PSEUDOSCALAR-MESON DECAY CONSTANTS

(by M. Suzuki, LBNL)

Charged mesons

The decay constant f_P for a charged pseudoscalar meson P is defined by

$$\langle 0|A_\mu(0)|P(\mathbf{q})\rangle = i f_P \, q_\mu \ ,$$

where A_μ is the axial-vector part of the charged weak current after a Cabibbo-Kobayashi-Maskawa mixing-matrix element $V_{qq'}$ has been removed. The state vector is normalized by $\langle P(\mathbf{q})|P(\mathbf{q}')\rangle = (2\pi)^3 \, 2E_q \, \delta(\mathbf{q}-\mathbf{q}')$, and its phase is chosen to make f_P real and positive. Note, however, that in many theoretical papers our $f_P/\sqrt{2}$ is denoted by f_P.

In determining f_P experimentally, radiative corrections must be taken into account. Since the photon-loop correction introduces an infrared divergence that is canceled by soft-photon emission, we can determine f_P only from the combined rate for $P^\pm \to \ell^\pm \nu_\ell$ and $P^\pm \to \ell^\pm \nu_\ell \gamma$. This rate is given by

$$\Gamma\,(P \to \ell\nu_\ell + \ell\nu_\ell\gamma) =$$

$$\frac{G_F^2 |V_{qq'}{}^2|}{8\pi} f_P^2 \, m_\ell^2 \, m_P \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 [1 + \mathscr{O}(\alpha)] \ .$$

Radiative corrections include inner bremsstrahlung, which is independent of the structure of the meson [1–3], and also a structure-dependent term [4,5]. After radiative corrections are made, there are ambiguities in extracting f_P from experimental measurements. In fact, the definition of f_P is no longer unique.

It is desirable to define f_P such that it depends only on the properties of the pseudoscalar meson, not on the final decay products. The short-distance corrections to the fundamental electroweak constants like $G_F|V_{qq'}|$ should be separated out. Following Marciano and Sirlin [6], we define f_P with the following form for the $\mathscr{O}(\alpha)$ corrections:

$$1 + \mathscr{O}(\alpha) = \left[1 + \frac{2\alpha}{\pi}\ln\left(\frac{m_Z}{m_\rho}\right)\right]\left[1 + \frac{\alpha}{\pi}F(x)\right]$$

$$\times\left\{1 - \frac{\alpha}{\pi}\left[\frac{3}{2}\ln\left(\frac{m_\rho}{m_P}\right) + C_1 + C_2\frac{m_\ell^2}{m_\rho^2}\ln\left(\frac{m_\rho^2}{m_\ell^2}\right) + C_3\frac{m_\ell^2}{m_\rho^2} + \ldots\right]\right\} \ .$$

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Here

$$F(x) = 3\ln x + \frac{13-19x^2}{8(1-x^2)} - \frac{8-5x^2}{2(1-x^2)^2}\, x^2 \ln x$$

$$-2\Big(\frac{1+x^2}{1-x^2}\ln x + 1\Big)\ln(1-x^2) + 2\Big(\frac{1+x^2}{1-x^2}\Big)L(1-x^2)\ ,$$

with

$$x \equiv m_\ell/m_P\ , \qquad L(z) \equiv \int_0^z \frac{\ln(1-t)}{t}\, dt\ .$$

The first bracket in the expression for $1+\mathcal{O}(\alpha)$ is the short-distance electroweak correction. The QCD correction reduces this factor by 0.00033. The second bracket together with the term $-(3\alpha/2\pi)\ln(m_\rho/m_P)$ in the third bracket corresponds to the radiative corrections to the point-like pion decay ($\Lambda_{\rm cutoff} \approx m_\rho$) [2]. The rest of the corrections in the third bracket are expanded in powers of m_ℓ/m_ρ. The expansion coefficients C_1, C_2, and C_3 depend on the hadronic structure of the pseudoscalar meson and in most cases cannot be computed accurately. In particular, C_1 absorbs the uncertainty in the matching energy scale between short- and long-distance strong interactions and thus is the main source of uncertainty in determining f_{π^+} accurately.

With the experimental value for the decay $\pi \to \mu\nu_\mu + \mu\nu_\mu\gamma$, one obtains

$$f_{\pi^+} = 130.7 \pm 0.1 \pm 0.36 \text{ MeV}\ ,$$

where the first error comes from the experimental uncertainty on $|V_{ud}|$ and the second comes from the uncertainty on C_1 $(= 0 \pm 0.24)$ [6]. Similarly, one obtains from the decay $K \to \mu\nu_\mu + \mu\nu_\mu\gamma$ the decay constant

$$f_{K^+} = 159.8 \pm 1.4 \pm 0.44 \text{ MeV}\ ,$$

where the first error is due to the uncertainty on $|V_{us}|$.

For the heavy pseudoscalar mesons, uncertainties in the experimental values for the decay rates are much larger than the radiative corrections. For the D^+, only an upper bound can be obtained from the published data:

$$f_{D^+} < 310 \text{ MeV (CL} = 90\%)\ .$$

Three groups have measured the $D_s^+ \to \mu^+\nu_\mu$ branching fraction, leading to the following values of the decay constant:

$$
\begin{aligned}
f_{D_s^+} =& 232 \pm 45 \pm 20 \pm 48 \text{ MeV} \quad [7] \ , \\
f_{D_s^+} =& 344 \pm 37 \pm 52 \pm 42 \text{ MeV} \quad [8] \ , \\
f_{D_s^+} =& 430^{+150}_{-130} \pm 40 \text{ MeV} \qquad\quad [9] \ ,
\end{aligned}
$$

where the first errors are statistical, the second errors are systematic, and the third errors are uncertainties involved in extracting the branching fraction $\mathrm{B}(D_s^+ \to \mu^+\nu_\mu)$. We must wait for more data before drawing a conclusion on $f_{D_s^+}$.

There have been many attempts to extract f_P from spectroscopy and nonleptonic decays using theoretical models. Since it is difficult to estimate uncertainties for them, we have listed here only values of decay constants that are obtained directly from the observation of $P^\pm \to \ell^\pm\nu_\ell$.

Light neutral mesons

The decay constants for the light neutral pseudoscalar mesons π^0, η, and η' are defined by

$$\langle 0|A_\mu(0)|P^0(\mathbf{q})\rangle = \mathrm{i}(f_P/\sqrt{2})q_\mu \ ,$$

where A_μ is a neutral axial-vector current of octet or singlet. Values of f_P can be obtained from the two-photon decay $P^0 \to \gamma\gamma$, since in the $m_P = 0$ limit the decay matrix element is determined by the Adler-Bell-Jackiw anomaly [10,11]. However, large uncertainties enter values of f_P through extrapolation to the physical mass and, in the case of η and η', through the mixing angle, too.

The CELLO Collaboration has obtained the values [12]

$$
\begin{aligned}
f_{\pi^0} &= 119 \pm 4 \text{ MeV} \\
f_\eta &= 133 \pm 10 \text{ MeV} \\
f_{\eta'} &= 126 \pm 7 \text{ MeV} \ ,
\end{aligned}
$$

while the TPC/2γ Collaboration has obtained [13]

$$
\begin{aligned}
f_\eta &= 129 \pm 8 \text{ MeV} \\
f_{\eta'} &= 110 \pm 7 \text{ MeV} \ .
\end{aligned}
$$

(We have multiplied the published values by $\sqrt{2}$ to be in accord with our definition of f_P.)

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