11. THE CABIBBO-KOBAYASHI-MASKAWA MIXING MATRIX

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In the Standard Model with $\mathrm{SU}(2) \times \mathrm{U}(1)$ as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix relating these bases was defined for six quarks and given an explicit parametrization by Kobayashi and Maskawa [1] in 1973. It generalizes the four-quark case, where the matrix is parametrized by a single angle, the Cabibbo angle [2].

By convention, the mixing is often expressed in terms of a 3×3 unitary matrix V operating on the charge $-e/3$ quarks (d, s, and b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{11.1}$$

The values of individual matrix elements can in principle all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Using the constraints discussed below together with unitarity, and assuming only three generations, the 90% confidence limits on the magnitude of the elements of the complete matrix are:

$$\begin{pmatrix} 0.9745 \text{ to } 0.9760 & 0.217 \text{ to } 0.224 & 0.0018 \text{ to } 0.0045 \\ 0.217 \text{ to } 0.224 & 0.9737 \text{ to } 0.9753 & 0.036 \text{ to } 0.042 \\ 0.004 \text{ to } 0.013 & 0.035 \text{ to } 0.042 & 0.9991 \text{ to } 0.9994 \end{pmatrix} . \tag{11.2}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of others.

There are several parametrizations of the Cabibbo-Kobayashi-Maskawa matrix. We advocate a "standard" parametrization [3] of V that utilizes angles θ_{12}, θ_{23}, θ_{13}, and a phase, δ_{13}:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{11.3}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for the "generation" labels $i, j = 1, 2, 3$. This has distinct advantages of interpretation, for the rotation angles are defined and labeled in a way which relate to the mixing of two specific generations and if one of these angles vanishes, so does the mixing between those two generations; in the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified with the Cabibbo angle [2]. The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases.

The matrix elements in the first row and third column, which can be directly measured in decay processes, are all of a simple form, and as c_{13} is known to deviate from unity only in the sixth decimal place, $V_{ud} = c_{12}$, $V_{us} = s_{12}$, $V_{ub} = s_{13}\ e^{\delta_{13}}$, $V_{cb} = s_{23}$, and $V_{tb} = c_{23}$ to an excellent approximation. The phase δ_{13} lies in the range $0 \le \delta_{13} < 2\pi$, with non-zero values generally breaking CP invariance for the weak interactions. The generalization to the n generation case contains $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases. The range of matrix elements in Eq. (11.2) corresponds to 90% CL limits on the sines of the angles of $s_{12} = 0.217$ to 0.222, $s_{23} = 0.036$ to 0.042, and $s_{13} = 0.0018$ to 0.0044.

Kobayashi and Maskawa [1] originally chose a parametrization involving the four angles, θ_1, θ_2, θ_3, δ:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} , \tag{11.4}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1, 2, 3$. In the limit $\theta_2 = \theta_3 = 0$, this reduces to the usual Cabibbo mixing with θ_1 identified (up to a sign) with the Cabibbo angle [2]. Several different forms of the Kobayashi-Maskawa parametrization are found in the literature. Since all these parametrizations are referred to as "the" Kobayashi-Maskawa form, some care about which one is being used is needed when the quadrant in which δ lies is under discussion.

A popular approximation that emphasizes the hierarchy in the size of the angles, $s_{12} \gg s_{23} \gg s_{13}$, is due to Wolfenstein [4], where one sets $\lambda \equiv s_{12}$, the sine of the Cabibbo angle, and then writes the other elements in terms of powers of λ:

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \tag{11.5}$$

with A, ρ, and η real numbers that were intended to be of order unity. No physics can depend on which of the above parametrizations (or any other) is used as long as a single one is used consistently and care is taken to be sure that no other choice of phases is in conflict.

Our present knowledge of the matrix elements comes from the following sources:

(1) $|V_{ud}|$ – Analyses have been performed comparing nuclear beta decays that proceed through a vector current to muon decay. Radiative corrections are essential to extracting the value of the matrix element. They already include [5] effects of order $Z\alpha^2$, and most of the theoretical argument centers on the nuclear mismatch and structure-dependent radiative corrections [6,7]. New data have been obtained on superallowed $0^+ \rightarrow 0^+$ beta decays [8]. Taking the complete data set for nine decays, the values obtained in analyses by two groups are:

$$\begin{aligned} ft =& 3146.0 \pm 3.2 \quad \text{(Ref. 8)} \\ ft =& 3150.8 \pm 2.8 \quad \text{(Ref. 9)}\ . \end{aligned} \tag{11.6}$$

Averaging these results (essentially for $|V_{ud}|^{-2}$), but keeping the same error bar, we obtain $|V_{ud}| = 0.9735 \pm 0.0005$. It has been argued [10] that the change in charge-symmetry-violation for quarks inside nucleons that are in nuclear matter results in a further increase of the ft value by 0.075 to 0.2%, leading to a systematic underestimate of $|V_{ud}|$. While more work needs to be done to clarify the structure-dependent effects, for now we add linearly a further $0.1 \pm 0.1\%$ to the ft values coming from nuclear decays, obtaining a value:

$$|V_{ud}| = 0.9740 \pm 0.0010 \ . \tag{11.7}$$

(2) $|V_{us}|$ – Analysis of K_{e3} decays yields [11]

$$|V_{us}| = 0.2196 \pm 0.0023 \ . \tag{11.8}$$

With isospin violation taken into account in K^+ and K^0 decays, the extracted values of $|V_{us}|$ are in agreement at the 1% level. A reanalysis [7] obtains essentially the same value, but quotes a somewhat smaller error which is only statistical. The analysis [12] of hyperon decay data has larger theoretical uncertainties because of first order SU(3) symmetry breaking effects in the axial-vector couplings. This has been redone incorporating second order SU(3) symmetry breaking corrections in models [13] applied to the WA2 data [14] to give a value of $|V_{us}| = 0.2176 \pm 0.0026$ with the "best-fit" model, which is consistent with Eq. (11.8). Since the values obtained in the models differ outside the errors and generally do not give good fits, we retain the value in Eq. (11.8) for $|V_{us}|$.

(3) $|V_{cd}|$ – The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. The dimuon production cross sections of the CDHS group [15] yield $\overline{B}_c \, |V_{cd}|^2 = 0.41 \pm 0.07 \times 10^{-2}$, where $\overline{B}_c$ is the semileptonic branching fraction of the charmed hadrons produced. The corresponding value from a more recent Tevatron experiment [16], where a next-to-leading-order QCD analysis has been carried out, is $0.534 \pm 0.021^{+0.025}_{-0.051} \times 10^{-2}$, where the last error is from the scale uncertainty. Assuming a similar scale error for the CDHS result and averaging these two results gives $0.49 \pm 0.05 \times 10^{-2}$. Supplementing this with data [17] on the mix of charmed particle species produced by neutrinos and PDG values for their semileptonic branching fractions to give [16] $\overline{B}_c = 0.099 \pm 0.012$, then yields

$$|V_{cd}| = 0.224 \pm 0.016 \tag{11.9}$$

(4) $|V_{cs}|$ – Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange quark density in the parton-sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an $SU(3)$ symmetric sea, leads to a lower bound [15], $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of D_{e3} decay with the expression [18] that follows from the standard weak interaction amplitude:

$$\Gamma(D \to \overline{K} e^+ \nu_e) = |f_+^D(0)|^2 \ |V_{cs}|^2 \ (1.54 \times 10^{11} \ \mathrm{s}^{-1}) \ . \tag{11.10}$$

Here $f_+^D(q^2)$, with $q = p_D - p_K$, is the form factor relevant to D_{e3} decay; its variation has been taken into account with the parametrization $f_+^D(t)/f_+^D(0) = M^2/(M^2 - t)$ and $M = 2.1$ GeV$/c^2$, a form and mass consistent with direct measurements [19]. Combining data on branching ratios for D_{e3} decays with accurate values for the D lifetimes [19] yields a value of $(0.818 \pm 0.041) \times 10^{11}\ \ \mathrm{s}^{-1}$ for $\Gamma(D \to \overline{K} e^+ \nu_e)$. Therefore

$$|f_+^D(0)|^2\ |V_{cs}|^2 = 0.531 \pm 0.027\ . \tag{11.11}$$

A very conservative assumption is that $|f_+^D(0)| < 1$, from which it follows that $|V_{cs}| > 0.62$. Calculations of the form factor either performed [20,21] directly at $q^2 = 0$, or done [22] at the maximum value of $q^2 = (m_D - m_K)^2$ and interpreted at $q^2 = 0$ using the measured q^2 dependence, gives the value $f_+^D(0) = 0.7 \pm 0.1$. It follows that

$$|V_{cs}| = 1.04 \pm 0.16\ . \tag{11.12}$$

The constraint of unitarity when there are only three generations gives a much tighter bound (see below).

(5) $|V_{cb}|$ – The heavy quark effective theory [23](HQET) provides a nearly model-independent treatment of B semileptonic decays to charmed mesons, assuming that both the b and c quarks are heavy enough for the theory to apply. From measurements of the exclusive decay $B \to \overline{D}^* \ell^+ \nu_\ell$, the value $|V_{cb}| = 0.0387 \pm 0.0021$ has been extracted [24] using corrections based on the HQET. Exclusive $B \to \overline{D} \ell^+ \nu_\ell$ decays give a consistent but less precise result. Analysis of inclusive decays, where the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual $V - A$ interaction, depends on going from the quark to hadron level. This is also understood within the context of the HQET [25], and the results for $|V_{cb}|$ are again consistent with those from exclusive decays. Combining all these results [24]:

$$|V_{cb}| = 0.0395 \pm 0.0017\ , \tag{11.13}$$

which is now the third most accurately measured CKM matrix element.

(6) $|V_{ub}|$ – The decay $b \to u \ell \overline{\nu}$ and its charge conjugate can be observed from the semileptonic decay of B mesons produced on the $\Upsilon(4S)$ $(b\overline{b})$ resonance by measuring the lepton energy spectrum above the endpoint of the $b \to c \ell \overline{\nu}_\ell$ spectrum. There the $b \to u \ell \overline{\nu}_\ell$ decay rate can be obtained by subtracting the background from nonresonant e^+e^- reactions. This continuum background is determined from auxiliary measurements off the $\Upsilon(4S)$. The interpretation of the result in terms of $|V_{ub}/V_{cb}|$ depends fairly strongly on the theoretical model used to generate the lepton energy spectrum, especially for $b \to u$ transitions [21,22,26]. Combining the experimental and theoretical uncertainties, we quote

$$|V_{ub}/V_{cb}| = 0.08 \pm 0.02\ . \tag{11.14}$$

This result is supported by the first exclusive determinations of $|V_{ub}|$ from the decays $B \to \pi \ell \nu_\ell$ and $B \to \rho \ell \nu_\ell$ by the CLEO experiment [27] to obtain $|V_{ub}| = 3.3 \pm 0.4 \pm 0.7 \times 10^{-3}$, where the first error is experimental and the second reflects systematic uncertainty from different theoretical models of the exclusive decays. While this result is consistent with Eq. (11.14) and has a similar error bar, given the theoretical model dependence of both results we do not combine them, and retain the inclusive result for V_{ub}.

(7) V_{tb} – The discovery of the top quark by the CDF and DØ collaborations utilized in part the semileptonic decays of t to b. One can set a (still rather crude) limit on the fraction of decays of the form $t \to b\ \ell^+\ \nu_\ell$, as opposed to semileptonic t decays that involve s or d quarks, of [28]

$$\frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.99 \pm 0.29\ . \tag{11.15}$$

For many of these CKM matrix elements, the primary source of error is no longer statistical, but rather theoretical. This arises from explicit model dependence in interpreting data or in the use of specific hadronic matrix elements to relate experimental measurements to weak transitions of quarks. This is even more the case in extracting CKM matrix elements from loop diagrams discussed below. Such errors are generally not Gaussian. We have taken a "1σ" range to correspond to a 68% likelihood that the true value lies within "$\pm 1\sigma$" of the central value.

The results for three generations of quarks, from Eqs. (11.7), (11.8), (11.9), (11.12), (11.13), (11.14), and (11.15) plus unitarity, are summarized in the matrix in Eq. (11.2). The ranges given there are different from those given in Eqs. (11.7)–(11.15) because of the inclusion of unitarity, but are consistent with the one-standard-deviation errors on the input matrix elements. Note in particular that the unitarity constraint has pushed $|V_{ud}|$ about one standard deviation higher than given in Eq. (11.7).

The data do not preclude there being more than three generations. Moreover, the entries deduced from unitarity might be altered when the CKM matrix is expanded to accommodate more generations. Conversely, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that any additional element in the first row have a magnitude $|V_{ub'}| < 0.08$. When there are more than three generations, the allowed ranges (at 90% CL) of the matrix elements connecting the first three generations are

$$\begin{pmatrix} 0.9724 \text{ to } 0.9755 & 0.217 \text{ to } 0.223 & 0.0018 \text{ to } 0.0044 & \ldots \\ 0.199 \text{ to } 0.232 & 0.847 \text{ to } 0.975 & 0.036 \text{ to } 0.042 & \ldots \\ 0 \text{ to } 0.10 & 0 \text{ to } 0.36 & 0.05 \text{ to } 0.9994 & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}, \tag{11.16}$$

where we have used unitarity (for the expanded matrix) and the same measurements of the magnitudes of the CKM matrix elements, except for Eq. (11.15), which assumed three generations, and is replaced by the much weaker result [28] $|V_{tb}| > 0.055$ at 95% CL.

Further information, particularly on CKM matrix elements involving the top quark, can be obtained from flavor-changing processes that occur at the one-loop level. We have not used this information in the discussion above since the derivation of values for V_{td} and V_{ts}

in this manner from, for example, B mixing or $b \to s\gamma$, require an additional assumption that the top-quark loop, rather than new physics, gives the dominant contribution to the process in question. Conversely, the agreement of CKM matrix elements extracted from loop diagrams with the values based on direct measurements and three generations can be used to place restrictions on new physics.

The measured value [24] of $\Delta M_{B_d} = 0.472 \pm 0.018$ ps^{-1} from $B_d^0 - \overline{B}_d^0$ mixing can be turned in this way into information on $|V_{tb}{}^* V_{td}|$, assuming that the dominant contribution to the mass difference arises from the matrix element between a B_d and a $\overline{B}_d$ of an operator that corresponds to a box diagram with W bosons and top quarks as sides. Using the characteristic hadronic matrix element that then occurs, $\widehat{B}_{B_d} f_{B_d}{}^2 = (1.4 \pm 0.1)(175 \pm 25 \text{ MeV})^2$ from lattice QCD calculations [29], which we regard as having become the most reliable source of such matrix elements, next-to-leading-order QCD corrections ($\eta_{\rm QCD} = 0.55$) [30], and the running top-quark mass, $\overline{m}_t(m_t) = 166 \pm 5$ GeV, as input,

$$|V_{tb}{}^* \cdot V_{td}| = 0.0084 \pm 0.0018 \;, \tag{11.17}$$

where the uncertainty comes primarily from that in the hadronic matrix elements, whose estimated errors are combined linearly.

In the ratio of B_s to B_d mass differences, many common factors (such as the QCD correction and dependence on the top-quark mass) cancel, and we have

$$\frac{\Delta M_{B_s}}{\Delta M_{B_d}} = \frac{M_{B_s}}{M_{B_d}} \, \frac{\widehat{B}_{B_s} f_{B_s}^2}{\widehat{B}_{B_d} f_{B_d}^2} \, \frac{|V_{tb}^* \cdot V_{ts}|^2}{|V_{tb}^* \cdot V_{td}|^2} \;. \tag{11.18}$$

With the experimentally measured masses [19], $\widehat{B}_{B_s}/\widehat{B}_{B_d} = 1.01 \pm 0.04$ and $f_{B_s}/f_{B_d} = 1.15 \pm 0.05$ from lattice QCD [29], and the improved experimental lower limit [24] at 95% CL of $\Delta M_{B_s} > 10.2$ ps^{-1},

$$|V_{td}|/|V_{ts}| < 0.27 \;. \tag{11.19}$$

Since with three generations, $|V_{ts}| \approx |V_{cb}|$, this result converts to $|V_{td}| < 0.011$, which is a significant constraint by itself (see Fig. 11.2).

The CLEO observation [31] of $b \to s\gamma$ can be translated [32] similarly into $|V_{ts}|/|V_{cb}| = 1.1 \pm 0.43$, where the large uncertainty is again dominantly theoretical. In $K^+ \to \pi^+ \nu\overline{\nu}$ there are significant contributions from loop-diagrams involving both charm and top quarks. Experiment is just begining to probe the level predicted in the Standard Model [33]. All these additional indirect constraints are consistent with the matrix elements obtained from the direct measurements plus unitarity, assuming three generations; with the recent results on B mixing and theoretical improvements in lattice calculations, adding the indirect constraints to the fit reduces the range allowed for $|V_{td}|$.

Direct and indirect information on the CKM matrix is neatly summarized in terms of the "unitarity triangle." The name arises since unitarity of the 3×3 CKM matrix applied to the first and third columns yields

$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td} V_{tb}^* = 0 \;. \tag{11.20}$$

The unitarity triangle is just a geometrical presentation of this equation in the complex plane [34]. We can always choose to orient the triangle so that $V_{cd}\ V_{cb}{}^*$ lies along the horizontal; in the parametrization we have chosen, V_{cb} is real, and V_{cd} is real to a very good approximation in any case. Setting cosines of small angles to unity, Eq. (11.20) becomes

$$V_{ub}{}^* + V_{td} = s_{12}\ V_{cb}{}^*\ , \tag{11.21}$$

which is shown as the unitarity triangle in Fig. 11.1(a). Rescaling the triangle by a factor $[1/|s_{12}\ V_{cb}|]$ so that the base is of unit length, the coordinates of the vertices become

$$A\big(\mathrm{Re}(V_{ub})/|s_{12}\ V_{cb}|\ ,\quad -\mathrm{Im}(V_{ub})/|s_{12}\,V_{cb}|\big)\ ,\ B(1,0)\ ,\ C(0,0)\ . \tag{11.22}$$

In the Wolfenstein parametrization [4], the coordinates of the vertex A of the unitarity triangle are simply (ρ, η), as shown in Fig. 11.1(b).

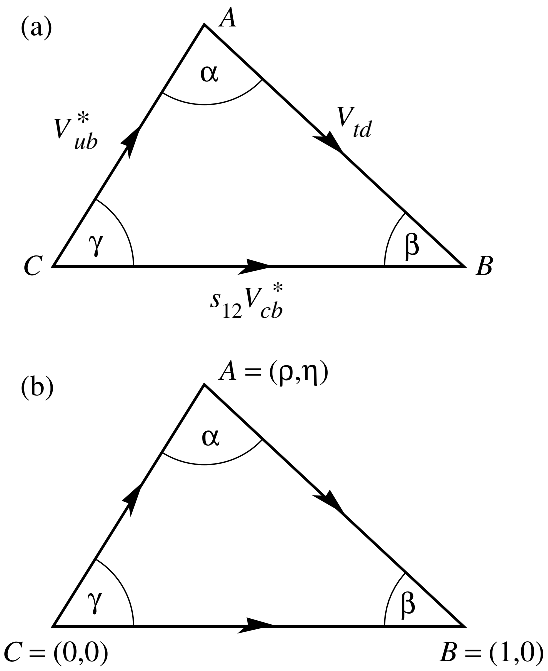

Figure 11.1: (a) Representation in the complex plane of the triangle formed by the CKM matrix elements $V_{ub}{}^*$, V_{td}, and $s_{12}\ V_{cb}{}^*$. (b) Rescaled triangle with vertices $A(\rho, \eta)$, $B(1,0)$, and $C(0,0)$.

CP-violating processes will involve the phase in the CKM matrix, assuming that the observed CP violation is solely related to a nonzero value of this phase. This allows additional constraints to be brought to bear. More specifically, a necessary and sufficient condition for CP violation with three generations can be formulated in a parametrization-independent manner in terms of the non-vanishing of the determinant of the commutator of the mass matrices for the charge $2e/3$ and charge $-e/3$ quarks [35]. CP violating amplitudes or differences of rates are all proportional to the CKM factor in this quantity. This is the product of factors $s_{12}s_{13}s_{23}c_{12}c_{13}^2c_{23}s_{\delta_{13}}$ in the parametrization adopted above, and is $s_1^2 s_2 s_3 c_1 c_2 c_3 s_\delta$ in that of Ref. 1. With the approximation of setting cosines to unity, this is just twice the area of the unitarity triangle.

While hadronic matrix elements whose values are imprecisely known generally enter the calculations, the constraints from CP violation in the neutral kaon system, taken together with the restrictions on the magnitudes of the CKM matrix elements shown above, are tight enough to restrict considerably the range of angles and the phase of the CKM matrix. For example, the constraint obtained from the CP-violating parameter ϵ in the neutral K system corresponds to the vertex A of the unitarity triangle lying on a hyperbola for fixed values of the hadronic matrix elements [36,37]. The constraints on the vertex of the unitarity triangle that follow from $|V_{ub}|$, B mixing, and ϵ are shown in Fig. 11.2. The improved limit in Eq. (11.19) that arises from the ratio of B_s to B_d mixing eliminates a significant region for the vertex A of the unitarity triangle, otherwise allowed by direct measurements of the CKM matrix elements. This limit is more robust theoretically since it depends on ratios (rather than absolute values) of hadronic matrix elements and is independent of the top mass or QCD corrections (which cancel in the ratio). Ultimately in the Standard Model, the CP-violating process $K_L \to \pi^0\nu\overline{\nu}$ offers high precision in measuring the imaginary part of $V_{td} \cdot V_{ts}{}^*$ to yield Im V_{td}, the altitude of the unitarity triangle. However, the experimental upper limit is presently many orders of magnitude away from the requisite sensitivity.

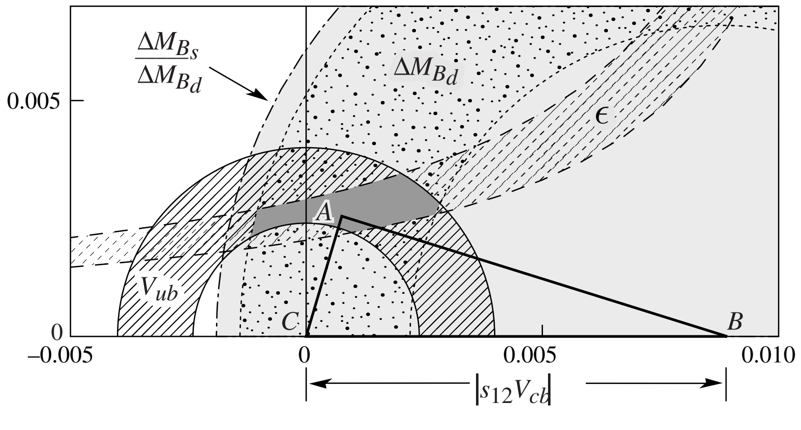

Figure 11.2: Constraints on the position of the vertex, A, of the unitarity triangle following from $|V_{ub}|$, B-mixing, and ϵ. A possible unitarity triangle is shown with A in the preferred region.

For CP-violating asymmetries of neutral B mesons decaying to CP eigenstates, there is a direct relationship between the magnitude of the asymmetry in a given decay and $\sin 2\phi$, where $\phi = \alpha,\ \beta,\ \gamma$ is an appropriate angle of the unitarity triangle [34]. The combination of all the direct and indirect information can be used to find the implications for future measurements of CP violation in the B system. (See Sec. 12 on CP Violation and the review on "CP Violation in B Decay – Standard Model Predictions" in the B Listings.)

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