QCD

To reduce the size of this section's PostScript file, we have divided it into two PostScript files. We present the following index:

PART 1

PART 2

9. QUANTUM CHROMODYNAMICS

9.1. The QCD Lagrangian

Revised September 1997 by I. Hinchliffe (LBNL).

Quantum Chromodynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons, is one of the components of the SU(3)×SU(2)×U(1) Standard Model. A quark of specific flavor (such as a charm quark) comes in 3 colors; gluons come in eight colors; hadrons are color-singlet combinations of quarks, anti-quarks, and gluons. The Lagrangian describing the interactions of quarks and gluons is (up to gauge-fixing terms)

$$\begin{aligned} L_{\rm QCD} = &-\frac{1}{4}F^{(a)}_{\mu\nu}\; F^{(a)\mu\nu} + i\sum_q \overline{\psi}^i_q\, \gamma^\mu\, (D_\mu)_{ij}\, \psi^j_q \\ &- \sum_q m_q\, \overline{\psi}^i_q\, \psi_{qi}\ , \end{aligned} \tag{9.1}$$

$$F^{(a)}_{\mu\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu + g_s\, f_{abc}\, A^b_\mu\, A^c_\nu\ , \tag{9.2}$$

$$(D_\mu)_{ij} = \delta_{ij}\,\partial_\mu - i g_s\, \sum_a \frac{\lambda^a_{i,j}}{2} A^a_\mu\ , \tag{9.3}$$

where g_s is the QCD coupling constant, and the f_{abc} are the structure constants of the SU(3) algebra (the λ matrices and values for f_{abc} can be found in "SU(3) Isoscalar Factors and Representation Matrices," Sec. 32 of this *Review*). The $\psi^i_q(x)$ are the 4-component Dirac spinors associated with each quark field of (3) color i and flavor q, and the $A^a_\mu(x)$ are the (8) Yang-Mills (gluon) fields. A complete list of the Feynman rules which derive from this Lagrangian, together with some useful color-algebra identities, can be found in Ref. 1.

The principle of "asymptotic freedom" (see below) determines that the renormalized QCD coupling is small only at high energies, and it is only in this domain that high-precision tests—similar to those in QED—can be performed using perturbation theory. Nonetheless, there has been in recent years much progress in understanding and quantifying the predictions of QCD in the nonperturbative domain, for example, in soft hadronic processes and on the lattice [2]. This short review will concentrate on QCD at short distances (large momentum transfers), where perturbation theory is the standard tool. It will discuss the processes that are used to determine the coupling constant of QCD. Other recent reviews of the coupling constant measurements may be consulted for a different perspective [3].

9.2. The QCD coupling and renormalization scheme

The renormalization scale dependence of the effective QCD coupling $\alpha_s = g_s^2/4\pi$ is controlled by the β-function:

$$\mu\frac{\partial\alpha_s}{\partial\mu} = -\frac{\beta_0}{2\pi}\,\alpha_s^2 - \frac{\beta_1}{4\pi^2}\,\alpha_s^3 - \frac{\beta_2}{64\pi^3}\,\alpha_s^4 - \cdots , \tag{9.4a}$$

$$\beta_0 = 11 - \frac{2}{3}n_f , \tag{9.4b}$$

$$\beta_1 = 51 - \frac{19}{3}n_f , \tag{9.4c}$$

$$\beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2 ; \tag{9.4d}$$

where n_f is the number of quarks with mass less than the energy scale μ. The expression for the next term in this series (β_3) can be found in Ref. 4. In solving this differential equation for α_s, a constant of integration is introduced. This constant is the one fundamental constant of QCD that must be determined from experiment. The most sensible choice for this constant is the value of α_s at a fixed-reference scale μ_0, but it is more conventional to introduce the dimensional parameter Λ, since this provides a parametrization of the μ dependence of α_s. The definition of Λ is arbitrary. One way to define it (adopted here) is to write a solution of Eq. (9.4) as an expansion in inverse powers of $\ln(\mu^2)$:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0\,\ln(\mu^2/\Lambda^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\,\frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4\ln^2(\mu^2/\Lambda^2)} \times\left(\left(\ln\left[\ln(\mu^2/\Lambda^2)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right] . \tag{9.5a}$$

The last term in this expansion is

$$\mathcal{O}\left(\frac{\ln^2\left[\ln(\mu^2/\Lambda^2)\right]}{\ln^3(\mu^2/\Lambda^2)}\right) , \tag{9.5b}$$

and is usually neglected in the definition of Λ. We choose to include it. For a fixed value of $\alpha_s(M_Z)$, the inclusion of this term shifts the value of Λ by ~ 15 MeV. This solution illustrates the *asymptotic freedom* property: $\alpha_s \to 0$ as $\mu \to \infty$. Alternative definitions of Λ are possible. We adopt this as the standard. Values given by experiments using other definitions are adjusted as needed to meet our definition.

Consider a "typical" QCD cross section which, when calculated perturbatively, starts at $\mathcal{O}(\alpha_s)$:

$$\sigma = A_1\ \alpha_s + A_2\ \alpha_s^2 + \ \cdots\ . \tag{9.6}$$

The coefficients A_1, A_2 come from calculating the appropriate Feynman diagrams. In performing such calculations, various divergences arise, and these must be regulated in a consistent way. This requires a particular renormalization scheme (RS). The most commonly used one is the modified minimal subtraction ($\overline{\rm MS}$) scheme [5]. This involves continuing momentum integrals from 4 to 4–2ϵ dimensions, and then subtracting off the resulting $1/\epsilon$ poles and also $(\ln 4\pi - \gamma_E)$, which is another artifact of continuing the dimension. (Here γ_E is the Euler-Mascheroni constant.) To preserve the dimensionless nature of the coupling, a mass scale μ must also be introduced: $g \to \mu^\epsilon g$. The finite coefficients A_i $(i > 2)$ thus obtained depend implicitly on the renormalization convention used and explicitly on the scale μ.

The first two coefficients (β_0, β_1) in Eq. (9.4) are independent of the choice of RS's. In contrast, the coefficients of terms proportional to α_s^n for $n > 3$ *are* RS-dependent. The form given above for β_2 is in the $\overline{\rm MS}$ scheme. It has become conventional to use the $\overline{\rm MS}$ scheme for calculating QCD cross sections beyond leading order.

The fundamental theorem of RS dependence is straightforward. Physical quantities, in particular the cross section, calculated to all orders in perturbation theory, do not depend on the RS. It follows that a truncated series *does* exhibit RS dependence. In practice, QCD cross sections are known to leading order (LO), or to next-to-leading order (NLO), or in a few cases, to next-to-next-to-leading order (NNLO); and it is only the latter two cases, which have reduced RS dependence, that are useful for precision tests. At NLO the RS dependence is completely given by one condition which can be taken to be the value of the renormalization scale μ. At NNLO this is not sufficient, and μ is no longer equivalent to a choice of scheme; both must now be specified. One, therefore, has to address the question of what is the "best" choice for μ within a given scheme, usually $\overline{\rm MS}$. There is no definite answer to this question—higher-order corrections do not "fix" the scale, rather they render the theoretical predictions less sensitive to its variation.

One could imagine that choosing a scale μ characteristic of the typical energy scale (E) in the process would be most appropriate. In general, a poor choice of scale generates terms of order $\ln(E/\mu)$ in the A_i's. Various methods have been proposed including choosing: the scale for which the next-to-leading-order correction vanishes ("Fastest Apparent Convergence [6]"); the scale for which the next-to-leading-order prediction is stationary [7], (*i.e.*, the value of μ where $d\sigma/d\mu = 0$); or the scale dictated by the effective charge scheme [8] or by the BLM scheme [9]. By comparing the values of α_s that different reasonable schemes give, an estimate of theoretical errors can be obtained. It has also been suggested to replace the perturbation series by its Pade approximant [10]. Results obtained using this method have, in certain cases, a reduced scale dependence [11,12].

An important corollary is that if the higher-order corrections are naturally small, then the additional uncertainties introduced by the μ dependence are likely to be less than the experimental measurement errors. There are some processes, however, for which the choice of scheme *can* influence the extracted value of $\Lambda_{\overline{\rm MS}}$. There is no resolution to this problem other than to try to calculate even more terms in the perturbation series. It is important to note that, since the perturbation series is an asymptotic expansion, there is a limit to the precision with which any theoretical quantity can be calculated. In some processes, the highest-order perturbative terms may be comparable in size to nonperturbative corrections (sometimes called higher-twist or renormalon effects, for a discussion see [13]); an estimate of these terms and their uncertainties is required if a value of α_s is to be extracted.

In the cases where the higher-order corrections to a process are known and are large, some caution should be exercised when quoting the value of α_s. In what follows, we will attempt to indicate the size of the theoretical uncertainties on the extracted value of α_s. There are two simple ways to determine this error. First, we can estimate it by comparing the value of $\alpha_s(\mu)$ obtained by fitting data using the QCD formula to highest known order in α_s, and then comparing it with the value obtained using the next-to-highest-order formula (μ is chosen as the typical energy scale in the process). The corresponding Λ's are then obtained by evolving $\alpha_s(\mu)$ to $\mu = M_Z$ using Eq. (9.4) to the same order in α_s as the fit. Alternatively, we can vary the value of μ over a reasonable range, extracting a value of Λ for each choice of μ. This method is of its nature imprecise, since "reasonable" involves a subjective judgment. In either case, if the perturbation series is well behaved, the resulting error on $\alpha_s(M_Z)$ will be small.

In the above discussion we have ignored quark-mass effects, *i.e.*, we have assumed an idealized situation where quarks of mass greater than μ are neglected completely. In this picture, the β-function coefficients change by discrete amounts as flavor thresholds are crossed when integrating the differential equation for α_s. It follows that, for a relationship such as Eq. (9.5) to remain valid for all values of μ, Λ must also change as flavor thresholds are crossed. This leads to the concept of a different Λ for each range of μ corresponding to an effective number of massless quarks: $\Lambda \to \Lambda^{(n_f)}$. There is some arbitrariness in how this relationship is set up. As an idealized case, consider QCD with $n_f - 1$ massless quarks and one quark of mass M. Now imagine an experiment at energy scale μ; for example, this could be $e^+e^- \to$ hadrons at center-of-mass energy μ. If $\mu \gg M$, the mass M is negligible and the process is well described by QCD with n_f massless flavors and its parameter $\Lambda^{(n_f)}$ up to terms of order M^2/μ^2. Conversely if $\mu \ll M$, the heavy quark plays no role and the process is well described by QCD with $n_f - 1$ massless flavors and its parameter $\Lambda^{(n_f-1)}$ up to terms of order μ^2/M^2. If $\mu \sim M$, the effects of the quark mass are process-dependent and cannot be absorbed into the running coupling.

A mass scale μ' is chosen where the relationship between $\Lambda^{(n_f-1)}$ and $\Lambda^{(n_f)}$ will be fixed. μ' should be of order M and the relationship should not depend on it. A prescription has been given [14] which has this property. We use this procedure choosing $\mu' = M_Q$, where M_Q is the mass of the value of the running quark mass defined in the $\overline{\mathrm{MS}}$ scheme (see the note on "Quark Masses" in the Particle Listings for more details), *i.e.*, where $M_{\overline{\mathrm{MS}}}(M_Q) = M_Q$. Then [14]

$$\begin{aligned}
\beta_0^{n_f-1} \ln\Big(\frac{\Lambda^{(n_f)}}{\Lambda^{(n_f-1)}}\Big)^2 &= (\beta_0^{n_f} - \beta_0^{n_f-1})\ \ln\Big(\frac{M_Q}{\Lambda^{(n_f)}}^2\Big) \\
&+ 2\Big(\frac{\beta_1^{n_f}}{\beta_0^{n_f}} - \frac{\beta_1^{n_f-1}}{\beta_0^{n_f-1}}\Big)\ \ln\Big[\ln\Big(\frac{M_Q}{\Lambda^{(n_f)}}\Big)^2\Big] \\
&- \frac{2\beta_1^{n_f-1}}{\beta_0^{n_f-1}}\ \ln\Big(\frac{\beta_0^{n_f}}{\beta_0^{n_f-1}}\Big) \\
&+ \frac{4\dfrac{\beta_1^{n_f}}{(\beta_0^{n_f})^2}\Big(\dfrac{\beta_1^{n_f}}{\beta_0^{n_f}} - \dfrac{\beta_1^{n_f-1}}{\beta_0^{n_f-1}}\Big)\ \ln\Big[\ln\Big(\dfrac{M_Q}{\Lambda^{n_f}}\Big)^2\Big]}{\ln\Big(\dfrac{M_Q}{\Lambda^{(n_f)}}\Big)^2} \\
&+ \frac{\dfrac{1}{\beta_0^{n_f}}\Big[\Big(\dfrac{2\beta_1^{n_f}}{\beta_0^{n_f}}\Big)^2 - \Big(\dfrac{2\beta_1^{n_f-1}}{\beta_0^{n_f-1}}\Big)^2 - \dfrac{\beta_2^{n_f}}{2\beta_0^{n_f}} + \dfrac{\beta_2^{n_f-1}}{2\beta_0^{n_f-1}} - \dfrac{22}{9}\Big]}{\ln\Big(\dfrac{M_Q}{\Lambda^{(n_f)}}\Big)^2} .
\end{aligned} \tag{9.7}$$

This result is valid to order α_s^3 (or alternatively to terms of order $1/\ln^2[(M_Q/\Lambda^{(n_f)})^2]$). The order α_s^4 expression is also available [15].

An alternative matching procedure can be used [16]. This procedure requires the equality $\alpha_s(\mu)^{(n_f)} = \alpha_s(\mu)^{(n_f-1)}$ for $\mu = M_Q$. This matching is somewhat arbitrary; a different relation between $\Lambda^{(n_f)}$ and $\Lambda^{(n_f-1)}$ would result if $\mu = M_Q/2$ were used. In practice, the differences between these procedures are very small. $\Lambda^{(5)} = 200$ MeV corresponds to $\Lambda^{(4)} = 289$ MeV in the scheme of Ref. 16 and $\Lambda^{(4)} = 280$ MeV in the scheme adopted above. Note that the differences between $\Lambda^{(5)}$ and $\Lambda^{(4)}$ are numerically very significant.

Data from deep-inelastic scattering are in a range of energy where the bottom quark is not readily excited, and hence, these experiments quote $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$. Most data from PEP, PETRA, TRISTAN, LEP, and SLC quote a value of $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$ since these data are in an energy range where the bottom quark is light compared to the available energy. We have converted it to $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$ as required. A few measurements, including the lattice gauge theory values from the ψ system and from τ decay are at sufficiently low energy that $\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ is appropriate.

In order to compare the values of α_s from various experiments, they must be evolved using the renormalization group to a common scale. For convenience, this is taken to be the mass of the Z boson. This evolution uses third-order perturbation theory and can introduce additional errors particularly if extrapolation from very small scales is used. The variation in the charm and bottom quark masses ($m_b = 4.3 \pm 0.2$ and $m_c = 1.3 \pm 0.3$ are used) can also introduce errors. These result in a fixed value of $\alpha_s(2\ \text{GeV})$ giving an uncertainty in $\alpha_s(M_Z) = \pm 0.001$ if only perturbative evolution is used. There could be additional errors from nonperturbative effects that enter at low energy. All values are in the $\overline{\text{MS}}$ scheme unless otherwise noted.

9.3. QCD in deep-inelastic scattering

The original and still one of the most powerful quantitative tests of perturbative QCD is the breaking of Bjorken scaling in deep-inelastic lepton-hadron scattering. In the leading-logarithm approximation, the measured structure functions $F_i(x, Q^2)$ are related to the quark distribution functions $q_i(x, Q^2)$ according to the naive parton model, by the formulae in "Cross-section Formulae for Specific Processes," Sec. 35 of this *Review*. (In that section, q_i is denoted by the notation f_q). In describing the way in which scaling is broken in QCD, it is convenient to define nonsinglet and singlet quark distributions:

$$F^{NS} = q_i - q_j \qquad\qquad F^S = \sum_i (q_i + \overline{q}_i) \ . \tag{9.8}$$

The nonsinglet structure functions have nonzero values of flavor quantum numbers such as isospin or baryon number. The variation with Q^2 of these is described by the so-called DGLAP equations [17,18]:

$$Q^2 \, \frac{\partial F^{NS}}{\partial Q^2} = \frac{\alpha_s(|Q|)}{2\pi} P^{qq} * F^{NS} \tag{9.9a}$$

$$Q^2 \, \frac{\partial}{\partial Q^2} \begin{pmatrix} F^S \\ G \end{pmatrix} = \frac{\alpha_s(|Q|)}{2\pi} \begin{pmatrix} P^{qq} & 2n_f P^{qg} \\ P^{gq} & P^{gg} \end{pmatrix} * \begin{pmatrix} F^S \\ G \end{pmatrix} \tag{9.9b}$$

where $*$ denotes a convolution integral:

$$f * g = \int_x^1 \frac{dy}{y} \, f(y) \, g\left(\frac{x}{y}\right) \ . \tag{9.10}$$

The leading-order Altarelli-Parisi [18] splitting functions are

$$P^{qq} = \frac{4}{3} \left[\frac{1 + x^2}{(1-x)_+} \right] + 2\delta(1-x) \ , \tag{9.11a}$$

$$P^{qg} = \frac{1}{2} \left[x^2 + (1-x)^2 \right] \ , \tag{9.11b}$$

$$P^{gq} = \frac{4}{3} \left[\frac{1 + (1-x)^2}{x} \right] \ , \tag{9.11c}$$

$$\begin{aligned} P^{gg} = 6 \left[\frac{1-x}{x} + x(1-x) + \frac{x}{(1-x)_+} + \frac{11}{12} \delta(1-x) \right] \\ - \frac{n_f}{3} \delta(1-x) \ . \end{aligned} \tag{9.11d}$$

Here the gluon distribution $G(x,Q^2)$ has been introduced and $1/(1-x)_+$ means

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx\ \frac{f(x)-f(1)}{(1-x)}\ . \tag{9.12}$$

The precision of contemporary experimental data demands that higher-order corrections also be included [19]. The above results are for massless quarks. At low Q^2 values, there are also important "higher-twist" (HT) contributions of the form:

$$F_i(x,Q^2) = F_i^{(LT)}\ (x,Q^2) + \frac{F_i^{(HT)}(x,Q^2)}{Q^2} + \ \cdots\ . \tag{9.13}$$

Leading twist (LT) indicates a term whose behavior is predicted by perturbative QCD. These corrections are numerically important only for $Q^2 < \mathcal{O}(\text{few GeV}^2)$ except for x very close to 1. At very large values of x corrections proportional to $\log(1-x)$ can become important [20].

A detailed review of the current status of the experimental data can be found, for example, in Refs. [21–23], and only a brief summary will be presented here. We shall only include determinations of Λ from the recently published results; the earlier editions of this *Review* should be consulted for the earlier data. In any event, the recent results will dominate the average since their errors are smaller. Data now exist from HERA at much smaller values of x than the fixed-target data. They provide valuable information about the shape of the antiquark and gluon distribution functions at $x \sim 10^{-4}$ [24].

From Eq. (9.9), it is clear that a nonsinglet structure function offers in principle the most precise test of the theory, since the Q^2 evolution is independent of the unmeasured gluon distribution. The CCFR collaboration fit to the Gross-Llewellyn Smith sum rule [25] is known to order α_s^3 [26]

$$\begin{aligned} &\int_0^1 dx \left(F_3^{\overline{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2) \right) = \\ &\qquad 3\left[\left(1 - \frac{\alpha_s}{\pi}(1 + 3.58\frac{\alpha_s}{\pi} + 19.0(\frac{\alpha_s}{\pi})^2 \right) - \Delta HT \right]\ \ , \end{aligned} \tag{9.14}$$

where the higher-twist contribution $\Delta HT = (0.09 \pm 0.045)/Q^2$ [26,27]. Using the CCFR data [28], this gives α_s (1.76 GeV) $= 0.26 \pm 0.035$ (expt.) $\pm\ 0.03$ (theory). The error from higher-twist terms dominates the theoretical error, the higher-twist term being approximately 50% larger than the α_s^3 term. The CCFR data have been recalibrated since this result was published [29] so this result can be expected to change; it should not therefore be included in an average. An experiment at Serpukov [30] has measured the sum rule at $< Q^2 >= 1.7$ GeV^2 and obtains α_s (1.7 GeV) $= 0.35 \pm 0.03$ (expt.) or $\Lambda^{(4)}_{\overline{\rm MS}} = 359 \pm 59$(expt.) MeV. The error does not include (theoretical) errors arising from the choice of μ and the higher-twist terms. Estimating the uncertainty from the higher-twist terms as 50% of their effect gives ± 60 MeV of additional error in the extracted value of $\Lambda^{(4)}_{\overline{\rm MS}}$.

Measurements involving singlet-dominated structure functions, such as F_2, result in correlated measurements of $\Lambda^{(4)}_{\overline{\rm MS}}$ and the gluon distribution. By utilizing high-statistics data at large x (> 0.25) and large Q^2, where F_2 behaves like an nonsinglet and F_3 at smaller x, a nonsinglet fit can be performed with better statistical precision, and hence, the error on the measured value of $\Lambda^{(4)}_{\overline{\rm MS}}$ is much reduced. Recently, CCFR gives $\Lambda^{(4)}_{\overline{\rm MS}} = 337 \pm 28 \pm 13$(higher-twist) MeV [29] from $F_2(\nu N)$ and $F_3(\nu N)$. There is an additional uncertainty of ± 59 MeV from the choice of scale. The NMC collaboration [31] gives $\alpha_s(7\ \mathrm{GeV}^2) = 0.264 \pm 0.018(\mathrm{stat.}) \pm 0.070(\mathrm{syst.}) \pm 0.013(\ \mathrm{higher\text{-}twist})$. The systematic error is larger than the CCFR result, partially because the data are at smaller values of x and the gluon distribution is more important. A reanalysis [32] of EMC data [33] gives $\Lambda^{(4)}_{\overline{\rm MS}} = 211 \pm 80 \pm 80$ MeV from $F_2(\nu N)$. Finally a combined analysis [34] of SLAC [35] and BCDMS [36] data gives $\Lambda^{(4)}_{\overline{\rm MS}} = 263 \pm 42 \pm 55$ MeV. Here the systematic error is an estimate of the uncertainty due to the choice of Q^2 used in the argument of α_s, and in the scale at which the structure functions (factorization scale) used in the QCD calculation are evaluated.

The results from Refs. [29–32], [34], and [37] can be combined to give $\Lambda^{(4)}_{\overline{\rm MS}} = 305 \pm 25 \pm 50$ MeV which corresponds to $\alpha_s(M_Z) = 0.117 \pm 0.002 \pm 0.004$, Here the first error is a combination of statistical and systematic errors, and the second error is due to the scale uncertainty. This result is an average of the results weighted by their statistical and systematic errors. The scale error, which is common to all, is then reapplied to the average.

The spin-dependent structure functions, measured in polarized lepton nucleon scattering, can also be used to determine α_s. Here the values of $Q^2 \sim 2.5\ \mathrm{GeV}^2$ are small and higher-twist corrections are important. A fit [38] using the measured spin dependent structure functions themselves [39] gives $\alpha_s(M_Z) = 0.120^{+0.004}_{-0.005}(\mathrm{expt.})^{+0.009}_{-0.006}(\mathrm{theory})$. These authors also determine α_s from the Bjorken sum rule [40] and obtain $\alpha_s(M_Z) = 0.118^{+0.010}_{-0.024}$; consistent with an earlier determination [41], the larger error being due to the extrapolation into the (unmeasured) small x region. Theoretically, the sum rule is preferable as the perturbative QCD result is known to higher order and these terms are important at the low Q^2 involved. It has been shown that the theoretical errors associated with the choice of scale are considerably reduced by the use of Pade approximants [11] which results in $\alpha_s(1.7\ \mathrm{GeV}) = 0.328 \pm 0.03(\mathrm{expt.}) \pm 0.025(\mathrm{theory})$ corresponding to $\alpha_s(M_Z) = 0.116^{+0.003}_{-0.005}(\mathrm{expt.}) \pm 0.003(\mathrm{theory})$. No error is included from the extrapolation into the region of x that is unmeasured. If data were to become available at smaller values of x so that this extrapolation could be more tightly constrained, the sum rule method would provide the best determination of α_s; the more conservative result from the structure functions themselves is used in the average.

At very small values of x and Q^2, the x and Q^2 dependence of the structure functions is predicted by perturbative QCD [42]. Here terms to all orders in $\alpha_s \ln(1/x)$ are summed. The data from HERA [24] on $F_2^{ep}(x, Q^2)$ can be fitted to this form [43], including the NLO terms which are required to fix the Q^2 scale. The data are dominated by $4\ \mathrm{GeV}^2 < Q^2 < 100\ \mathrm{GeV}^2$. The fit [45] using H1 data [46] gives

$\alpha_s(M_Z) = 0.122 \pm 0.004$ (expt.) $\pm\, 0.009$ (theory). (The theoretical error is taken from Ref. 43.) The dominant part of the theoretical error is from the scale dependence; errors from terms that are suppressed by $1/\log(1/x)$ in the quark sector are included [44] while those from the gluon sector are not.

Typically, Λ is extracted from the deep inelastic scattering data by parameterizing the parton densities in a simple analytic way at some Q_0^2, evolving to higher Q^2 using the next-to-leading-order evolution equations, and fitting globally to the measured structure functions to obtain $\Lambda^{(4)}_{\overline{\rm MS}}$. Thus, an important by-product of such studies is the extraction of parton densities at a fixed-reference value of Q_0^2. These can then be evolved in Q^2 and used as input for phenomenological studies in hadron-hadron collisions (see below). To avoid having to evolve from the starting Q_0^2 value each time, a parton density is required; it is useful to have available a simple analytic approximation to the densities valid over a range of x and Q^2 values. A package is available from the CERN computer library that includes an exhaustive set of fits [47]. Most of these fits are obsolete. In using a parameterization to predict event rates, a next-to-leading order fit must be used if the process being calculated is known to next-to-leading order in QCD perturbation theory. In such a case, there is an additional scheme dependence; this scheme dependence is reflected in the $\mathcal{O}(\alpha_s)$ corrections that appear in the relations between the structure functions and the quark distribution functions. There are two common schemes: a deep-inelastic scheme where there are no order α_s corrections in the formula for $F_2(x, Q^2)$ and the minimal subtraction scheme. It is important when these next-to-leading order fits are used in other processes (see below), that the same scheme is used in the calculation of the partonic rates.

9.4. QCD in decays of the τ lepton

The semi-leptonic branching ratio of the tau ($\tau \to \nu_\tau +$ hadrons, R_τ) is an inclusive quantity. It is related to the contribution of hadrons to the imaginary part of the W self energy $\big(\Pi(s)\big)$. However, it is more inclusive than R since it involves an integral

$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}(1 - \frac{s}{m_\tau^2})^2 \;\; \mathrm{Im}\,(\Pi(s)) \;\; .$$

Since the scale involved is low, one must take into account nonperturbative (higher-twist) contributions which are suppressed by powers of the τ mass.

$$\begin{aligned} R_\tau =& 3.058\Big[1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.2\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^2 + 26.4\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^3 \\ &+ a\frac{m^2}{m_\tau^2} + b\frac{m\psi\overline{\psi}}{m_\tau^4} + c\frac{\psi\overline{\psi}\psi\overline{\psi}}{m_\tau^6} + \cdots\Big] \;. \end{aligned} \tag{9.15}$$

Here a, b, and c are dimensionless constants and m is a light quark mass. The term of order $1/m_\tau^2$ is a kinematical effect due to the light quark masses and is consequently very small. The nonperturbative terms are estimated using sum rules [48]. In total, they are

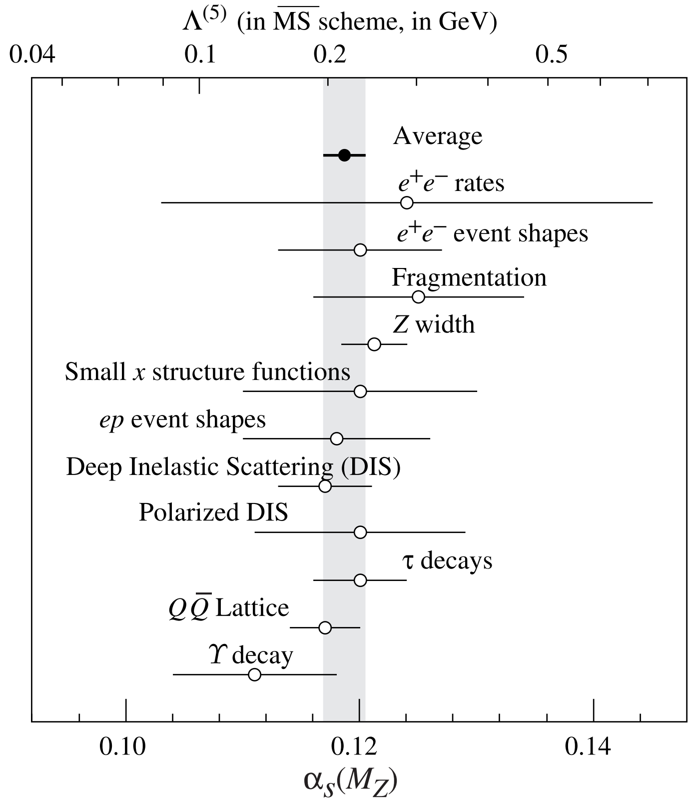

Figure 9.1: Summary of the values of $\alpha_s(M_Z)$ and $\Lambda^{(5)}$ from various processes. The values shown indicate the process and the measured value of α_s extrapolated up to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties.

estimated to be -0.014 ± 0.005 [49,50]. This estimate relies on there being no term of order Λ^2/m_τ^2 $\left(\text{note that } \frac{\alpha_s(m_\tau)}{\pi} \sim (\frac{0.5 \text{ GeV}}{m_\tau})^2\right)$. The a, b, and c can be determined from the data [51] by fitting to moments of the $\Pi(s)$. The values so extracted [52,53] are consistent with the theoretical estimates. If the nonperturbative terms are omitted from the fit, the extracted value of $\alpha_s(m_\tau)$ decreases by ~ 0.02.

For $\alpha_s(m_\tau) = 0.35$ the perturbative series for R_τ is $R_\tau \sim 3.058(1{+}0.112{+}0.064{+}0.036)$. The size (estimated error) of the nonperturbative term is 20% (7%) of the size of the order α_s^3 term. The perturbation series in not very well convergent; if the order α_s^3 term is omitted, the extracted value of $\alpha_s(m_\tau)$ increases by 0.05. The order α_s^4 term has been estimated [54] and attempts made to resum the entire series [55,56]. These estimates can be used to obtain an estimate of the errors due to these unknown terms [57,58]. We assign an uncertainty of ± 0.02 to $\alpha_s(m_\tau)$ from these sources.

R_τ can be extracted from the semi-leptonic branching ratio from the relation $R_\tau = 1/(\mathrm{B}(\tau \to e\nu\overline{\nu}) - 1.97256$; where $\mathrm{B}(\tau \to e\nu\overline{\nu})$ is measured directly or extracted from the lifetime, the muon mass, and the muon lifetime assuming universality of lepton couplings. Using the average lifetime of 290.7 ± 1.3 fs and a τ mass of 1777.00 ± 0.30 MeV from the PDG fit gives $R_\tau = 3.642 \pm 0.024$. The direct measurement of $\mathrm{B}(\tau \to e\nu\overline{\nu})$ can be combined with $\mathrm{B}(\tau \to \mu\nu\overline{\nu})$ to give $\mathrm{B}(\tau \to e\nu\overline{\nu}) = 0.1783 \pm 0.0007$ which $R_\tau = 3.636 \pm 0.021$. Averaging these yields $\alpha_s(m_\tau) = 0.350 \pm 0.008$ using the experimental error alone. We assign a theoretical error equal to 40% of the contribution from the order α^3 term and all of the nonperturbative contributions. This then gives $\alpha_s(m_\tau) = 0.35 \pm 0.03$ for the final result.

9.5. QCD in high-energy hadron collisions

There are many ways in which perturbative QCD can be tested in high-energy hadron colliders. The quantitative tests are only useful if the process in question has been calculated beyond leading order in QCD perturbation theory. The production of hadrons with large transverse momentum in hadron-hadron collisions provides a direct probe of the scattering of quarks and gluons: $qq \to qq$, $qg \to qg$, $gg \to gg$, *etc.* Recent higher–order QCD calculations of the jet rates [59] and shapes are in impressive agreement with data [60].This agreement has led to the proposal that these data could be used to provide a determination of α_s [61]. Data are also available on the angular distribution of jets; these are also in agreement with QCD expectations [62,63].

QCD corrections to Drell-Yan type cross sections (*i.e.*, the production in hadron collisions by quark-antiquark annihilation of lepton pairs of invariant mass Q from virtual photons, or of real W or Z bosons), are known [64]. These $\mathcal{O}(\alpha_s)$ QCD corrections are sizable at small values of Q. It is interesting to note that the corresponding correction to W and Z production, as measured in $p\overline{p}$ collisions at $\sqrt{s} = 0.63$ TeV and $\sqrt{s} = 1.8$ TeV, has essentially the same theoretical form and is of order 30%.

The production of W and Z bosons and photons at large transverse momentum can also be used to test QCD. The leading-order QCD subprocesses are $q\overline{q} \to \gamma g$ and $qg \to \gamma q$. If the parton distributions are taken from other processes and a value of $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$ assumed, then an absolute prediction is obtained. Conversely, the data can be used to extract information on quark and gluon distributions and on the value of $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$. The next-to-leading-order QCD corrections are known [65,66] (for photons), and for W/Z production [67], and so a precision test is possible in principle. Data exist from the CDF and DØ collaborations [68,69]. The UA2 collaboration [70] has extracted a value of $\alpha_s(M_W) = 0.123 \pm 0.018(\mathrm{stat.}) \pm 0.017(\mathrm{syst.})$ from the measured ratio $R_W = \dfrac{\sigma(W + 1\mathrm{jet})}{\sigma(W + 0\mathrm{jet})}$. The result depends on the algorithm used to define a jet, and the dominant systematic errors due to fragmentation and corrections for underlying events (the former causes jet energy to be lost, the latter causes it to be increased) are connected to the algorithm. The scale at which $\alpha_s(M)$ is to be evaluated is not clear. A change from $\mu = M_W$ to $\mu = M_W/2$ causes a shift of 0.01 in the extracted α_s. The quoted error should be increased to take this into account. There is dependence on the parton distribution functions, and hence, α_s appears explicitly in the formula for R_W, and implicitly in the distribution functions. The DØ collaboration has performed an analysis similar to UA2. They are unable to obtain a fit where the two values of α_s are consistent with one another, and do not quote a value of α_s [71]. The values from this process are no longer used in determining the overall average value of α_s.