THE Z BOSON

Precision measurements at the Z-boson resonance using electron-positron colliding beams began in 1989 at the SLC and at LEP. During 1989–93, the four CERN experiments have made high-statistics studies of the Z. The availability of longitudinally polarized electron beams at the SLC in 1993 has enabled a precision determination of the effective electroweak mixing angle $\sin^2\overline{\theta}_W$ that is competitive with the CERN results on this parameter.

The Z-boson properties reported in this section may broadly be categorized as:

- The standard 'lineshape' parameters of the Z consisting of its mass, M_Z, its total width, Γ_Z, and its partial decay widths, Γ(hadrons) and $\Gamma(\ell\overline{\ell})$ where $\ell = e, \mu, \tau, \nu$;
- The b- and c-quark-related partial widths and charge asymmetries which require special techniques;
- Determination of rare Z decay modes and the search for modes that violate known conservation laws.

Details on Z-parameter determination and the study of $Z \to b\overline{b}, c\overline{c}$ at LEP are given in this note.

The standard 'lineshape' parameters of the Z are determined with increasing precision from an analysis of the production cross sections of these final states in e^+e^- collisions at LEP. The $Z \to \nu\overline{\nu}(\gamma)$ state is identified directly by detecting single photon production and indirectly by subtracting the visible partial widths from the total width. Inclusion in this analysis of the forward-backward asymmetry of charged leptons, $A_{FB}^{(0,\ell)}$, of the τ polarization, $P(\tau)$, and its forward-backward asymmetry, $P(\tau)^{fb}$, enables the separate determination of the effective vector ($\overline{g}_V$) and axial vector ($\overline{g}_A$) couplings of the Z to these leptons and the ratio ($\overline{g}_V/\overline{g}_A$) which is related to the effective electroweak mixing angle $\sin^2\overline{\theta}_W$ (see the Standard Model review).

Determination of the b- and c-quark-related partial widths and charge asymmetries involves tagging the b and c quarks. Traditionally this was done by requiring the presence of a prompt lepton in the event with high momentum and high transverse momentum (with respect to the accompanying jet). Precision vertex measurement with silicon detectors enables one to do impact parameter and lifetime tagging. Lately, sophisticated neural-network techniques have been used to classify events as b or non-b on a statistical basis using event-shape variables.

***Z*-parameter determination at LEP**

LEP is run at a few energy points on and around the Z mass constituting an energy 'scan.' The shape of the cross-section variation around the Z peak can be described by a Breit-Wigner *ansatz* with an energy-dependent total width [1]. The **three** main properties of this distribution, viz., the **position** of the peak, the **width** of the distribution, and the **height** of the peak, determine respectively the values of M_Z, Γ_Z, and $\Gamma(e^+e^-)\times\Gamma(f\overline{f})$, where $\Gamma(e^+e^-)$ and $\Gamma(f\overline{f})$ are the electron and fermion partial widths of the Z. The quantitative determination of these parameters is done by writing analytic expressions for these cross sections in terms of the parameters and fitting the calculated cross sections to the measured ones by varying these parameters, taking properly into account all the errors. Single-photon exchange and γ-Z interference are included, and the large ($\sim$30 %) initial-state radiation (ISR) effects are taken into account by convoluting the analytic expressions over a 'Radiator Function [1,2]' $H(s, s')$. Thus for the process $e^+e^- \to f\overline{f}$:

$$\sigma_f(s) = \int H(s,s')\ \sigma_f^0(s')\ ds' \tag{1}$$

$$\sigma_f^0(s) = \sigma_Z^0 + \sigma_\gamma^0 + \sigma_{\gamma Z}^0 \tag{2}$$

$$\sigma_Z^0 = \frac{12\pi}{M_Z^2}\ \frac{\Gamma(e^+e^-)\Gamma(f\overline{f})}{\Gamma_Z^2}\ \frac{s\ \Gamma_Z^2}{(s-M_Z^2)^2\ +\ s^2\Gamma_Z^2/M_Z^2} \tag{3}$$

$$\sigma_\gamma^0 = \frac{4\pi\alpha^2(s)}{3s}\ Q_f^2 N_c^f \tag{4}$$

$$\sigma_{\gamma Z}^0 = -\frac{2\sqrt{2}\alpha(s)}{3}\ (Q_f G_F N_c^f g_{Ve} g_{Vf}) \times \frac{(s-M_Z^2)M_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \tag{5}$$

where Q_f is the charge of the fermion, $N_c^f = 3(1)$ for quark (lepton) and g_{Vf} is the neutral vector coupling of the Z to the fermion-antifermion pair $f\overline{f}$.

Since $\sigma^0_{\gamma Z}$ is expected to be much less than σ^0_Z, the LEP collaborations have generally calculated the interference term in the framework of the Standard Model using the best known values of g_V. This fixing of $\sigma^0_{\gamma Z}$ leads to a tighter constraint on M_Z and consequently a smaller error on its fitted value.

Defining

$$A_f = 2\frac{g_{Vf}\cdot g_{Af}}{(g^2_{Vf}+g^2_{Af})} \tag{6}$$

where g_{Af} is the neutral axial-vector coupling of the Z to $f\overline{f}$, the lowest-order expressions for the various lepton-related asymmetries on the Z pole are [3] $A_{FB}^{(0,\ell)} = (3/4)A_e A_f$, $P(\tau) = -A_\tau$, $P(\tau)^{fb} = -(3/4)A_e$, $A_{LR} = A_e$. The full analysis takes into account the energy dependence of the asymmetries. Experimentally A_{LR} is defined as $(\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$ where $\sigma_{L(R)}$ are the $e^+e^- \to Z$ production cross sections with left- (right)-handed electrons.

In terms of g_A and g_V, the partial decay width of the Z to $f\overline{f}$ can be written as

$$\Gamma(f\overline{f}) = \frac{G_F M_Z^3}{6\sqrt{2}\pi}\,(g^2_{Vf}+g^2_{Af})\; N_c^f(1+\delta_{\rm QED})(1+\delta_{\rm QCD}) \tag{7}$$

where $\delta_{\rm QED} = 3\alpha Q_f^2/4\pi$ accounts for final-state photonic corrections and $\delta_{\rm QCD} = 0$ for leptons and $\delta_{\rm QCD} = (\alpha_s/\pi) + 1.409(\alpha_s/\pi)^2 - 12.805(\alpha_s/\pi)^3$ for quarks, α_s being the strong coupling constant at $\mu = M_Z$.

In the above framework, the QED radiative corrections have been explicitly taken into account by convoluting over the ISR and allowing the electromagnetic coupling constant to run [4]: $\alpha(s) = \alpha/(1-\Delta\alpha)$. On the other hand, weak radiative corrections that depend upon the assumptions of the electroweak theory and on the values of the unknown $M_{\rm top}$ and $M_{\rm Higgs}$ are accounted for by **absorbing them into the couplings**, which are then called the *effective* couplings $\overline{g}_V$ and

$\overline{g}_A$ (or alternatively the effective parameters of the $\star$ scheme of Kennedy and Lynn [5]).

S-matrix approach to the Z

While practically all experimental analyses of LEP/SLC data have followed the 'Breit-Wigner' approach described above, an alternative S-matrix-based analysis is also possible. The Z, like all unstable particles, is associated with a complex pole in the S matrix. The pole position is process independent and gauge invariant. The mass, $\overline{M}_Z$, and width, $\overline{\Gamma}_Z$, can be defined in terms of the pole in the energy plane via [6]

$$\overline{s} = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z \tag{8}$$

leading to the relations

$$\overline{M}_Z = M_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \tag{9}$$

$$\approx M_Z \;\; - 34 \text{ MeV} \tag{10}$$

$$\overline{\Gamma}_Z = \Gamma_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \tag{11}$$

$$\approx \Gamma_Z \;\; - 0.9 \text{ MeV} \tag{12}$$

Some authors [7] choose to define the Z mass and width via

$$\overline{s} = (\overline{M}_Z - \frac{i}{2}\overline{\Gamma}_Z)^2 \tag{13}$$

which yields $\overline{M}_Z \approx M_Z - 26$ MeV, $\overline{\Gamma}_Z \approx \Gamma_Z - 1.2$ MeV.

The L3 collaboration at LEP (ADRIANI 93H, ADRIANI 93M) have analyzed their data using the S-matrix approach as defined in Eq. (8), in addition to the conventional one. As expected, they observe a downward shift of ≈ 34 MeV in the Z mass.

Handling the large-angle e^+e^- final state

Unlike other $f\overline{f}$ decay final states of the Z, the e^+e^- final state has a contribution not only from the s-channel but also from the t-channel and s-t interference. The full amplitude is not amenable to fast calculation, which is essential if one has to carry out minimization fits within reasonable computer time. The usual procedure is to calculate the non-s channel part of the cross section separately using the Standard Model program

ALIBABA [8] using the 'most reasonable' values of $M_{\rm top}$, and $M_{\rm Higgs}$ and add it to the s-channel cross section calculated as for other channels. This leads to two additional sources of error in the analysis: firstly, the theoretical calculation in ALIBABA itself is known to be accurate to ~ 0.5 %, and secondly, there is uncertainty due to unknown $M_{\rm top}$ and $M_{\rm Higgs}$. These additional errors are propagated into the analysis by including them in the systematic error on the e^+e^- final state.

Errors due to uncertainty in LEP energy determination [9]

The systematic errors related to the LEP energy measurement can be classified as:

- The absolute energy scale error;
- Energy-point-to-energy-point errors due to the non-linear response of the magnets to the exciting currents;
- Energy-point-to-energy-point errors due to possible higher-order effects in the relationship between the dipole field and beam energy;
- Energy reproducibility errors due to various unknown uncertainties in temperatures, tidal effects, corrector settings, RF status *etc.* Since one groups together data taken at 'nominally same' energies in different fills, it can be assumed that these errors are uncorrelated and are reduced by $\sqrt{\overline{N}_{\rm fill}}$ where $\overline{N}_{\rm fill}$ is the (luminosity weighted) effective number of fills at a particular energy point.

At each energy point the last two errors can be summed into one point-to-point error.

Choice of fit parameters

The LEP collaborations have chosen the following primary set of parameters for fitting: M_Z, Γ_Z, $\sigma^0_{\rm hadron}$, $R({\rm lepton})$, $A^{(0,\ell)}_{FB}$, where $R({\rm lepton}) = \Gamma({\rm hadrons})/\Gamma({\rm lepton})$, $\sigma^0_{\rm hadron} = 12\pi\Gamma(e^+e^-)\Gamma({\rm hadrons})/M_Z^2\Gamma_Z^2$. With a knowledge of these fitted parameters and their covariance matrix, any other parameter can be derived. The main advantage of these parameters is that they form the **least correlated** set of parameters, so that it becomes easy to combine results from the different LEP experiments.

Thus, the most general fit carried out to cross section and asymmetry data determines the **nine parameters**: M_Z, Γ_Z, σ^0_{hadron}, $R(e)$, $R(\mu)$, $R(\tau)$, $A^{(0,e)}_{FB}$, $A^{(0,\mu)}_{FB}$, $A^{(0,\tau)}_{FB}$. Assumption of lepton universality leads to a **five-parameter fit** determining M_Z, Γ_Z, σ^0_{hadron}, $R(\text{lepton})$, $A^{(0,\ell)}_{FB}$. The use of **only** cross-section data leads to six- or four-parameter fits if lepton universality is or is not assumed, *i.e.*, $A^{(0,\ell)}_{FB}$ values are not determined.

In order to determine the best values of the effective vector and axial vector couplings of the charged leptons to the Z, the above mentioned nine- and five-parameter fits are carried out with added constraints from the measured values of A_τ and A_e obtained from τ polarization studies at LEP and the determination of A_{LR} at SLC.

Combining results from the four LEP experiments [10]

Each LEP experiment provides the values of the parameters mentioned above together with the full covariance matrix. The statistical and experimental systematic errors are assumed to be uncorrelated among the four experiments. The sources of **common** systematic errors are i) the LEP energy uncertainties, and ii) the effect of theoretical uncertainty in calculating the small-angle Bhabha cross section for luminosity determination and in estimating the non-s channel contribution to the large-angle Bhabha cross section. Using this information, a full covariance matrix, V, of all the input parameters is constructed and a combined parameter set is obtained by minimizing $\chi^2 = \Delta^T V^{-1} \Delta$, where Δ is the vector of residuals of the combined parameter set to the results of individual experiments.

Study of $Z \to b\overline{b}, c\overline{c}$ at LEP

These studies lead to the experimental determination of the ratios of the partial widths $\Gamma(b\overline{b})/\Gamma(\text{hadrons})$, $\Gamma(c\overline{c})/\Gamma(\text{hadrons})$, and the forward-backward (charge) asymmetries $A^{(0,b)}_{FB}$ and $A^{(0,c)}_{FB}$. Each LEP experiment has used more than one technique of b,c tagging, and then used certain assumptions, some model dependent, to extract the final numbers. This makes the task of **combining** these LEP results quite daunting. A first step in this direction was made recently [10] for a joint LEP presentation for the 1993 summer conferences. In principle, the task is the same as for combining results on the lineshape: identification of the **common systematic errors**, which in this case exist not only among different LEP experiments, but also within one collaboration if different (partially correlated) methods are employed to extract the same quantity.

Extraction of $\mathbf{\Gamma(b\overline{b})/\Gamma}$***(hadrons)***

Three methods have been used to tag a b quark: lepton tagging, analysis of event shape, and lifetime tagging. The latter includes impact-parameter or decay-length double tags, or in combination with the lepton or event-shape tags. The various sources of common systematic errors are:

- Semileptonic decay model;
- Semileptonic branching ratios of b and c quarks;
- $c\overline{c}$ contamination, $\Gamma(c\overline{c})/\Gamma(\text{hadrons})$;
- Variation of Monte Carlo parameters, in particular the b-fragmentation function;
- Hemisphere correlations.

Some other possible common errors due to charmed hadron composition, b,c production in uds events, *etc.* are expected to be small.

Extraction of $\mathbf{\Gamma(c\overline{c})/\Gamma}$***(hadrons)***

In addition to the methods of lepton tagging, event-shape analysis, and lifetime tags, $D^*(2010)$ tagging is also used to identify c quarks. The common systematic errors are basically the same as for the b-tagging case, where instead of $c\overline{c}$ contamination, it is $b\overline{b}$ contamination.

Extraction of $\boldsymbol{A_{FB}^{(0,b)}}$ ***and*** $\boldsymbol{A_{FB}^{(0,c)}}$

For determining the asymmetry, it is essential to identify the charge of the quark in addition to tagging an event as due to $b\overline{b}$ or $c\overline{c}$. Thus, asymmetry measurements have utilized a lepton tag and a lifetime tag in combination with a jet-charge measurement and a $D^*(2010)$ tag. The values of $A_{FB}^{(0,b)}$ are corrected for the effect of $B^0\overline{B}^0$ mixing. The common errors in $A_{FB}^{(0,b)}$ are due to:

- Semileptonic decay model and branching ratios;
- Fragmentation;
- $\Gamma(b\overline{b})$ and $\Gamma(c\overline{c})$;
- $B^0\overline{B}^0$ mixing;
- $A_{FB}^{(0,c)}$.

For $A_{FB}^{(0,c)}$ the first four common errors are the same as for $A_{FB}^{(0,b)}$. The fifth error is due to $A_{FB}^{(0,b)}$; the sixth is due to the uncertainty in the probability of producing a $D^*(2010)$ in $c\overline{c}$ and $b\overline{b}$ events and in the $D^*(2010)$ branching ratio.

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