ref. 7.

A second type of CP violation, referred to as indirect CP violation, or CP violation in the mixing, would arise from any difference in the widths $\Delta\Gamma$ of the two mass eigenstates, or more precisely from complex mixing effects $\mathrm{Arg}(\Gamma_{12}M_{12}^*) \neq 0$, that would give $|q/p| \neq 1$ and also give a nonvanishing lifetime difference for the two B mass eigenstates [8]. Indirect CP violation in the K system is responsible for Re $\epsilon \neq 0$, which give CP-violating asymmetries in leptonic decay rates. Such effects are expected to be tiny in the B_d system, where both $|q/p| - 1$ and the difference of lifetimes $\Delta\Gamma/\Gamma$ are expected to be of order 10^{-2} [8]. For B_s a difference in the widths is possible, due to the fact that a number of the simplest two-body channels contribute only to a single CP. The difference in widths could be as much as 20% of the total width in the B_s system [9]. However the quantity $|q/p| - 1$ is expected to be even smaller in the B_s system than in the B_d system. An indirect CP-violating asymmetry would be seen as an charge asymmetry in the same-sign dilepton events produced via mixing from an incoherent state that initially contains a $B^0\overline{B}^0$ pair. This asymmetry vanishes with $\Delta\Gamma$; it is expected to be no larger than 1% in B_d decays [10].

There are additional CP-violating effects in neutral B decays which arise from interference between the two paths to a given final state f

$$B \to f \text{ or } B \to \overline{B} \to f \tag{5}$$

This effect, an interference between decay with and without mixing, is seen also in K decays where it contributes to the parameter Im ϵ. This interference can produce rate differences between B decay to a CP-eigenstate and the CP-conjugate $\overline{B}$ decay. Such asymmetries can be directly related to the CKM phases, provided there is no direct CP violation in addition to this effect. In channels where there is also direct CP violation, the relationship between the measured asymmetry and the CKM parameters is more complicated.

A simple way to distinguish the three types of CP violation is to note that direct CP violation occurs when $|\overline{\mathcal{A}}/\mathcal{A}| \neq 1$ while indirect CP violation requires $|q/p| \neq 1$ (see the review on B^0–$\overline{B}^0$ Mixing). CP violation due to the interference between direct decay and decay after mixing can occur when both quantities have unit absolute value; it requires only that their product have a nonzero weak phase [11].

Neutral B decays to CP eigenstates: The decays of neutral B mesons into CP eigenstates are of particular interest because many of these decays allow clean theoretical interpretation in terms of the parameters of the Standard Model [12]. We denote such a state by f_{CP}, for example $f_{CP} = J/\psi(1S)K_S$ or $f_{CP} = \pi\pi$, and define the amplitudes

$$\mathcal{A}_{f_{CP}} \equiv \left\langle f_{CP}|B^0\right\rangle , \quad \overline{\mathcal{A}}_{f_{CP}} \equiv \left\langle f_{CP}|\overline{B}^0\right\rangle \ . \tag{6}$$

For convenience let us introduce the quantity $\lambda_{f_{CP}}$

$$\lambda_{f_{CP}} \equiv \frac{q}{p}\,\frac{\overline{\mathcal{A}_{f_{CP}}}}{\mathcal{A}_{f_{CP}}} \ . \tag{7}$$

In the limit of no CP violation, $\lambda_{f_{CP}} = \pm 1$, where the sign is given by the CP eigenvalue of the particular state f_{CP}.

When the small difference in width of the two B_d states is ignored we can write

$$(q/p)_{B_d} = \frac{(V_{tb}^* V_{td})}{(V_{tb} V_{td}^*)} = e^{-2i\phi_M} \quad , \tag{8}$$

where $2\phi_M$ denotes the CKM phase of the B–$\overline{B}$ mixing diagram (see the review on B^0–$\overline{B}^0$ Mixing). The time-dependent decay width for an initial $B^0(\overline{B}^0)$ state to decay to a state f is then given by

$$\begin{aligned}
&\Gamma(B^0_{\rm phys}(t) \to f_{CP}) = \\
&\quad |\mathcal{A}_{f_{CP}}|^2 e^{-\Gamma t} \Big[\frac{1 + |\lambda_{f_{CP}}|^2}{2} + \frac{1 - |\lambda_{f_{CP}}|^2}{2} \\
&\qquad \times \cos(\Delta M t) - {\rm Im}\, \lambda_{f_{CP}} \sin(\Delta M t) \Big], \\
&\Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP}) = \\
&\quad |\mathcal{A}_{f_{CP}}|^2 e^{-\Gamma t} \Big[\frac{1 + |\lambda_{f_{CP}}|^2}{2} - \frac{1 - |\lambda_{f_{CP}}|^2}{2} \\
&\qquad \times \cos(\Delta M t) + {\rm Im}\, \lambda_{f_{CP}} \sin(\Delta M t) \Big] \quad .
\end{aligned} \tag{9}$$

The time-dependent CP asymmetry is thus

$$\begin{aligned}
a_{f_{CP}}(t) &\equiv \frac{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) - \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})}{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) + \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})} \\
&= \frac{(1 - |\lambda_{f_{CP}}|^2) \cos(\Delta M t) - 2{\rm Im}\, (\lambda_{f_{CP}}) \sin(\Delta M t)}{1 + |\lambda_{f_{CP}}|^2} \quad .
\end{aligned} \tag{10}$$

Further, when there is no direct CP violation in a channel, that is when all amplitudes that contribute have the same CKM decay-phase, ϕ_D, then $|\mathcal{A}_{f_{CP}}/\overline{\mathcal{A}_{f_{CP}}}| = 1$. In that case $\lambda_{f_{CP}}$ depends on CKM-matrix parameters only, without hadronic uncertainties, and can be written $\lambda_{f_{CP}} = \pm e^{-2i(\phi_D + \phi_M)}$. Then Eq. (10) simplifies to

$$\begin{aligned}
a_{f_{CP}}(t) &= -{\rm Im}\, (\lambda_{f_{CP}}) \sin(\Delta M t) \\
&= \pm \sin(2(\phi_M + \phi_D)) \sin(\Delta M t) \quad .
\end{aligned} \tag{11}$$

where the overall sign is given by the CP eigenvalue, ± 1, of the final state f_{CP}. The mixing phase ϕ_M and the decay phase ϕ_D are each convention dependent, that is their value can be changed by redefining the phases of some of the quark fields. However Im $\lambda_{f_{CP}}$ depends on convention-independent combinations of CKM parameters only. From Eq. (11) one can directly relate the measured CP-violating asymmetry to the phase of particular combination of CKM-matrix elements in the Standard Model.

Extracting CKM parameters from measured asymmetries: In order make this relationship one looks at the CKM elements that appear in the relevant decay amplitudes and in the mixing diagrams. If the final state of the decay includes a K_S, an additional contribution from the K-mixing phase must be included in relating the measured asymmetry to the CKM parameters.

Whenever a penguin amplitude can contribute there are three separate diagrams, corresponding to the three flavors of up-type quarks in the loop. Each of these has a different CKM coefficient. We use the Unitarity condition Eq. (1) to express one coefficient as minus the sum of the other two. This regroups the three terms as a sum of two terms each of which involves a difference of two penguin diagrams (and thus is an ultra-violet finite quantity). As we will see below, the most convenient regrouping is different for $b \rightarrow q\overline{q}s$ decays and for $b \rightarrow q\overline{q}d$ decays.

When there is a tree diagram one of the two penguin terms will have the same CKM coefficient (and hence the same weak phase) as the tree diagram. Terms with the same weak phase can always be treated as a single contribution, from the perspective of looking for CP violations, although one must be sure to include all the relevant operators when estimating the expected size of such a term. In what follows we use the term "tree-dominated contribution" to describe a tree contribution plus any penguin contribution with the same weak phase. We label the second penguin term, which has a different CKM coefficient from the tree diagram as a "pure penguin contribution." Where no tree diagrams contribute there are two pure penguin terms. With this convention there are at most two terms with different weak decay phases that contribute for any decay in the Standard Model. It is instructive to note that any beyond-Standard-Model contribution, whatever its weak phase, can always be written as a sum of two terms with the weak phases of the two Standard Model terms, thus it is the pattern of relative strengths, and isospin structure, of the two terms that is peculiar to the Standard Model. (Care should be taken when comparing the terms defined by this grouping with statements in the literature about the sizes of terms made using definitions that do not include this regrouping.)

Table 1 gives the CKM factors for the various $b \to q\overline{q}'s$-quark decay channels. Here we choose to group penguin terms by eliminating the coefficient $V_{ts}V_{tb}^*$. Note that the two penguin terms in this arrangement are each the difference between a top quark contribution and a lighter (c or u) quark contribution, so they differ only by the mass dependent factors in this second contribution and by their overall sign and the CKM factors. One is suppressed by the CKM factor $\lambda^2(\rho - i\eta)$ compared to the other.

The columns labeled "Sample B_d Modes" and "Sample B_s Modes" list some of the simplest CP-study modes for each case. (These are either CP eigenstates, or modes from which CP-eigenstate contributions can be isolated, for example by angular analysis.) The columns labeled"Angle" show the angle of the unitarity triangle measured by $\phi_M + \phi_D$ where ϕ_M is the weak phase due to mixing, and ϕ_D that of the dominant decay amplitude (only the sum of these quantities is convention independent). Any Cabibbo-suppressed pure-penguin terms gives a negligible correction to this result. For the decay $b \to s\overline{s}s$ there is no tree contribution so the angle given is that due to the dominant penguin term, ignoring the Cabibbo-suppressed penguin term.

The quark decays to $u\overline{u}s$ and $d\overline{d}s$ contribute to the same set of final state hadrons and so must be combined. Here the tree diagram contributes to the Cabibbo-suppressed amplitude, so that the net result is that the two terms are expected to give comparable contributions with different CKM phases. For these decays, as with other direct CP-violating processes, there is no simple relationship between the measured asymmetry and a CKM phase, and thus no entry in the “Angle” columns in Table 1.

Table 1: $B \to q\overline{q}s$ decay modes

Quark process	Leading term	Secondary term	Sample B_d modes	B_d angle	Sample B_s modes	B_s angle
$b \to c\overline{c}s$	$V_{cb}V_{cs}^* = A\lambda^2$ tree + penguin$(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only$(u-t)$	$J/\psi\ K_S$	β	$\psi\eta$ $D_s\overline{D}_s$	0
$b \to s\overline{s}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only$(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only$(u-t)$	$\phi\, K_S$	β	$\phi\eta'$	0
$b \to u\overline{u}s$ $b \to d\overline{d}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only$(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ tree + penguin$(u-t)$	$\pi^0\ K_S$ $\rho\, K_S$	competing terms	$\phi\pi^0$ $K_S\overline{K}_S$	competing terms

Table 2: $B \to q\overline{q}d$ decay modes

Quark process	Leading term	Secondary term	Sample B_d modes	B_d angle	Sample B_s modes	B_s angle
$b \to c\overline{c}d$	$V_{cb}V_{cd}^* = -A\lambda^3$ tree + penguin$(c-u)$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only$(t-u)$	D^+D^-	$^*\beta$	$\psi\, K_S$	$^*\beta_s$
$b \to s\overline{s}d$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only$(t-u)$	$V_{cb}V_{cd}^* = A\lambda^3$ penguin only$(c-u)$	$\phi\pi$ $K_S\overline{K}_S$	competing terms	$\phi\, K_S$	competing terms
$b \to u\overline{u}d$ $b \to d\overline{d}d$	$V_{ub}V_{ud}^* = A\lambda^3(\rho - i\eta)$ tree + penguin$(u-c)$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only$(t-c)$	$\pi\pi$; $\pi\rho$ $\pi\, a_1$	$^*\alpha$	π^0K_S ρ^0K_S	competing terms
$b \to c\overline{u}d$	$V_{cb}V_{ud}^* = A\lambda^2$	0	$D^0\pi^0$, $D^0\rho^0$ → CP eigenstate	β	D^0K_S → CP eigenstate	0

*Leading terms only.

In addition to the neutral CP-eigenstate methods to determine the angles of the unitarity triangle listed in the tables, there are a number of other methods that involve decays that self-tag B-flavor, such as $DK^*(892)$ in either neutral [13] or charged [14] B decays. Further methods to measure γ in charged $B \to DK$ or $B \to D\pi$ have been suggested [15], which use interferences between a suppressed B decay followed by an allowed D decay and an allowed B decay followed by a suppressed D decay. However the relationship between the decay asymmetry and the angle is not as simple as Eq. (11) in this case. These methods require accurate measurements of several branching ratios, including a number that are quite small.

In Table 2 we list decays $b \to q\overline{q}'d$ decays. Here we choose to eliminate whichever of the two terms $V_{ud}V_{ub}^*$ or $V_{cd}V_{cb}^*$ is not present in the tree diagrams, so that the two penguin terms are one with the same weak phase as the tree and a second with CKM coefficient $V_{td}V_{tb}^*$ which has the opposite weak phase as the dominant mixing term in the Standard Model and hence a known value, zero, for $\phi_M + \phi_D$.

Here the competition between the tree-dominated and pure-penguin amplitudes is stronger because there is no Cabibbo suppression of the latter. The pure-penguin contributions are expected to be somewhat smaller because of the $\alpha(m_b)/\pi$ suppression factor. Table 2 lists the angle $\phi_M + \phi_D$, using ϕ_D for the tree-dominated terms as the angle measured. However the measured angle may be significantly shifted from this value if the pure-penguin terms turn out to be large. In certain cases one still may be able to extract a measurement of an angle, for example of $\sin(2\alpha)$ from the $\pi^+\pi^-$ asymmetry by measuring the rates in several isospin-related channels and using a multiparameter fit to separate a tree-only contribution [16]. The impact of electroweak penguins, which will not be removed by this analysis [17] is quite small in this channel [18]. This isospin analysis requires measuring the decay rate for channel $\pi^0\pi^0$, which will be a challenge. For the $\rho\pi$ decays the restrictions due to isospin can again be used to make a multiparameter fit to the ρ-regions of the Dalitz plot for $\pi^+\pi^-\pi^0$ distribution [6]. The interference between different ρ-charge channels is significant and may provide sufficient information to allow the separation of tree-dominated and pure-penguin effects and thus extraction of the parameter α. Isospin analyses at the very least can be used to test whether the penguin contributions are indeed small enough to be neglected in the determination of α.

In the case $b \to s\bar{s}d$ there are no tree graph contributions. The phase of the dominant penguin contribution is such that, combined with mixing effects, it gives a zero asymmetry for B_d decays and an asymmetry proportional to β for B_s decays. However, Gérard and Hou [19] have pointed out that interference with the sub-dominant penguin terms, proportional to $V_{ub}V_{ud}^*$ can give significant direct CP-violation asymmetries for such channels. Fleischer [20] has estimated that this asymmetry is possibly as large as 50%. While the sub-dominant term in this case would vanish if the masses of the up quark and the charm quark were equal, these estimates, which are based on the actual quark mass values and extreme values of operator matrix elements estimated using models, cannot be excluded. Thus, contrary to some comments in the literature, observation of CP-violating asymmetries in channels such as $B_d \to \phi\pi^0$ or $K^0\overline{K}^0$ would not necessarily require beyond-Standard-Model effects to explain them.

The entry for $b \to c\bar{u}d$ where the D^0 decays to a CP eigenstate ignores the small effect of doubly-Cabibbo-suppressed D-decays [21]. In contrast, the last entry indicates that one can select modes reached only by doubly-Cabibbo-suppressed decays from $D^0\pi$ and observe their interference with unsuppressed decays to the same channel from $\overline{D}^0\pi$ states, and thereby obtain a measurement of gamma [22].

There are some decay channels which are common to the B^0 and $\overline{B}^0$ but which are not CP eigenstates. For example the channel $J/\psi(1S)K^*(892)$ where the $K^*(892) \to K_S\pi^0$, the final state is not a CP eigenstate because both even and odd relative angular momenta between the $J/\psi(1S)$ and the $K^*(892)$ are allowed. One can use angular analysis to separate the different CP final states and measure the asymmetry in each [23]. The method applies in many quasi-two-body decays, such as other vector-vector channels, or those with higher-spin particles in final states. The branching ratio to these channels may be significantly larger than the CP-eigenstate (vector-scalar or scalar-scalar) channels with the same quark content. Such angular analyses may therefore be important in achieving accurate values for the parameters α and β.

Additional ways to extract CKM parameters by relationships between rates for channels such as $\pi\pi$, πK that can be extracted using SU(3) invariance have received considerable attention in the literature [24]. While these relationships will be interesting to investigate, the uncertainties introduced by SU(3) corrections may be significant. The review by Buras [5] gives a good summary of these ideas.

References

1. A.B. Carter and A.I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. **D23**, 1567 (1981);
I.I. Bigi and A.I. Sanda Nucl. Phys. **B193**, 85 (1981) and Nucl. Phys. **B281**, 41 (1987).
2. I.I. Bigi, V.A. Khoze, N.G. Uraltsev, and A.I. Sanda, in *CP Violation*, ed. C. Jarlskog (World Scientific, Singapore, 1989), p. 175.
3. L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1984).
4. See our review on "CP Violation" by L. Wolfenstein the full *Review*.
5. A. Buras, Proceedings of "Beauty 95" meeting, Oxford, hep-ph/9509329.
6. H.R. Quinn and A.E. Snyder, Phys. Rev. **D48**, 2139 (1993).
7. D. Atwood and A. Soni, Z. Phys. **C64**, 241 (1994); Phys. Rev. Lett. **74**, 220 (1995);
G. Eilam, M. Gronau, and R. Mendel, Phys. Rev. Lett. **74**, 4984 (1995);
R. Enomoto and M. Tanabashi, hep-ph/9706340.
8. See our review on "B^0–$\overline{B}^0$ Mixing" by H. Quinn in the B^0 Listings in the full *Review*;
Y. Grossman, Y. Nir, S. Plaszczynski, and M-H. Schune, SLAC-PUB-7622, hep-ph/9709288.
9. R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pene, and J.C. Raynal, Phys. Lett. **B316**, 567 (1993), M. Beneke, G. Buchalla, I. Dunietz Phys. Rev. **D54**, 4419 (1996).
10. T. Altomari, L. Wolfenstein, and J.D. Bjorken, Phys. Rev. **D37**, 1860 (1988).

11. For a review of CP violation in B decays, Y. Nir and H.R. Quinn, Ann. Rev. Nucl. and Part. Sci. **42**, 211 (1992) and references contained therein.
12. I. Dunietz and J.L. Rosner, Phys. Rev. **D34**, 1404 (1986).
13. I. Dunietz, Phys. Lett. **B270**, 75 (1991).
14. M. Gronau and D. Wyler, Phys. Lett. **B265**, 172 (1991).
15. D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997).
16. M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
17. N.G. Deshpande and X.-G. He, Phys. Rev. Lett. **74**, 26 (1995).
18. M. Gronau, O.F. Hernandez, D. London, and J.L. Rosner hep-ph/5904327(1995).
19. J.M. Gérard and W.S. Hou, Phys. Rev. **D43**, 2909 (1991) and Phys. Lett. **B253**, 478 (1991);
H. Simma, G. Eilam, and D. Wyler, Nucl. Phys. **B352**, 367 (1991).
20. R. Fleischer, Phys. Lett. **B341**, 205 (1994).
21. Y. Grossman and M. Worah Phys. Lett. **B395**, 241 (1997).
22. D. Atwood, I. Dunietz, and A Soni Phys. Rev. Lett. **78**, 3257 (1997).
23. J.R. Dell'Aquila and C.A. Nelson, Phys. Rev. **D33**, 101 (1986);
B. Kayser *et al.*, Phys. Lett. **B237**, 3339 (1990);
I. Dunietz *et al.*, Phys. Rev. **D43**, 2193 (1991).
24. See for example M. Gronau, J.L. Rosner, and D. London, Phys. Rev. Lett. **73**, 21 (1994);
M. Gronau, O.F. Hernandez, D. London, and J.L. Rosner Phys. Rev. **D50**, 4529 (1994);
M. Gronau and J.L. Rosner, Phys. Rev. Lett. **76**, 1200 (1996).