17. THE HUBBLE CONSTANT

Revised August 1997 by C.J. Hogan (University of Washington).

In a uniform expanding universe, the position $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ of any particle relative to another obey Hubble's relation $\boldsymbol{v} = H_0 \boldsymbol{r}$, where H_0 is Hubble's constant.* As cosmological distances are measured in Mpc, the natural unit for H_0 is km s^{-1} Mpc^{-1}, which has the dimensions of inverse time: $[100 \text{ km s}^{-1} \text{ Mpc}^{-1}]^{-1} = 9.78 \times 10^9$ yr.

The real universe is nonuniform on small scales, and its motion obeys the Hubble relation only as a large scale average. As typical non-Hubble motions ("peculiar velocities") are less than about 500 km s^{-1}, on scales more than about 5,000 km s^{-1} the deviations from Hubble flow are less than about 10%, so the notion of a global Hubble constant is well defined. The value of H_0 averaged over the local 15,000 km s^{-1} volume is known to lie within 10% of its global value even if H_0 itself is not known this precisely [1–3].

Measurement of H_0 thus entails measuring large absolute distances. Traditionally, certain astronomical systems ("Standard Candles") are used to measure relative distance, and are tied to an absolute trigonometric parallax scale by a series of distance ratios (or "distance ladder") [4–9]. Several relatively new techniques now allow direct absolute calibration using physical models.

Table 17.1 lists several candles and calibrators with a typical range of distance accessible to each. The ranges are not precisely defined; the near end suffers from small numbers of accessible objects and the far end from faint signal. The precision quoted is in units of astronomical "distance modulus," given by $\mu = 5\log_{10}(\textit{distance in parsecs}) - 5.0$; a ± 0.1 magnitude error in magnitude or distance modulus corresponds to a 5% error in distance. In the case of distance ratios the precision is estimated by cross-checking indicators on a galaxy-by-galaxy basis. Some options often used for verification and absolute calibration are listed. The Hubble relation itself is included, as it is the most precise indication of relative distance for large distances, and is used to verify the standardization of several candles.

Table 17.1: Selected extragalactic distance indicators.[†]

Technique	Range of distance	Accuracy (1σ)	Verification/ calibration
Cepheids	<LMC to 25 Mpc	0.15 mag	LMC/parallax
SNIa	4 Mpc to $>$ 2 Gpc	0.2 mag	Hubble/Cepheid
EPM/SNII	LMC to 200 Mpc	0.4 mag	Hubble/Cepheid
PNLF	LMC to 20 Mpc	0.1 mag	SBF/Cepheid
SBF	1 Mpc to 100 Mpc	0.1 mag	PNLF/Cepheid
TF	1 Mpc to 100 Mpc	0.3 mag	Hubble/Cepheid
BCG	50 Mpc to 1 Gpc	0.3 mag	Hubble/SBF
GCLF	1 Mpc to 100 Mpc	0.4 mag	SBF/MWG
SZ	100 Mpc to $>$ 1 Gpc	0.4 mag	Hubble/Model
GL	$\sim$5 Gpc	0.4 mag	Model
Hubble	20 Mpc to $\gtrsim$1Gpc	500 km s^{-1} $\div$ $H_0 D$	BCG, SNeIa/H_0

MWG = Milky Way Galaxy

[†]Extracted from [4-9].

17.1. Calibration of Cepheid variables

Using stars as standard candles and the Earth's orbit as a baseline, distances in the nearby Galaxy are tied directly to trigonometric parallax measurements. With the release in 1997 of the first results of the Hipparcos satellite, the range, precision, and size of calibrating samples have greatly improved. The early recalibrations of the absolute scale of the Galaxy indicate an increase in the distance scale for Cepheid variables which propagates to all larger scale measurements, reducing previous measurements of H_0 by 0.1 to 0.2 mag [10,11]. (Note that the RRLyrae distance scale, used to calibrate the distances to old globular clusters within the Galaxy, has also increased [12], which increases the stellar brightness and decreases their estimated age, possibly reconciling the cosmic age and Hubble parameter for a wider range of cosmological models [11,13].) The revised distance scale however would also increase the distance to the Large Magellanic Cloud (LMC) which is constrained by geometrical arguments from SN 1987A [14]. Another promising method is based on detailed knowledge of orbits of gas in N4258 precisely constrained by observations of maser gas emission. This has the potential to calibrate the Cepheid scale independently [23].

The best studied and most trusted of the absolute calibrators, Cepheids are bright stars undergoing overstable oscillations driven by the variation of helium opacity with temperature. The period of oscillation is tightly correlated with the absolute brightness of the star, though this "period-luminosity relation" [15] may vary with metallicity [16,17]. With Hubble Space Telescope (HST), Cepheids are now measured in over a dozen galaxies out to 25 Mpc ($\mu = 32$) allowing direct absolute calibration of many other indicators to better than 10% accuracy [18–22].

17.2. Type Ia supernovae (SNIa)

A SNIa occurs when a degenerate dwarf, of the order of a solar mass and of CNO composition, undergoes explosive detonation or deflagration by nuclear burning to iron-group elements (Ni, Co, Fe). Their uniformity arises because the degenerate material only becomes unstable when it is gravitationally compressed to where the electrons become close to relativistic, which requires approximately a Chandrasekhar mass (1.4 solar masses). Theoretical models of the explosion predict approximately the right peak brightness, but cannot give a precise calibration. SNIa are very bright, so their brightness distribution can be studied using the distant Hubble flow as a reference. Indeed, the Hubble diagram of distant SNIa shows that they can yield remarkably precise relative distances; even though they display large variations in brightness, with detailed knowledge of the shape of the light curve and colors, the relative intrinsic brightness of a single SNIa can be predicted to $\Delta m \simeq 0.2$ mag and its distance estimated to $\simeq 10\%$ accuracy [24–26]. Distant SNIa constrain the global deviations from a linear Hubble law including those from cosmic deceleration [27–28].

17.3. Type II supernovae (SNII)

A SNII occurs when a massive star has accumulated 1.4 solar masses of iron group elements in its core; there is then no source of nuclear energy and the core collapses by the Chandrasekhar instability. The collapse to a neutron star releases a large gravitational binding energy, some of which powers an explosion. The large variety of envelopes around collapsing cores means that SNII are not at all uniform in their properties. However, their distances can be calibrated absolutely by the fairly reliable "expanding photosphere method" (EPM). In principle the spectral temperature and absolute flux yield the source angular size; spectral lines yield the expansion velocity, which combined with elapsed time gives a physical size; and the two sizes yield a distance. Models of real photospheres are not so simple but yield individual distances accurate to about 20% [29]. This is in principle an independent absolute distance, but is verified by comparison with Cepheids in several cases, the distant Hubble diagram and Tully-Fisher distance ratios in several others, and by multiple-epoch fits of the same object.

17.4. Planetary nebula luminosity function (PNLF)

A planetary nebula (PN) forms when the gaseous envelope is ejected from a low-mass star as its core collapses to a white dwarf. We see bright fluorescent radiation from the ejected gas shell, excited by UV light from the hot proto-white dwarf. The line radiation makes PN's easy to find and measure even in far-away galaxies; a bright galaxy can have tens of thousands, of which hundreds are bright enough to use to construct a PNLF. It is found empirically that the range of PN brightnesses has a sharp upper cutoff possibly as a consequence of the very narrow range in core masses that result from normal stellar evolution. The cutoff appears to provide a good empirical standard candle [30], verified by comparison with SBF distance ratios.

17.5. Surface brightness fluctuations (SBF)

In images of galaxies, individual stars are generally too crowded to resolve. However, with modern linear detectors, it is still possible to measure the moments of the distribution of stellar brightness in a population (in particular, the brightness-weighted average stellar brightness) from surface brightness fluctuations. Stellar populations in elliptical galaxies appear to be universal enough for this to be one of the most precise standard candles, as verified by comparison with PNLF and Cepheids, although absolute calibration must be done on the bulge components of spiral galaxies. With HST data it can now be applied into the far Hubble flow [31–32].

17.6. Tully-Fisher (TF) and diameter-dispersion (D_n–σ)

The TF relation refers to a correlation of the properties of whole spiral galaxies, between rotational velocity and total luminosity. In rough terms, the relation can be understood as a relation between mass and luminosity, but given the variation in structural properties and stellar populations the narrow relation is a surprisingly good relative distance indicator. The TF distance ratios and precision have been verified by cross-checking against all of the above methods, and against the Hubble flow, particularly galaxy cluster averages, which permit greater precision. HST has permitted absolute calibration of TF in a larger, more representative, and more distant sample, including galaxies in the Virgo and Fornax clusters [33]. For elliptical galaxies, a similar relation ("D_n–σ") is particularly useful for verifying distance ratios of galaxy clusters, whose cores contain almost no spirals.

17.7. Brightest cluster galaxies (BCG)

As a result of agglomeration, rich clusters of galaxies have accumulated the largest and brightest galaxies in the universe in their centers, which are remarkably homogenous. They provide a check on the approach to uniform Hubble flow on large scales [2–3] and are now tied to an absolute scale via SBF [34].

17.8. Globular cluster luminosity function (GCLF)

Many galaxies have systems of globular clusters orbiting them, each of which contain hundreds of thousands of stars and hence is visible at large distances. Empirically it appears that similar galaxies have similar distributions of globular cluster luminosity [35]

17.9. Sunyaev-Zeldovich effect (SZ)

The electron density and temperature of the hot plasma in a cluster of galaxies can be measured in two ways which depend differently on distance: the thermal x-ray emission, which is mostly bremsstrahlung by hot electrons, and the Sunyaev-Zeldovich effect on the microwave background, caused by Compton scattering off the same electrons. This provides in principle an absolute calibration. Although the model has other unconstrained parameters, such as the gas geometry, which limit the precision and reliability of distances, in the handful of cases which have been studied most recently the distances are broadly in accord with those obtained by the other techniques. [36–38]

17.10. Gravitational lenses (GL)

The time delay δt between different images of a high redshift gravitationally lensed quasar is $\delta t = C\delta\theta^2/H_0 \approx 1$ yr for image separations $\delta\theta$ of the order of arcseconds, with a numerical factor C typically of order unity determined by the specific lens geometry (the angular distribution of the lensing matter) and background cosmology. Variability of the double quasar 0957+561 has permitted measurements of δt from time series correlation, 417 ± 3 days [39–40], with well controlled theoretical errors in deriving constraints on H_0 [41]; measurements of other lens systems are also improving [42]. It is an amazing sanity check that this technique, which relies on no other intermediate steps for its calibration, gives estimates on the scale of the Hubble length which are consistent with local measures of H_0.

17.11. Estimates of H_0

The central idea is to find "landmark" systems whose distance is given by more than one technique. The number of techniques and the range of each has now increased enough for reliable overlapping calibration at each stage of the distance scale. The reason for the diversity of estimates of the Hubble constant lies in the many different ways to combine these techniques to obtain an absolute distance calibration in the Hubble flow. There is now broad agreement within the errors among a wide variety of semi-independent ladders with different systematics. As examples, we cite a variety of (somewhat arbitrarily chosen) independent methods, which illustrate some of the choices and tradeoffs, summarized in Table 17.2. Note that most of the quoted values depend in common on the absolute Cepheid calibration.

1. Expanding photosphere method (EPM) distances give an absolute calibration to objects in the distant Hubble flow. A small sample of these direct distances with small flow corrections gives $H_0 = 73 \pm 6$ (statistical) $\pm\, 7$ (systematic).The distance estimates and limits on the systematic error component are verified by Cepheid distances in three cases, where the Cepheid/EPM distances come out to 1.02 ± 0.08 (LMC), $1.01^{+0.23}_{-0.17}$ (M101) and 1.13 ± 0.28 (M100).
2. With HST, it is now possible to calibrate SNIa directly with Cepheid distances to host galaxies. The light from brighter SNIa decays more slowly than from faint ones, so the best fits to the distant Hubble diagram include information about the light curve shape rather than simply assuming uniformity.
3. The distance to Virgo or any other local cluster is tied to H_0 via the distant Hubble diagram for TF or D_n–σ distances for galaxies in distant clusters. This can be done with a large scale flow model fit to many clusters or using the distance ratio to a fiducial reference such as the Coma cluster.
4. TF comparison with distant field galaxies in the Hubble flow (after corrections for Malmquist bias in the samples, which is worse than in cluster samples) yield $H_0 = 80 \pm 10$ km s^{-1} Mpc^{-1}.
5. The distant BCG sample is now calibrated with SBF directly.

6. Recent SZ and GL estimates lie squarely in the range of the other techniques and are completely independent of them, although errors are not yet well constrained with such small samples.

The central values by most reliably calibrated methods lie in the range $H_0 = 60$ to 80 km s^{-1} Mpc^{-1}, and indeed this corresponds roughly with the range of estimates expected from the internally estimated errors. Thus systematic errors are at least not overwhelming, although there are still discrepencies which are not understood.

Footnote and References:

∗ This simple Newtonian description is valid to first order in $\boldsymbol{v}$; the role of the Hubble constant in relativistic world-models is summarized in the Big-Bang Cosmology section (Sec. 15).

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Table 17.2: Some recent estimates of Hubble's constant

Technique	Calibration*	Ties to Hubble flow	Result* (km s^{-1} Mpc^{-1})	Ref.
EPM	EPM model, Cepheids	Direct EPM Hubble Diagram + Flow model or TF	$73 \pm 6 \pm 7$	[29,19]
SNeIa	Host galaxy Cepheids	Direct SNIa Hubble Diagram	63 ± 3.4	[25]
			58 ± 8	[21]
Clusters	Virgo mean (M100 Cepheids)	Virgo infall model	$81 \pm 11^{\dagger}$	[19]
	+ local + M101 Cepheids	Virgo/Coma ratio	$73–77 \pm 10^{\dagger}$	[19]
		Cluster TF + LS flow model fit	$82 \pm 11^{\dagger}$	[19]
	M96 Cepheids	LeoI to Virgo and Coma	69 ± 8	[22]
Field TF	Local Cepheids	Field TF Hubble Diagram + Malmquist bias correction	80 ± 10	[43]
BCG	SBF, Cepheids	BCG	82 ± 8	[34]
SZ	SZ model + X-ray maps + SZ maps	Single cluster velocities A478,A2142,A2256	54 ± 14	[38]
		Coma	74 ± 29	[37]
GL	Lens model, time delay	Direct, Q0957+561	63 ± 12	[40]

* For all methods except SZ and GL, add a common multiplicative error of ± 0.15 mag or 7% in H_0 for absolute calibration of Cepheids. These values assume the pre-Hipparcos calibration of the Cepheid PL relation.

† Plus Virgo depth uncertainty (scales with M100/Virgo ratio).