K^0_L – THIS IS PART 3 OF 4

To reduce the size of this section's PostScript file, we have divided it into four PostScript files. We present the following index:

PART 1

PART 2

PART 3

PART 4

CP VIOLATION IN K^0_L DECAY

(by L. Wolfenstein, Carnegie-Mellon University and
T. Trippe, LBNL)

Experimentally Measured Parameters

CP violation has been observed in the semi-leptonic decays $K^0_L \rightarrow \pi^{\mp}\ell^{\pm}\nu$ and in the nonleptonic decay $K^0_L \rightarrow 2\pi$. The experimental numbers that have been measured are [1]

$$\delta = \frac{\Gamma(K^0_L \rightarrow \pi^-\ell^+\nu) - \Gamma(K^0_L \rightarrow \pi^+\ell^-\nu)}{\Gamma(K^0_L \rightarrow \pi^-\ell^+\nu) + \Gamma(K^0_L \rightarrow \pi^+\ell^-\nu)} \tag{1a}$$

$$\begin{aligned} \eta_{+-} &= A(K^0_L \rightarrow \pi^+\pi^-)/A(K^0_S \rightarrow \pi^+\pi^-) \\ &= |\eta_{+-}|\, e^{i\phi_{+-}} \end{aligned} \tag{1b}$$

$$\begin{aligned} \eta_{00} &= A(K^0_L \rightarrow \pi^0\pi^0)/A(K^0_S \rightarrow \pi^0\pi^0) \\ &= |\eta_{00}|\, e^{i\phi_{00}} \ . \end{aligned} \tag{1c}$$

Thus there are five real numbers, three magnitudes, and two phases. We list $\delta(\mu)$ for $K^0_L \rightarrow \pi\mu\nu$ and $\delta(e)$ for $K^0_L \rightarrow \pi e\nu$ separately and a weighted average δ. Experimentally for the $K^0_L \rightarrow \pi^0\pi^0$ decay the quantities directly measured (and also of greatest theoretical interest) are $|\eta_{00}/\eta_{+-}|$ and $\phi_{00} - \phi_{+-}$.

Analysis Based on CPT Invariance [2]

CP violation can occur either in the K^0–$\overline{K}^0$ mixing or in the decay amplitudes. Assuming CPT invariance, the CP violation in the mixing is described by a single parameter ϵ:

$$\begin{aligned} |\, K^0_L\rangle &= \Big[(1+\epsilon)\,|\, K^0\,\rangle - (1-\epsilon)\,|\,\overline{K}^0\,\rangle\Big] \\ &\qquad / \left[2(1+|\,\epsilon\,|^2)\right]^{1/2} \end{aligned} \tag{2a}$$

$$\begin{aligned} |\, K^0_S\rangle &= \Big[(1+\epsilon)\,|\, K^0\,\rangle + (1-\epsilon)\,|\,\overline{K}^0\,\rangle\Big] \\ &\qquad / \left[2(1+|\,\epsilon\,|^2)\right]^{1/2} \ . \end{aligned} \tag{2b}$$

The decay amplitudes are written

$$\langle\, I=0\,|\,T\,|\,K^0\rangle = e^{i\delta_0}A_0 \tag{3a}$$

$$\langle\, I=2\,|\,T\,|\,K^0\rangle = e^{i\delta_2}A_2 \tag{3b}$$

where δ_I are the $\pi\pi$ scattering phase shifts at the K^0 mass and I is the isospin of the final state. CP violation is measured by (Im A_I/Re A_I). One can then write

$$\eta_{+-} = \epsilon + \epsilon' \tag{4a}$$

$$\eta_{00} = \epsilon - 2\epsilon' \tag{4b}$$

where

$$\epsilon' = \frac{i}{\sqrt{2}}\, e^{i(\delta_2-\delta_0)}\, \frac{\text{Re } A_2}{\text{Re } A_0} \left[\frac{\text{Im } A_2}{\text{Re } A_2} - \frac{\text{Im } A_0}{\text{Re } A_0}\right] \tag{5}$$

neglecting small corrections of order ϵ' times Re(A_2/A_0). It is possible by a choice of phase convention to set Im A_0 or Im A_2 or Im ϵ to 0, but none of these is 0 with the usual phase convention in the Standard Model. The choice Im A_0=0 is the Wu-Yang phase convention [3].

By applying CPT invariance and unitarity it is possible to relate δ to ϵ and to determine the phases of ϵ and ϵ'. If one assumes the $\Delta S = \Delta Q$ rule (see below note on the "$\Delta S = \Delta Q$ Rule in K^0 Decay") the expression for δ becomes

$$\delta = 2\text{Re } \epsilon/(1 + \mid \epsilon \mid^2) \approx 2\text{Re } \epsilon \ . \tag{6}$$

This quantity is independent of phase convention and is seen from Eq. (2) to equal $\langle K_L^0 \mid K_S^0\rangle$. The phase of ϵ is given by

$$\phi(\epsilon) \approx \tan^{-1} \frac{2(m_{K_L} - m_{K_S})}{\Gamma_{K_S} - \Gamma_{K_L}} = 43.49 \pm 0.08^\circ \tag{7a}$$

while Eq. (5) gives

$$\phi(\epsilon') = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 48 \pm 4^\circ \ . \tag{7b}$$

The approximation in Eq. ($7a$) depends on the assumption that direct CP violation is negligible in all K^0 decays and is expected to be good to a few tenths of a degree. Eq. ($7a$) is evaluated using the values of the $K_L^0 - K_S^0$ mass difference $\Delta m = (0.5304 \pm 0.0014) \times 10^{10}\hbar\text{s}^{-1}$ and the K_S^0 mean life $\tau_s = (0.8927 \pm 0.0009) \times 10^{-10}$s from the current edition. The value of the $\pi\pi$ phase shifts is taken from the fit given by Chell and Olsson [4]. The most important point for the analysis is that $\cos[\phi(\epsilon') - \phi(\epsilon)] \simeq 1$. The consequence of this analysis is that only two real quantities need be measured, the magnitude of ϵ and the value of (ϵ'/ϵ) including its sign. The measured quantity $|\eta_{00}/\eta_{+-}|^2$ which is very close to unity, is given to a good approximation by

$$|\eta_{00}/\eta_{+-}|^2 \approx 1 - 6\mathrm{Re}\,(\epsilon'/\epsilon)$$
$$= 1 - 6(\epsilon'/\epsilon)\cos\left[\phi(\epsilon') - \phi(\epsilon)\right] \ . \tag{8}$$

Since the cos in Eq. (8) is expected theoretically to be very close to unity it is customary to say that $|\eta_{00}/\eta_{+-}|^2$ determines ϵ'/ϵ.

It is possible to use the values of ϕ_{+-} and $\phi_{00} - \phi_{+-}$ to set limits on CPT violation. [See Tests of Conservation Laws.]

Models

In the superweak model [5] CP violation is restricted to the mass mixing so that to a high degree of accuracy one expects $\epsilon' = 0$. The phase $\phi(\epsilon)$ is given in this model exactly by Eq. ($7a$) so that this has sometimes been referred to as the superweak phase; however, as noted above, all CPT-invariant models give Eq. ($7a$) as a very good approximation. In the Standard Model CP violation is entirely due to the phase in the Cabibbo-Kobayashi-Maskawa mixing matrix [6](q.v.). Since CP violation occurs in first order in decay amplitudes and in second order in mass-matrix mixing, one expects a significant nonzero value of ϵ'. The calculation is uncertain partly because m_t and V_{td} are not well known and primarily because of the difficulty of estimating hadronic matrix elements [7]. The theoretical results for ϵ'/ϵ in the Standard Model are generally in the range from 10^{-4} to 3×10^{-3}.

Fit for ϕ_{+-}, ϕ_{00}, ϕ_{00}–ϕ_{+-}, Δm, and τ_s

The Fermilab E773 experiment has published new results on the CP-violation phases ϕ_{+-} and ϕ_{00}, the $K^0_L - K^0_S$ mass difference Δm, and the K^0_S mean life τ_s (Document ID in our listings: SCHWINGENHEUER 95; reference [8]). The CPLEAR experiment has published new results on ϕ_{+-} (ADLER 95B [9]) and Δm (ADLER 95 [10]).

Fermilab E773 (SCHWINGENHEUER 95 [8]) and E731 (GIBBONS 93 [11]) measure $\phi_{+-} - \phi_f$ and calculate the regeneration phase ϕ_f from the power law momentum dependence of the regeneration amplitude using analyticity and dispersion relations. In the E731 result, a systematic error of ± 0.5 degrees for departures from a pure power-law is included. For the E773 result, they modeled a variety of effects that do distort the amplitude from a pure power law and ascribed a $\pm 0.35°$ systematic error from uncertainties in these effects. Even so, the E731 result remains valid within its quoted errors. KLEINKNECHT 94 [12] and KLEINKNECHT 95 [13] argue that these systematic errors should be around $3°$, primarily because of the absence of data on the momentum dependence of the regeneration amplitude above 160 GeV/c. BRIERE 95 [14] and BRIERE 95C [15] reply that the current understanding of regeneration is sufficient to allow a precise and reliable correction for the region above 160 GeV/c. The question is one of judgement about the reliability of the assumptions used. In the absence of any contradictory evidence, we choose to accept the judgement of the E731/E773 experimenters in setting their systematic errors.

In this edition we give a joint fit to the data on ϕ_{+-}, ϕ_{00}, $\phi_{00} - \phi_{+-}$, Δm, and τ_s, including the effects of correlations. Measurements of ϕ_{+-} and ϕ_{00} are highly correlated with Δm and τ_s. The correlations are given in the footnotes of the ϕ_{+-} and ϕ_{00} data listings. In earlier editions of the Review we adjusted the experimental values of ϕ_{+-} and ϕ_{00} to account for correlations with Δm and τ_s but did not include the effects of these correlations when evaluating Δm and τ_s. When a joint fit is done, the ϕ_{+-} measurements and their correlations have a strong influence on the fitted value of Δm. This is because the CERN NA31 vacuum regeneration experiments (CAROSI 90 [16] and GEWENIGER 74B [17]), the Fermilab E773/E731 regenerator experiments (SCHWINGENHEUER 95 [8] and GIBBONS 93 [11]), and the CPLEAR $K^0 - \overline{K}^0$ asymmetry experiment (ADLER 95B [9]) have very different dependences of ϕ_{+-} on Δm, as can be seen in Fig. 1. The correlations move the fitted Δm lower so that the ϕ_{+-} measurements are in good agreement with each other and with ϕ(superweak).

The (ϕ_{+-}, τ_s) correlations influence the τ_s fit result in a similar manner, as can be seen in Fig. 2. The influence of the ϕ_{+-} experiments is not as great on τ_s as it is on Δm because the indirect measurements of τ_s derived from the diagonal crossing bands in Fig. 2 are not as precise as the direct measurements of τ_s from E773/E731 (SCHWINGENHEUER 95 [8] and GIBBONS 93 [11]).

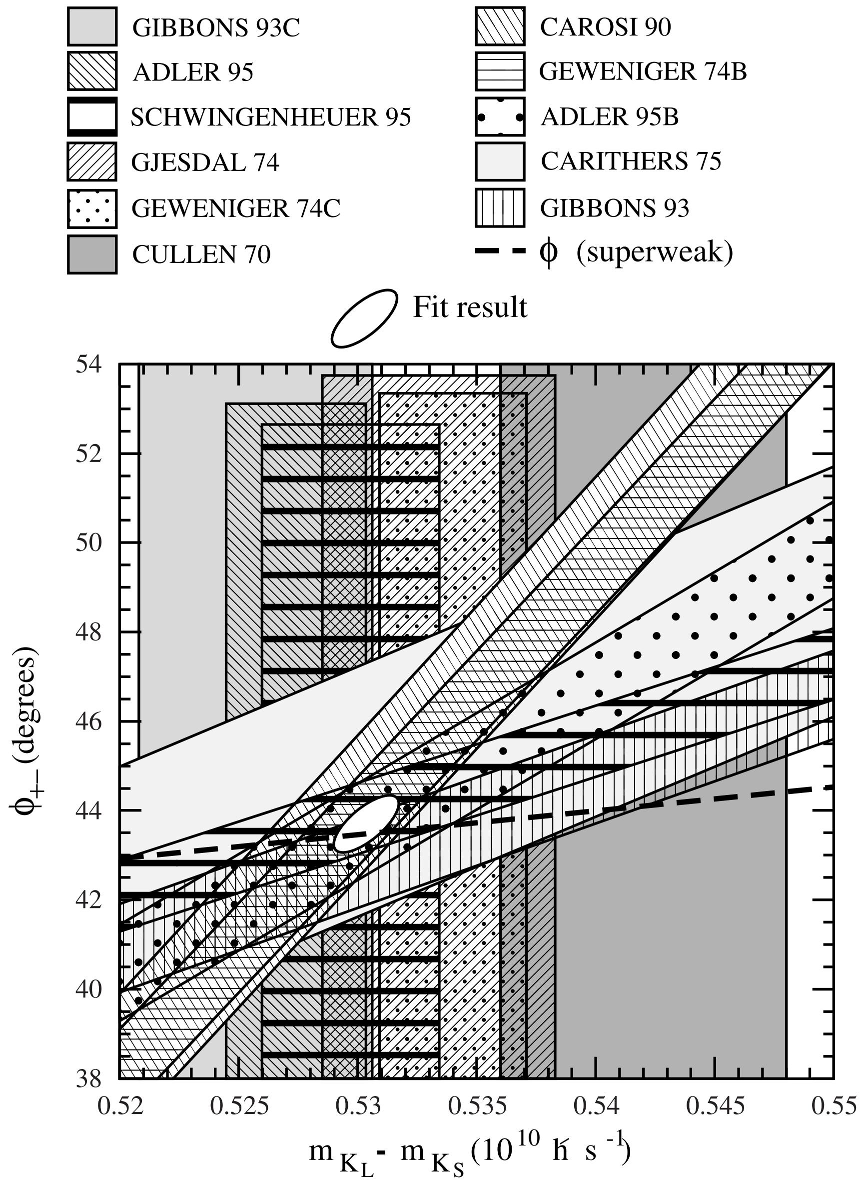

Figure 1: ϕ_{+-} vs Δm. Δm measurements appear as vertical bands spanning $\Delta m \pm 1\sigma$, some of which are cut near the top to aid the eye. The ϕ_{+-} measurements appear as diagonal bands spanning $\phi_{+-} \pm \sigma_\phi$. The dashed line shows ϕ(superweak). The ellipse shows the 1σ contour of the fit result. See Table 1 for data references.

In Fig. 1 [Fig. 2] the slope of the diagonal ϕ_{+-} bands shows the Δm [τ_s] dependence; the unseen τ_s [Δm] dependent term is evaluated using the fitted τ_s [Δm]. The vertical half-width σ_ϕ of each band is the ϕ_{+-} error for fixed Δm [τ_s] and includes the systematic error due to the error in the fitted τ_s [Δm].

Table 2 gives the resulting fit values for the parameters and Table 3 gives the correlation matrix.

Table 1: References for Fig. 1 and Fig. 2

Meas. / Fig. No.				
ϕ_{+-}	Δm	τ_s	PDG Document ID	Ref.
1,2			CAROSI 90	[16]
1		2	GEWENIGER 74B	[17]
1,2			ADLER 95B	[9]
1		2	CARITHERS 75	[18]
1,2	1	2	SCHWINGENHEUER 95	[8]
1,2		2	GIBBONS 93	[11]
	1		GIBBONS 93C	[19]
	1		ADLER 95	[10]
	1		GJESDAL 74	[20]
	1		GEWENIGER 74C	[21]
	1		CULLEN 70	[22]
		2	ARONSON 76	[23]
		2	GROSSMAN 87	[24]
		2	SKJEGGESTAD 72	[25]

A similar analysis has been done by the CPLEAR Collaboration [26]. The small differences between their results and ours are due primarily to different treatments of τ_s. Their fit constrains τ_s to the PDG 1994 value, while our fit includes the more recent SCHWINGENHEUER 95 [8] τ measurement.

Table 2: Results of the fit for ϕ_{+-}, ϕ_{00}, $\phi_{00} - \phi_{+-}$, Δm, and τ_s. The fit has $\chi^2 = 12.0$ for 18 degrees of freedom (22 measurements -5 parameters $+1$ constraint).

Quantity	Fit Result
ϕ_{+-}	$43.7 \pm 0.6°$
Δm	$(0.5304 \pm 0.0014) \times 10^{10}\hbar\ \mathrm{s}^{-1}$
τ_s	$(0.8927 \pm 0.0009) \times 10^{-10}\mathrm{s}$
ϕ_{00}	$43.5 \pm 1.0°$
$\Delta\phi$	$-0.2 \pm 0.8°$

Fit for ϵ'/ϵ, $|\eta_{+-}|$, $|\eta_{00}|$, and $\mathrm{B}(K_L \to \pi\pi)$

We list measurements of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$ and ϵ'/ϵ. Independent information on $|\eta_{+-}|$ and $|\eta_{00}|$ can be obtained

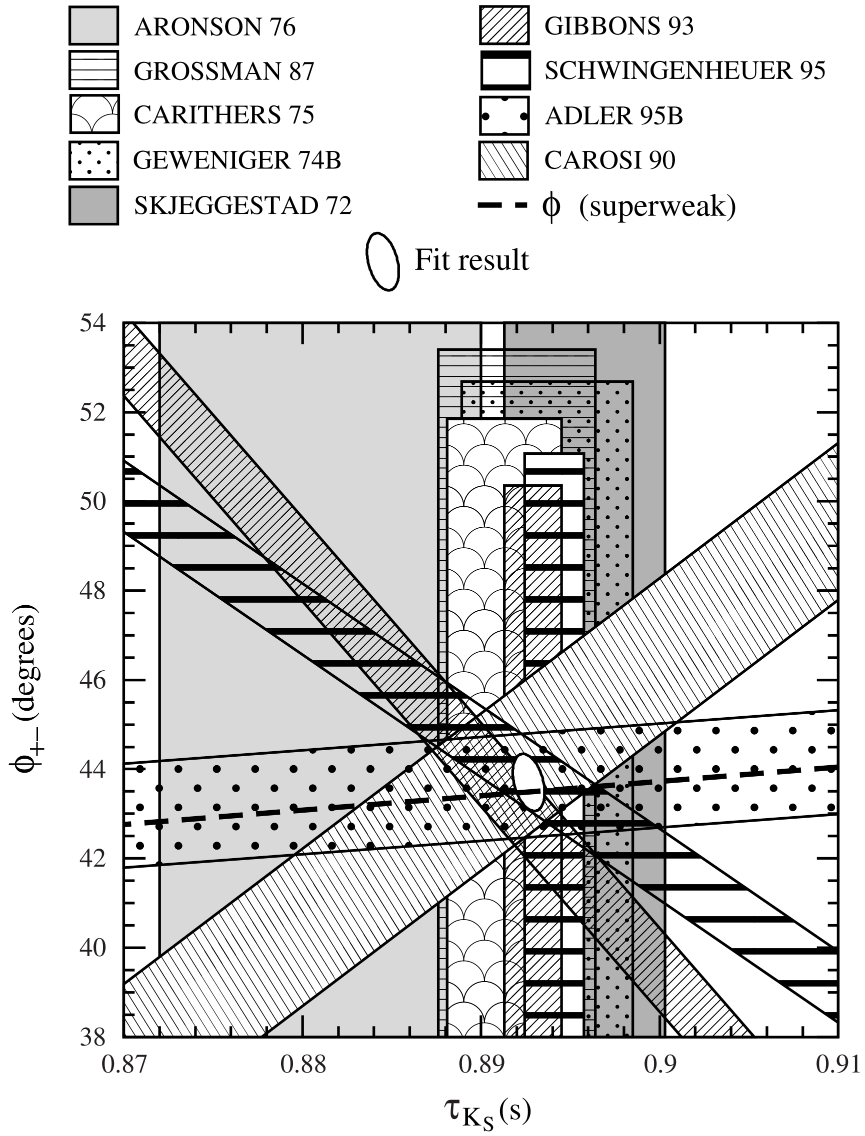

Figure 2: ϕ_{+-} vs τ_s. τ_s measurements appear as vertical bands spanning $\tau_s \pm 1\sigma$, some of which are cut near the top to aid the eye. The ϕ_{+-} measurements appear as diagonal bands spanning $\phi_{+-} \pm \sigma_\phi$. The dashed line shows ϕ(superweak). The ellipse shows the fit result's 1σ contour. See Table 1 for data references.

from measurements of the K_L^0 and K_S^0 lifetimes (τ_s) and branching ratios (B) to $\pi\pi$, using the relations

$$|\eta_{+-}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^+\pi^-)}{\tau(K_L^0)}\ \frac{\tau(K_S^0)}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}\right]^{1/2}, \qquad (9a)$$

$$|\eta_{00}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\tau(K_L^0)}\ \frac{\tau(K_S^0)}{\mathrm{B}(K_S^0 \to \pi^0\pi^0)}\right]^{1/2}. \qquad (9b)$$

For historical reasons the branching ratio fits and the CP-violation fits are done separately, but we want to include the

Table 3: Correlation matrix for the fitted parameters.

	ϕ_{+-}	Δm	τ_s	ϕ_{00}	$\Delta\phi$
ϕ_{+-}	1.00	0.71	−0.36	0.60	−0.02
Δm	0.71	1.00	−0.21	0.48	0.04
τ_s	−0.36	−0.21	1.00	−0.19	0.04
ϕ_{00}	0.60	0.48	−0.19	1.00	0.79
$\Delta\phi$	−0.02	0.04	0.04	0.79	1.00

influence of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$, and ϵ'/ϵ measurements on $B(K_L^0 \to \pi^+\pi^-)$ and $B(K_L^0 \to \pi^0\pi^0)$ and vice versa. We approximate a global fit to all of these measurements by first performing two independent fits: 1) BRFIT, a fit to the K_L^0 branching ratios, rates, and mean life, and 2) ETAFIT, a fit to the $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{+-}/\eta_{00}|$, and ϵ'/ϵ measurements. The results from fit 1, along with the K_S^0 values from this edition are used to compute values of $|\eta_{+-}|$ and $|\eta_{00}|$ which are included as measurements in the $|\eta_{00}|$ and $|\eta_{+-}|$ sections with a document ID of BRFIT 96. Thus the fit values of $|\eta_{+-}|$ and $|\eta_{00}|$ given in this edition include both the direct measurements and the results from the branching ratio fit.

The process is reversed in order to include the direct $|\eta|$ measurements in the branching ratio fit. The results from fit 2 above (before including BRFIT 96 values) are used along with the K_L^0 and K_S^0 mean lives and the $K_S^0 \to \pi\pi$ branching fractions to compute the K_L^0 branching ratios $\Gamma(K_L^0 \to \pi^+\pi^-)/\Gamma(\text{total})$ and $\Gamma(K_L^0 \to \pi^0\pi^0)/\Gamma(K_L^0 \to \pi^+\pi^-)$. These branching ratio values are included as measurements in the branching ratio section with a document ID of ETAFIT 96. Thus the K_L^0 branching ratio fit values in this edition include the results of direct measurements of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$, and ϵ'/ϵ. A more detailed discussion of these fits is given in the 1990 edition of this *Review* [27].

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