B^0–$\overline{B}^0$ MIXING

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There are two neutral B meson systems which are like the neutral kaon system, in that two CP-conjugate states exist: the states $B^0 = \overline{b}d$, and $\overline{B}^0 = \overline{d}b$, which we will call the B_d system; and the states $B^0_s = \overline{b}s$, and $\overline{B}^0_s = \overline{s}b$, which we call the B_s system. For early work on CP violation in the B systems, chiefly the B_d system, see Ref. 1. In both these systems the mass eigenstates are not CP eigenstates, but are mixtures of the two CP-conjugate quark states. The fact that the mixing, due to box diagrams, shown in Fig. 1, produces non-CP eigenstates means that there is a CP-violating phase that enters in the amplitude for these diagrams. The two mass eigenstates can be written, for example for the B_d system,

$$
\begin{aligned}
|B_L\rangle &= p|B^0\rangle + q|\overline{B}^0\rangle \ , \\
|B_H\rangle &= p|B^0\rangle - q|\overline{B}^0\rangle \ .
\end{aligned}
\tag{1}
$$

Here H and L stand for Heavy and Light, respectively.

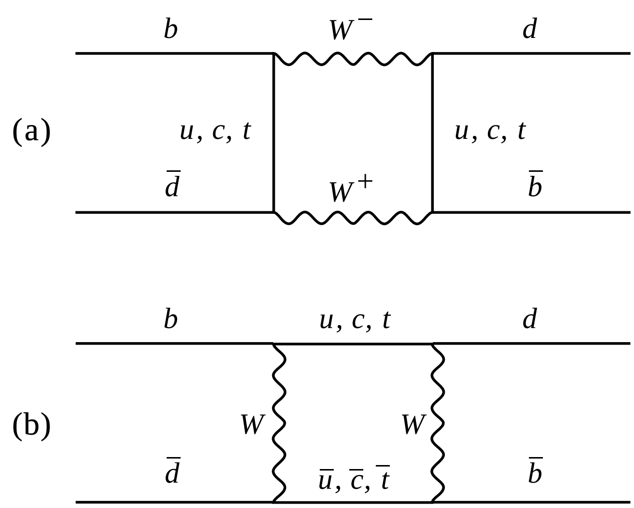

Figure 1: Mixing Diagrams.

The complex coefficients p and q obey the normalization condition

$$|q|^2 + |p|^2 = 1 \ . \tag{2}$$

We define the mass difference ΔM and width difference $\Delta\Gamma$ between the neutral B mesons:

$$\begin{aligned} \Delta M &\equiv M_H - M_L \ , \\ \Delta\Gamma &\equiv \Gamma_H - \Gamma_L \ , \end{aligned} \tag{3}$$

so that ΔM is positive by definition. Finding the eigenvalues of the mass-mixing matrix, one gets

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4(|M_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2) \tag{4}$$

and

$$\Delta M \Delta\Gamma = 4\mathrm{Re}\,(M_{12}\Gamma_{12}^*) \ , \tag{5}$$

where the off-diagonal term of the mixing matrix is written as $M_{12} + i\Gamma_{12}$. Note that both M_{12} and Γ_{12} may be complex quantities; the separation is defined by the fact that Γ_{12} is given by the absorbtive part of the diagrams (cut contributions). The ratio q/p is given by

$$\frac{q}{p} = -\frac{\Delta M - \frac{i}{2}\Delta\Gamma}{2(M_{12} - \frac{i}{2}\Gamma_{12})} = -\frac{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}{\Delta M - \frac{i}{2}\Delta\Gamma} \ . \tag{6}$$

Whereas in the kaon case the lifetimes of the two eigenstates are significantly different and the difference in masses between them is small, in the B_d system it is the mass differences that dominate the physics, and the two states have nearly equal predicted widths (and thus lifetimes). We define, for $q = d, s$

$$x_q = \frac{\Delta M_q}{\Gamma_q} \ , \qquad\qquad y_q = \frac{\Delta\Gamma_q}{\Gamma_q} \ . \tag{7}$$

The value of x_d is about 0.7, not very different from the similar quantity for the K^0 which is 0.48. The difference between the widths of the two B_d eigenstates is produced by the contributions from channels to which both B^0 and $\overline{B}^0$ can decay. These have branching ratios of $\mathcal{O}(10^{-3})$ [2]. Furthermore there are contributions of both signs to the difference, so there is no reason that the net effect should be much larger than the individual terms. Conservatively, one expects $y_d \leq 10^{-2}$and thus also $|q/p|_d$ equal to 1 to a very good approximation. Experimentally no effect of a difference in lifetimes has been observed.

For B_s there is currently only a lower bound on the value of x_s. Theoretical expectation is that it may be as large as 20 or more, which makes it quite difficult to measure. A significant difference in widths is possible, due to the fact that a number of the simplest two-body channels contribute only to a single CP (like the two-pion state which dominates K-decays and is the source of the large width difference in that system). The difference in widths could be as much as 20% of the total width in the B_s system [3]. Note that this still gives a small ratio, of order a few percent, for $\Delta\Gamma/\Delta M$.

The proper time evolution of an initially ($t = 0$) pure B^0 or $\overline{B}^0$ is given by

$$
\begin{aligned}
|B^0_{\rm phys}(t)\rangle &= g_+(t)|B^0\rangle + (q/p)g_-(t)|\overline{B}^0\rangle \ , \\
|\overline{B}^0_{\rm phys}(t)\rangle &= (p/q)g_-(t)|B^0\rangle + g_+(t)|\overline{B}^0\rangle \ . \qquad (8)
\end{aligned}
$$

where

$$
\begin{aligned}
&g_\pm = \tfrac{1}{2}\exp(-\Gamma t/2)\exp(-iMt) \\
&\times \left\{ e^{-(\Delta\Gamma/2-\Delta M)t} \pm e^{+(\Delta\Gamma/2-\Delta M)t} \right\} \ . \qquad (9)
\end{aligned}
$$

The rate at which an initial B^0_q ($\overline{B}^0_q$) decays as a $\overline{B}^0_q$ (B^0_q) is thus

$$R_q(t) = q/p \ (\text{or } p/q)\Gamma|g_-(t)|^2 \ . \qquad (10)$$

The quantity χ_q measures the total probability that a created B^0 decays as a $\overline{B}^0$; it is given by

$$\chi_q = \int_0^\infty R_q(t)dt = \tfrac{1}{2}|q/p|^2 \frac{x_q^2 - y_q^2/4}{(1+x_q^2)(1-y_q^2/4)} \ , \qquad (11)$$

Time-dependent mixing measurements are now being done for the B_d system; earlier experiments measured only the time-integrated mixing, which is parameterized by a parameter χ_d. In this case to a good approximation we can set $|q/p| = 1$ and

$|y_d| \ll x_d < 1$ so that the simpler form $\chi_d = \frac{1}{2}\frac{x_d^2}{1+x_d^2}$ applies, and a measurement of χ_d implies a value of x_d .

In the B^0–$\overline{B}^0$ mixing section of the B^0 Particle Listings, we list the χ_d measurements, most of which come from $\Upsilon(4S)$ data, and the Δm_{B^0} measurements, which come from Z data. We average these sections separately, but then include the results from both sections in "OUR EVALUATION" of χ_s and $\Delta M_{B_s^0}$. We convert both of these sets of measurements and list them in the x_d section. The x_d values obtained from Δm_{B^0} measurements have a common systematic error due to the error on τ_{B^0}. The averaging takes this common systematic error into account.

Because of the large value of x_s the quantity χ_s will be close to its upper limit of 0.5. This means that one cannot determine x_s accurately by measuring χ_s. It will require excellent time resolution to resolve the time-dependent mixing of the B_s^0 system, and thereby determine $\Delta M_{B_s^0}$.

In the B_s^0–$\overline{B}_s^0$ mixing section of the B_s^0 Particle Listings, we give measurements of χ_B, the mixing parameter for a high-energy admixture of b-hadrons

$$\chi_B = f_d \, \frac{\mathcal{B}_d}{\langle\mathcal{B}\rangle}\chi_d + f_s \, \frac{\mathcal{B}_s}{\langle\mathcal{B}\rangle}\chi_s \; . \tag{12}$$

Here f_d and f_s are the fractions of b hadrons that are produced as B^0 and B_s^0 mesons respectively, and $\mathcal{B}_d$, $\mathcal{B}_s$, and $\langle\mathcal{B}\rangle$ are branching fractions for B_d, B_s, and the b-hadron admixture respectively decaying to the observed mode. If we assume that $\chi_s = 0.5$ and $\mathcal{B}_d/\langle\mathcal{B}\rangle = \mathcal{B}_s/\langle\mathcal{B}\rangle = 1$, Eq. (12) can be used to determine f_s as discussed in the note on "Production and Decay of b-Flavored Hadrons."

References

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