29. STANDARD MODEL OF ELECTROWEAK INTERACTIONS

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The standard electroweak model is based on the gauge group [1] SU(2) $\times$ U(1), with gauge bosons W_μ^i, $i = 1,2,3$, and B_μ for the SU(2) and U(1) factors, respectively, and the corresponding gauge coupling constants g and g'. The left-handed fermion fields $\psi_i = \begin{pmatrix} \nu_i \\ \ell_i^- \end{pmatrix}$ and $\begin{pmatrix} u_i \\ d_i' \end{pmatrix}$ of the i^{th} fermion family transform as doublets under SU(2), where $d_i' \equiv \sum_j V_{ij}\, d_j$, and V is the Cabibbo-Kobayashi-Maskawa mixing matrix. (Constraints on V are discussed in the section on the Cabibbo-Kobayashi-Maskawa mixing matrix.) The right-handed fields are SU(2) singlets. In the minimal model there are three fermion families and a single complex Higgs doublet $\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.

After spontaneous symmetry breaking the Lagrangian is

$$\begin{aligned}
\mathscr{L}_F = & \sum_i \overline{\psi}_i \left(i \not{\partial} - m_i - \frac{g m_i H}{2 M_W} \right) \psi_i \\
& - \frac{g}{2\sqrt{2}} \sum_i \overline{\psi}_i\, \gamma^\mu\, (1-\gamma^5)(T^+\, W_\mu^+ + T^-\, W_\mu^-)\, \psi_i \\
& - e \sum_i q_i\, \overline{\psi}_i\, \gamma^\mu\, \psi_i\, A_\mu \\
& - \frac{g}{2\cos\theta_W} \sum_i \overline{\psi}_i\, \gamma^\mu (g_V^i - g_A^i \gamma^5)\, \psi_i\, Z_\mu\ .
\end{aligned} \tag{29.1}$$

$\theta_W \equiv \tan^{-1}(g'/g)$ is the weak angle; $e = g\sin\theta_W$ is the positron electric charge; and $A \equiv B\cos\theta_W + W^3 \sin\theta_W$ is the (massless) photon field. $W^\pm \equiv (W^1 \mp iW^2)/\sqrt{2}$ and $Z \equiv -B\sin\theta_W + W^3\cos\theta_W$ are the massive charged and neutral weak boson fields, respectively. T^+ and T^- are the weak isospin raising and lowering operators. The vector and axial couplings are

$$g_V^i \equiv t_{3L}(i) - 2q_i \sin^2\theta_W \tag{29.2}$$

$$g_A^i \equiv t_{3L}(i)\ , \tag{29.3}$$

where $t_{3L}(i)$ is the weak isospin of fermion i ($+1/2$ for u_i and ν_i; $-1/2$ for d_i and e_i) and q_i is the charge of ψ_i in units of e.

The second term in $\mathscr{L}_F$ represents the charged-current weak interaction [2]. For example, the coupling of a W to an electron and a neutrino is

$$-\frac{e}{2\sqrt{2}\sin\theta_W} \left[W_\mu^-\ \overline{e}\ \gamma^\mu (1-\gamma^5)\nu + W_\mu^+\ \overline{\nu}\ \gamma^\mu\ (1-\gamma^5) e \right]\ . \tag{29.4}$$

For momenta small compared to M_W, this term gives rise to the effective four-fermion interaction with the Fermi constant given (at tree level, *i.e.*, lowest order in perturbation theory) by $G_F/\sqrt{2} = g^2/8M_W^2$. CP violation is incorporated in the Standard Model by a single observable phase in V_{ij}. The third term in $\mathscr{L}_F$ describes electromagnetic interactions (QED), and the last is the weak neutral-current interaction.

In Eq. (29.1), m_i is the mass of the i^{th} fermion ψ_i. For the quarks these are the current masses. For the light quarks, as described in the Full Listings, $m_u \approx$ 2–8 MeV, $m_d \approx$ 5–15 MeV, and $m_s \approx$ 100–300 MeV (these are running masses evaluated at 1 GeV). For the heavier quarks, the "pole" masses are $m_c \approx$ 1.2–1.9 GeV and $m_b \approx$ 4.5–4.9 GeV. The average of the recent CDF [4] and DØ [5] values for m_t is 180 ± 12 GeV. See "The Note on Quark Masses" in the Full Listings for more information.

H is the physical neutral Higgs scalar which is the only remaining part of ϕ after spontaneous symmetry breaking. The Yukawa coupling of H to ψ_i, which is flavor diagonal in the minimal model, is $g m_i/2M_W$. The H mass is not predicted by the model. Experimental limits are given in the Higgs section. In nonminimal models there are additional charged and neutral scalar Higgs particles [6].

29.1. Renormalization and radiative corrections

The Standard Model has three parameters (not counting M_H and the fermion masses and mixings). A particularly useful set is:

(a) The fine structure constant $\alpha = 1/137.036$, determined from the quantum Hall effect. In most electroweak-renormalization schemes, it is convenient to define a running α dependent on the energy scale of the process, with $\alpha^{-1} \sim 137$ appropriate at low energy. At energies of order M_Z, $\alpha^{-1} \sim 128$. For example, in the modified minimal subtraction ($\overline{\text{MS}}$) scheme, one has $\widehat{\alpha}(M_Z)^{-1} = 127.90 \pm 0.09$ [7], while the conventional (on-shell) QED renormalization yields [8] $\alpha(M_Z)^{-1} = 128.90 \pm 0.09$, which differs by finite constants from $\widehat{\alpha}(M_Z)^{-1}$. The uncertainty, due to the low-energy hadronic contribution to vacuum polarization, is the dominant theoretical uncertainty in the interpretation of precision data. The values include recent reevaluations [8–12] of this effect, which, following a correction to [11], are now in reasonable agreement. Further improvement will require improved measurements of the cross section for $e^+e^- \rightarrow$ hadrons at low energy.

(b) The Fermi constant, $G_F = 1.16639(2) \times 10^{-5}$ GeV^{-2}, determined from the muon lifetime formula [13]:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right) \times \left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right] , \tag{29.5a}$$

where

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \tag{29.5b}$$

and

$$\alpha(m_\mu)^{-1} = \alpha^{-1} - \frac{2}{3\pi} \ln\left(\frac{m_\mu}{m_e}\right) + \frac{1}{6\pi} \approx 136 \ . \tag{29.5c}$$

The uncertainty in G_F from the input quantities is 1.1×10^{-10} GeV^{-2}. The quoted uncertainty of 2×10^{-10} is dominated by second order radiative corrections, estimated from the magnitude of the known $\alpha^2 \ln(m_\mu/m_e)$ term to be $\sim 1.8 \times 10^{-10}$ (alternately, one can view Eq. (29.5) as the exact definition of G_F; then the theoretical uncertainty appears instead in the formulae for quantities derived from G_F).

(c) $\sin^2\theta_W$, determined from the Z mass and other Z-pole observables, the W mass, and neutral-current processes [14]. The value of $\sin^2\theta_W$ depends on the renormalization prescription. There are a number of popular schemes [16–21] leading to $\sin^2\theta_W$ values which differ by small factors which depend on m_t and M_H. The notation for these schemes is shown in Table 29.1. Discussion of the schemes follows the table.

Table 29.1: Notations used to indicate the various schemes discussed in the text. Each definition of $\sin\theta_W$ leads to values that differ by small factors depending on m_t and M_H.

Scheme	Notation
On-shell	$s_W = \sin\theta_W$
NOV	$s_{M_Z} = \sin\theta_W$
$\overline{\text{MS}}$	$\widehat{s}_Z = \sin\theta_W$
$\overline{\text{MS}}$ ND	$\widehat{s}_{ND} = \sin\theta_W$
Effective angle	$\overline{s}_f = \sin\theta_W$

(i) The on-shell scheme promotes the tree-level formula $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ to a definition of the renormalized $\sin^2\theta_W$ to all orders in perturbation theory, *i.e.*, $\sin^2\theta_W \to s_W^2 \equiv 1 - M_W^2/M_Z^2$. This scheme is simple conceptually. However, M_W is known much less precisely than M_Z and in practice one extracts s_W^2 from M_Z alone using

$$M_W = \frac{A_0}{s_W(1-\Delta r)^{1/2}} \tag{29.6a}$$

$$M_Z = \frac{M_W}{c_W} \ , \tag{29.6b}$$

where $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} =$ 37.2802 GeV, and Δr includes the radiative corrections relating α, $\alpha(M_Z)$, G_F, M_W, and M_Z. One finds $\Delta r \sim \Delta r_0 - \rho_t/\tan^2\theta_W$, where $\Delta r_0 \approx 1 - \alpha/\alpha(M_Z) \approx 0.06$ is due to the running of α and $\rho_t = 3G_F\, m_t^2/8\sqrt{2}\pi^2 \approx 0.0100$ $(m_t/180\ \mathrm{GeV})^2$ represents the dominant (quadratic) m_t dependence. There are additional contributions to Δr from bosonic loops, including those which depend logarithmically on the Higgs mass M_H. One has $\Delta r = 0.0376 \pm 0.0025 \pm 0.0007$ for $(m_t, M_H) = (180 \pm 7, 300)$, where the second uncertainty is from $\alpha(M_Z)$. Thus the value of s_W^2 extracted from M_Z includes a large uncertainty (~ 0.0008) from the currently allowed range of m_t.

(*ii*) A more precisely determined quantity $s^2_{M_Z}$ can be obtained from M_Z by removing the (m_t, M_H) dependent term from Δr [17], *i.e.*,

$$s^2_{M_Z} c^2_{M_Z} \equiv \frac{\pi\alpha(M_Z)}{\sqrt{2}\, G_F\, M_Z^2}\ . \tag{29.7}$$

This yields $s^2_{M_Z} = 0.2311 \pm 0.0002$, with most of the uncertainty from α rather than M_Z. Scheme (*ii*) is equivalent to using M_Z rather than $\sin^2\theta_W$ as the third fundamental parameter. However, it recognizes that $s^2_{M_Z}$ is still a useful derived quantity. The small uncertainty in $s^2_{M_Z}$ compared to other schemes is because the m_t dependence has been removed by definition. However, the m_t uncertainty reemerges when other quantities (*e.g.*, M_W or other Z-pole observables) are predicted in terms of M_Z.

Both s_W^2 and $s^2_{M_Z}$ depend not only on the gauge couplings but also on the spontaneous-symmetry breaking, and both definitions are awkward in the presence of any extension of the Standard Model which perturbs the value of M_Z (or M_W). Other definitions are motivated by the tree-level coupling constant definition $\theta_W = \tan^{-1}(g'/g)$.

(*iii*) In particular, the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme introduces the quantity $\sin^2\widehat{\theta}_W(\mu) \equiv \widehat{g}\,'^2(\mu)/\left[\widehat{g}\,^2(\mu) + \widehat{g}\,'^2(\mu)\right]$, where the couplings $\widehat{g}$ and $\widehat{g}'$ are defined by modified minimal subtraction and the scale μ is conveniently chosen to be M_Z for electroweak processes. The value of $\widehat{s}\,^2_Z = \sin^2\widehat{\theta}_W(M_Z)$ extracted from M_Z is less sensitive than s_W^2 to m_t (by a factor of $\tan^2\theta_W$), and is less sensitive to most types of new physics than s_W^2 or $s^2_{M_Z}$. It is also very useful for comparing with the predictions of grand unification. There are actually several variant definitions of $\sin^2\widehat{\theta}_W(M_Z)$, differing according to whether or how finite $\alpha\ln(m_t/M_Z)$ terms are decoupled (subtracted from the couplings). One cannot entirely decouple the $\alpha\ln(m_t/M_Z)$ terms from all electroweak quantities because $m_t \gg m_b$ breaks SU(2) symmetry. The scheme that will be adopted here decouples the $\alpha \ln(m_t/M_Z)$ terms from the $\gamma - Z$ mixing [7,18], essentially eliminating any $\ln(m_t/M_Z)$ dependence in the formulae for asymmetries at the Z pole when written in terms of $\widehat{s}^{\,2}_Z$. The various definitions are related by

$$\widehat{s}^{\,2}_Z = c\,(m_t, M_H) s^2_W = \overline{c}\,(m_t, M_H)\, s^2_{M_Z} \ , \tag{29.8}$$

where $c = 1.035 \pm 0.003$ for $m_t = 180 \pm 7$ GeV and $M_H = 300$ GeV. Similarly $\overline{c} = 1.002 \pm 0.001$. The quadratic m_t dependence is given by $c \sim 1 + \rho_t / \tan^2\theta_W$. The expressions for M_W and M_Z in the $\overline{\rm MS}$ scheme are

$$M_W = \frac{A_0}{\widehat{s}_Z(1 - \Delta\widehat{r}_W)^{1/2}} \tag{29.9a}$$

$$M_Z = \frac{M_W}{\widehat{\rho}^{1/2}\widehat{c}_Z} \ . \tag{29.9b}$$

One predicts $\Delta\widehat{r}_W = 0.0705 \pm 0.0001 \pm 0.0007$ for $m_t = 180 \pm 7$ GeV and $M_H = 300$ GeV. $\Delta\widehat{r}_W$ has no quadratic m_t dependence, because shifts in M_W are absorbed into the observed G_F, so that $\Delta\widehat{r}_W$ is dominated by $\Delta r_0 = 1 - \alpha/\alpha(M_Z)$. Similarly, $\widehat{\rho} \sim 1 + \rho_t$. Including bosonic loops, $\widehat{\rho} = 0.0103 \pm 0.0008$ for $m_t = 180 \pm 7$ GeV.

(iv) A variant $\overline{\rm MS}$ quantity $\widehat{s}^{\,2}_{\rm ND}$ (used in the 1992 edition of this *Review*) does not decouple the $\alpha \ln(m_t/M_Z)$ terms [19]. It is related to $\widehat{s}^{\,2}_Z$ by

$$\widehat{s}^{\,2}_Z = \widehat{s}^{\,2}_{\rm ND} \Big/ \left(1 + \frac{\widehat{\alpha}}{\pi} d\right) \tag{29.10a}$$

$$d = \frac{1}{3}\left(\frac{1}{\widehat{s}^{\,2}} - \frac{8}{3}\right)\left[(1 + \frac{\widehat{\alpha}_s}{\pi}) \ln\frac{m_t}{M_Z} - \frac{15\widehat{\alpha}_s}{\pi}\right] \ , \tag{29.10b}$$

where $\widehat{\alpha}_s$ is the QCD coupling at M_Z. Thus, $\widehat{s}^{\,2}_Z - \widehat{s}^{\,2}_{\rm ND} \sim -0.0002$ for $(m_t,\, M_H) = (180, 300)$ GeV.

(v) Yet another definition, the effective angle [20,21] $\overline{s}^2_f$ for Z coupling to fermion f, is described below.

Experiments are now at such a level of precision that complete $\mathcal{O}(\alpha)$ radiative corrections must be applied. For neutral-current and Z-pole processes, these corrections are conveniently divided into two classes:

1. QED diagrams involving the emission of real photons or the exchange of virtual photons in loops, but not including vacuum polarization diagrams. These graphs yield finite and gauge-invariant contributions to observable processes. However, they are dependent on energies, experimental cuts, *etc.*, and must be calculated individually for each experiment.

2. Electroweak corrections, including $\gamma\gamma$, γZ, ZZ, and WW vacuum polarization diagrams, as well as vertex corrections, box graphs, *etc.*, involving virtual W's and Z's. Many of these corrections are absorbed into the renormalized Fermi constant defined in Eq. (29.5). Others modify the tree-level expressions for Z-pole observables and neutral-current amplitudes in several ways [14]. One-loop corrections are included for all processes. In addition, certain two-loop corrections are also important. In particular, two-loop corrections involving the top-quark [22] modify ρ_t in $\widehat{\rho}$, Δr, and elsewhere by

$$\rho_t \to \rho_t[1 + R(M_H/m_t)\rho_t/3] , \tag{29.11}$$

where $-3.8 > R > -11.8$ is strongly dependent on M_H/m_t: $R = -3.8$ for M_H at its lower direct limit and $R = -7.8$ for $M_H = 1.7m_t \approx 300$ GeV. -11.8 is in absolute lower bound for R which is assumed for large M_H. Mixed QCD-electroweak loops of order $\alpha\alpha_s m_t^2$ [23] and $\alpha\alpha_s^2 m_t^2$ [24] multiply ρ_t by $1 - 2$ $\alpha_s(0.3m_t)(\pi^2 + 3)/9\pi \sim 0.88$, where the three-loop result is included through the use of a lower scale for α_s. These mixed corrections increase the predicted value of m_t by 6%. Analogous electroweak and mixed two-loop terms are also known for the $Z \to b\overline{b}$ vertex [22,25].

29.2. Cross section and asymmetry formulas

It is convenient to write the four-fermion interactions relevant to ν-hadron, νe, and parity-violating e-hadron neutral-current processes in a form that is valid in an arbitrary gauge theory (assuming massless left-handed neutrinos). One has

$$-\mathscr{L}^{\nu\text{Hadron}} = \frac{G_F}{\sqrt{2}}\, \overline{\nu}\, \gamma^\mu\, (1-\gamma^5)\nu$$

$$\times \sum_i \left[\epsilon_L(i)\, \overline{q}_i\, \gamma_\mu(1-\gamma^5)q_i + \epsilon_R(i)\, \overline{q}_i\, \gamma_\mu(1+\gamma^5)q_i\right] , \tag{29.12}$$

$$-\mathscr{L}^{\nu e} = \frac{G_F}{\sqrt{2}}\, \overline{\nu}_\mu\, \gamma^\mu(1-\gamma^5)\nu_\mu\, \overline{e}\, \gamma_\mu(g_V^{\nu e} - g_A^{\nu e}\gamma^5)e \tag{29.13}$$

(for $\nu_e e$ or $\overline{\nu}_e e$, the charged-current contribution must be included), and

$$-\mathscr{L}^{e\text{Hadron}} = -\frac{G_F}{\sqrt{2}}$$

$$\times \sum_i \left[C_{1i}\, \overline{e}\, \gamma_\mu\, \gamma^5\, e\, \overline{q}_i\, \gamma^\mu\, q_i + C_{2i}\, \overline{e}\, \gamma_\mu\, e\, \overline{q}_i\, \gamma^\mu\, \gamma^5\, q_i\right] \tag{29.14}$$

(One must add the parity-conserving QED contribution.)

The Standard Model expressions for $\epsilon_{L,R}(i)$, $g_{V,A}^{\nu e}$, and C_{ij} are given in Table 29.2. Note that $g_{V,A}^{\nu e}$ and the other quantities are coefficients of effective four-fermi operators, which differ from the quantities defined in Eq. (29.2) and Eq. (29.3) in the radiative corrections and in the presence of possible physics beyond the Standard Model.

A precise determination of the on-shell s_W^2, which depends only very weakly on m_t and M_H, is obtained from deep inelastic neutrino scattering from (approximately) isoscalar targets [26]. The ratio $R_\nu \equiv \sigma_{\nu N}^{NC}/\sigma_{\nu N}^{CC}$ of neutral- to charged-current cross sections has been measured to 1% accuracy by the CDHS [27] and CHARM [28] collaborations [29,30] at CERN, and the CCFR collaboration at Fermilab [31] has obtained an even more precise result, so it is important to obtain theoretical expressions for R_ν and $R_{\overline{\nu}} \equiv \sigma_{\overline{\nu}N}^{NC}/\sigma_{\overline{\nu}N}^{CC}$ (as functions of $\sin^2\theta_W$) to comparable accuracy. Fortunately, most of the uncertainties from the strong interactions and neutrino spectra cancel in the ratio.

A simple zeroth-order approximation is

$$R_\nu = g_L^2 + g_R^2 r \tag{29.15a}$$

$$R_{\overline{\nu}} = g_L^2 + \frac{g_R^2}{r} \ , \tag{29.15b}$$

where

$$g_L^2 \equiv \epsilon_L\,(u)^2 + \epsilon_L\,(d)^2 \approx \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W \tag{29.16a}$$

$$g_R^2 \equiv \epsilon_R\,(u)^2 + \epsilon_R\,(d)^2 \approx \frac{5}{9}\sin^4\theta_W \ , \tag{29.16b}$$

and $r \equiv \sigma_{\overline{\nu}N}^{CC}/\sigma_{\nu N}^{CC}$ is the ratio of $\overline{\nu}$ and ν charged-current cross sections, which can be measured directly. [In the simple parton model, ignoring hadron energy cuts, $r \approx (\frac{1}{3}+\epsilon)/(1+\frac{1}{3}\epsilon)$, where $\epsilon \sim 0.125$ is the ratio of the fraction of the nucleon's momentum carried by antiquarks to that carried by quarks.] In practice, Eq. (29.15) must be corrected for quark mixing, the s and c seas, c-quark threshold effects, nonisoscalar target effects, W-Z propagator differences, and radiative corrections (which lower the extracted value of $\sin^2\theta_W$ by ~ 0.009). Details of the neutrino spectra, experimental cuts, x and Q^2 dependence of structure functions, and longitudinal structure functions enter only at the level of these corrections and therefore lead to very small uncertainties. The largest theoretical uncertainty is associated with the c threshold, which mainly affects σ^{CC}. Using the slow rescaling prescription [14] the central value of $\sin^2\theta_W$ varies as 0.013 [m_c(GeV)–1.3], where m_c is the effective mass. For $m_c = 1.31 \pm 0.24$ GeV (determined from ν-induced dimuon production [31]) this contributes ± 0.003 to the total theoretical uncertainty $\Delta\sin^2\theta_W \sim \pm 0.004$. This would require a high-energy neutrino beam for improvement. (The experimental uncertainty is ± 0.003). The CCFR group quotes $s_W^2 = 0.2218 \pm 0.0059$ for $(m_t, M_H) = (150, 100)$, but this result is insensitive to (m_t, M_H). Combining all of the precise deep-inelastic measurements, one obtains $s_W^2 = 0.2259 \pm 0.0043$ for (m_t, M_H) in the allowed range.

Table 29.2: Standard Model expressions for the neutral-current parameters for ν-hadron, νe, and e-hadron processes. If radiative corrections are ignored, $\rho = \kappa = 1$, $\lambda = 0$. At $\mathscr{O}(\alpha)$ in the on-shell scheme, $\rho^{NC}_{\nu N} = 1.0095$, $\kappa_{\nu N} = 1.0382$, $\lambda_{u_L} = -0.0032$, $\lambda_{d_L} = -0.0026$, and $\lambda_{u_R} = 1/2\,\lambda_{d_R} = 3.6 \times 10^{-5}$ for $m_t = 180$ GeV, $M_H = 300$ GeV, $M_Z = 91.1884$ GeV, and $\langle Q^2 \rangle = 20$ GeV^2. For νe scattering, $\kappa_{\nu e} = 1.0385$ and $\rho_{\nu e} = 1.0143$ (at $\langle Q^2 \rangle = 0.$). For atomic parity violation, $\rho'_{eq} = 0.9884$ and $\kappa'_{eq} = 1.036$. For the SLAC polarized electron experiment, $\rho'_{eq} = 0.979$, $\kappa'_{eq} = 1.034$, $\rho_{eq} = 1.002$, and $\kappa_{eq} = 1.06$ after incorporating additional QED corrections, while $\lambda_{2u} = -0.013$, $\lambda_{2d} = 0.003$. The dominant m_t dependence is given by $\rho \sim 1 + \rho_t$, while $\kappa \sim 1 + \rho_t / \tan^2\theta_W$ (on-shell) or $\kappa \sim 1(\overline{\text{MS}})$.

Quantity	Standard Model Expression
$\epsilon_L(u)$	$\rho^{NC}_{\nu N}\left(\frac{1}{2} - \frac{2}{3}\kappa_{\nu N}\ \sin^2\theta_W + \lambda_{uL}\right)$
$\epsilon_L(d)$	$\rho^{NC}_{\nu N}\left(-\frac{1}{2} + \frac{1}{3}\kappa_{\nu N}\ \sin^2\theta_W + \lambda_{dL}\right)$
$\epsilon_R(u)$	$\rho^{NC}_{\nu N}\left(-\frac{2}{3}\kappa_{\nu N}\ \sin^2\theta_W + \lambda_{uR}\right)$
$\epsilon_R(d)$	$\rho^{NC}_{\nu N}\left(\frac{1}{3}\kappa_{\nu N}\ \sin^2\theta_W + \lambda_{dR}\right)$
$g^{\nu e}_V$	$\rho_{\nu e}\left(-\frac{1}{2} + 2\kappa_{\nu e}\ \sin^2\theta_W\right)$
$g^{\nu e}_A$	$\rho_{\nu e}\left(-\frac{1}{2}\right)$
C_{1u}	$\rho'_{eq}\left(-\frac{1}{2} + \frac{4}{3}\kappa'_{eq}\ \sin^2\theta_W\right)$
C_{1d}	$\rho'_{eq}\left(\frac{1}{2} - \frac{2}{3}\kappa'_{eq}\ \sin^2\theta_W\right)$
C_{2u}	$\rho_{eq}\left(-\frac{1}{2} + 2\kappa_{eq}\ \sin^2\theta_W\right) + \lambda_{2u}$
C_{2d}	$\rho_{eq}\left(\frac{1}{2} - 2\kappa_{eq}\ \sin^2\theta_W\right) + \lambda_{2d}$

The laboratory cross section for $\nu_\mu e \to \nu_\mu e$ or $\overline{\nu}_\mu e \to \overline{\nu}_\mu e$ elastic scattering is

$$\frac{d\sigma_{\nu_\mu,\overline{\nu}_\mu}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi} \times \left[(g_V^{\nu e} \pm g_A^{\nu e})^2 + (g_V^{\nu e} \mp g_A^{\nu e})^2 (1-y)^2 - (g_V^{\nu e 2} - g_A^{\nu e 2}) \frac{y\, m_e}{E_\nu}\right], \tag{29.17}$$

where the upper (lower) sign refers to $\nu_\mu(\overline{\nu}_\mu)$, and $y \equiv E_e/E_\nu$ [which runs from 0 to $(1 + m_e/2E_\nu)^{-1}$] is the ratio of the kinetic energy of the recoil electron to the incident ν or $\overline{\nu}$ energy. For $E_\nu \gg m_e$ this yields a total cross section

$$\sigma = \frac{G_F^2\, m_e\, E_\nu}{2\pi}\left[(g_V^{\nu e} \pm g_A^{\nu e})^2 + \frac{1}{3}(g_V^{\nu e} \mp g_A^{\nu e})^2\right] . \tag{29.18}$$

The most accurate leptonic measurements [32–34] of $\sin^2\theta_W$ are from the ratio $R \equiv \sigma_{\nu_\mu e}/\sigma_{\overline{\nu}_\mu e}$ in which many of the systematic uncertainties cancel. Radiative corrections (other than m_t effects) are small compared to the precision of present experiments and have negligible effect on the extracted $\sin^2\theta_W$. The most precise (CHARM II) experiment [34] determined not only $\sin^2\theta_W$ but $g_{V,A}^{\nu e}$ as well. The cross sections for $\nu_e e$ and $\overline{\nu}_e e$ may be obtained from Eq. (29.17) by replacing $g_{V,A}^{\nu e}$ by $g_{V,A}^{\nu e} + 1$, where the 1 is due to the charged-current contribution.

The SLAC polarized-electron experiment [35] measured the parity-violating asymmetry

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} , \tag{29.19}$$

where $\sigma_{R,L}$ is the cross section for the deep-inelastic scattering of a right- or left-handed electron: $e_{R,L}N \to e\mathrm{X}$. In the quark parton model

$$\frac{A}{Q^2} = a_1 + a_2\,\frac{1-(1-y)^2}{1+(1-y)^2} , \tag{29.20}$$

where $Q^2 > 0$ is the momentum transfer and y is the fractional energy transfer from the electron to the hadrons. For the deuteron or other isoscalar target, one has, neglecting the s quark and antiquarks,

$$a_1 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{1u} - \frac{1}{2}C_{1d}\right) \approx \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(-\frac{3}{4} + \frac{5}{3}\sin^2\theta_W\right) \tag{29.21a}$$

$$a_2 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{2u} - \frac{1}{2}C_{2d}\right) \approx \frac{9G_F}{5\sqrt{2}\pi\alpha}\left(\sin^2\theta_W - \frac{1}{4}\right) . \tag{29.21b}$$

Radiative corrections (other than m_t effects) lower the extracted value of $\sin^2\theta_W$ by $\sim$ 0.005.

There are now precise experiments measuring atomic parity violation [36] in cesium [37], bismuth [38], lead [39], and thallium [40]. The uncertainties associated with atomic wave functions are quite small for cesium, for which the theoretical uncertainty is $\sim 1\%$ [41] but somewhat larger for the other atoms. For heavy atoms one determines the "weak charge"

$$\begin{aligned} Q_W &= -2\,[C_{1u}\,(2Z+N) + C_{1d}(Z+2N)] \\ &\approx Z(1-4\sin^2\theta_W) - N . \end{aligned} \tag{29.22}$$

Radiative corrections increase the extracted $\sin^2\theta_W$ by ~ 0.008.

In the future it should be possible to reduce the theoretical wave function uncertainties by taking the ratios of parity violation in different isotopes [36,42]. There would still be some residual uncertainties from differences in the neutron charge radii, however [43].

The forward-backward asymmetry for $e^+e^- \to \ell\overline{\ell}$, $\ell = \mu$ or τ, is defined as

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \ , \tag{29.23}$$

where $\sigma_F(\sigma_B)$ is the cross section for ℓ^- to travel forward (backward) with respect to the e^- direction. A_{FB} and R, the total cross section relative to pure QED, are given by

$$R = F_1 \tag{29.24}$$

$$A_{FB} = 3F_2/4F_1 \ , \tag{29.25}$$

where

$$F_1 = 1 - 2\chi_0 \, g_V^e \, g_V^\ell \ \cos\delta_R + \chi_0^2 \left(g_V^{e2} + g_A^{e2}\right)\left(g_V^{\ell 2} + g_A^{\ell 2}\right) \tag{29.26a}$$

$$F_2 = -2\chi_0 \, g_A^e \, g_A^\ell \ \cos\delta_R + 4\chi_0^2 \, g_A^e \, g_A^\ell \, g_V^e \, g_V^\ell \ , \tag{29.26b}$$

where

$$\tan\delta_R = \frac{M_Z\Gamma_Z}{M_Z^2 - s} \tag{29.27}$$

$$\chi_0 = \frac{G_F}{2\sqrt{2}\pi\alpha}\frac{sM_Z^2}{\left[(M_Z^2 - s)^2 + M_Z^2\Gamma_Z^2\right]^{1/2}} \tag{29.28}$$

and $\sqrt{s}$ is the CM energy. Eq. (29.26) is valid at tree level. If the data are radiatively corrected for QED effects (as described above), then the remaining electroweak corrections can be incorporated [44] (in an approximation adequate for existing PEP, PETRA, and TRISTAN data, which are well below the Z pole) by replacing χ_0 by $\chi(s) \equiv (1+\rho_t)\chi_0(s)\alpha/\alpha(s)$, where $\alpha(s)$ is the running QED coupling, and evaluating g_V in the $\overline{\text{MS}}$ scheme. Formulas for $e^+e^- \to$ *hadrons* may be found in Ref. 45.

At LEP and SLC, there are high-precision measurements of various Z-pole observables [46–49]. These include the Z mass and total width Γ_Z, and partial widths $\Gamma(f\overline{f})$ for $Z \to f\overline{f}$ for fermion f ($f = e$, μ, τ, hadrons, b, c, and ν). The data is consistent with lepton-family universality $\Gamma(e^+e^-) = \Gamma(\mu^+\mu^-) = \Gamma(\tau^+\tau^-)$, so one may work with an average width $\Gamma(\ell\overline{\ell})$. It is convenient to use the variables M_Z, Γ_Z, $R \equiv \Gamma(\text{had})/\Gamma(\ell\overline{\ell})$, $\sigma_{\text{had}} \equiv 12\pi\Gamma(e^+e^-)\Gamma(\text{had})/M_Z^2\,\Gamma_Z^2$, $R_b \equiv \Gamma(b\overline{b})/\Gamma(\text{had})$, and $R_c \equiv \Gamma(c\overline{c})/\Gamma(\text{had})$, most of which are weakly correlated experimentally. ($\Gamma(\text{had})$ is the partial width into hadrons.) The largest correlation coefficient of -0.35 occurs between R_b and R_c. R is insensitive to m_t except for $Z \to b\overline{b}$ vertex and final state corrections and the implicit dependence through $\sin^2\theta_W$. Thus it is especially useful for constraining α_s. The width for invisible decays, $\Gamma(\text{inv}) = \Gamma_Z - 3\Gamma(\ell\overline{\ell}) - \Gamma(\text{had}) = 499.9 \pm 2.5$ MeV, can be used to determine the number of neutrino flavors lighter than $M_Z/2$, $N_\nu = \Gamma_{\text{inv}}/\Gamma(\nu\overline{\nu}) = 2.991 \pm 0.016$.

There are also measurements of various asymmetries. These include the polarization or left-right asymmetry

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \ , \tag{29.29}$$

where $\sigma_L(\sigma_R)$ is the cross section for a left- (right)-handed incident electron. A_{LR} has been measured precisely by the SLD collaboration at SLC [48] and has the advantages of being extremely sensitive to $\sin^2\theta_W$ and insensitive to QED radiative corrections. Other asymmetries are the forward-backward asymmetries $A_{FB}^{(0,f)}$ for $f = e$, μ, τ, b, c ($A_{FB}^{(0,e)}$, $A_{FB}^{(0,\mu)}$, $A_{FB}^{(0,\tau)}$ are consistent with lepton-family universality, allowing an average value $A_{FB}^{(0,\ell)}$), the hadronic-charge asymmetry, the τ polarization P_τ, and its angular distribution. Further details, including references to the data from the LEP experiments (ALEPH, DELPHI, L3, OPAL) may be found in the Full Listings in the 'Note on the Z Boson' and in [46–49]. At tree level and neglecting QED effects and terms of order $(\Gamma_Z/M_Z)^2$, one has

$$A_{FB}^{(0,f)} \approx \frac{3}{4} A_f \, \frac{A_e + P_e}{1 + P_e A_e} \tag{29.30}$$

$$A_{LR} \approx A_e P_e \ , \tag{29.31}$$

where P_e is the initial e^- polarization and

$$A_f \equiv \frac{2 g_V^f g_A^f}{g_V^{f2} + g_A^{f2}} \ . \tag{29.32}$$

Similarly, A_τ is given by the negative total τ polarization, and A_e can be extracted from the angular distribution of the polarization. In addition, the SLD collaboration [49] has extracted the final-state couplings A_b and A_c from the left-right forward-backward asymmetry, using

$$\frac{\sigma_{LF} - \sigma_{LB} - \sigma_{RF} + \sigma_{RB}}{\sigma_{LF} + \sigma_{LB} + \sigma_{RF} + \sigma_{RB}} = A_f \ , \tag{29.33}$$

where, for example, σ_{LF} is the cross section for a left-handed incident electron to produce a fermion f traveling in the forward hemisphere.

It has become customary for the experimental groups to present corrected asymmetries A^0, in which photon exchange and γ-Z interference, QED corrections, and corrections for $\sqrt{s} \neq M_Z$ are removed from the data, leaving the pure electroweak asymmetries. Ignoring negligible electroweak boxes, these corrected asymmetries are expressed using effective tree-level expression *e.g.*, $A_{FB}^{(0,f)} = \frac{3}{4}\overline{A}_f\overline{A}_e$ (for $P_e = 0$) and $A_{LR}^0 = \overline{A}_e$, where

$$\overline{A}_f = \frac{2\overline{g}_V^f\,\overline{g}_A^f}{\overline{g}_V^{f2} + \overline{g}_A^{f2}}\ , \tag{29.34a}$$

and

$$\overline{g}_V^f = \sqrt{\rho_f}\,(t_{3L}^{(f)} - 2q_f\kappa_f\sin^2\theta_W) \tag{29.34b}$$

$$\overline{g}_A^f = \sqrt{\rho_f}\,t_{3L}^{(f)}\ . \tag{29.34c}$$

The electroweak-radiative corrections have been absorbed into corrections $\rho_f - 1$ and $\kappa_f - 1$, which depend on the fermion f and on the renormalization scheme. In the on-shell scheme, the quadratic m_t dependence is given by $\rho_f \sim 1 + \rho_t$, $\kappa_f \sim \kappa_f^{os} \sim 1 + \rho_t/\tan^2\theta_W$, while in $\overline{\text{MS}}$, $\rho_f \sim \widehat{\rho}$, $\kappa_f \equiv \widehat{\kappa}_f \sim 1$. In practice, additional bosonic loops, vertex corrections, *etc.*, must be included. For example, in the $\overline{\text{MS}}$ scheme one has, for $(m_t, M_H) = (180, 300)$, $\rho_\ell = 1.0053$ and $\widehat{\kappa}_\ell = 1.0012$. It is convenient to define an effective angle $\overline{s}_f^2 \equiv \sin^2\overline{\theta}_{Wf} \equiv \widehat{\kappa}_f\widehat{s}_Z^2 = \kappa_f^{os}\,s_W^2$, in terms of which $\overline{g}_V^f$ and $\overline{g}_A^f$ are given by $\sqrt{\rho_f}$ times their tree-level formulae. Because $\overline{g}_V^\ell$ is very small, not only A_{LR}^0 $A_{FB}^{(0,\ell)}$, and P_τ^0, but also $A_{FB}^{(0,b)}$, $A_{FB}^{(0,c)}$, and the hadronic-charge asymmetry are mainly sensitive to $\overline{s}_\ell^2$. One finds that $\widehat{\kappa}_f$ is almost independent of (m_t, M_H), so that

$$\overline{s}_\ell^2 \sim \widehat{s}_Z^2 + 0.00028 \tag{29.35}$$

using Ref. 20, or $\overline{s}_\ell^2 \sim \widehat{s}_Z^2 + 0.0002$ from Ref. 21 (the small difference is an indication of theoretical uncertainties from higher-order terms, *etc.*). In any case, the asymmetries determine values of $\overline{s}_\ell^2$ and $\widehat{s}_Z^2$ almost independent of m_t, while the κ's for the other schemes are m_t dependent.

29.3. *W* and *Z* decays

The partial decay width for gauge bosons to decay into massless fermions $f_1\overline{f}_2$ is

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226 \pm 1 \text{ MeV} \tag{29.36a}$$

$$\Gamma(W^+ \to u_i\overline{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi}\,|V_{ij}|^2 \approx (705 \pm 4)\,|V_{ij}|^2 \text{ MeV} \tag{29.36b}$$

$$\Gamma(Z \to \psi_i\overline{\psi}_i) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi}\left[g_V^{i2} + g_A^{i2}\right] \tag{29.36c}$$

$$\approx \begin{cases} 167.2 \pm 0.1 \text{ MeV } (\nu\overline{\nu}), & 84.0 \pm 0.1 \text{ MeV } (e^+e^-), \\ 300.6 \pm 0.3 \text{ MeV } (u\overline{u}), & 383.3 \pm 0.3 \text{ MeV } (d\overline{d}), \\ 375.9 \mp 0.2 \text{ MeV } (b\overline{b}). & \end{cases}$$

For leptons $C = 1$, while for quarks $C = 3\Big(1 + \alpha_s(M_V)/\pi + 1.409\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3\Big)$, where the 3 is due to color and the factor in parentheses represents the universal QCD corrections for massless quarks [50]. The $Z \to f\overline{f}$ widths contain a number of additional corrections [51]: universal (non-singlet) top-mass contributions [52]; fermion mass effects and further QCD corrections proportional to m_q^2 [53] (m_q is the running quark mass evaluated at the Z scale) which are different for vector and axial-vector partial widths; and singlet contributions starting from two loop order which are large, strongly top-mass dependent, family universal and flavor non-universal [54]. All QCD effects are known and included up to three loop order with the exception of order $\alpha_s^3 m_b^2$ corrections which are very small. The QED factor $1 + 3\alpha q_f^2/4\pi$ and order $\alpha\alpha_s$ corrections [55] have to be included, as well. Expressing the widths in terms of $G_F M_{W,Z}^3$ incorporates the bulk of the low-energy radiative corrections [16,56]. The electroweak corrections are incorporated by replacing $g_{V,A}^{i2}$ by $\overline{g}_{V,A}^{i2}$. Hence, the widths are proportional to $\rho_i \sim 1 + \rho_t$. There is additional (negative) quadratic m_t dependence in the $Z \to b\overline{b}$ vertex corrections [57] which causes $\Gamma(b\overline{b})$ to decrease with m_t. The dominant effect is to multiply $\Gamma(b\overline{b})$ by the vertex correction $1 + \delta\rho_{b\overline{b}}$, where $\delta\rho_{b\overline{b}} \sim 10^{-2}(-\frac{1}{2}\frac{m_t^2}{M_Z^2} + \frac{1}{5})$. In practice, the corrections are included in ρ_b and κ_b.

For 3 fermion families the total widths are predicted to be

$$\Gamma_Z \approx 2.497 \pm 0.002 \text{ GeV} \tag{29.37}$$

$$\Gamma_W \approx 2.09 \pm 0.01 \text{ GeV} \quad . \tag{29.38}$$

The numerical values for the widths assume $M_Z = 91.1884 \pm 0.0022$ GeV, $M_W = 80.26 \pm 0.16$ GeV, $\alpha_s = 0.123$, and $m_t = 180 \pm 7$ GeV, where the α_s and m_t values are predicted by the global fits for $M_H = 300$ GeV. The uncertainties for Γ_W and Γ_Z are dominated by ΔM_W and Δm_t, respectively. The uncertainty in α_s, ± 0.004, introduces an additional uncertainty of 0.13% in the hadronic widths, corresponding to ± 2 MeV in Γ_Z.

These predictions are to be compared with the experimental results $\Gamma_Z = 2.4963 \pm 0.0032$ GeV and $\Gamma_W = 2.08 \pm 0.07$ GeV.

29.4. Experimental results

The values of the principal Z-pole observables are listed in Table 29.3, along with the Standard Model predictions for $M_Z = 91.1884 \pm 0.0022$, $m_t = 180 \pm 7$ GeV (for $M_H = 300$ GeV), 60 GeV $< M_H < 1$ TeV, and $\alpha_s = 0.123 \pm 0.004$. Note that, the values of the Z-pole observables (as well as M_W) differ from those in the Full Listings because they include recent preliminary results [47,49,59]. The values and predictions of M_W [59], the Q_W for cesium [36,41], and recent results from deep inelastic and $\nu_\mu e$ scattering are also listed. The agreement is generally excellent. Major exceptions are $R_b = \Gamma(b\bar{b})/\Gamma(\text{had})$ which is 3.7σ above the Standard Model prediction, and $R_c = \Gamma(c\bar{c})/\Gamma(\text{had})$ which is 2.4σ below. These are strongly correlated: if R_c is fixed at the Standard Model value of 0.172, then one obtains [47] $R_b = 0.2205 \pm 0.0016$, which is still 3.0σ too high. Within the Standard Model framework, these values must be considered large statistical fluctuations or systematic errors. However, R_b tends to favor small values of m_t, and when combined with other observables, small values for M_H. Many types of new physics could contribute to R_b (see also Sec. 30 on "Constraints on New Physics from Electroweak Analyses" in this *Review*). The implications of this possiblity for the value of $\alpha_s(M_Z)$ extracted from the fits are discussed below. The left-right asymmetry $A^0_{LR} = 0.1551 \pm 0.0040$ [49] based on all data from 1992–1995 has moved closer to the Standard Model expectation of 0.144 ± 0.003 than the previous value 0.1637 ± 0.0075, from 1992–1993. However, because of the smaller error A^0_{LR} is still 2.3σ above the Standard Model prediction. There is also an experimental difference of $\sim 1.5\sigma$ between the SLD value of $A^0_e = A^0_{LR}$ and the LEP value $A^0_{\ell\text{LEP}} \sim 0.147 \pm 0.004$ obtained from $A^{(0,\ell)}_{FB}$, $A^0_e(P_\tau)$, $A^0_\tau(P_\tau)$ assuming lepton family universality. Finally, the forward-backward asymmetry into τ's, $A^{0\tau}_{FB} = 0.0206 \pm 0.0023$ [47], is 2.2σ above the Standard Model prediction and 1.6σ above the average 0.0162 ± 0.0014 of A^{0e}_{FB} and $A^{0\mu}_{FB}$. This is small enough to be a fluctuation, so lepton-family universality will be assumed. The observables in Table 29.3 (including correlations on the LEP observables), as well as all low-energy neutral-current data [14,15], are used in the global fits described below. The parameter $\sin^2\theta_W$ can be determined from the Z-pole observables and M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results [14], shown in Table 29.4, are in impressive agreement with each other, indicating the quantitative success of the Standard Model. The one discrepancy is the value $\widehat{s}^{\,2}_Z = 0.2302 \pm 0.0005$ from A^0_{LR} which is 2.1σ below the value (0.2315 ± 0.0004) from the global fit to all data and 2.6σ below the value 0.2318 ± 0.0004 obtained from all data other than A^0_{LR}.

The data allow a simultaneous determination of $\sin^2\theta_W$, m_t, and the strong coupling $\alpha_s(M_Z)$. The latter is determined mainly from Γ_Z and R, and is only weaky correlated with the other variables. The global fit to all data, including the CDF/DØ value $m_t = 180 \pm 12$ GeV, yields

$$\begin{aligned}
&\widehat{s}^{\,2}_Z = 0.2315 \pm 0.0002 \pm 0.0003 \\
&m_t = 180 \pm 7^{+12}_{-13}\ \text{GeV} \\
&\alpha_s(M_Z) = 0.123 \pm 0.004 \pm 0.002\ ,
\end{aligned} \tag{29.39}$$

where the central values are for a Higgs mass of 300 GeV, and the second error bars are for $M_H \to 1000(+)$ or $60(-)$ GeV. In all fits, the

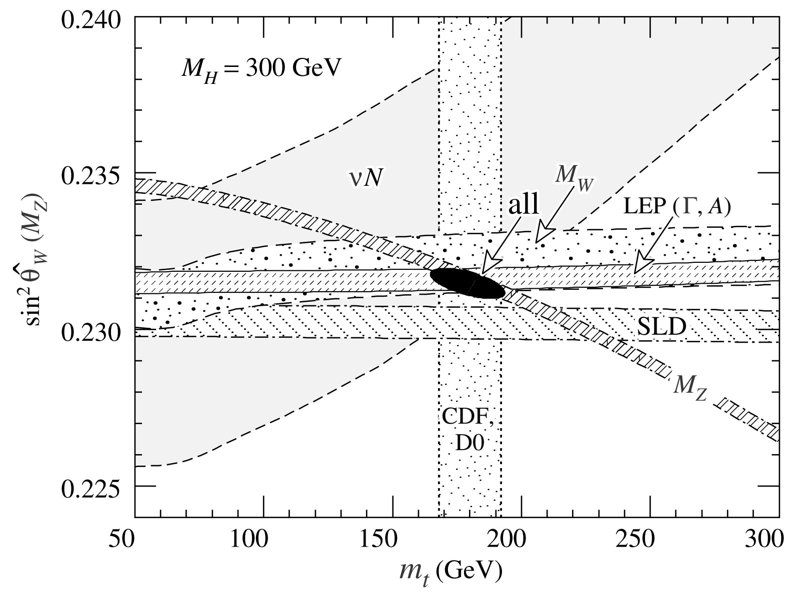

Figure 29.1: One-standard-deviation uncertainties in $\sin^2\hat{\theta}_W$ as a function of m_t, the direct CDF and DØ range 180 ± 12 GeV, and the 90% CL region in $\sin^2\hat{\theta}_W - m_t$ allowed by all data, assuming $M_H = 300$ GeV.

errors include full statistical, systematic, and theoretical uncertainties. The $\hat{s}^2_Z$ error is dominated by m_t, and $\hat{s}^2_Z$ and m_t have a strong negative correlation of ~ -0.62. In the on-shell scheme one has $s^2_W = 0.2236 \pm 0.0008$, the larger error due to the stronger sensitivity to m_t. The extracted value of α_s is based on a formula which has almost no theoretical uncertainty (if one assumes the exact validity of the Standard Model), and is in excellent agreement with the values 0.122 ± 0.007 from jet-event shapes in e^+e^- annihilation, and the average 0.118 ± 0.003 from all data (including the Z-lineshape data), as described in our Section 28 on "Quantum Chromodynamics" in this *Review*. However, it is higher than some of the individual values extracted from low-energy data, such as deep-inelastic scattering (0.112 ± 0.002 (exp) ± 0.004 (scale)) or lattice calculations of the $b\bar{b}$ and $c\bar{c}$ spectra (0.115 ± 0.003). It has been suggested [60] that there is a real discrepancy. However, caution is requred since most of the determinations are dominated by theory errors.

The value of R_b is more than 3σ above the Standard Model expectation. If this is not just a fluctuation but is due to a new physics contribution to the $Z \to b\bar{b}$ vertex (many types would couple preferentially to the third family), the value of $\alpha_s(M_Z)$ extracted from the hadronic Z width would be reduced [15]. Allowing for this possibility one obtains $\alpha_s(M_Z) = 0.101 \pm 0.008$. (See also Sec. 30 on "Constraints on New Physics from Electroweak Analyses." in this *Review*)

In principle the low value of R_c could also be due to new physics. However, allowing for new physics contributions to R_c alone, one obtains $\alpha_s(M_Z) = 0.19 \pm 0.03$, which is clearly inconsistent with low-energy determinations. Allowing new contributions to both R_b and R_c yields the slightly lower but still high value of $\alpha_s(M_Z) = 0.16 \pm 0.04$. We will, therefore, take the view that the R_c value is a fluctuation. We keep the experimental values $R_b = 0.2219(17)$ and $R_c = 0.1540(74)$ and their correlation (-0.35) in all fits, but do not allow any special vertex corrections for $Z \to c\bar{c}$. This is effectively equivalent to using the lower value 0.2205(16) that the LEP experimenters obtain for R_b when they constrain R_c to the Standard Model value of 0.172.

One can also carry out a fit to the indirect data alone, *i.e.*, without including the value $m_t = 180 \pm 12$ GeV observed directly by CDF and DØ. (The indirect prediction is for the pole mass, which should correspond approximately to the kinematic mass extracted from the collider events.) One obtains $m_t = 179 \pm 8^{+17}_{-20}$ GeV, with little change in the $\sin^2\theta_W$ and α_s values, in remarkable agreement with the direct CDF/DØ value. The results of fits to various combinations of the data are shown in Table 29.5 and the relation between $\widehat{s}^{\,2}_Z$ and m_t for various observables in Fig. 29.1.

The data indicate a preference for a small Higgs mass. This is because there is a strong correlation between the quadratic ρ_t terms and logarithmic M_H effects in all of the indirect data except the $Z \to b\bar{b}$ vertex. The latter favor a smaller m_t and therefore a smaller M_H. The difference in χ^2 for the global fit is $\Delta\chi^2 = \chi^2(M_H = 1000 \text{ GeV}) - \chi^2(M_H = 60 \text{ GeV}) = 7.9$. Hence, the data favor a small value of M_H, as in supersymmetric extensions of the Standard Model, and m_t on the lower side of the allowed range; including the direct constraint $M_H \geq 60$ GeV, the best fit is for $M_H = 60$ GeV, with the limit $M_H < 320(430)$ GeV at 90(95)% CL. However, one should be cautious because the M_H constraint is driven almost entirely by R_b and A_{LR}, both of which deviate from the Standard Model prediction. Using $\alpha(M_Z)$ and $\widehat{s}^{\,2}_Z$ as inputs, one can predict $\alpha_s(M_Z)$ assuming grand unification. One predicts [61] $\alpha_s(M_Z) = 0.130 \pm 0.001 \pm 0.01$ for the simplest theories based on the minimal supersymmetric extension of the Standard Model, where the first (second) uncertainty is from the inputs (thresholds). This is consistent with the experimental $\alpha_s(M_Z) = 0.121(4)(1)$ from the Z-lineshape (using the lower M_H range appropriate for supersymmetry) and with the average 0.118 ± 0.003 (see our Section 28 on "Quantum Chromodynamics" in this *Review*), but is high compared to some low-energy determinations of α_s [60]. Nonsupersymmetric unified theories predict the low value $\alpha_s(M_Z) = 0.073 \pm 0.001 \pm 0.001$.

One can also determine the radiative correction parameters Δr: including the CDF and DØ data, one obtains $\Delta r = 0.039 \pm 0.003$ and $\Delta\widehat{r}_W = 0.068 \pm 0.0013$, where the error includes m_t and M_H, in excellent agreement with the predictions 0.038 ± 0.005 and 0.0705 ± 0.0007.

Table 29.3: Principal LEP and other recent observables, compared with the Standard Model predictions for $M_Z = 91.1884 \pm 0.0022$ GeV, 60 GeV $< M_H <$ 1 TeV, the global best fit value $m_t = 180 \pm 7$ GeV (for $M_H = 300$ GeV), $\alpha_s = 0.123 \pm 0.004$, and $\alpha_s(M_Z)^{-1} = 128.90 \pm 0.09$. The LEP averages [58] of the ALEPH, DELPHI, L3, and OPAL results include common systematic errors and correlations [58]. $\bar{s}^2_\ell(A^{(0,q)}_{FB})$ is the effective angle extracted from the hadronic-charge asymmetry. A^0_{LR} includes all data from 1992–1995 [48,49]. The values of $\Gamma(\ell\bar{\ell})$, $\Gamma(\mathrm{had})$, and $\Gamma(\mathrm{inv})$ are not independent of Γ_Z, R, and σ_{had}. The M_W value is from CDF, UA2, and DØ [59]. M_W and M_Z are correlated, but the effect is negligible due to the tiny M_Z error. The two values of s^2_W from deep-inelastic scattering are from CCFR [31] and the global average, respectively. The $g^{\nu e}_{V,A}$ are from CHARM II [34]. The second error in Q_W (for cesium) is theoretical [41]. Older low-energy results are not listed but are included in the fits. In the Standard Model predictions, the first uncertainty is from M_Z and Δr, while the second is from m_t and M_H. The $\Delta\alpha_s = 0.004$ uncertainty leads to additional errors of 0.002 (Γ_Z), 0.02 (R), 0.02 (σ), 2.0 ($\Gamma(\mathrm{had})$).

Quantity	Value	Standard Model
M_Z (GeV)	91.1884 ± 0.0022	input
Γ_Z (GeV)	2.4963 ± 0.0032	$2.497 \pm 0.001 \pm 0.002$
R	20.788 ± 0.032	$20.77 \pm 0.004 \pm 0.002$
$\sigma_{\mathrm{had}}(nb)$	41.488 ± 0.078	$41.45 \pm 0.002 \pm 0.004$
R_b	0.2219 ± 0.0017	$0.2156 \pm 0 \pm 0.0003$
R_c	0.1540 ± 0.0074	$0.172 \pm 0 \pm 0$
$A^{(0,\ell)}_{FB}$	0.0172 ± 0.0012	$0.0155 \pm 0.0004 \pm 0.0004$
$A^0_\tau(P_\tau)$	0.1418 ± 0.0075	$0.144 \pm 0.002 \pm 0.002$
$A^0_e(P_\tau)$	0.1390 ± 0.0089	$0.144 \pm 0.002 \pm 0.002$
$A^{(0,b)}_{FB}$	0.0997 ± 0.0031	$0.101 \pm 0.001 \pm 0.001$
$A^{(0,c)}_{FB}$	0.0729 ± 0.0058	$0.072 \pm 0.001 \pm 0.001$
A^0_{LR}	0.1551 ± 0.0040	$0.144 \pm 0.002 \pm 0.002$
$\overline{A}_b$	0.841 ± 0.053	$0.934 \pm 0 \pm 0$
$\overline{A}_c$	0.606 ± 0.090	$0.667 \pm 0.001 \pm 0.001$
$\bar{s}^2_\ell(A^{(0,q)}_{FB})$	0.2325 ± 0.0013	$0.2319 \pm 0.0002 \pm 0.0002$
$\Gamma(\ell\bar{\ell})$ (MeV)	83.93 ± 0.14	$83.97 \pm 0.01 \pm 0.06$
$\Gamma(\mathrm{had})$ (MeV)	1744.8 ± 3.0	$1743.8 \pm 0.2 \pm 1.2$
$\Gamma(\mathrm{inv})$ (MeV)	499.9 ± 2.5	$501.6 \pm 0 \pm 0.3$
M_W (GeV)	80.26 ± 0.16	$80.34 \pm 0.01 \pm 0.04$
Q_W	$-71.04 \pm 1.58 \pm 0.88$	$-72.88 \pm 0.05 \pm 0.03$
$s^2_W = 1 - \frac{M^2_W}{M^2_Z}$	0.2218 ± 0.0059 0.2260 ± 0.0048	$0.2237 \pm 0.0002 \pm 0.0008$
$g^{\nu e}_A$	-0.503 ± 0.017	$-0.507 \pm 0 \pm 0.0004$
$g^{\nu e}_V$	-0.035 ± 0.017	$-0.037 \pm 0.0005 \pm 0.0003$

Table 29.4: Values obtained for s^2_W (on-shell) and $\widehat{s}^{\,2}_Z(\overline{\rm MS})$ from various reactions assuming the global best fit value $m_t = 180 \pm 07$ GeV (for $M_H = 300$ GeV), and $\alpha_s = 0.123 \pm 0.004$. The uncertainties include the effect of 60 GeV $< M_H <$ 1 TeV. The determination from Γ_Z, R, and $\sigma_{\rm had}$ uses the experimental value of M_Z, so that the values obtained are from the vertices and not the overall scale.

Reaction	s^2_W	$\widehat{s}^{\,2}_Z$
M_Z	0.2237 ± 0.0010	0.2316 ± 0.0005
M_W	0.2242 ± 0.0011	0.2321 ± 0.0009
$\Gamma_Z,\ R, \sigma_{\rm had}$	0.2239 ± 0.0013	0.2317 ± 0.0013
$A^{(0,\ell)}_{FB}$	0.2228 ± 0.0009	0.2307 ± 0.0007
LEP asymmetries	0.2237 ± 0.0007	0.2316 ± 0.0003
A^0_{LR}	0.2223 ± 0.0008	0.2302 ± 0.0005
$\overline{A}_b, \overline{A}_c$	0.250 ± 0.021	0.259 ± 0.022
Deep inelastic (isocalar)	0.226 ± 0.004	0.234 ± 0.005
$\nu_\mu(\overline{\nu}_\mu)p \to \nu_\mu(\overline{\nu}_\mu)p$	0.205 ± 0.030	0.212 ± 0.031
$\nu_\mu(\overline{\nu}_\mu)e \to \nu_\mu(\overline{\nu}_\mu)e$	0.221 ± 0.007	0.228 ± 0.008
atomic parity violation	0.216 ± 0.008	0.223 ± 0.008
SLAC eD	0.216 ± 0.017	0.223 ± 0.018
All data	0.2236 ± 0.0008	0.2315 ± 0.0004

29.5. Deviations from the Standard Model

The Z pole, W mass, and neutral-current data can be used to search for and set limits on deviations from the Standard Model.

For example, the relation between M_W and M_Z is modified if there are Higgs multiplets with weak isospin $> 1/2$ with significant vacuum expectation values. In order to calculate to higher orders in such theories one must define a set of four fundamental renormalized parameters. It is convenient to take these as α, G_F, M_Z, and M_W, since M_W and M_Z are directly measurable. Then $\widehat{s}^{\,2}_Z$ and ρ_0 can be considered dependent parameters defined by

$$\widehat{s}^{\,2}_Z \equiv A_0^2/M_W^2(1-\Delta\widehat{r}_W) \tag{29.40}$$

and

$$\rho_0 \equiv M_W^2/(M_Z^2\,\widehat{c}^{\,2}_Z\,\widehat{\rho})\ . \tag{29.41}$$

Table 29.5: Values of $\widehat{s}^2_Z$ and s^2_W (in parentheses), α_s, and m_t for various combinations of observables. The central values are for $M_H = 300$ GeV, and the second set of errors is for $M_H \to 1000(+)$, $60(-)$.

Data	$\widehat{s}^2_Z$ (s^2_W)	α_s (M_Z)	m_t (GeV)
Indirect + CDF + DØ	0.2315(2)(3)	0.123(4)(2)	$180 \pm 7^{+12}_{-13}$
	(0.2236 ± 0.0008)		
All indirect	0.2315(2)(2)	0.123(4)(2)	$179 \pm 8^{+17}_{-20}$
	(0.2236 ± 0.0009)		
All LEP	0.2318(3)(2)	0.124(4)(2)	$171 \pm 10^{+18}_{-20}$
	(0.2246 ± 0.0011)		
SLD + M_Z	0.2302(5)(0)	—	220^{+14+19}_{-15-24}
	(0.2184 ± 0.0020)		
Z pole	0.2314(3)(1)	0.123(4)(2)	181^{+8+18}_{-9-20}
(LEP + SLD)	(0.2234 ± 0.0010)		

Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect the radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (29.12)–(29.14), (29.28), and Γ_Z in Eq. (29.36). (Also, the expression for M_Z is divided by $\sqrt{\rho_0}$; the M_W formula is unchanged.) There is now enough data to determine ρ_0, $\sin^2\theta_W$, m_t, and α_s simultaneously. In particular, R_b and the direct CDF and DØ events yield m_t independent of ρ_0, the asymmetries yield $\widehat{s}^2_Z$, R gives α_s, and M_Z and the widths constrain ρ_0. From the global fit (including CDF and DØ),

$$\rho_0 = 1.0012 \pm 0.0013 \pm 0.0018 \tag{29.42}$$

$$\widehat{s}^2_Z = 0.2314 \pm 0.0002 \pm 0.0002 \tag{29.43}$$

$$\alpha_s = 0.121 \pm 0.004 \pm 0.001 \tag{29.44}$$

$$m_t = 171 \pm 12 \ , \tag{29.45}$$

where the second error is from M_H. This is in remarkable agreement with the Standard Model expectation $\rho_0 = 1$, and constrains any higher-dimensional Higgs representation to have vacuum expectation values of less than a few percent of those of the doublets. The allowed regions in the $\rho_0 - \widehat{s}^2_Z$ plane are shown in Fig. 29.2. Allowing for new physics in R_b, one obtains $\rho_0 = 1.0002(14)(18)$ and $\alpha_s = 0.101(8)(1)$. The effects of other types of new physics are described in Sec. 30

on "Constraints on New Physics from Electroweak Analyses" in this *Review*.

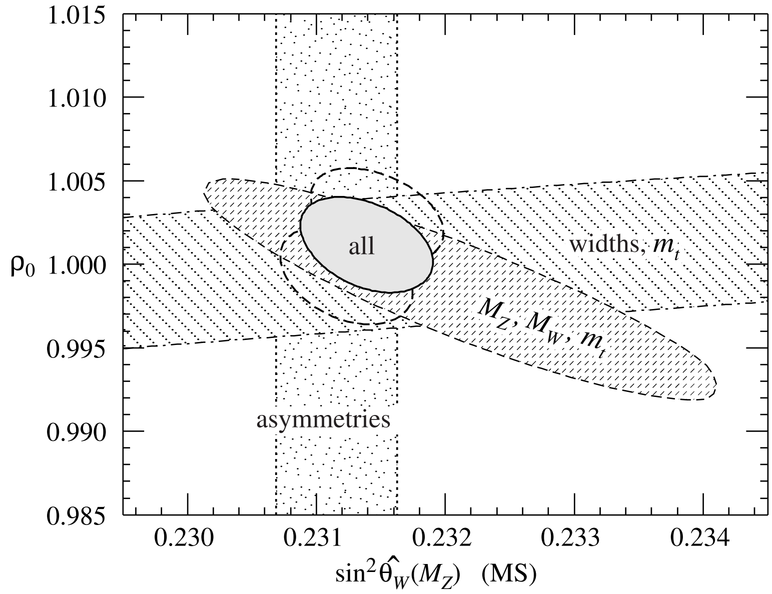

Figure 29.2: The allowed regions in $\sin^2\widehat{\theta}_W - \rho_0$ at 90% CL. m_t is a free parameter and $M_H = 300$ GeV is assumed. (The upper (lower) dashed contours are for $M_H = 1000$ (60) GeV.) The horizontal (width) band uses the experimental value of M_Z in Eq. (29.36).

Most of the parameters relevant to ν-hadron, νe, e-hadron, and e^+e^- processes are determined uniquely and precisely from the data in "model independent" fits (*i.e.*, fits which allow for an arbitrary electroweak gauge theory). The values for the parameters defined in Eqs. (29.12)–(29.14) are given in Table 29.6 along with the predictions of the Standard Model. The agreement is excellent. The low-energy e^+e^- results are difficult to present in a model-independent way because Z-propagator effects are non-negligible at TRISTAN, PETRA, and PEP energies. However, assuming e-μ-τ universality, the lepton asymmetries imply [45] $4(g_A^e)^2 = 0.99 \pm 0.05$, in good agreement with the Standard Model prediction $\simeq 1$. The much more precisely measured Z-pole parameters in Table 29.3 are in excellent agreement with the Standard Model.

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Table 29.6: Values of the model-independent neutral-current parameters, compared with the Standard Model prediction using $M_Z = 91.1884$ GeV for $m_t = 180 \pm 7$ GeV and $M_H = 300$ GeV. There is a second $g_{V,A}^{\nu e}$ solution, given approximately by $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$, which is eliminated by e^+e^- data under the assumption that the neutral current is dominated by the exchange of a single Z. θ_i, $i = L$ or R, is defined as $\tan^{-1}[\epsilon_i(u)/\epsilon_i(d)]$.

Quantity	Experimental Value	Standard Model Prediction		Correlation
$\epsilon_L(u)$	0.332 ± 0.016	0.345 ± 0.0003		
$\epsilon_L(d)$	-0.438 ± 0.012	-0.429 ± 0.0004		non-
$\epsilon_R(u)$	-0.178 ± 0.013	-0.156		Gaussian
$\epsilon_R(d)$	$-0.026\ ^{+0.075}_{-0.048}$	0.078		
g_L^2	0.3017 ± 0.0033	0.303 ± 0.0005		
g_R^2	0.0326 ± 0.0033	0.030		small
θ_L	2.50 ± 0.035	2.46		
θ_R	$4.58\ ^{+0.46}_{-0.28}$	5.18		
$g_A^{\nu e}$	-0.507 ± 0.014	-0.507 ± 0.0004		-0.04
$g_V^{\nu e}$	-0.041 ± 0.015	-0.037 ± 0.0003		
C_{1u}	-0.214 ± 0.046	-0.190 ± 0.0005	-0.995	-0.79
C_{1d}	0.359 ± 0.041	0.342 ± 0.0004		0.79
$C_{2u} - \frac{1}{2}C_{2d}$	-0.04 ± 0.13	-0.052 ± 0.0009		

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