SEARCHES FOR QUARK AND LEPTON COMPOSITENESS

If quarks and leptons are made of constituents, then at the scale of constituent binding energies, there should appear new interactions among quarks and leptons. At energies much below the compositeness scale (Λ), these interactions are suppressed by inverse powers of Λ. The dominant effect should come from the lowest dimensional interactions with four fermions (contact terms), whose most general chirally invariant form reads [1]

$$L = \frac{g^2}{2\Lambda^2}\Big[\eta_{LL}\,\overline{\psi}_L\gamma_\mu\psi_L\,\overline{\psi}_L\gamma^\mu\psi_L + \eta_{RR}\,\overline{\psi}_R\gamma_\mu\psi_R\,\overline{\psi}_R\gamma^\mu\psi_R + 2\eta_{LR}\,\overline{\psi}_L\gamma_\mu\psi_L\,\overline{\psi}_R\gamma^\mu\psi_R\Big]\ . \quad (1)$$

Chiral invariance provides a natural explanation why quark and lepton masses are much smaller than their inverse size Λ. We may determine the scale Λ unambiguously by using the above form of the effective interactions; the conventional method [1] is to fix its scale by setting $g^2/4\pi = g^2(\Lambda)/4\pi = 1$ for the new strong interaction coupling and by setting the largest magnitude of the coefficients $\eta_{\alpha\beta}$ to be unity. In the following, we denote

$$\begin{aligned}
\Lambda &= \Lambda^{\pm}_{LL} \ \text{ for } \ (\eta_{LL}, \eta_{RR}, \eta_{LR}) = (\pm1,\, 0,\, 0)\ ,\\
\Lambda &= \Lambda^{\pm}_{RR} \ \text{ for } \ (\eta_{LL}, \eta_{RR}, \eta_{LR}) = (0,\, \pm1,\, 0)\ ,\\
\Lambda &= \Lambda^{\pm}_{VV} \ \text{ for } \ (\eta_{LL}, \eta_{RR}, \eta_{LR}) = (\pm1,\, \pm1,\, \pm1)\ ,\\
\Lambda &= \Lambda^{\pm}_{AA} \ \text{ for } \ (\eta_{LL}, \eta_{RR}, \eta_{LR}) = (\pm1,\, \pm1,\, \mp1)\ , \qquad (2)
\end{aligned}$$

as typical examples. Such interactions can arise by constituent interchange (when the fermions have common constituents, e.g., for $ee \to ee$) and/or by exchange of the binding quanta (whenever binding quanta couple to constituents of both particles).

Another typical consequence of compositeness is the appearance of excited leptons and quarks (ℓ^* and q^*). Phenomenologically, an excited lepton is defined to be a heavy lepton which shares leptonic quantum number with one of the existing leptons (an excited quark is defined similarly). For example, an excited electron e^* is characterized by a nonzero transition-magnetic coupling with electrons. Smallness of the lepton mass and the success of QED prediction for g–2 suggest chirality conservation, *i.e.*, an excited lepton should not couple to both left- and right-handed components of the corresponding lepton.

Excited leptons may be classified by SU(2)×U(1) quantum numbers. Typical examples are:

1. Sequential type

$$\begin{pmatrix} \nu^* \\ \ell^* \end{pmatrix}_L , \qquad [\nu_R^*] , \qquad \ell_R^* .$$

   ν_R^* is necessary unless ν^* has a Majorana mass.

2. Mirror type

$$[\nu_L^*] , \qquad \ell_L^* , \qquad \begin{pmatrix} \nu^* \\ \ell^* \end{pmatrix}_R .$$

3. Homodoublet type

$$\begin{pmatrix} \nu^* \\ \ell^* \end{pmatrix}_L , \qquad \begin{pmatrix} \nu^* \\ \ell^* \end{pmatrix}_R .$$

Similar classification can be made for excited quarks.

Excited fermions can be pair produced via their gauge couplings. The couplings of excited leptons with Z are listed in the following table (for notation see Eq. (1) in "Standard Model of Electroweak Interactions"):

	Sequential type	Mirror type	Homodoublet type
V^{ℓ^*}	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2} + 2\sin^2\theta_W$	$-1 + 2\sin^2\theta_W$
A^{ℓ^*}	$-\frac{1}{2}$	$+\frac{1}{2}$	0
$V^{\nu_D^*}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+1$
$A^{\nu_D^*}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0
$V^{\nu_M^*}$	0	0	—
$A^{\nu_M^*}$	$+1$	-1	—

Here ν^*_D (ν^*_M) stands for Dirac (Majorana) excited neutrino. The corresponding couplings of excited quarks can be easily obtained. Although form factor effects can be present for the gauge couplings at $q^2 \neq 0$, they are usually neglected.

In addition, transition magnetic type couplings with a gauge boson are expected. These couplings can be generally parametrized as follows:

$$
\begin{aligned}
\mathcal{L} = & \frac{\lambda_\gamma^{(f^*)} e}{2m_{f^*}} \overline{f}^* \sigma^{\mu\nu} (\eta_L \tfrac{1-\gamma_5}{2} + \eta_R \tfrac{1+\gamma_5}{2}) f F_{\mu\nu} \\
& + \frac{\lambda_Z^{(f^*)} e}{2m_{f^*}} \overline{f}^* \sigma^{\mu\nu} (\eta_L \tfrac{1-\gamma_5}{2} + \eta_R \tfrac{1+\gamma_5}{2}) f Z_{\mu\nu} \\
& + \frac{\lambda_W^{(\ell^*)} g}{2m_{\ell^*}} \overline{\ell}^* \sigma^{\mu\nu} \tfrac{1-\gamma_5}{2} \nu W_{\mu\nu} \\
& + \frac{\lambda_W^{(\nu^*)} g}{2m_{\nu^*}} \overline{\nu}^* \sigma^{\mu\nu} (\eta_L \tfrac{1-\gamma_5}{2} + \eta_R \tfrac{1+\gamma_5}{2}) \ell \, W^\dagger_{\mu\nu} \\
& + \text{h.c.} \,,
\end{aligned} \tag{3}
$$

where $g = e/\sin\theta_W$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the photon field strength, $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, *etc.* The normalization of the coupling is chosen such that

$$
\max(|\eta_L|, |\eta_R|) = 1 \ .
$$

Chirality conservation requires

$$
\eta_L \eta_R = 0 \ . \tag{4}
$$

These couplings can arise from SU(2)×U(1)-invariant higher-dimensional interactions. A well-studied model is the interaction of homodoublet type ℓ^* with the Lagrangian [2,3]

$$
\mathcal{L} = \frac{1}{2\Lambda} \overline{L}^* (g f \tfrac{\tau^a}{2} W^a_{\mu\nu} + g' f' Y B_{\mu\nu}) \tfrac{1-\gamma_5}{2} L + \text{h.c.} \,, \tag{5}
$$

where L denotes the lepton doublet (ν, ℓ), Λ is the compositeness scale, g, g' are SU(2) and U(1)$_Y$ gauge couplings, and $W^a_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths for SU(2) and U(1)$_Y$ gauge fields. The same interaction occurs for mirror-type excited leptons. For sequential-type excited leptons, the ℓ^* and ν^* couplings become unrelated, and the couplings receive the extra suppression of $(250\,\mathrm{GeV})/\Lambda$ or m_{L^*}/Λ. In any case, these couplings satisfy the relation

$$\lambda_W = -\sqrt{2}\sin^2\theta_W(\lambda_Z \cot\theta_W + \lambda_\gamma)\ . \tag{6}$$

Additional coupling with gluons is possible for excited quarks:

$$\mathcal{L} = \frac{1}{2\Lambda}\overline{Q}^*\sigma^{\mu\nu}\left(g_s f_s \tfrac{\lambda^a}{2} G^a_{\mu\nu} + g\, f \tfrac{\tau^a}{2} W^a_{\mu\nu} + g'\, f' Y B_{\mu\nu}\right) \times \tfrac{1-\gamma_5}{2} Q + \mathrm{h.c.}\ , \tag{7}$$

where Q denotes a quark doublet, g_s is the QCD gauge coupling, and $G^a_{\mu\nu}$ the gluon field strength.

Some experimental analyses assume the relation $\eta_L = \eta_R = 1$, which violates chiral symmetry. We encode the results of such analyses if the crucial part of the cross section is proportional to the factor $\eta_L^2 + \eta_R^2$ and the limits can be reinterpreted as those for chirality conserving cases $(\eta_L, \eta_R) = (1,0)$ or $(0,1)$ after rescaling λ.

Several different conventions are used by LEP experiments to express the transition magnetic couplings. To facilitate comparison, we reexpress these in terms of λ_Z and λ_γ using the following relations and taking $\sin^2\theta_W = 0.23$. We assume chiral couplings, *i.e.*, $|c| = |d|$ in the notation of Ref. 2.

1. ALEPH (charged lepton and neutrino)

$$\lambda_Z^{\mathrm{ALEPH}} = \frac{1}{2}\lambda_Z \quad \text{(1990 papers)} \tag{8a}$$

$$\frac{2c}{\Lambda} = \frac{\lambda_Z}{m_{\ell^*}[\text{or } m_{\nu^*}]} \quad (\text{for } |c| = |d|) \tag{8b}$$

2. ALEPH (quark)

$$\lambda_u^{\mathrm{ALEPH}} = \frac{\sin\theta_W\cos\theta_W}{\sqrt{\dfrac{1}{4} - \dfrac{2}{3}\sin^2\theta_W + \dfrac{8}{9}\sin^4\theta_W}}\,\lambda_Z = 1.11\lambda_Z \tag{9}$$

3. L3 and DELPHI (charged lepton)

$$\lambda^{\mathrm{L3}} = \lambda_Z^{\mathrm{DELPHI}} = -\frac{\sqrt{2}}{\cot\theta_W - \tan\theta_W}\,\lambda_Z = -1.10\lambda_Z \tag{10}$$

4. L3 (neutrino)

$$f_Z^{\mathrm{L3}} = \sqrt{2}\lambda_Z \tag{11}$$

5. OPAL (charged lepton)

$$\frac{f^{\mathrm{OPAL}}}{\Lambda} = -\frac{2}{\cot\theta_W - \tan\theta_W}\,\frac{\lambda_Z}{m_{\ell^*}} = -1.56\,\frac{\lambda_Z}{m_{\ell^*}} \tag{12}$$

6. OPAL (quark)

$$\frac{f^{\mathrm{OPAL}}c}{\Lambda} = \frac{\lambda_Z}{2m_{q^*}} \quad (\mathrm{for}\ |c| = |d|) \tag{13}$$

7. DELPHI (charged lepton)

$$\lambda_\gamma^{\mathrm{DELPHI}} = -\frac{1}{\sqrt{2}}\,\lambda_\gamma \tag{14}$$

If leptons are made of color triplet and antitriplet constituents, we may expect their color-octet partners. Transitions between the octet leptons (ℓ_8) and the ordinary lepton (ℓ) may take place via the dimension-five interactions

$$\mathcal{L} = \frac{1}{2\Lambda}\sum_\ell \left\{ \overline{\ell}_8^\alpha\, g_S\, F_{\mu\nu}^\alpha\, \sigma^{\mu\nu} \left(\eta_L\, \ell_L + \eta_R\, \ell_R\right) + h.c. \right\} \tag{15}$$

where the summation is over charged leptons and neutrinos. The leptonic chiral invariance implies $\eta_L\,\eta_R = 0$ as before.

References

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3. N. Cabibbo, L. Maiani, and Y. Srivastava, Phys. Lett. **139B**, 459 (1984).