13. QUARK MODEL

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13.1. Quantum numbers of the quarks

Each quark has spin 1/2 and baryon number 1/3. Table 13.1 gives the additive quantum numbers (other than baryon number) of the three generations of quarks. Our convention is that the *flavor* of a quark (I_z, S, C, B, or T) has the same sign as its *charge*. With this convention, any flavor carried by a *charged* meson has the same sign as its charge; *e.g.*, the strangeness of the K^+ is +1, the bottomness of the B^+ is +1, and the charm *and* strangeness of the D_s^- are each −1.

By convention, each quark is assigned positive parity. Then each antiquark has negative parity.

Table 13.1: Additive quantum numbers of the quarks.

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

13.2. Mesons: $q\overline{q}$ states

Nearly all known mesons are bound states of a quark q and an antiquark $\overline{q}'$ (the flavors of q and q' may be different). If the orbital angular momentum of the $q\overline{q}'$ state is L, then the parity P is $(-1)^{L+1}$. A state $q\overline{q}$ of a quark and its own antiquark is also an eigenstate of charge conjugation, with $C = (-1)^{L+S}$, where the spin S is 0 or 1. The $L = 0$ states are the pseudoscalars, $J^P = 0^-$, and the vectors, $J^P = 1^-$. Assignments for many of the known mesons are given in Table 13.2. States in the "normal" spin-parity series, $P = (-1)^J$, must, according to the above, have $S = 1$ and hence $CP = +1$. Thus mesons with normal spin-parity and $CP = -1$ are forbidden in the $q\overline{q}'$ model. The $J^{PC} = 0^{--}$ state is forbidden as well. Mesons with such J^{PC} may exist, but would lie outside the $q\overline{q}'$ model.

The nine possible $q\overline{q}'$ combinations containing u, d, and s quarks group themselves into an octet and a singlet:

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1} \tag{13.1}$$

States with the same IJ^P and additive quantum numbers can mix. (If they are eigenstates of charge conjugation, they must also have the same value of C.) Thus the $I = 0$ member of the ground-state pseudoscalar octet mixes with the corresponding pseudoscalar singlet to produce the η and η'. These appear as members of a nonet, which is shown as the middle plane in Fig. 13.1(a). Similarly, the ground-state vector nonet appears as the middle plane in Fig. 13.1(b).

A fourth quark such as charm can be included in this scheme by extending the symmetry to SU(4), as shown in Fig. 13.1. Bottom extends the symmetry to SU(5); to draw the multiplets would require four dimensions.

For the pseudoscalar mesons, the Gell-Mann-Okubo formula is

$$m_\eta^2 = \frac{1}{3}(4m_K^2 - m_\pi^2) \ , \tag{13.2}$$

assuming no octet-singlet mixing. However, the octet η_8 and singlet η_1 mix because of SU(3) breaking. In general, the mixing angle is mass dependent and becomes complex for resonances of finite width. Neglecting this, the physical states η and η' are given in terms of a mixing angle θ_P by

$$\eta = \eta_8 \cos\theta_P - \eta_1 \sin\theta_P \tag{13.3a}$$

$$\eta' = \eta_8 \sin\theta_P + \eta_1 \cos\theta_P \ . \tag{13.3b}$$

These combinations diagonalize the mass-squared matrix

$$M^2 = \begin{pmatrix} M_{11}^2 & M_{18}^2 \\ M_{18}^2 & M_{88}^2 \end{pmatrix} \ , \tag{13.4}$$

where $M_{88}^2 = \frac{1}{3}(4m_K^2 - m_\pi^2)$. It follows that

$$\tan^2\theta_P = \frac{M_{88}^2 - m_\eta^2}{m_{\eta'}^2 - M_{88}^2} \ . \tag{13.5}$$

The sign of θ_P is meaningful in the quark model. If

$$\eta_1 = (u\overline{u} + d\overline{d} + s\overline{s})/\sqrt{3} \tag{13.6a}$$

$$\eta_8 = (u\overline{u} + d\overline{d} - 2s\overline{s})/\sqrt{6} \ , \tag{13.6b}$$

then the matrix element M_{18}^2, which is due mostly to the strange quark mass, is negative. From the relation

$$\tan\theta_P = \frac{M_{88}^2 - m_\eta^2}{M_{18}^2} \ , \tag{13.7}$$

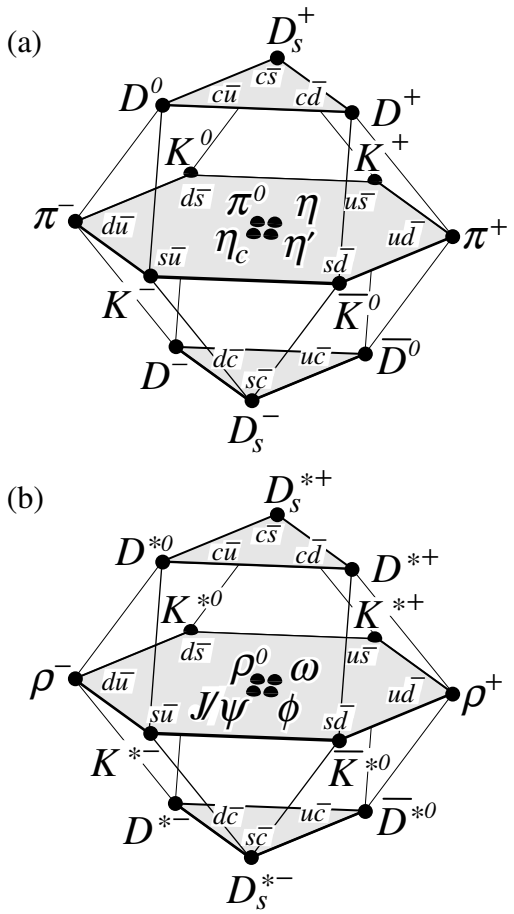

Figure 13.1: SU(4) 16-plets for the (a) pseudoscalar and (b) vector mesons made of u, d, s, and c quarks. The nonets of light mesons occupy the central planes, to which the $c\overline{c}$ states have been added. The neutral mesons at the centers of these planes are mixtures of $u\overline{u}$, $d\overline{d}$, $s\overline{s}$, and $c\overline{c}$ states.

we find that $\theta_P < 0$. However, caution is suggested in the use of the η-η' mixing-angle formulas, as they are extremely sensitive to SU(3) If we allow $M_{88}^2 = \frac{1}{3}(4m_K^2 - m_\pi^2)(1+\Delta)$, the mixing angle is determined by

$$\tan^2\theta_P = 0.0319(1+17\Delta) \tag{13.8}$$

$$\theta_P = -10.1^\circ(1+8.5\Delta) \tag{13.9}$$

to first order in Δ. A small breaking of the Gell-Mann-Okubo relation can produce a major modification of θ_P.

For the vector mesons, $\pi \to \rho$, $K \to K^*$, $\eta \to \phi$, and $\eta' \to \omega$, so that

$$\phi = \omega_8 \cos\theta_V - \omega_1 \sin\theta_V \tag{13.10}$$

$$\omega = \omega_8 \sin\theta_V + \omega_1 \cos\theta_V \ . \tag{13.11}$$

Table 13.2: Suggested $q\overline{q}$ quark-model assignments for most of the known mesons. Some assignments, especi multiplet and for some of the higher multiplets, are controversial. Mesons in bold face are included in the M Table. Of the light mesons in the Summary Table, the $f_1(1420)$, $f_0(1500)$, $f_J(1710)$, $f_2(2300)$, $f_2(2340)$, and t the $\eta(1440)$ entry are not in this table. Within the $q\overline{q}$ model, it is especially hard to find a place for the firs f mesons and for one of the $\eta(1440)$ peaks. See the "Note on Non-$q\overline{q}$ Mesons" at the end of the Meson Listing

$N\,^{2S+1}L_J$	J^{PC}	$u\overline{d}$, $u\overline{u}$, $d\overline{d}$ $I=1$	$u\overline{u}$, $d\overline{d}$, $s\overline{s}$ $I=0$	$c\overline{c}$ $I=0$	$b\overline{b}$ $I=0$	$\overline{s}u$, $\overline{s}d$ $I=1/2$	$c\overline{u}$, $c\overline{d}$ $I=1/2$	$c\overline{s}$ $I=0$
$1\,^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{\eta}$, $\boldsymbol{\eta'}$	$\boldsymbol{\eta_c}$		$\boldsymbol{K}$	$\boldsymbol{D}$	$\boldsymbol{D_s}$
$1\,^3S_1$	1^{--}	$\boldsymbol{\rho}$	$\boldsymbol{\omega}$, $\boldsymbol{\phi}$	$\boldsymbol{J/\psi(1S)}$	$\boldsymbol{\Upsilon(1S)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{D^*(2010)}$	$\boldsymbol{D_s^*(2110)}$
$1\,^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{h_1(1170)}$, $h_1(1380)$	$h_c(1P)$		$\boldsymbol{K_{1B}}^\dagger$	$\boldsymbol{D_1(2420)}$	$\boldsymbol{D_{s1}(2536)}$
$1\,^3P_0$	0^{++}	*	*	$\boldsymbol{\chi_{c0}(1P)}$	$\boldsymbol{\chi_{b0}(1P)}$	$\boldsymbol{K_0^*(1430)}$		
$1\,^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{f_1(1285)}$, $\boldsymbol{f_1(1510)}$	$\boldsymbol{\chi_{c1}(1P)}$	$\boldsymbol{\chi_{b1}(1P)}$	$\boldsymbol{K_{1A}}^\dagger$		
$1\,^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{f_2(1270)}$, $\boldsymbol{f_2'(1525)}$	$\boldsymbol{\chi_{c2}(1P)}$	$\boldsymbol{\chi_{b2}(1P)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{D_2^*(2460)}$	
$1\,^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$				$\boldsymbol{K_2(1770)}$		
$1\,^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{\omega(1600)}$	$\boldsymbol{\psi(3770)}$		$\boldsymbol{K^*(1680)}^\ddagger$		
$1\,^3D_2$	2^{--}					$\boldsymbol{K_2(1820)}$		
$1\,^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{\omega_3(1670)}$, $\boldsymbol{\phi_3(1850)}$			$\boldsymbol{K_3^*(1780)}$		
$1\,^3F_4$	4^{++}	$a_4(2040)$	$\boldsymbol{f_4(2050)}$, $f_4(2220)$			$\boldsymbol{K_4^*(2045)}$		
$2\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1300)}$	$\boldsymbol{\eta(1295)}$	$\boldsymbol{\eta_c(2S)}$		$K(1460)$		
$2\,^3S_1$	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{\omega(1420)}$, $\boldsymbol{\phi(1680)}$	$\boldsymbol{\psi(2S)}$	$\boldsymbol{\Upsilon(2S)}$	$\boldsymbol{K^*(1410)}^\ddagger$		
$2\,^3P_2$	2^{++}		$f_2(1810)$, $\boldsymbol{f_2(2010)}$		$\boldsymbol{\chi_{b2}(2P)}$	$K_2^*(1980)$		
$3\,^1S_0$	0^{-+}	$\pi(1770)$	$\eta(1760)$			$K(1830)$		

* See our scalar minireview in the Particle Listings. The candidates for the $I=1$ states are $a_0(980)$ and $a_0(145$ while for $I=0$ they are: $f_0(400$–$1200)$, $f_0(980)$, and $f_0(1370)$. The light scalars are problematic, since there n two poles for one $q\overline{q}$ state and $a_0(980)$, $f_0(980)$ may be $K\overline{K}$ bound states.

† The K_{1A} and K_{1B} are nearly equal (45°) mixes of the $K_1(1270)$ and $K_1(1400)$.

‡The $K^*(1410)$ could be replaced by the $K^*(1680)$ as the $2\,^3S_1$ state.

For "ideal" mixing, $\phi = s\overline{s}$, so $\tan\theta_V = 1/\sqrt{2}$ and $\theta_V = 35.3°$. Experimentally, θ_V is near 35°, the sign being determined by a formula like that for $\tan\theta_P$. Following this procedure we find the mixing angles given in Table 13.3.

Table 13.3: Singlet-octet mixing angles for several nonets, neglecting possible mass dependence and imaginary parts. The sign conventions are given in the text. The values of $\theta_{\rm quad}$ are obtained from the equations in the text, while those for $\theta_{\rm lin}$ are obtained by replacing m^2 by m throughout. Of the two isosinglets in a nonet, the mostly octet one is listed first.

J^{PC}	Nonet members	$\theta_{\rm quad}$	$\theta_{\rm lin}$
0^{-+}	π, K, η, η'	$-10°$	$-23°$
1^{--}	ρ, $K^*(892)$, ϕ, ω	$39°$	$36°$
2^{++}	$a_2(1320)$, $K_2^*(1430)$, $f_2'(1525)$, $f_2(1270)$	$28°$	$26°$
3^{--}	$\rho_3(1690)$, $K_3^*(1780)$, $\phi_3(1850)$, $\omega_3(1670)$	$29°$	$28°$

In the quark model, the coupling of neutral mesons to two photons is proportional to $\sum_i Q_i^2$, where Q_i is the charge of the i-th quark. This provides an alternative characterization of mixing. For example, defining

$$\text{Amp}\,[P \to \gamma(k_1)\,\gamma(k_2)] = M\epsilon^{\mu\nu\alpha\beta}\,\epsilon^*_{1\mu}\,k_{1\nu}\,\epsilon^*_{2\alpha}\,k_{2\beta}\ , \qquad (13.12)$$

where $\epsilon_{i\lambda}$ is the λ component of the polarization vector of the i^{th} photon, one finds

$$\begin{aligned}\frac{M(\eta\to\gamma\gamma)}{M(\pi^0\to\gamma\gamma)} &= \frac{1}{\sqrt{3}}(\cos\theta_P - 2\sqrt{2}\sin\theta_P)\\ &= \frac{1.73\pm0.18}{\sqrt{3}} \qquad (13.13a)\\ \frac{M(\eta'\to\gamma\gamma)}{M(\pi^0\to\gamma\gamma)} &= 2\sqrt{2/3}\left(\cos\theta_P + \frac{\sin\theta_P}{2\sqrt{2}}\right)\\ &= 2\sqrt{2/3}\,(0.78\pm0.04)\ , \qquad (13.13b)\end{aligned}$$

where the numbers with errors are experimental. These data favor $\theta_P \approx -20°$, which is compatible with the quadratic mass mixing formula with about 12% SU(3) breaking in M_{88}^2.

13.3. Baryons: *qqq* states

All the established baryons are apparently 3-quark (qqq) states, and each such state is an SU(3) color singlet, a completely antisymmetric state of the three possible colors. Since the quarks are fermions, the state function for any baryon must be antisymmetric under interchange of any two equal-mass quarks (up and down quarks in the limit of isospin symmetry). Thus the state function may be written as

$$|\, qqq \,\rangle_A = |\,\text{color}\,\rangle_A \times |\,\text{space, spin, flavor}\,\rangle_S \ , \tag{13.14}$$

where the subscripts S and A indicate symmetry or antisymmetry under interchange of any two of the equal-mass quarks. Note the contrast with the state function for the three nucleons in ^{3}H or ^{3}He:

$$|\, NNN \,\rangle_A = |\,\text{space, spin, isospin}\rangle_A \ . \tag{13.15}$$

This difference has major implications for internal structure, magnetic moments, *etc.* (For a nice discussion, see Ref. 1.)

The "ordinary" baryons are made up of u, d, and s quarks. The three flavors imply an approximate flavor SU(3), which requires that baryons made of these quarks belong to the multiplets on the right side of

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_M \oplus \mathbf{1}_A \tag{13.16}$$

(see Sec. 33, on "SU(n) Multiplets and Young Diagrams"). Here the subscripts indicate symmetric, mixed-symmetry, or antisymmetric states under interchange of any two quarks. The **1** is a uds state (Λ_1) and the octet contains a similar state (Λ_8). If these have the same spin and parity they can mix. An example is the mainly octet D_{03} $\Lambda(1690)$ and mainly singlet D_{03} $\Lambda(1520)$. In the ground state multiplet, the SU(3) flavor singlet Λ is forbidden by Fermi statistics. The mixing formalism is the same as for η-η' or ϕ-ω (see above), except that for baryons the mass M instead of M^2 is used. Section 32, on "SU(3) Isoscalar Factors and Representation Matrices", shows how relative decay rates in, say, $\mathbf{10} \to \mathbf{8} \otimes \mathbf{8}$ decays may be calculated. A summary of results of fits to the observed baryon masses and decay rates for the best-known SU(3) multiplets is given in Appendix II of our 1982 edition [2].

The addition of the c quark to the light quarks extends the flavor symmetry to SU(4). Figures 13.2(a) and 13.2(b) show the (badly broken) SU(4) baryon multiplets that have as their "ground floors" the SU(3) octet that contains the nucleons and the SU(3) decuplet that contains the $\Delta(1232)$. All the particles in a given SU(4) multiplet have the same spin and parity. The only charmed baryons that have

been discovered each contain one charmed quark. These belong to the first floor of the multiplet shown in Fig. 13.2(a); for details, see the "Note on Charmed Baryons" in the Baryon Particle Listings. The addition of a b quark extends the flavor symmetry to SU(5); it would require four dimensions to draw the multiplets.

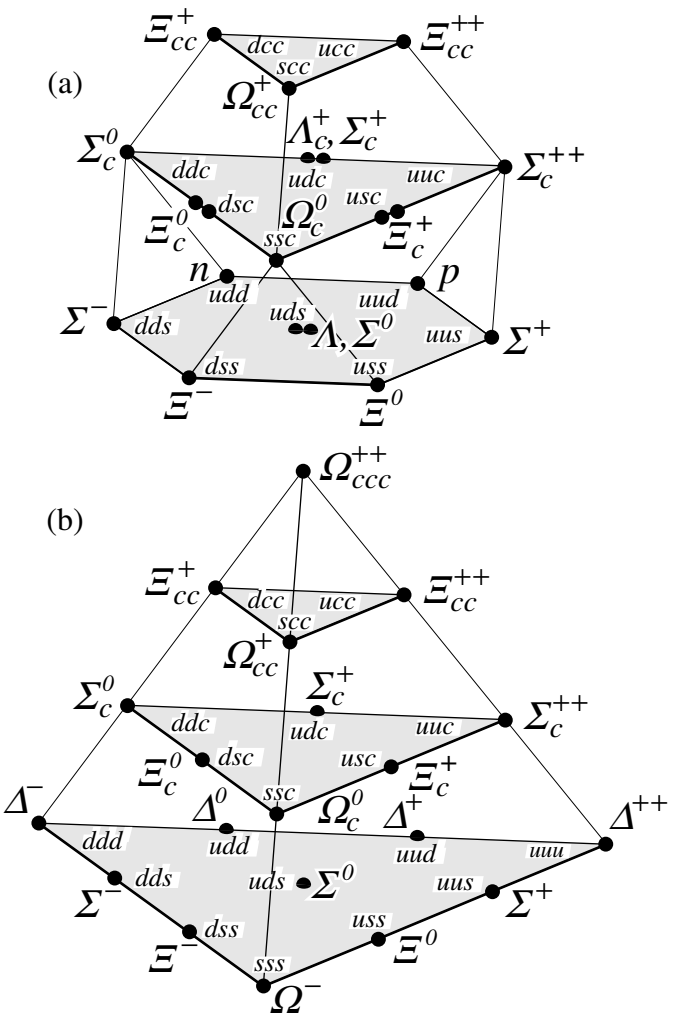

Figure 13.2: SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

For the "ordinary" baryons, flavor and spin may be combined in an approximate flavor-spin SU(6) in which the six basic states are $d\uparrow$, $d\downarrow$, $\cdots$, $s\downarrow$ ($\uparrow$, $\downarrow$ = spin up, down). Then the baryons belong to the multiplets on the right side of

$$\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A \ . \tag{13.17}$$

These SU(6) multiplets decompose into flavor SU(3) multiplets as follows:

$$\mathbf{56} = {}^4\mathbf{10} \oplus {}^2\mathbf{8} \tag{13.18a}$$

$$\mathbf{70} = {}^2\mathbf{10} \oplus {}^4\mathbf{8} \oplus {}^2\mathbf{8} \oplus {}^2\mathbf{1} \tag{13.18b}$$

$$\mathbf{20} = {}^2\mathbf{8} \oplus {}^4\mathbf{1} \ , \tag{13.18c}$$

where the superscript $(2S+1)$ gives the net spin S of the quarks for each particle in the SU(3) multiplet. The $J^P = 1/2^+$ octet containing the nucleon and the $J^P = 3/2^+$ decuplet containing the $\Delta(1232)$ together make up the "ground-state" 56-plet in which the orbital angular momenta between the quark pairs are zero (so that the spatial part of the state function is trivially symmetric). The **70** and **20** require some excitation of the spatial part of the state function in order to make the overall state function symmetric. States with nonzero orbital angular momenta are classified in SU(6)$\otimes$O(3) supermultiplets. Physical baryons with the same quantum numbers do not belong to a single supermultiplet, since SU(6) is broken by spin-dependent interactions, differences in quark masses, *etc.* Nevertheless, the SU(6)$\otimes$O(3) basis provides a suitable framework for describing baryon state functions.

It is useful to classify the baryons into bands that have the same number N of quanta of excitation. Each band consists of a number of supermultiplets, specified by (D, L_N^P), where D is the dimensionality of the SU(6) representation, L is the total quark orbital angular momentum, and P is the total parity. Supermultiplets contained in bands up to $N = 12$ are given in Ref. 3. The $N = 0$ band, which contains the nucleon and $\Delta(1232)$, consists only of the $(56,0_0^+)$ supermultiplet. The $N = 1$ band consists only of the $(70,1_1^-)$ multiplet and contains the negative-parity baryons with masses below about 1.9 GeV. The $N = 2$ band contains five supermultiplets: $(56,0_2^+)$, $(70,0_2^+)$, $(56,2_2^+)$, $(70,2_2^+)$, and $(20,1_2^+)$. Baryons belonging to the $(20,1_2^+)$ supermultiplet are not ever likely to be observed, since a coupling from the ground-state baryons requires a two-quark excitation. Selection rules are similarly responsible for the fact that many other baryon resonances have not been observed [4].

In Table 13.4, quark-model assignments are given for many of the established baryons whose SU(6)$\otimes$O(3) compositions are relatively

unmixed. We note that the unestablished resonances $\Sigma(1480)$, $\Sigma(1560)$, $\Sigma(1580)$, $\Sigma(1770)$, and $\Xi(1620)$ in our Baryon Particle Listings are too low in mass to be accommodated in most quark models [4,5].

Table 13.4: Quark-model assignments for many of the known baryons in terms of a flavor-spin SU(6) basis. Only the dominant representation is listed. Assignments for some states, especially for the $\Lambda(1810)$, $\Lambda(2350)$, $\Xi(1820)$, and $\Xi(2030)$, are merely educated guesses.

J^P	(D, L_N^P)	S	Octet members				Singlets
$1/2^+$	$(56,0_0^+)$	1/2	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0_2^+)$	1/2	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(?)$	
$1/2^-$	$(70,1_1^-)$	1/2	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$	$\Lambda(1405)$
$3/2^-$	$(70,1_1^-)$	1/2	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1_1^-)$	3/2	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$	
$3/2^-$	$(70,1_1^-)$	3/2	$N(1700)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^-$	$(70,1_1^-)$	3/2	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(?)$	
$1/2^+$	$(70,0_2^+)$	1/2	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(?)$
$3/2^+$	$(56,2_2^+)$	1/2	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2_2^+)$	1/2	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3_3^-)$	1/2	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3_3^-)$	3/2	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4_4^+)$	1/2	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	
			Decuplet members				
$3/2^+$	$(56,0_0^+)$	3/2	$\Delta(1232)$	$\Sigma(1385)$	$\Xi(1530)$	$\Omega(1672)$	
$1/2^-$	$(70,1_1^-)$	1/2	$\Delta(1620)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$3/2^-$	$(70,1_1^-)$	1/2	$\Delta(1700)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$5/2^+$	$(56,2_2^+)$	3/2	$\Delta(1905)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$7/2^+$	$(56,2_2^+)$	3/2	$\Delta(1950)$	$\Sigma(2030)$	$\Xi(?)$	$\Omega(?)$	
$11/2^+$	$(56,4_4^+)$	3/2	$\Delta(2420)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	

The quark model for baryons is extensively reviewed in Ref. 6 and 7.

13.4. Dynamics

Many specific quark models exist, but most contain the same basic set of dynamical ingredients. These include:

i) A confining interaction, which is generally spin-independent.

ii) A spin-dependent interaction, modeled after the effects of gluon exchange in QCD. For example, in the S-wave states, there is a spin-spin hyperfine interaction of the form

$$H_{HF} = -\alpha_S M \sum_{i>j} (\overrightarrow{\sigma}\lambda_a)_i (\overrightarrow{\sigma}\lambda_a)_j \ , \tag{13.19}$$

where M is a constant with units of energy, λ_a $(a = 1, \cdots, 8,)$ is the set of SU(3) unitary spin matrices, defined in Sec. 32, on "SU(3) Isoscalar Factors and Representation Matrices," and the sum runs over constituent quarks or antiquarks. Spin-orbit interactions, although allowed, seem to be small.

iii) A strange quark mass somewhat larger than the up and down quark masses, in order to split the SU(3) multiplets.

iv) In the case of isoscalar mesons, an interaction for mixing $q\overline{q}$ configurations of different flavors (*e.g.*, $u\overline{u} \leftrightarrow d\overline{d} \leftrightarrow s\overline{s}$), in a manner which is generally chosen to be flavor independent.

These four ingredients provide the basic mechanisms that determine the hadron spectrum.

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