# 24. SU(3) ISOSCALAR FACTORS AND REPRESENTATION MATRICES

The most commonly used SU(3) isoscalar factors, corresponding to the singlet, octet, and decuplet content of $8 \otimes 8$ and $10 \otimes 8$, are shown at the right. The notation uses particle names to identify the coefficients, so that the pattern of relative couplings may be seen at a glance. We illustrate the use of the coefficients below. See J.J de Swart, Rev. Mod. Phys. **35**, 916 (1963) for detailed explanations and phase conventions.

*A* $\sqrt{\ }$ *is to be understood over every integer in the matrices;* the exponent 1/2 on each matrix is a reminder of this. For example, the $\Xi \to \Omega K$ element of the $10 \to 10 \otimes 8$ matrix is $-\sqrt{6}/\sqrt{24} = -1/2$.

Intramultiplet relative decay strengths may be read directly from the matrices. For example, in decuplet → octet + octet decays, the ratio of $\Omega^* \to \Xi \overline{K}$ and $\Delta \to N\pi$ partial widths is, from the $10 \to 8 \times 8$ matrix,

$$\frac{\Gamma\,(\Omega^* \to \Xi \overline{K})}{\Gamma\,(\Delta \to N\pi)} = \frac{12}{6} \times \text{ (phase space factors)} \ . \tag{24.1}$$

Including isospin Clebsch-Gordan coefficients, we obtain, *e.g.*,

$$\frac{\Gamma(\Omega^{*-} \to \Xi^0 K^-)}{\Gamma(\Delta^+ \to p\,\pi^0)} = \frac{1/2}{2/3} \times \frac{12}{6} \times p.s.f. = \frac{3}{2} \times p.s.f. \tag{24.2}$$

Partial widths for $8 \to 8 \otimes 8$ involve a linear superposition of $8_1$ (symmetric) and $8_2$ (antisymmetric) couplings. For example,

$$\Gamma(\Xi^* \to \Xi\pi) \sim \left(-\sqrt{\frac{9}{20}}\, g_1 + \sqrt{\frac{3}{12}}\, g_2\right)^2 \ . \tag{24.3}$$

The relations between $g_1$ and $g_2$ (with de Swart's normalization) and the standard $D$ and $F$ couplings that appear in the interaction Lagrangian,

$$\mathscr{L} = -\sqrt{2}\; D\; Tr\,(\{\overline{B}, B\}M) + \sqrt{2}\; F\; Tr\,([\overline{B}, B]\,M) \ , \tag{24.4}$$

where $[\overline{B}, B] \equiv \overline{B}B - B\overline{B}$ and $\{\overline{B}, B\} \equiv \overline{B}B + B\overline{B}$, are

$$D = \frac{\sqrt{30}}{40}\, g_1 \ , \qquad F = \frac{\sqrt{6}}{24}\, g_2 \ . \tag{24.5}$$

Thus, for example,

$$\Gamma(\Xi^* \to \Xi\pi) \sim (F - D)^2 \sim (1 - 2\alpha)^2 \ , \tag{24.6}$$

where $\alpha \equiv D/(D+F)$.

The generators of SU(3) transformations, $\lambda_a$ $(a = 1, 8)$, are $3 \times 3$ matrices that obey the following commutation and anticommutation relationships:

$$[\lambda_a,\ \lambda_b] \equiv \lambda_a \lambda_b - \lambda_b \lambda_a = 2 i f_{abc} \lambda_c \tag{24.7}$$

$$\{\lambda_a,\ \lambda_b\} \equiv \lambda_a \lambda_b + \lambda_b \lambda_a = \frac{4}{3} \delta_{ab} I + 2 d_{abc} \lambda_c\ , \tag{24.8}$$

where $I$ is the $3 \times 3$ identity matrix, and $\delta_{ab}$ is the Kronecker delta symbol. The $f_{abc}$ are odd under the permutation of any pair of indices, while the $d_{abc}$ are even. The nonzero values are

**$\mathbf{1} \to \mathbf{8} \otimes \mathbf{8}$**

$$(\,\Lambda\,) \;\to\; (\,N\overline{K}\;\;\Sigma\pi\;\;\Lambda\eta\;\;\Xi K\,) \qquad = \frac{1}{\sqrt{8}}\,(\,2\quad 3\quad -1\quad -2\,)^{1/2}$$

**$\mathbf{8_1} \to \mathbf{8} \otimes \mathbf{8}$**

$$\begin{pmatrix} N \\ \Sigma \\ \Lambda \\ \Xi \end{pmatrix} \to \begin{pmatrix} N\pi\;\;N\eta\;\;\Sigma K\;\;\Lambda K \\ N\overline{K}\;\;\Sigma\pi\;\;\Lambda\pi\;\;\Sigma\eta\;\;\Xi K \\ N\overline{K}\;\;\Sigma\pi\;\;\Lambda\eta\;\;\Xi K \\ \Sigma\overline{K}\;\;\Lambda\overline{K}\;\;\Xi\pi\;\;\Xi\eta \end{pmatrix} = \frac{1}{\sqrt{20}} \begin{pmatrix} 9\quad -1\quad -9\quad -1 \\ -6\quad 0\quad 4\quad 4\quad -6 \\ 2\quad -12\quad -4\quad -2 \\ 9\quad -1\quad -9\quad -1 \end{pmatrix}^{1/2}$$

**$\mathbf{8_2} \to \mathbf{8} \otimes \mathbf{8}$**

$$\begin{pmatrix} N \\ \Sigma \\ \Lambda \\ \Xi \end{pmatrix} \to \begin{pmatrix} N\pi\;\;N\eta\;\;\Sigma K\;\;\Lambda K \\ N\overline{K}\;\;\Sigma\pi\;\;\Lambda\pi\;\;\Sigma\eta\;\;\Xi K \\ N\overline{K}\;\;\Sigma\pi\;\;\Lambda\eta\;\;\Xi K \\ \Sigma\overline{K}\;\;\Lambda\overline{K}\;\;\Xi\pi\;\;\Xi\eta \end{pmatrix} = \frac{1}{\sqrt{12}} \begin{pmatrix} 3\quad 3\quad 3\quad -3 \\ 2\quad 8\quad 0\quad 0\quad -2 \\ 6\quad 0\quad 0\quad 6 \\ 3\quad 3\quad 3\quad -3 \end{pmatrix}^{1/2}$$

**$\mathbf{10} \to \mathbf{8} \otimes \mathbf{8}$**

$$\begin{pmatrix} \Delta \\ \Sigma \\ \Xi \\ \Omega \end{pmatrix} \to \begin{pmatrix} N\pi\;\;\Sigma K \\ N\overline{K}\;\;\Sigma\pi\;\;\Lambda\pi\;\;\Sigma\eta\;\;\Xi K \\ \Sigma\overline{K}\;\;\Lambda\overline{K}\;\;\Xi\pi\;\;\Xi\eta \\ \Xi\overline{K} \end{pmatrix} = \frac{1}{\sqrt{12}} \begin{pmatrix} -6\quad 6 \\ -2\quad 2\quad -3\quad 3\quad 2 \\ 3\quad -3\quad 3\quad 3 \\ 12 \end{pmatrix}^{1/2}$$

**$\mathbf{8} \to \mathbf{10} \otimes \mathbf{8}$**

$$\begin{pmatrix} N \\ \Sigma \\ \Lambda \\ \Xi \end{pmatrix} \to \begin{pmatrix} \Delta\pi\;\;\Sigma K \\ \Delta\overline{K}\;\;\Sigma\pi\;\;\Sigma\eta\;\;\Xi K \\ \Sigma\pi\;\;\Xi K \\ \Sigma\overline{K}\;\;\Xi\pi\;\;\Xi\eta\;\;\Omega K \end{pmatrix} = \frac{1}{\sqrt{15}} \begin{pmatrix} & -12 & 3 & \\ 8 & -2 & -3 & 2 \\ & -9 & 6 & \\ 3 & -3 & -3 & 6 \end{pmatrix}^{1/2}$$

**$\mathbf{10} \to \mathbf{10} \otimes \mathbf{8}$**

$$\begin{pmatrix} \Delta \\ \Sigma \\ \Xi \\ \Omega \end{pmatrix} \to \begin{pmatrix} \Delta\pi\;\;\Delta\eta\;\;\Sigma K \\ \Delta\overline{K}\;\;\Sigma\pi\;\;\Sigma\eta\;\;\Xi K \\ \Sigma\overline{K}\;\;\Xi\pi\;\;\Xi\eta\;\;\Omega K \\ \Xi\overline{K}\;\;\Omega\eta \end{pmatrix} = \frac{1}{\sqrt{24}} \begin{pmatrix} 15\quad 3\quad -6 \\ 8\quad 8\quad 0\quad -8 \\ 12\quad 3\quad -3\quad -6 \\ 12\quad -12 \end{pmatrix}^{1/2}$$

| $abc$ | $f_{abc}$ |
|---|---|
| 123 | 1 |
| 147 | 1/2 |
| 156 | $-1/2$ |
| 246 | 1/2 |
| 257 | 1/2 |
| 345 | 1/2 |
| 367 | $-1/2$ |
| 458 | $\sqrt{3}/2$ |
| 678 | $\sqrt{3}/2$ |

| $abc$ | $d_{abc}$ | $abc$ | $d_{abc}$ |
|---|---|---|---|
| 118 | $1/\sqrt{3}$ | 355 | 1/2 |
| 146 | 1/2 | 366 | $-1/2$ |
| 157 | 1/2 | 377 | $-1/2$ |
| 228 | $1/\sqrt{3}$ | 448 | $-1/(2\sqrt{3})$ |
| 247 | $-1/2$ | 558 | $-1/(2\sqrt{3})$ |
| 256 | 1/2 | 668 | $-1/(2\sqrt{3})$ |
| 338 | $1/\sqrt{3}$ | 778 | $-1/(2\sqrt{3})$ |
| 344 | 1/2 | 888 | $-1/\sqrt{3}$ |

The $\lambda_a$'s are

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Equation (24.7) defines the Lie algebra of SU(3). A general $d$-dimensional representation is given by a set of $d \times d$ matrices satisfying Eq. (24.7) with the $f_{abc}$ given above. Equation (24.8) is specific to the defining 3-dimensional representation.