9. QUANTUM CHROMODYNAMICS

9.1. The QCD Lagrangian

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Quantum Chromodynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons, is one of the components of the SU(3)×SU(2)×U(1) Standard Model. A quark of specific flavor (such as a charm quark) comes in 3 colors; gluons come in eight colors; hadrons are color-singlet combinations of quarks, anti-quarks, and gluons. The Lagrangian describing the interactions of quarks and gluons is (up to gauge-fixing terms)

$$\begin{aligned} L_{\rm QCD} = &-\frac{1}{4} F^{(a)}_{\mu\nu}\, F^{(a)\mu\nu} + i \sum_q \overline{\psi}^i_q\, \gamma^\mu\, (D_\mu)_{ij}\, \psi^j_q \\ &- \sum_q m_q\, \overline{\psi}^i_q\, \psi_{qi}\ , \end{aligned} \tag{9.1}$$

$$F^{(a)}_{\mu\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu + g_s\, f_{abc}\, A^b_\mu\, A^c_\nu\ , \tag{9.2}$$

$$(D_\mu)_{ij} = \delta_{ij}\, \partial_\mu - i g_s\, \sum_a \frac{\lambda^a_{i,j}}{2} A^a_\mu\ , \tag{9.3}$$

where g_s is the QCD coupling constant, and the f_{abc} are the structure constants of the SU(3) algebra (the λ matrices and values for f_{abc} can be found in "SU(3) Isoscalar Factors and Representation Matrices," Sec. 32 of this *Review*). The $\psi^i_q(x)$ are the 4-component Dirac spinors associated with each quark field of (3) color i and flavor q, and the $A^a_\mu(x)$ are the (8) Yang-Mills (gluon) fields. A complete list of the Feynman rules which derive from this Lagrangian, together with some useful color-algebra identities, can be found in Ref. 1.

The principle of "asymptotic freedom" (see below) determines that the renormalized QCD coupling is small only at high energies, and it is only in this domain that high-precision tests—similar to those in QED—can be performed using perturbation theory. Nonetheless, there has been in recent years much progress in understanding and quantifying the predictions of QCD in the nonperturbative domain, for example, in soft hadronic processes and on the lattice [2]. This short review will concentrate on QCD at short distances (large momentum transfers), where perturbation theory is the standard tool. It will discuss the processes that are used to determine the coupling constant of QCD. Other recent reviews of the coupling constant measurements may be consulted for a different perspective [3].

9.2. The QCD coupling and renormalization scheme

The renormalization scale dependence of the effective QCD coupling $\alpha_s = g_s^2/4\pi$ is controlled by the β-function:

$$\mu\frac{\partial\alpha_s}{\partial\mu} = -\frac{\beta_0}{2\pi}\,\alpha_s^2 - \frac{\beta_1}{4\pi^2}\,\alpha_s^3 - \frac{\beta_2}{64\pi^3}\,\alpha_s^4 - \cdots , \tag{9.4a}$$

$$\beta_0 = 11 - \frac{2}{3}n_f , \tag{9.4b}$$

$$\beta_1 = 51 - \frac{19}{3}n_f , \tag{9.4c}$$

$$\beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2 ; \tag{9.4d}$$

where n_f is the number of quarks with mass less than the energy scale μ. In solving this differential equation for α_s, a constant of integration is introduced. This constant is the one fundamental constant of QCD that must be determined from experiment. The most sensible choice for this constant is the value of α_s at a fixed-reference scale μ_0, but it is more conventional to introduce the dimensional parameter Λ, since this provides a parametrization of the μ dependence of α_s. The definition of Λ is arbitrary. One way to define it (adopted here) is to write a solution of Eq. (9.4) as an expansion in inverse powers of $\ln(\mu^2)$:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0\,\ln(\mu^2/\Lambda^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\,\frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4\ln^2(\mu^2/\Lambda^2)}\right.$$
$$\left.\times\left(\left(\ln\left[\ln(\mu^2/\Lambda^2)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right] . \tag{9.5a}$$

The last term in this expansion is

$$\mathcal{O}\left(\frac{\ln^2\left[\ln(\mu^2/\Lambda^2)\right]}{\ln^3(\mu^2/\Lambda^2)}\right) , \tag{9.5b}$$

and is usually neglected in the definition of Λ. We choose to include it even though its effect on $\alpha_s(\mu)$ is smaller than the experimental errors. For a fixed value of $\alpha_s(M_Z)$, the inclusion of this term shifts the value of Λ by ~ 15 MeV. This solution illustrates the *asymptotic freedom* property: $\alpha_s \to 0$ as $\mu \to \infty$. Alternative definitions of Λ are possible. We adopt this as the standard. Values given by experiments using other definitions are adjusted as needed to meet our definition.

Consider a "typical" QCD cross section which, when calculated perturbatively, starts at $\mathcal{O}(\alpha_s)$:

$$\sigma = A_1\ \alpha_s + A_2\ \alpha_s^2 + \ \cdots\ . \tag{9.6}$$

The coefficients A_1, A_2 come from calculating the appropriate Feynman diagrams. In performing such calculations, various divergences arise, and these must be regulated in a consistent way. This requires a particular renormalization scheme (RS). The most commonly used one is the modified minimal subtraction ($\overline{\text{MS}}$) scheme [4]. This involves continuing momentum integrals from 4 to 4–2ϵ dimensions, and then subtracting off the resulting $1/\epsilon$ poles and also $(\ln 4\pi - \gamma_E)$, which is another artifact of continuing the dimension. (Here γ_E is the Euler-Mascheroni constant.) To preserve the dimensionless nature of the coupling, a mass scale μ must also be introduced: $g \rightarrow \mu^\epsilon g$. The finite coefficients A_i thus obtained depend implicitly on the renormalization convention used and explicitly on the scale μ.

The first two coefficients (β_0, β_1) in Eq. (9.4) are independent of the choice of RS's. In contrast, the coefficients of terms proportional to α_s^n for $n > 3$ *are* RS-dependent. The form given above for β_2 is in the $\overline{\text{MS}}$ scheme. It has become conventional to use the $\overline{\text{MS}}$ scheme for calculating QCD cross sections beyond leading order.

The fundamental theorem of RS dependence is straightforward. Physical quantities, in particular the cross section, calculated to all orders in perturbation theory, do not depend on the RS. It follows that a truncated series *does* exhibit RS dependence. In practice, QCD cross sections are known to leading order (LO), or to next-to-leading order (NLO), or in a few cases, to next-to-next-to-leading order (NNLO); and it is only the latter two cases, which have reduced RS dependence, that are useful for precision tests. At NLO the RS dependence is completely given by one condition which can be taken to be the value of the renormalization scale μ. At NNLO this is not sufficient, and μ is no longer equivalent to a choice of scheme; both must now be specified. One, therefore, has to address the question of what is the "best" choice for μ. There is no definite answer to this question—higher-order corrections do not "fix" the scale, rather they render the theoretical predictions less sensitive to its variation.

One could imagine that choosing a scale μ characteristic of the typical energy scale (E) in the process would be most appropriate. In general, a poor choice of scale generates terms of order $\ln(E/\mu)$ in the A_i's. Various methods have been proposed including choosing: the scale for which the next-to-leading-order correction vanishes ("Fastest Apparent Convergence [5]"); the scale for which the next-to-leading-order prediction is stationary [6], (*i.e.*, the value of μ where $d\sigma/d\mu = 0$); or the scale dictated by the effective charge scheme [7] or by the BLM scheme [8]. By comparing the values of α_s that different reasonable schemes give, an estimate of theoretical errors can be obtained.

An important corollary is that if the higher-order corrections are naturally small, then the additional uncertainties introduced by the μ dependence are likely to be less than the experimental measurement errors. There are some processes, however, for which the choice of scheme *can* influence the extracted value of $\Lambda_{\overline{\text{MS}}}$. There is no resolution to this problem other than to try to calculate even more terms in the perturbation series.

It is important to note that, since the perturbation series is an asymptotic expansion, there is a limit to the precision with which any theoretical quantity can be calculated. In some processes, the highest-order perturbative terms may be comparable in size to nonperturbative corrections (sometimes called higher-twist or renormalon effects, for a discussion see [9]); an estimate of these terms and their uncertainties is required if a value of α_s is to be extracted.

In the cases where the higher-order corrections to a process are known and are large, some caution should be exercised when quoting the value of α_s. In what follows, we will attempt to indicate the size of the theoretical uncertainties on the extracted value of α_s. There are two simple ways to determine this error. First, we can estimate it by comparing the value of $\alpha_s(\mu)$ obtained by fitting data using the QCD formula to highest known order in α_s, and then comparing it with the value obtained using the next-to-highest-order formula (μ is chosen as the typical energy scale in the process). The corresponding Λ's are then obtained by evolving $\alpha_s(\mu)$ to $\mu = m_Z$ using Eq. (9.4) to the same order in α_s as the fit, and then converting to $\Lambda^{(4)}$ using Eq. (9.7). Alternatively, we can vary the value of μ over a reasonable range, extracting a value of Λ for each choice of μ. This method is of its nature imprecise, since "reasonable" involves a subjective judgment. In either case, if the perturbation series is well behaved, the resulting error on Λ will be small.

In the above discussion we have ignored quark-mass effects, *i.e.*, we have assumed an idealized situation where quarks of mass greater than μ are neglected completely. In this picture, the β-function coefficients change by discrete amounts as flavor thresholds are crossed when integrating the differential equation for α_s. It follows that, for a relationship such as Eq. (9.5) to remain valid for all values of μ, Λ must also change as flavor thresholds are crossed. This leads to the concept of a different Λ for each range of μ corresponding to an effective number of massless quarks: $\Lambda \rightarrow \Lambda^{(n_f)}$. There is some arbitrariness in how this relationship is set up. As an idealized case, consider QCD with $n_f - 1$ massless quarks and one quark of mass M. Now imagine an experiment at energy scale μ; for example, this could be $e^+e^- \rightarrow hadrons$ at center-of-mass energy μ. If $\mu \gg M$, the mass M is negligible and the process is well described by QCD with n_f massless flavors and its parameter $\Lambda^{(n_f)}$ up to terms of order M^2/μ^2. Conversely if $\mu \ll M$, the heavy quark plays no role and the process is well described by QCD with $n_f - 1$ massless flavors and its parameter $\Lambda^{(n_f-1)}$ up to terms of order μ^2/M^2. If $\mu \sim M$, the effects of the quark mass are process-dependent and cannot be absorbed into the running coupling.

A mass scale μ' is chosen where the relationship between $\Lambda^{(nf-1)}$ and $\Lambda^{(nf)}$ will be fixed. μ' should be of order M and the relationship should not depend on it. A prescription has been given [10] which has this property. We use this procedure choosing $\mu' = M_Q$, where M_Q is the mass of the value of the running quark mass defined in the $\overline{\mathrm{MS}}$ scheme (see the note on "Quark Masses" in the Particle Listings for more details), *i.e.*, where $M_{\overline{\mathrm{MS}}}(M_Q) = M_Q$. Then [10]

$$
\begin{aligned}
\beta_0^{nf-1} \ln\left(\frac{\Lambda^{(nf)}}{\Lambda^{(nf-1)}}\right)^2 &= (\beta_0^{nf} - \beta_0^{nf-1})\ \ln\left(\frac{M_Q}{\Lambda^{(nf)}}^2\right) \\
&+ 2\left(\frac{\beta_1^{nf}}{\beta_0^{nf}} - \frac{\beta_1^{nf-1}}{\beta_0^{nf-1}}\right)\ \ln\left[\ln\left(\frac{M_Q}{\Lambda^{(nf)}}\right)^2\right] \\
&- \frac{2\beta_1^{nf-1}}{\beta_0^{nf-1}}\ \ln\left(\frac{\beta_0^{nf}}{\beta_0^{nf-1}}\right) \\
&+ \frac{4\dfrac{\beta_1^{nf}}{(\beta_0^{nf})^2}\left(\dfrac{\beta_1^{nf}}{\beta_0^{nf}} - \dfrac{\beta_1^{nf-1}}{\beta_0^{nf-1}}\right)\ \ln\left[\ln\left(\dfrac{M_Q}{\Lambda^{nf}}\right)^2\right]}{\ln\left(\dfrac{M_Q}{\Lambda^{(nf)}}\right)^2} \\
&+ \frac{\dfrac{1}{\beta_0^{nf}}\left[\left(\dfrac{2\beta_1^{nf}}{\beta_0^{nf}}\right)^2 - \left(\dfrac{2\beta_1^{nf-1}}{\beta_0^{nf-1}}\right)^2 - \dfrac{2\beta_2^{nf}}{\beta_0^{nf}} + \dfrac{2\beta_2^{nf-1}}{\beta_0^{nf-1}} - \dfrac{14}{9}\right]}{\ln\left(\dfrac{M_Q}{\Lambda^{(nf)}}\right)^2} \ .
\end{aligned}
\tag{9.7}
$$

This result is valid to order α_s^3 (or alternatively to terms of order $1/\ln^2[(M_Q/\Lambda^{(nf)})^2]$).

An alternative matching procedure can be used [11]. This procedure requires the equality $\alpha_s(\mu)^{(nf)} = \alpha_s(\mu)^{(nf-1)}$ for $\mu = M_Q$. This matching is somewhat arbitrary; a different relation between $\Lambda^{(nf)}$ and $\Lambda^{(nf-1)}$ would result if $\mu = M_Q/2$ were used. In practice, the differences between these procedures are very small. $\Lambda^{(5)} = 200$ MeV corresponds to $\Lambda^{(4)} = 289$ MeV in the scheme of Ref. 11 and $\Lambda^{(4)} = 280$ MeV in the scheme adopted above. Note that the differences between $\Lambda^{(5)}$ and $\Lambda^{(4)}$ are numerically very significant.

Data from deep-inelastic scattering are in a range of energy where the bottom quark is not readily excited, and hence, these experiments quote $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$. Most data from PEP, PETRA, TRISTAN, LEP, and SLC quote a value of $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$ since these data are in an energy range where the bottom quark is light compared to the available energy. We have converted it to $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$ as required. A few measurements, including the lattice gauge theory values from the ψ system and from τ decay are at sufficiently low energy that $\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ is appropriate.

We turn now to a discussion of renormalization-scheme dependence in QCD. Although necessarily rather technical, this discussion is vital to understanding how α_s (or Λ) values can be measured and compared. See the review by Duke and Roberts [12] for further details.

In order to compare the values of α_s from various experiments, they must be evolved using the renormalization group to a common scale. For convenience, this is taken to be the mass of the Z boson. This evolution uses third-order perturbation theory and can introduce additional errors particularly if extrapolation from very small scales is used. The variation in the charm and bottom quark masses ($m_b = 4.3 \pm 0.2$ and $m_c = 1.3 \pm 0.3$ are used) can also introduce errors. These result in a fixed value of $\alpha_s(2\ \text{GeV})$, giving an uncertainty in $\alpha_s(M_Z) = \pm 0.001$ if only perturbative evolution is used. There could be additional errors from nonperturbative effects that enter at low energy. All values are in the $\overline{\text{MS}}$ scheme unless otherwise noted.

9.3. QCD in deep-inelastic scattering

The original and still one of the most powerful quantitative tests of perturbative QCD is the breaking of Bjorken scaling in deep-inelastic lepton-hadron scattering. In the leading-logarithm approximation, the measured structure functions $F_i(x, Q^2)$ are related to the quark distribution functions $q_i(x, Q^2)$ according to the naive parton model, by the formulae in "Cross-section Formulae for Specific Processes," Sec. 35 of this *Review.* (In that section, q_i is denoted by the notation f_q). In describing the way in which scaling is broken in QCD, it is convenient to define nonsinglet and singlet quark distributions:

$$F^{NS} = q_i - q_j \qquad\qquad F^S = \sum_i (q_i + \overline{q}_i) \ . \tag{9.8}$$

The nonsinglet structure functions have nonzero values of flavor quantum numbers such as isospin or baryon number. The variation with Q^2 of these is described by the so-called DGLAP equations [13,14]:

$$Q^2 \frac{\partial F^{NS}}{\partial Q^2} = \frac{\alpha_s(|Q|)}{2\pi} P^{qq} * F^{NS} \tag{9.9a}$$

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} F^S \\ G \end{pmatrix} = \frac{\alpha_s(|Q|)}{2\pi} \begin{pmatrix} P^{qq} & 2n_f P^{qg} \\ P^{gq} & P^{gg} \end{pmatrix} * \begin{pmatrix} F^S \\ G \end{pmatrix} \tag{9.9b}$$

where $*$ denotes a convolution integral:

$$f * g = \int_x^1 \frac{dy}{y} \, f(y) \, g\left(\frac{x}{y}\right) \ . \tag{9.10}$$

The leading-order Altarelli-Parisi [14] splitting functions are

$$P^{qq} = \frac{4}{3} \left[\frac{1 + x^2}{(1 - x)_+} \right] + 2\delta(1 - x) \ , \tag{9.11a}$$

$$P^{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right] \ , \tag{9.11b}$$

$$P^{gq} = \frac{4}{3}\left[\frac{1+(1-x)^2}{x}\right] \ , \tag{9.11c}$$

$$P^{gg} = 6\left[\frac{1-x}{x} + x(1-x) + \frac{x}{(1-x)_+} + \frac{11}{12}\delta(1-x)\right] - \frac{n_f}{3}\delta(1-x) \ . \tag{9.11d}$$

Here the gluon distribution $G(x,Q^2)$ has been introduced and $1/(1-x)_+$ means

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \ \frac{f(x)-f(1)}{(1-x)} \ . \tag{9.12}$$

The precision of contemporary experimental data demands that higher-order corrections also be included [15]. The above results are for massless quarks. Algorithms exist for the inclusion of nonzero quark masses [16]. At low Q^2 values, there are also important "higher-twist" (HT) contributions of the form:

$$F_i(x,Q^2) = F_i^{(LT)}\ (x,Q^2) + \frac{F_i^{(HT)}(x,Q^2)}{Q^2} + \ \cdots \ . \tag{9.13}$$

Leading twist (LT) indicates a term whose behavior is predicted by perturbative QCD. These corrections are numerically important only for $Q^2 < \mathcal{O}(10\,\mathrm{GeV}^2)$ except for x very close to 1.

A detailed review of the current status of the experimental data can be found, for example, in Refs. [17–20], and only a brief summary will be presented here. We shall only include determinations of Λ from the recently published results; the earlier editions of this *Review* should be consulted for the earlier data. In any event, the recent results will dominate the average since their errors are smaller. Data have now appeared from HERA at much smaller values of x than the previous data. They provide valuable information about the shape of the antiquark and gluon distribution functions at $x \sim 10^{-3}$ [21].

From Eq. (9.9), it is clear that a nonsinglet structure function offers in principle the most precise test of the theory, since the Q^2 evolution is independent of the unmeasured gluon distribution. The CCFR collaboration fit to the Gross-Llewellyn Smith sum rule [22] is known to order α_s^3 [23]

$$\int_0^1 dx(F_3^{\overline{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2)) = 3\left[(1 - \frac{\alpha_s}{\pi}(1 + 3.58\frac{\alpha_s}{\pi} + 19.0(\frac{\alpha_s}{\pi})^2) - \mathit{\Delta} HT\right] \ , \tag{9.14}$$

where the higher-twist contribution $\Delta HT = (0.09 \pm 0.045)/Q^2$ [23,24]. Using the CCFR data [25], this gives α_s (1.76 GeV) $= 0.26 \pm 0.035$ (expt.) $\pm$ 0.03 (theory). The error from higher-twist terms dominates the theoretical error, the higher-twist term being approximately 50% larger than the α_s^3 term.

A measurement of Λ has been made using F_3 in neutrino scattering [27]. The result is $\Lambda^{(4)}_{\overline{\rm MS}} = 179 \pm 36 \pm 41$ MeV. The errors are statistical and systematic but do not include (theoretical) errors arising from the choice of μ^2. Measurements involving singlet-dominated structure functions, such as F_2, result in correlated measurements of $\Lambda^{(4)}_{\overline{\rm MS}}$ and the gluon distribution. By utilizing high-statistics data at large x (> 0.25) and large Q^2, where F_2 behaves like an nonsinglet and F_3 at smaller x, a nonsinglet fit can be performed with better statistical precision, and hence, the error on the measured value of $\Lambda^{(4)}_{\overline{\rm MS}}$ is much reduced. CCFR gives $\Lambda^{(4)}_{\overline{\rm MS}} = 210 \pm 28 \pm 41$ MeV [27] from $F_2(\nu N)$ and $F_3(\nu N)$. There is an additional uncertainty of ± 59 MeV from the choice of scale. The NMC collaboration [28] gives $\alpha_s(7\ \text{GeV}^2) = 0.264 \pm 0.018(\text{stat.}) \pm 0.070(\text{syst.}) \pm 0.013(\text{ higher-twist})$. The systematic error is larger than the CCFR result, partially because the data are at smaller values of x and the gluon distribution is more important. A reanalysis [29] of EMC data [30] gives $\Lambda^{(4)}_{\overline{\rm MS}} = 211 \pm 80 \pm 80$ MeV from $F_2(\nu N)$. Finally a combined analysis [31] of SLAC [32] and BCDMS [33] data gives $\Lambda^{(4)}_{\overline{\rm MS}} = 263 \pm 42 \pm 55$ MeV. Here the systematic error is an estimate of the uncertainty due to the choice of Q^2 used in the argument of α_s, and in the scale at which the structure functions (factorization scale) used in the QCD calculation are evaluated.

The results from Refs. [27–29] and [31] can be combined to give $\alpha_s(M_Z) = 0.112 \pm 0.002 \pm 0.004$, or equivalently $\Lambda^{(4)}_{\overline{\rm MS}} = 234 \pm 26 \pm 50$ MeV. Here the first error is a combination of statistical and systematic errors, and the second error is due to the scale uncertainty. This result is an average of the results weighted by their statistical and systematic errors. The scale error which is common to all is then reapplied to the average.

The spin-dependent structure functions can also be used to determine α_s. Here the values of $Q^2 \sim 2.5$ GeV^2 are small and higher-twist corrections are again important. The values extracted are consistent with the average quote below [26].

At very small values of x and large Q^2, the x-dependence of the structure functions is predicted by perturbative QCD [34]. Here terms to all orders in $\alpha_s \ln(1/x)$ are summed. The data from HERA [21] on $F_2^{ep}(x, Q^2)$ have been fitted to the this form [35], including the NLO terms which are required to fix the Q^2 scale. The data are dominated by $4\ \text{GeV}^2 < Q^2 < 100\ \text{GeV}^2$. The fit gives $\alpha_s(M_Z) = 0.120 \pm 0.005$ (expt.)± 0.009 (theory). The dominant part of the theoretical error is from the scale dependence. The fit neglects terms which are suppressed by $1/\ln(1/x)$. Hence, the uncertainties from this source cannot be estimated and are not included in the quoted error. This result is not averaged with the other ones from scaling violations, since the values there are derived from the Q^2 dependence alone, and this possible source of error is not present.

Typically, Λ is extracted from the data by parametrizing the parton densities in a simple analytic way at some Q_0^2, evolving to higher Q^2 using the next-to-leading-order evolution equations, and fitting globally to the measured structure functions to obtain $\Lambda^{(4)}_{\overline{\rm MS}}$. Thus, an important by-product of such studies is the extraction of parton densities at a fixed-reference value of Q_0^2. These can then be evolved in Q^2 and used as input for phenomenological studies in hadron-hadron collisions (see below). To avoid having to evolve from the starting Q_0^2 value each time, a parton density is required; it is useful to have available a simple analytic approximation to the densities valid over a range of x and Q^2 values. A package is available from the CERN computer library that includes an exhaustive set of fits [36]. Some of these fits are obsolete. In using a parameterization to predict event rates, a next-to-leading order fit must be used if the process being calculated is known to next-to-leading order in QCD perturbation theory. In such a case, there is an additional scheme dependence; this scheme dependence is reflected in the $\mathcal{O}(\alpha_s)$ corrections that appear in the relations between the structure functions and the quark distribution functions. There are two common schemes: a deep-inelastic scheme where there are no order α_s corrections in the formula for $F_2(x,Q^2)$ and the minimal subtraction scheme. It is important when these next-to-leading order fits are used in other processes (see below), that the same scheme is used in the calculation of the partonic rates.

9.4. QCD in decays of the τ lepton

The semi-leptonic branching ratio of the tau ($\tau \to \nu_\tau + hadrons$, R_τ) is an inclusive quantity. It is related to the contribution of hadrons to the imaginary part of the W self energy $\big(\Pi(s)\big)$. However, it is more inclusive than R since it involves an integral

$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}(1-\frac{s}{m_\tau^2})^2 \; {\rm Im}\,(\Pi(s)) \ .$$

Since the scale involved is low, one must take into account nonperturbative (higher-twist) contributions which are suppressed by powers of the τ mass.

$$\begin{aligned} R_\tau = & 3.058\Bigg[1+\frac{\alpha_s(m_\tau)}{\pi}+5.2\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^2+26.4\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^3 \\ & + a\frac{m^2}{m_\tau^2}+b\frac{m\psi\overline{\psi}}{m_\tau^4}+c\frac{\psi\overline{\psi}\psi\overline{\psi}}{m_\tau^6}+\cdots\Bigg] \ . \end{aligned} \tag{9.15}$$

Here a, b, and c are dimensionless constants and m is a light quark mass. The term of order $1/m_\tau^2$ is a kinematical effect due to the light quark masses and is consequently very small. The nonperturbative terms are estimated using sum rules [37]. In total, they are estimated to be -0.007 ± 0.004 [38]. This estimate relies on there being no term of order Λ^2/m_τ^2 $\left(\text{note that } \frac{\alpha_s(m_\tau)}{\pi} \sim (\frac{0.5 \ \text{GeV}}{m_\tau})^2\right)$. The a, b, and c can be determined from the data [39] by fitting to moments of the $\Pi(s)$. The values so extracted [40,41] are consistent with the theoretical estimates. If the nonperturbative terms are omitted from the fit, the extracted value of $\alpha_s(m_\tau)$ decreases by ~ 0.02.

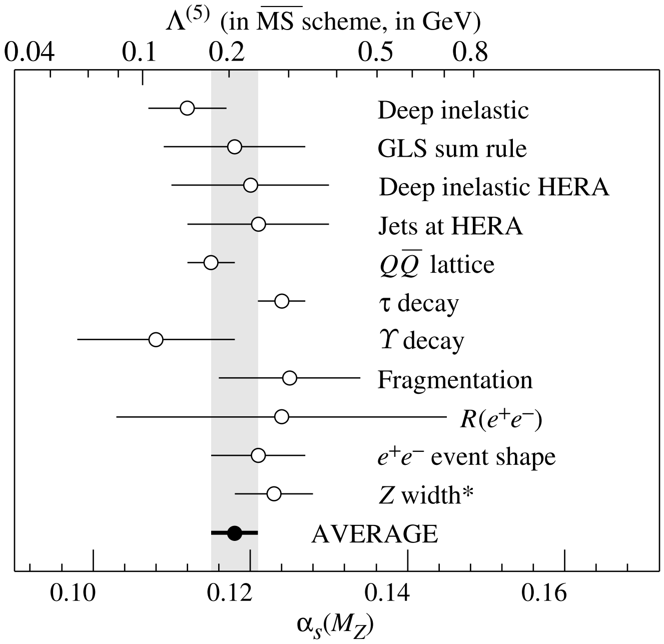

Figure 9.1: Summary of the values of $\alpha_s(M_Z)$ and $\Lambda^{(5)}$ from various processes ordered from top to bottom by increasing energy scale of the measurements. The values shown indicate the process and the measured value of α_s extrapolated up to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties. The value denoted by '*' is not used in the average (see text).

For $\alpha_s(m_\tau) = 0.37$ the perturbative series for R_τ is $R_\tau \sim 3.058(1+0.118+0.072+0.043)$. The size (estimated error) of the nonperturbative term is 20% (7%) of the size of the order α_s^3 term. The perturbation series in not very well convergent; if the order α_s^3 term is omitted, the extracted value of $\alpha_s(m_\tau)$ increases by 0.05. R_τ, can be extracted from the semi-leptonic branching ratio from the relation $R_\tau = 1/(\mathrm{B}(\tau \to e\nu\overline{\nu}) - 1.97256$; where $\mathrm{B}(\tau \to e\nu\overline{\nu})$ is measured directly or extracted from the lifetime, the muon mass and the muon lifetime assuming universality of lepton couplings. Using the average lifetime of 291.3 ± 1.6 fs [42] and a τ mass of $1.776.96 \pm 0.30$ [43] gives $R_\tau = 3.633 \pm 0.031$. Assuming e/μ universality, the data give $\mathrm{B}(\tau \to e\nu\overline{\nu}) = 0.1780 \pm 0.0006$ [44]. Averaging these yields $\alpha_s(m_\tau) = 0.370 \pm 0.008$ using the experimental error alone. This result is consistent with measurements reported recently by other collaborations [45,46]. The value of $\alpha_s(m_\tau) = 0.306 \pm 0.017$ quoted by CLEO [41] uses the measured moments and the average value $\mathrm{B}(\tau \to e\nu\overline{\nu}) = 0.1810 \pm 0.0012$ from the 1992 edition of this review. We assign a theoretical error equal to 1/2 of the contribution from the order α^3 term and all of the nonperturbative contributions. This then gives $\alpha_s(m_\tau) = 0.370 \pm 0.033$ for the final result. Note that the theoretical errors are dominant. The small theoretical errors have been criticized [47].

9.5. QCD in high-energy hadron collisions

There are many ways in which perturbative QCD can be tested in high-energy hadron colliders. The quantitative tests are only useful if the process in question has been calculated beyond leading order in QCD perturbation theory. The production of hadrons with large transverse momentum in hadron-hadron collisions provides a direct probe of the scattering of quarks and gluons: $qq \to qq$, $qg \to qg$, $gg \to gg$, *etc.* The present generation of $p\overline{p}$ colliders provide center-of-mass energies which are sufficiently high that these processes can be unambiguously identified in two-jet production at large transverse momentum. Recent higher–order QCD calculations of the jet rates [48] and shapes are in impressive agreement with data [49]. As an example, Fig. 36.7 in this *Review* shows the inclusive jet cross section at zero pseudorapidity as a function of the jet transverse momentum for $p\overline{p}$ collisions. The QCD prediction combines the parton distributions with the leading-order $2 \to 2$ parton scattering amplitudes. Data are also available on the angular distribution of jets; these are also in agreement with QCD expectations [50,51].

QCD corrections to Drell-Yan type cross sections (*i.e.*, the production in hadron collisions by quark-antiquark annihilation of lepton pairs of invariant mass Q from virtual photons, or of real W or Z bosons), are known [52]. These $\mathcal{O}(\alpha_s)$ QCD corrections are sizable at small values of Q.

It is interesting to note that the corresponding correction to W and Z production, as measured in $p\overline{p}$ collisions at $\sqrt{s} = 0.63$ TeV and $\sqrt{s} = 1.8$ TeV, has essentially the same theoretical form and is of order 30%.

The production of W and Z bosons and photons at large transverse momentum can also be used to determine α_s. The leading-order QCD subprocesses are $q\overline{q} \to \gamma g$ and $qg \to \gamma q$. If the parton distributions are taken from other processes and a value of $\Lambda^{(4)}_{\overline{\rm MS}}$ assumed, then an absolute prediction is obtained. Conversely, the data can be used to extract information on quark and gluon distributions and on the value of $\Lambda^{(4)}_{\overline{\rm MS}}$. The next-to-leading-order QCD corrections are known [53,54] (for photons), and for W/Z production [55], and so a precision test is possible in principle. Data exist from the CDF and DØ collaborations [56,57]. The UA2 collaboration [58] has extracted a value of $\alpha_s(M_W) = 0.123 \pm 0.018(\text{stat.}) \pm 0.017(\text{syst.})$ from the measured ratio $R_W = \dfrac{\sigma(W + 1jet)}{\sigma(W + 0jet)}$. The result depends on the algorithm used to define a jet, and the dominant systematic errors due to fragmentation and corrections for underlying events (the former causes jet energy to be lost, the latter causes it to be increased) are connected to the algorithm. The scale at which $\alpha_s(M)$ is to be evaluated is not clear. A change from $\mu = M_W$ to $\mu = M_W/2$ causes a shift of 0.01 in the extracted α_s. The quoted error should be increased to take this into account. There is dependence on the parton distribution functions, and hence, α_s appears explicitly in the formula for R_W, and implicitly in the distribution functions. The DØ collaboration has performed an analysis similar to UA2. They are unable to obtain a fit where the two values of α_s are consistent with one another, and do not quote a value of α_s [59]. The values from this process are no longer used in determining the overall average value of α_s.

9.6. QCD in heavy-quarkonium decay

Under the assumption that the hadronic and leptonic decay widths of heavy $Q\overline{Q}$ resonances can be factorized into a nonperturbative part—dependent on the confining potential—and a calculable perturbative part, the ratios of partial decay widths allow measurements of α_s at the heavy-quark mass scale. The most precise data come from the decay widths of the 1^{--} $J/\psi(1S)$ and Υ resonances. The total decay width of the Υ is predicted by perturbative QCD [60]

$$
\begin{aligned}
R_\Upsilon &= \frac{\Gamma(\Upsilon \to \text{hadrons})}{\Gamma(\Upsilon \to \mu^+\mu^-)} \\
&= \frac{10(\pi^2-9)\alpha_s^3(M)}{9\pi\alpha_{\text{em}}^2} \\
&\quad \times \left[1 + \frac{\alpha_s}{\pi}\left(-19.4 + \frac{3\beta_0}{2}\left(1.162 + \ln\left(\frac{2M}{M_\Upsilon}\right)\right)\right)\right] \, . \qquad (9.16)
\end{aligned}
$$

Data are available for the Υ, Υ', Υ'' and ψ. The result is very sensitive to α_s and the data are sufficiently precise ($R_\mu(\Upsilon) = 32.5 \pm 0.9$) [61] that the theoretical errors will dominate. There are theoretical corrections to this simple formula due to the relativistic nature of the $Q\overline{Q}$ system; $v^2/c^2 \sim 0.1$ for the Υ. They are more severe for the ψ. There are also nonperturbative corrections of the form Λ^2/m_Υ^2; again these are more severe for the ψ. A fit to Υ, Υ', and Υ'' [62] gives $\alpha_s(M_Z) = 0.108 \pm 0.001$ (expt.). The results from each state separately and also from the ψ are consistent with each other. There is an uncertainty of order ± 0.005 from the choice of scale; the error from v^2/c^2 corrections is a little larger. $\alpha_s(M_Z) = 0.108 \pm 0.010$ is a fair representation of the total error including the possibility of nonperturbative corrections.

9.7. Perturbative QCD in e^+e^- collisions

The total cross section for $e^+e^- \to$ hadrons is obtained (at low values of $\sqrt{s}$) by multiplying the muon-pair cross section by the factor $R = 3\Sigma_q e_q^2$. The higher-order QCD corrections to this quantity have been calculated, and the results can be expressed in terms of the factor:

$$
R = R^{(0)}\left[1 + \frac{\alpha_s}{\pi} + C_2\left(\frac{\alpha_s}{\pi}\right)^2 + C_3\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right] , \qquad (9.17)
$$

where $C_2 = 1.411$ and $C_3 = -12.8$ [63].

$R^{(0)}$ can be obtained from the formula for $d\sigma/d\Omega$ for $e^+e^- \to f\overline{f}$ by integrating over Ω. The formula is given in Sec. 35.2 of this *Review*. This result is only correct in the zero-quark-mass limit. The $\mathcal{O}(\alpha_s)$ corrections are also known for massive quarks [64]. The principal advantage of determining α_s from R in e^+e^- annihilation is that there is no dependence on fragmentation models, jet algorithms, *etc.*

A comparison of the theoretical prediction of Eq. (9.17) (corrected for the b-quark mass), with all the available data at values of $\sqrt{s}$ between 20 and 65 GeV, gives [65] $\alpha_s(35\ \mathrm{GeV}) = 0.146 \pm 0.030$. The size of the order α_s^3 term is of order 40% of that of the order α_s^2 and 3% of the order α_s. If the order α_s^3 term is not included, a fit to the data yields α_s (34 GeV) $= 0.142 \pm 0.03$, indicating that the theoretical uncertainty is smaller than the experimental error.

Measurements of the ratio of hadronic to leptonic width of the Z at LEP and SLC, Γ_h/Γ_μ probe the same quantity as R. Using the average of $\Gamma_h/\Gamma_\mu = 20.788 \pm 0.032$ gives $\alpha_s(M_Z) = 0.123 \pm 0.004 \pm 0.002$ [66]. There are theoretical errors arising from the values of the top-quark and Higgs masses which enter due to electroweak corrections to the Z width and from the choice of scale.

While this method has small theoretical uncertainties from QCD itself, it relies sensitively on the electroweak couplings of the Z to quarks [67]. The experimental results on $\Gamma(Z \to b\overline{b})$ and $\Gamma(Z \to c\overline{c})$ are not in agreement with the Standard Model [68]. If these widths are taken from experiment (rather than from the Standard Model), the extracted vale of $\alpha_s(M_Z)$ is 0.183. If the Standard Model is used for $\Gamma(Z \to c\overline{c})$, $\alpha_s(M_Z) = 0.104$ results. In view of these problems, the value from Γ_h/Γ_μ is not included in the final average.

An alternative method of determining α_s in e^+e^- annihilation is from measuring quantities that are sensitive to the relative rates of two-, three-, and four-jet events. A recent review should be consulted for more details [69] of the issues mentioned briefly here. In addition to simply counting jets, there are many possible choices of such "shape variables": thrust [70], energy-energy correlations [71], planar triple-energy correlations [72], average jet mass, *etc.* All of these are infrared safe, which means they can be reliably calculated in perturbation theory. The starting point for all these quantities is the multijet cross section. For example, at order α_s, for the process $e^+e^- \to qqg$:

$$\frac{1}{\sigma}\,\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\ , \tag{9.18}$$

where

$$x_i = \frac{2E_i}{\sqrt{s}} \tag{9.19}$$

are the center-of-mass energy fractions of the final-state (massless) quarks. A distribution in a "three-jet" variable, such as those listed above, is obtained by integrating this differential cross section over an appropriate phase space region for a fixed value of the variable. The order α_s^2 corrections to this process have been computed, as well as the 4-jet final states such as $e^+e^- \to qqgg$ [73].

There are many methods used by the e^+e^- experimental groups to determine α_s from the event topology. The jet-counting algorithm, originally introduced by the JADE collaboration [74], has been used by the LEP groups. Here, particles of momenta p_i and p_j are combined into a pseudo-particle of momentum $p_i + p_j$ if the invariant mass of the pair is less than $y_0\sqrt{s}$. The process is then iterated until no more pairs of particles or pseudo-particles remain. The remaining number is then defined to be the number of jets in the event, and can be compared to the QCD prediction. The Durham algorithm is slightly different: in computing the mass of a pair of partons, it uses $M^2 = 2\min(E_1^2, E_2^2)(1 - \cos\theta_{ij})$ for partons of energies E_i and E_j separated by angle θ_{ij} [75].

There are theoretical ambiguities in the way this process is carried out. Quarks and gluons are massless, whereas the observed hadrons are not, so that the massive jets that result from this scheme (the so-called E-0 scheme) cannot be compared directly to the massless jets of perturbative QCD. Different recombination schemes have been tried, for example combining 3-momenta and then rescaling the energy of the cluster so that it remains massless (p scheme). These schemes result in the same data giving a slightly different values [76,77] of α_s. These differences can be used to determine a systematic error. In addition, since what is observed are hadrons rather than quarks and gluons, a model is needed to describe the evolution of a partonic final state into one involving hadrons, so that detector corrections can be applied. The QCDmatrix elements are combined with a parton-fragmentation model. This model can then be used to correct the data for a direct comparison with the parton calculation. The different hadronization models that are used [78–81] model the dynamics that are controlled by nonperturbative QCD effects which we cannot yet calculate. The fragmentation parameters of these Monte Carlos are tuned to get agreement with the observed data. The differences between these models contribute to the systematic errors. The systematic errors from recombination schemes and fragmentation effects dominate over the statistical and other errors of the LEP/SLD experiments.

The scale M at which $\alpha_s(M)$ is to be evaluated is not clear. The invariant mass of a typical jet (or $\sqrt{sy_0}$) is probably a more appropriate choice than the e^+e^- center-of-mass energy. If the value is allowed to float in the fit to the data, the data tend to prefer values of order $\sqrt{s}/10$ [82]; the exact value depends on the variable that is fitted. The dominant uncertainties arise from the choice of M and from the freedom in the fragmentation Monte Carlos.

The perturbative QCD formulae can break down in special kinematical configurations. For example, the thrust distribution contains terms of the type $\alpha_s \ln^2(1-T)$. The higher orders in the perturbation expansion contain terms of order $\alpha_s^n \ln^m(1-T)$. For $T \sim 1$ (the region populated by 2-jet events), the perturbation expansion is unreliable. The terms with $n \leq m$ can be summed to all orders in α_s [83]. If the jet recombination methods are used higher-order terms involve $\alpha_s^n \ln^m y_0$), these too can be resummed [84]. The resummed results give better agreement with the data at large values of T. Some caution should be exercised in using these resummed results because of the possibility of overcounting; the showering Monte Carlos that are used for the fragmentation corrections also generate some of these leading-log corrections. Different schemes for combining the order α_s^2 and the resummations are available [85]. These different schemes result in shifts in α_s of order ± 0.002 [86].

An average of the recent results from SLD [86], OPAL [87], L3 [88], ALEPH [89], and DELPHI [90], using the combined α_s^2 and resummation fitting to a large set of shape variables, gives $\alpha_s(M_Z) = 0.122 \pm 0.007$. The errors in the values of $\alpha_s(M_Z)$ from these shape variables are totally dominated by the theoretical uncertainties associated with the choice of scale, and the effects of hadronization Monte Carlos on the different quantities fitted.

Similar studies on event shapes have been undertaken at TRISTAN, at PEP/PETRA, and at CLEO. A combined result from various shape parameters by the TOPAZ collaboration gives α_s (58 GeV) $= 0.125 \pm 0.009$, using the fixed order QCD result, and α_s (58 GeV) $= 0.132 \pm 0.008$ (corresponding to $\alpha_s(M_Z) = 0.123 \pm 0.007$), using the same method as in the SLD and LEP average [91].

The measurements of event shapes at PEP/PETRA are summarized in earlier editions of this note. The results are consistent with those from Z decay, but have larger errors. We use α_s (34 GeV) $= 0.14 \pm 0.02$ [92]. A recent analysis by the TPC group [93] gives α_s (29 GeV) $= 0.160 \pm 0.012$, using the same method as TOPAZ. This value corresponds to $\alpha_s(M_Z) = 0.131 \pm 0.010$

The CLEO collaboration fits to the order α_s^2 results for the two jet fraction at $\sqrt{s} = 10.53$ GeV, and obtains $\alpha_s(10.93) = 0.164 \pm 0.004$ (expt.) $\pm\, 0.014$ (theory) [94]. The dominant systematic error arises from the choice of scale (μ), and is determined from the range of α_s that results from fit with $\mu = 10.53$ GeV, and a fit where μ is allowed to vary to get the lowest χ^2. The latter results in $\mu = 1.2$ GeV. Since the quoted result corresponds to $\alpha_s(1.2) = 0.35$, it is by no means clear that the perturbative QCD expression is reliable and the resulting error should, therefore, be treated with caution. A fit to many different variables as is done in the LEP/SLC analyses would give added confidence to the quoted error.

Since the errors in the event shape measurements are dominantly systematic, and are common to the experiments, the results from PEP/PETRA, TRISTAN, LEP, SLC, and CLEO are combined to give $\alpha_s(M_Z) = 0.122 \pm 0.007$. This result is used in forming the final average value of α_s.

The total cross section $e^+e^- \to b\bar{b} + X$ near threshold can be used to determine α_s [95]. The result quoted is $\alpha_s(M_Z) = 0.109 \pm 0.001$. The relevant process is only calculated to leading order and the BLM scheme [8] is used. This results in $\alpha_s(0.632\ m_b)$. If $\alpha_s(m_b)$ is used, the resulting $\alpha_s(M_Z)$ shifts to ~ 0.117. This result is not used in the average.

9.8. Scaling violations in fragmentation functions

Measurements of the fragmentation function $d_i(z, E)$, being the probability that a hadron of type i be produced with energy zE in e^+e^- collisions at $\sqrt{s} = 2E$, can be used to determine α_s. As in the case of scaling violations in structure functions, QCD predicts only the E dependence. Hence, measurements at different energies are needed to extract a value of α_s. Because the QCD evolution mixes the fragmentation functions for each quark flavor with the gluon fragmentation function, it is necessary to determine each of these before α_s can be extracted. The ALEPH collaboration has used data from energies ranging from $\sqrt{s} = 22$ GeV to $\sqrt{s} = 91$ GeV. A flavor tag is used to discriminate between different quark species, and the longitudinal and transverse cross sections are used to extract the gluon fragmentation function [96]. The result obtained is $\alpha_s(M_Z) = 0.126 \pm 0.007$ (expt.) $\pm\, 0.006$ (theory) [97]. The theory error is due mainly to the choice of scale. The OPAL collaboration [98] has also extracted the separate fragmentation functions. DELPHI [99] has also performed a similar analysis using data from other experiments at lower energy with the result $\alpha_s(M_Z) = 0.122 \pm 0.012 \pm 0.006$ (theory). An earlier analysis by this collaboration [100], is consistent with this result, but used fixed order QCD. The older result is not used in the average, which is determined to be $\alpha_s(M_Z) = 0.125 \pm 0.006 \pm 0.006$ (theory)

9.9. Jet rates in *ep* collisions

At lowest order in α_s, the *ep* scattering process produces a final state of (1+1) jets, one from the proton fragment and the other from the quark knocked out by the process $e + quark \rightarrow e + quark$. At next order in α_s, a gluon can be radiated, and hence a (2+1) jet final state produced. By comparing the rates for these (1+1) and (1+2) jet processes, a value of α_s can be obtained. A NLO QCD calculation is available [101]. The basic methodology is similar to that used in the jet counting experiments in e^+e^- annihilation discussed above. Unlike those measurements, the ones in *ep* scattering are not at a fixed value of Q^2. In addition to the systematic errors associated with the jet definitions, there are additional ones since the structure functions enter into the rate calculations. Results from H1 [102] and ZEUS [103] can be combined to give $\alpha_s(M_Z) = 0.121 \pm 0.004$ (stat.) $\pm$ 0.008 (syst.). The contributions to the systematic errors from experimental effects (mainly the hadronic energy scale) are comparable to the theoretical ones arising from scale choice, structure functions, and jet definitions. These errors are common to the two measurements; therefore, we have not reduced the systematic error after forming the average.

9.10. Lattice QCD

Lattice gauge theory calculations can be used to calculate the energy levels of a $Q\overline{Q}$ system and then extract α_s. The masses of the $Q\overline{Q}$ states depend only on the quark mass and on α_s. A limitation is that calculations cannot be performed for three light quark flavors. Results are available for zero (quenched approximation) and two light flavors, which allow extrapolation to three. The coupling constant so extracted is in a lattice renormalization scheme, and must be converted to the $\overline{\mathrm{MS}}$ scheme for comparison with other results. Using the mass differences of Υ and Υ' and Υ and χ_b, Davies *et al.* [104] extract a value of $\alpha_s(M_Z) = 0.115 \pm 0.002$. The result is consistent with an earlier result by the same group based on quenched approximation ($\alpha_s(M_Z) = 0.112 \pm 0.004$) [105]. The error is dominated by the conversion between the coupling constants, which is performed at next-to-leading order in perturbation theory. It is estimated by making an assumption about the size of the NNLO term in this conversion. If it is estimated as one-half of the NLO term, then the resulting value is $\alpha_s(M_Z) = 0.115 \pm 0.003$.

A similar result with larger errors is reported by [106], where results are consistent with $\alpha_s(M_Z) = 0.111 \pm 0.006$. This result confirms that obtained in quenched approximation by [107]. A calculation [108] using the strength of the force between two heavy quarks computed in the quenched approximation obtains a value of $\alpha_s(5\ \mathrm{GeV})$ that is consistent with these results.

The result with a more conservative error $\alpha_s(M_Z) = 0.115 \pm 0.003$ will be used in the average, although a recent reviewer quotes an error of ± 0.007 [109].

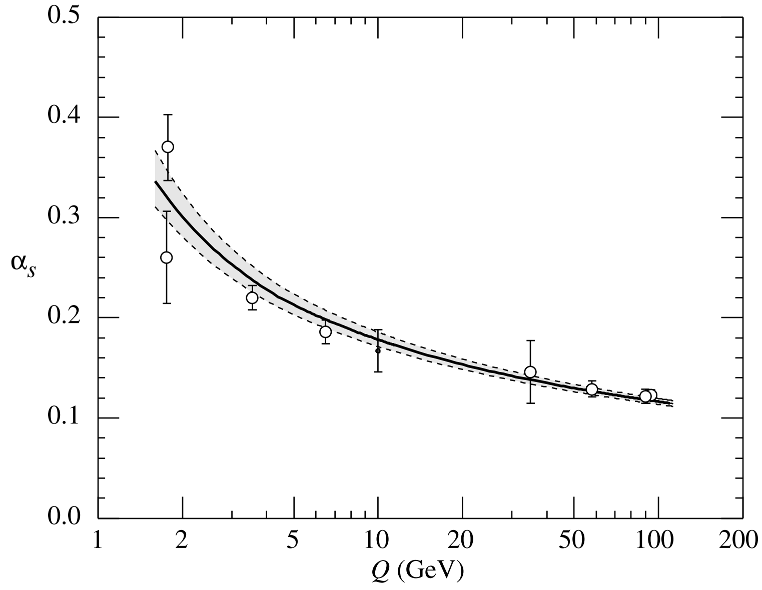

Figure 9.2: Summary of the values of $\alpha_s(Q)$ at the values of Q where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(Q)$ with increasing Q.

9.11. Conclusions

The need for brevity has meant that many other important topics in QCD phenomenology have had to be omitted from this review. One should mention in particular the study of exclusive processes (form factors, elastic scattering, ...), the behavior of quarks and gluons in nuclei, the spin properties of the theory, the interface of soft and hard QCD as manifest, for example, by minijet production and hard diffractive processes, and QCD effects in hadron spectroscopy.

In this short review, we have focused on those high-energy processes which currently offer the most quantitative tests of perturbative QCD. Figure 9.1 shows the values of $\alpha_s(M_Z)$ deduced from the various experiments. Figure 9.2 shows the values and the values of Q where they are measured. This figure clearly shows the experimental evidence for the variation of $\alpha_s(Q)$ with Q.

An average of the values in Fig. 9.1 (except the one from the width of the Z) gives $\alpha_s(M_z) = 0.118$, with a total χ^2 of 9.1 for ten fitted points, showing good consistency among the data. The error on the average, assuming that all of the errors in the contributing results are uncorrelated, is ± 0.0017, and is surely an underestimate. All the values are dominated by systematic, usually theoretical, errors. The two results with the smallest errors (± 0.003) are the ones from τ decay and lattice gauge theory. If these errors are increased to ± 0.006, the average is unchanged. There has been discussion of systematic differences in the data. The measurements which are dominated by low-energy (deep-inelastic scattering (not including HERA), τ decay, Υ width, lattice) average to $\alpha_s(M_z) = 0.118$ ($\chi^2 = 8.3$ for 5 points). Results from space-like momentum transfers (all ep results) average to $\alpha_s(M_z) = 0.114 \pm 0.004$, which might indicate some lack of theoretical understanding in comparing the data. Since, in most cases, the dominant error is systematic (mainly theoretical), a more conservative estimate of the final error is obtained by using the smallest of the individual errors on the experimental results, *i.e.*, ± 0.003. Our average value is then $\alpha_s(M_z) = 0.118 \pm 0.003$, which corresponds to $\Lambda^{(5)} = 209^{+39}_{-33}$ MeV.

References:

1. F.J. Yndurain, "Quantum Chromodynamics: an Introduction to the Theory of Quarks and Gluons", (Springer, New York, 1983);

   CTEQ Collaboration, Rev. Mod. Phys. **67**, 157 (1995).
2. For a recent review, see for example A.S. Kronfeld and P.B. Mackenzie, Ann. Rev. Nucl. and Part. Sci. **43**, 793 (1993).
3. For example see, B.R. Webber, in *Proceedings of the XXVII International Conference on High Energy Physics*, Glasgow, Scotland (July 1994);

   S. Bethke, at QCD94, Montpellier, France (July 1994).
4. W.A. Bardeen *et al.*, Phys. Rev. **D18**, 3998 (1978).
5. G. Grunberg, Phys. Lett. **95B**, 70 (1980), and Phys. Rev. **D29**, 2315 (1984).
6. P.M. Stevenson, Phys. Rev. **D23**, 2916 (1981), and Nucl. Phys. **B203**, 472 (1982).
7. S. Brodsky and H.J. Lu SLAC-PUB-6389 (Nov. 1993).
8. S. Brodsky, G.P. Lepage, and P.B. Mackenzie, Phys. Rev. **D28**, 228 (1983).
9. A.H. Mueller, Phys. Lett. **B308**, 355, (1993).
10. W. Bernreuther, Annals of Physics 151,127(1983).
11. W. Marciano, Phys. Rev. **D29**, 580 (1984).
12. D.W. Duke and R.G. Roberts, Phys. Reports **120**, 275 (1985).
13. V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972);

    Yu.L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977).
14. G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
15. G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 27 (1980);

    W. Furmanski and R. Petronzio, Phys. Lett. **97B**, 437 (1980), and Z. Phys. **C11**, 293 (1982);

    E.G. Floratos, C. Kounnas, and R. Lacaze, Phys. Lett. **98B**, 89 (1981), Phys. Lett. **98B**, 285 (1981), and Nucl. Phys. **B192**, 417 (1981);

    R.T. Herrod and S. Wada, Phys. Lett. **96B**, 195 (1981), and Z. Phys. **C9**, 351 (1981).
16. M. Glück *et al.*, Z. Phys. **C13**, 119 (1982).

17. M. Virchaux, Saclay preprint, DAPHNIA/SPP 92-30, in *Proceedings of the "QCD 20 years later" Workshop*, Aachen Germany, p. 205 (June 9–13, 1992).
18. R. Voss, in *Proceedings of the 1993 International Symposium on Lepton and Photon Interactions at High Energies, Ithaca, NY*, p. 144 (July 1993).
19. J. Feltese, in *Proceedings of the XXVII International Conference on High Energy Physics*, Glasgow, Scotland, July 1994.
20. F. Eisele, at the European Physical Society meeting, Brussels, (July 1995).
21. M. Derrick *et al.*, Phys. Lett. **B345**, 576 (1995);
    S. Aid *et al.*, Phys. Lett. **B354**, 494 (1995).
22. D. Gross, and Llewellyn Smith, Nucl. Phys. **B14**, 337 (1969).
23. J. Chyla and A.L. Kataev, Phys. Lett. **B297**, 385 (1992).
24. V.M. Braun and A.V. Kolesnichenko, Nucl. Phys. **B283**, 723 (1987).
25. W.C. Leung *et al.*, Phys. Lett. **B317**, 655 (1993);
    J. Kim at the European Physical Society meeting, Brussels, (July 1995).
26. J. Ellis and M. Karliner, Phys. Lett. **B341**, 397 (1995).
27. P.Z. Quintas *et al.*, Phys. Rev. Lett. **71**, 1307 (1993);
    J.P. Berge *et al.*, Z. Phys. **C49**, 187 (1991).
28. M. Arneodo *et al.*, Phys. Lett. **B309**, 222 (1993).
29. K. Bazizi and S.J. Wimpenny, UCR/DIS/91-02.
30. J.J. Aubert *et al.*, Nucl. Phys. **B333**, 1 (1990); Nucl. Phys. **293**, 740 (1987); Nucl. Phys. **B272**, 158 (1986); and Nucl. Phys. **B145**, 189 (1985).
31. M. Virchaux and A. Milsztajn, Phys. Lett. **B274**, 221 (1992).
32. L.W. Whitlow, Ph.D thesis, SLAC Report 357 (1990).
33. A.C. Benvenuti *et al.*, Phys. Lett. **B195**, 97 (1987); Phys. Lett. **B223**, 490 (1989); Phys. Lett. **B223**, 485, (1989); Phys. Lett. **B237**, 592 (1990); and Phys. Lett. **B237**, 599 (1990).
34. E.A. Kurayev, L.N. Lipatov, and V.S. Fadin, Sov. Phys. JETP **45**, 119 (1977);
    Ya.Ya. Balitsky, and L.N. Lipatov, Sov. J. Nucl. Phys. **28**, 882 (1978).
35. R.D. Ball and S. Forte, CERN-TH/95-148 (1995).
36. H. Plothow-Besch, Comp. Phys. Comm. **75**, 396 (1993).
37. M.A. Shifman, A.I Vainshtein, and V.I. Zakharov Nucl. Phys. **B147**, 385 (1979).
38. S. Narison and A. Pich, Phys. Lett. **B211**, 183 (1988);
    E. Braaten, S. Narison, and A. Pich, Nucl. Phys. **B373**, 581 (1992).
39. F. Le Diberder and A. Pich, Phys. Lett. **B289**, 165 (1992).
40. D. Buskulic *et al.*, Phys. Lett. **B307**, 209 (1993).
41. T. Coan *et al.*, CLNS 95/1332 (1995).

42. S.R. Wasserbaech, Aleph number PUB 95-06, Talk given at the Workshop on the Tau/Charm Factory, (June 1995, Argonne).
43. J.Z. Bai *et al.*, SLAC-PUB-95-6930 (1995).
44. G. Rahal-Callot, talk at European Physical Society meeting, Brussels (July 1995).
45. D. Buskulic *et al.*, Nucl. Phys. **B** (Proc. Suppl.) Vol. **40** (1995).
46. R. Akers *et al.*, CERN-PPE/95-06 (1995).
47. G. Altarelli, CERN-TH 7493/94 (1994);
    G. Altarelli, P. Nason, and G. Ridolfi, Z. Phys. **C68**, 257 (1995).
48. S.D. Ellis, Z. Kunszt,and D.E. Soper, Phys. Rev. Lett. **64**, 2121 (1991);
    W.T. Giele, E.W.N. Glover, and D. Kosower, Phys. Rev. Lett. **73**, 2019 (1994).
49. F. Abe *et al.*, Phys. Rev. Lett. **68**, 1104 (1992).
50. UA1 Collaboration: G. Arnison *et al.*, Phys. Lett. **B177**, 244 (1986).
51. F. Abe *et al.*, Phys. Rev. Lett. **64**, 157 (1990).
52. G. Altarelli, R.K. Ellis, and G. Martinelli, Nucl. Phys. **B143**, 521 (1978).
53. P. Aurenche, R. Baier, and M. Fontannaz, Phys. Rev. **D42**, 1440 (1990);
    P. Aurenche *et al.*, Phys. Lett. **140B**, 87 (1984);
    P. Aurenche *et al.*, Nucl. Phys. **B297**, 661 (1988).
54. H. Baer, J. Ohnemus, and J.F. Owens, Phys. Lett. **B234**, 127 (1990).
55. H. Baer and M.H. Reno, Phys. Rev. **D43**, 2892 (1991);
    P.B. Arnold and M.H. Reno, Nucl. Phys. **B319**, 37 (1989).
56. F. Abe *et al.*, Phys. Rev. Lett. **73**, 2662 (1994).
57. S. Abachi *et al.*, FERMILAB-CONF-95-215E (1995).
58. J. Alitti *et al.*, Phys. Lett. **B263**, 563 (1991).
59. DØ Collaboration, submitted to European Physical Society meeting, Brussels, (July 1995).
60. R. Barbieri *et al.*, Phys. Lett. **95B**, 93 (1980);
    B.P. Mackenzie and G.P. Lepage, Phys. Rev. Lett. **47**, 1244 (1981).
61. M. Kobel *et al.*, Z. Phys. **C53**, 193 (1992).
62. M. Kobel DESY-F31-91-03.
63. S.G. Gorishny, A. Kataev, and S.A. Larin, Phys. Lett. **B259**, 114 (1991);
    L.R. Surguladze and M.A. Samuel, Phys. Rev. Lett. **66**, 560 (1991).
64. K.G. Chetyrkin, J.H. Kuhn (Karlsruhe U., ITTP) Phys. Lett. **B308**, 127 (1993).
65. D. Haidt, in *Directions in High Energy Physics,* vol. 14, p. 201 Ed. P. Langacker (World Scientific, 1995).
66. A. Olchevsi, at the European Physical Society meeting, Brussels, (July 1995).
67. A. Blondel and C. Verzgrassi, Phys. Lett. **B311**, 346 (1993);
    G. Altarelli *et al.*, Nucl. Phys. **B405**, 3 (1993).

68. See the sections on "Constraints on New Physics from Electroweak Analyses" (Sec. 14) and "Standard Model of Electroweak Interactions" (Sec. 10) in this *Review*.
69. S. Bethke and J. Pilcher, Ann. Rev. Nucl. and Part. Sci. **42**, 251 (1992).
70. E. Farhi, Phys. Rev. Lett. **39**, 1587 (1977).
71. C.L. Basham *et al.*, Phys. Rev. **D17**, 2298 (1978).
72. F. Csikor *et al.*, Phys. Rev. **D31**, 1025 (1985).
73. R.K. Ellis, D.A. Ross, T. Terrano, Phys. Rev. Lett. **45**, 1226 (1980);
    Z. Kunszt and P. Nason, ETH-89-0836 (1989).
74. S. Bethke *et al.*, Phys. Lett. **B213**, 235 (1988).
75. S. Bethke *et al.*, Nucl. Phys. **B370**, 310 (1992).
76. M.Z. Akrawy *et al.*, Z. Phys. **C49**, 375 (1991).
77. K. Abe *et al.*, Phys. Rev. Lett. **71**, 2578 (1993).
78. B. Andersson *et al.*, Phys. Reports **97**, 33 (1983).
79. A. Ali *et al.*, Nucl. Phys. **B168**, 409 (1980);
    A. Ali and R. Barreiro, Phys. Lett. **118B**, 155 (1982).
80. B.R. Webber, Nucl. Phys. **B238**, 492 (1984).
81. T. Sjostrand and M. Bengtsson, Comp. Phys. Comm. **43**, 367 (1987).
82. O. Adriani *et al.*, Phys. Lett. **B284**, 471 (1992);
    M. Akrawy *et al.*, Z. Phys. **C47**, 505 (1990);
    B. Adeva *et al.*, Phys. Lett. **B248**, 473 (1990);
    P. Abreu *et al.*Z. Phys. **C54**, 55 (1992);
    D. Decamp *et al.*, Phys. Lett. **B255**, 623 (1991).
83. S. Catani *et al.*, Phys. Lett. **B263**, 491 (1991).
84. S. Catani *et al.*, Phys. Lett. **B269**, 432 (1991);
    S. Catani, B.R. Webber, and G. Turnock, Phys. Lett. **B272**, 368 (1991);
    N. Brown and J. Stirling, Z. Phys. **C53**, 629 (1992).
85. G. Turnock, Cavendish preprint HEP-92/3;
    G. Catani, G. Turnock, and B.R. Webber CERN-TH-6570/92 (1992);
    S. Catani *et al.*, Phys. Lett. **B269**, 432 (1991).
86. K. Abe *et al.*, Phys. Rev. **D51**, 962 (1995).
87. P.D. Acton *et al.*, Z. Phys. **C55**, 1 (1992).
88. O. Adriani *et al.*, Phys. Lett. **B284**, 471 (1992).
89. D. Decamp *et al.*, Phys. Lett. **B284**, 163 (1992).
90. P. Abreu *et al.*, Z. Phys. **C59**, 21 (1993).
91. Y. Onishi *et al.*, Phys. Lett. **B313**, 475 (1993).
92. W.J. Stirling and M.R. Whalley, Durham-RAL Database Publication RAL/87/107 (1987).

93. D.A. Bauer *et al.*, SLAC-PUB-5618.
94. L. Gibbons *et al.*, CLNS 95-1323 (1995).
95. M.B. Voloshin, Int. J. Mod. Phys. **A10**, 2865 (1995).
96. P. Nason and B.R. Webber Nucl. Phys. **B421**, 473 (1994).
97. D. Buskulic *et al.*, CERN-PPE-95-096, to appear in Phys. Lett. (1995).
98. OPAL Collaboaration, CERN-PPE/95-57 to appear in Zeit. fur Physik C (1995).
99. DELPHI Collaboration, DELPH 95-93, submitted to the European Physical Society meeting, Brussels, (July 1995).
100. P. Abreu *et al.*, Phys. Lett. **B311**, 408 (1993).
101. D. Graudenz, Phys. Rev. **D49**, 3921 (1994), J.G. Korner, E. Mirkes, and G.A. Schuler, Int. J. Mod. Phys. **A4**, 1781, (1989).
102. H1 Collaboration submitted to the European Physical Society meeting, Brussels, (July 1995).
103. ZEUS Collaboration submitted to the European Physical Society meeting, Brussels, (July 1995);

     S.L. Wu, in *Proceedings of the 1987 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg*, 27–31 July 1987, ed. by W. Bartel and R. Ruckl (North-Holland, Amsterdam, 1988), p. 39;

     W.J. Stirling, in *Proceedings of the 1987 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg*, 27–31 July 1987, ed. by W. Bartel and R. Ruckl (North-Holland, Amsterdam, 1988), p. 715.
104. C.T.H. Davies *et al.*, Phys. Lett. **B345**, 42 (1995).
105. G.P. Lepage and J. Sloan presented at 1993 Lattice conference, hep-lat/9312070.
106. S. Aoki *et al.*, Phys. Rev. Lett. **74**, 222 (1995).
107. A.X. El-Khadra *et al.*, Phys. Rev. Lett. **69**, 729 (1992);

     A.X. El-Khadra, presented at 1993 Lattice conference, OHSTPY-HEP-T-93-020 (1993);

     A.X. El-Khadra *et al.*, FNAL 94-091/T (1994).
108. M. Luscher *et al.*, CERN-TH 6996/93 (1993).
109. C. Michael presented at the 1995 Lepton-Photon Symposium, Beijing, China.