RARE KAON DECAYS

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A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–10]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+\nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focussed on the decays $K_L \to \pi^0\ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+\ell^+\ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0\gamma\gamma$, which also scales a CP-conserving background to CP violation in $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \gamma\ell^+\ell^-$, which could possibly shed light on long distance contributions to $K_L \to \mu^+\mu^-$.

B. Explicit violations of the Standard Model: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 3.3 \times 10^{-11}(91\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of nearly 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV, along with the expected near-future progress. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to axial-vector (or pseudoscalar) couplings and the latter is sensitive to vector (or scalar) couplings.

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.	(Near-) future aim
$K^+ \to \pi^+ e\mu$	$2.1 \cdot 10^{-10}$	BNL-777	90/11	$3 \cdot 10^{-12}$ (BNL-865)
$K_L \to \mu e$	$3.3 \cdot 10^{-11}$	BNL-791	93/12	$3 \cdot 10^{-12}$ (BNL-871)
$K_L \to \pi^0 e\mu$	$3.2 \cdot 10^{-9}$	FNAL-799	94/13	$5 \cdot 10^{-11}$ (KTeV)

Another forbidden decay currently being pursued is $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, etc.). Recently the upper limit on this process has been improved to 3×10^{-10} [15]. Data already collected by BNL-787 are expected to yield a further factor in sensitivity to this process.

C. Measurements of Standard Model parameters: Until recently, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ have been motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [16] and long-distance contributions were known to be negligible [2]. However, BNL-787 has attained the sensitivity at which the observation of an event can no longer be unambiguously attributed to non-SM physics. The previous 90% CL upper limit [14] is 2.4×10^{-9}, but running with an upgraded beam and detector BNL-787 recently observed one candidate event, corresponding to a branching ratio of $(4.2^{+9.7}_{-3.5}) \times 10^{-10}$ [15]. Further data already collected are expected to increase the sensitivity by more than a factor 2, and there are plans to collect data representing a further large increase in sensitivity. This reaction is now interesting from the point of view of constraining SM parameters. The branching ratio can be written in terms of the very well-measured rate of K_{e3} as [2]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [17]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [18], and X^{ℓ}_{NL} is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which are contained in X^{ℓ}_{NL}, are relatively small and now known [10] to $\leq 10\%$. Evaluating the constants in Eq. (1) with $m_t = 175$ GeV, one can cast this result in terms of the CKM parameters A, ρ and η (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [10]

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.0 \times 10^{-10} A^4 [\eta^2 + (\rho_o - \rho)^2] \qquad (2)$$

where $\rho_o \equiv 1 + (\frac{2}{3}X^e_{NL} + \frac{1}{3}X^{\tau}_{NL})/(A^2 V^4_{us} X(m_t)) \approx 1.4$. Thus, $\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the ρ, η plane with center $(\rho_o, 0)$ and radius $\approx \dfrac{1}{A^2}\sqrt{\frac{\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})}{1.0\times 10^{-10}}}$.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter ρ. For $m_t = 175$ GeV it is given by [10]:

$$\mathrm{B_{SD}}(K_L \to \mu^+\mu^-) \approx 1.7 \times 10^{-9} A^4 (\rho'_o - \rho)^2 \qquad (3)$$

where ρ'_o depends on the charm quark mass and is around 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is calculated in terms of the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B_{abs}}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (7.2 \pm 0.5) \times 10^{-9}$ listed in the current edition. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain ρ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present, it is not possible to compute this long-distance component reliably and, therefore, it is not possible to constrain ρ from this mode. It is expected that studies of the reactions $K_L \to \ell^+\ell^-\gamma$, and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ will improve our understanding of the long distance effects in $K_L \to \mu^+\mu^-$ (the current data is parameterized in terms of α_K^*, discussed on page 24 of the K_L^0 Particle Properties Listing in our 1997 WWW update).

D. Searches for CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,19]. The Standard Model predicts a branching ratio $\sim 10^{-11} - 10^{-10}$; for $m_t = 175$ GeV it is given approximately by [10]:

$$\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \approx 4.1 \times 10^{-10} A^4\eta^2 \ . \tag{4}$$

The current published upper bound is $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \leq 5.8 \times 10^{-5}$ [20] and KTeV (FNAL799II) is expected to place a bound of order 10^{-8} [21]. The KTeV group has recently quoted a preliminary result of 1.8×10^{-6} [22]. If lepton flavor is conserved, the 90% CL bound on $K^+ \to \pi^+\nu\bar{\nu}$ provides the model independent bound $B(K_L \to \pi^0\nu\bar{\nu}) < 1.1 \times 10^{-8}$ [23]. A recent proposal, BNL-926 [24], aims to make a $\sim 15\%$ measurement of $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu})$. There is also a Fermilab EOI [25] with comparable goals.

The decay $K_L \to \pi^0 e^+e^-$ also has sensitivity to the product $A^4\eta^2$. It has a direct CP-violating component that depends on the value of the top-quark mass, and that for $m_t = 175$ GeV is given by [10]:

$$\mathrm{B}_{\mathrm{dir}}(K_L \to \pi^0 e^+e^-) \approx 6.7 \times 10^{-11} A^4\eta^2 \ . \tag{5}$$

However, like $K_L \to \mu^+\mu^-$ this mode suffers from large theoretical uncertainties due to long distance strong interaction effects. It has an indirect CP-violating component given by:

$$\mathrm{B}_{\mathrm{ind}}(K_L \to \pi^0 e^+e^-) = |\epsilon|^2 \frac{\tau_{K_L}}{\tau_{K_S}} \mathrm{B}(K_S \to \pi^0 e^+e^-) \ , \tag{6}$$

that has been estimated to be less than 10^{-12} [26], but that will not be known precisely until a measurement of $K_S \to \pi^0 e^+e^-$ is available [4,27]. There is also a CP-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0\gamma\gamma$.

An analysis of $K_L \to \pi^0\gamma\gamma$ within chiral perturbation theory has been carried out in terms of a parameter a_V [28] that determines both the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$. A fit to the distribution has given $-0.32 < a_V < 0.19$ [29]; a value that suggests that the absorptive part of the CP-conserving contribution to $K_L \to \pi^0 e^+e^-$ is significantly smaller than the direct CP-violating component [29]. However, there remains some uncertainty in the interpretation of $K_L \to \pi^0\gamma\gamma$ in terms of a_V. Analyses that go beyond chiral perturbation theory have found larger values of a_V, helping with understanding the rate in that process [30]. This would indicate a sizeable CP-conserving component to $K_L \to \pi^0 e^+e^-$. The real part of the CP-conserving contribution to $K_L \to \pi^0 e^+e^-$ is also unknown. The related process, $K_L \to \pi^0\gamma e^+e^-$, is an additional background in some region of phase space [31].

Finally, BNL-845 observed a potential background to $K_L \to \pi^0 e^+e^-$ from the decay $K_L \to \gamma\gamma e^+e^-$ [32]. This was later confirmed with an order of magnitude larger sample by FNAL-799 [33], which measured additional kinematic quantities. It has been estimated that this background will enter at the level of 10^{-11} [34], comparable to the signal level. Because of this, the observation of $K_L \to \pi^0 e^+e^-$ will depend on background subtraction with good statistics.

The current upper bound for the process $K_L \to \pi^0 e^+e^-$ is 4.3×10^{-9} [35]. For the closely related muonic process, the upper bound is $\mathrm{B}(K_L \to \pi^0\mu^+\mu^-) \leq 5.1 \times 10^{-9}$ [36]. KTeV expects to reach a sensitivity of roughly 10^{-11} for both reactions [21].

E. Other long distance dominated modes: The decays $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e$ or μ) are described by chiral perturbation theory in terms of one parameter, ω^+ [37]. This parameter determines both the rate and distribution $d\Gamma/dm_{\ell\ell}$ for these processes. A careful study of these two reactions can provide a measurement of ω^+ and a test of the chiral perturbation theory description. A simultaneous fit to the rate and spectrum of $K^+ \to \pi^+ e^+ e^-$ gives: $\omega^+ = 0.89^{+0.24}_{-0.14}$; $\mathrm{B}(K^+ \to \pi^+ e^+ e^-) = (2.99 \pm 0.22) \times 10^{-7}$ [38]. These two results satisfy the prediction of chiral perturbation theory within two standard deviations [4]. Improved statistics for this mode and a measurement of the mode $K^+ \to \pi^+ \mu^+ \mu^-$ are thus desired. BNL-787 has recently measured $B(K^+ \to \pi^+ \mu^+ \mu^-) = (5.0 \pm 1.0) \times 10^{-8}$ [39] which is at about the predicted level, but the result is not yet accurate enough to provide additional constraints.

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