MUON DECAY PARAMETERS

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All measurements in direct muon decay, $\mu^- \to e^- + 2$ neutrals, and its inverse, $\nu_\mu + e^- \to \mu^- +$ neutral, are successfully described by the "V–A interaction," which is a particular case of a local, derivative-free, lepton-number-conserving, four-fermion interaction [1]. The matrix element is given below. The V–A form and the nature of the neutrals (ν_μ and $\overline{\nu}_e$), and hence the doublet assignments $(\nu_e\; e^-)_L$ and $(\nu_\mu\; \mu^-)_L$, can be determined from experiments [2,3].

All results in direct muon decay (energy spectra, polarizations, and angular distributions) and in inverse muon decay (the reaction cross section) at energies well below $m_W c^2$ may be parametrized in terms of amplitudes $g^\gamma_{\varepsilon\mu}$ and the Fermi coupling constant G_F, using the matrix element

$$\frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T\\ \epsilon,\mu=R,L}} g^\gamma_{\epsilon\mu} \langle \overline{e}_\epsilon \,|\, \Gamma^\gamma \,|\, (\nu_e)_n \rangle \, \langle (\overline{\nu}_\mu)_m \,|\, \Gamma_\gamma \,|\, \mu_\mu \rangle \ . \qquad (1)$$

We use the notation of Fetscher *et al.* [2], who in turn use the sign conventions and definitions of Scheck [4]. Here $\gamma = S, V, T$ indicate a scalar, vector, or tensor interaction; and ε, $\mu = R, L$ indicate a right- or left-handed chirality of the electron or muon. The chiralities n and m of the ν_e and $\overline{\nu}_\mu$ are then determined by the values of γ, ε, and μ. The particles are represented by fields of definite chirality [5].

As shown by Langacker and London [6], explicit lepton-number nonconservation still leads to a matrix element equivalent to Eq. (1). They conclude that it is not possible, even in principle, to test lepton-number conservation in (leptonic) muon decay if the final neutrinos are massless and are not observed.

The ten complex amplitudes $g^\gamma_{\varepsilon\mu}$ (g^T_{RR} and g^T_{LL} are identically zero) and G_F constitute 19 independent (real) parameters to be determined by experiment. The V–A interaction corresponds to the single amplitude g^V_{LL} being unity and all the others being zero.

C. Jarlskog [7] has noted that certain experiments observing the decay electron are especially informative if they yield the

V–A values. Indeed, all (direct) muon decay experiments are compatible with an arbitrary mix of the scalar and vector amplitudes g^S_{LL} and g^V_{LL}—in the extreme, even with the purely scalar $g^S_{LL} = 2$, $g^V_{LL} = 0$. The decision in favor of V–A comes from the quantitative observation of inverse muon decay, which would be forbidden for pure g^S_{LL} [2].

The differential decay probability to obtain an $e^{\pm}$ with (reduced) energy between x and $x + dx$, emitted in the direction $\widehat{z}$ at an angle between θ and $\theta + d\theta$ with respect to the muon polarization vector $\vec{P}_\mu$, and with its spin pointing in the arbitrary direction $\widehat{\zeta}$, is given by

$$\begin{aligned}\frac{d^2\Gamma}{dx\ d\cos\theta} = \frac{m_\mu}{4\pi^3}\, W^4_{e\mu}\, G^2_F\, \sqrt{x^2 - x_0^2} \\ \times \Big(F_{IS}(x) \pm P_\mu \cos\theta\, F_{AS}(x)\Big) \\ \times \,[1 + \vec{P}_e(x,\theta)\cdot\widehat{\zeta}\,]\ .\end{aligned}$$

Here $W_{e\mu} = \max(E_e) = (m^2_\mu + m^2_e)/2m_\mu$ is the maximum $e^{\pm}$ energy, $x = E_e/W_{e\mu}$ is the reduced energy, and $x_0 = m_e/W_{e\mu} = 9.67 \times 10^{-3}$. The quantity $P_\mu = |\vec{P}_\mu| \cdot \widehat{\zeta}$ has the significance of the direction in which a perfect polarization-sensitive electron detector would be most sensitive. The isotropic part of the spectrum, $F_{IS}(x)$, the anisotropic part, $F_{AS}(x)$, and the electron polarization, $\vec{P}_e(x,\theta)$, depend on bilinear combinations—called decay parameters—of the coupling constants $g^\gamma_{\varepsilon\mu}$. Neglecting possible nonzero neutrino masses, we have, in terms of the decay parameters ρ, η, ξ, δ, etc.,

$$\begin{aligned}F_{IS}(x) &= x(1-x) + \tfrac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta\, x_0(1-x) \\ F_{AS}(x) &= \tfrac{1}{3}\xi\,\sqrt{x^2 - x_0^2} \\ &\quad \times \left[1 - x + \tfrac{2}{3}\delta\Big(4x - 3 - (\sqrt{1 - x_0^2} - 1)\Big)\right] \\ \vec{P}_e(x,\theta) &= P_{T_1}\,\widehat{x} + P_{T_2}\,\widehat{y} + P_L\,\widehat{z}\ .\end{aligned}$$

Here $\widehat{x}$, $\widehat{y}$, and $\widehat{z}$ are orthogonal unit vectors defined as follows:

$$\begin{array}{rl} \widehat{z} & \text{is along the } e \text{ momentum} \\ \widehat{y} = [\widehat{z} \times \vec{P_\mu}]/|[\widehat{z} \times \vec{P_\mu}]| & \text{is transverse to the } e \text{ momentum and} \\ & \text{perpendicular to the "decay plane"} \\ \widehat{x} = \widehat{y} \times \widehat{z} & \text{is transverse to the } e \text{ momentum and} \\ & \text{in the "decay plane."} \end{array}$$

The components of $\vec{P}_e$ then are given by

$$\begin{aligned} P_{T_1}(x,\theta) &= P_\mu \sin\theta\, F_{T_1}(x) \Big/ \Big(F_{IS}(x) \pm P_\mu \cos\theta\, F_{AS}(x)\Big) \\ P_{T_2}(x,\theta) &= P_\mu \sin\theta\, F_{T_2}(x) \Big/ \Big(F_{IS}(x) \pm P_\mu \cos\theta\, F_{AS}(x)\Big) \\ P_L(x,\theta) &= \pm F_{IP}(x) + P_\mu \cos\theta \\ &\qquad \times F_{AP}(x) \Big/ \Big(F_{IS}(x) \pm P_\mu \cos\theta\, F_{AS}(x)\Big) \ , \end{aligned}$$

where

$$\begin{aligned} F_{T_1}(x) &= \tfrac{1}{12}\Big\{-2\Big[\xi'' + 12(\rho - \tfrac{3}{4})\Big](1-x)x_0 \\ &\qquad - 3\eta(x^2 - x_0^2) + \eta''(-3x^2 + 4x - x_0^2)\Big\} \\ F_{T_2}(x) &= \tfrac{1}{3}\sqrt{x^2 - x_0^2}\,\Big\{3\frac{\alpha'}{A}(1-x) + 2\frac{\beta'}{A}\sqrt{1 - x_0^2}\,\Big\} \\ F_{IP}(x) &= \tfrac{1}{54}\sqrt{x^2 - x_0^2}\,\Big\{9\xi'\Big(-2x + 2 + \sqrt{1 - x_0^2}\Big) \\ &\qquad + 4\xi(\delta - \tfrac{3}{4})\Big(4x - 4 + \sqrt{1 - x_0^2}\,\Big)\Big\} \\ F_{AP}(x) &= \tfrac{1}{6}\Big\{\xi''(2x^2 - x - x_0^2) + 4(\rho - \tfrac{3}{4})(4x^2 - 3x - x_0^2) \\ &\qquad + 2\eta''(1-x)x_0\Big\} \ . \end{aligned}$$

For the experimental values of the decay parameters ρ, ξ, ξ', ξ'', δ, η, η'', α/A, β/A, α'/A, β'/A, which are not all independent, see the Data Listings below. Experiments in the past have also been analyzed using the parameters a, b, c, a', b', c', α/A, β/A, α'/A, β'/A (and $\eta = (\alpha - 2\beta)/2A$), as defined by Kinoshita and Sirlin [8]. They serve as a model-independent summary of all possible measurements on the decay electron (see Listings below). The relations between the two sets of parameters are

$$\rho - \frac{3}{4} = \frac{3}{4}(-a+2c)/A \ ,$$
$$\eta = (\alpha - 2\beta)/A \ ,$$
$$\eta'' = (3\alpha + 2\beta)/A \ ,$$
$$\delta - \frac{3}{4} = \frac{9}{4} \cdot \frac{(a' - 2c')/A}{1 - [a + 3a' + 4(b+b') + 6c - 14c']/A} \ ,$$
$$1 - \xi\frac{\delta}{\rho} = 4\frac{[(b+b') + 2(c-c')]/A}{1-(a-2c)/A} \ ,$$
$$1 - \xi' = [(a+a') + 4(b+b') + 6(c+c')]/A \ ,$$
$$1 - \xi'' = (-2a + 20c)/A \ ,$$

where

$$A = a + 4b + 6c \ .$$

The relations to the coupling constants are:

$$a = 16(|g^V_{RL}|^2 + |g^V_{LR}|^2) + |g^S_{RL} + 6g^T_{RL}|^2 + |g^S_{LR} + 6g^T_{LR}|^2 \ ,$$
$$a' = 16(|g^V_{RL}|^2 - |g^V_{LR}|^2) + |g^S_{RL} + 6g^T_{RL}|^2 - |g^S_{LR} + 6g^T_{LR}|^2 \ ,$$
$$\alpha = 8\,\mathrm{Re}\left[g^V_{RL}(g^S_{LR} + 6g^T_{LR})^* + g^V_{LR}(g^S_{RL} + 6g^T_{RL})^*\right] \ ,$$
$$\alpha' = 8\,\mathrm{Im}\left[-g^V_{LR}(g^S_{RL} + 6g^T_{RL})^* - g^V_{RL}(g^S_{LR} + 6g^T_{LR})^*\right] \ ,$$
$$b = 4(|g^V_{RR}|^2 + |g^V_{LL}|^2) + |g^S_{RR}|^2 + |g^S_{LL}|^2 \ ,$$
$$b' = 4(|g^V_{RR}|^2 - |g^V_{LL}|^2) + |g^S_{RR}|^2 - |g^S_{LL}|^2 \ ,$$
$$\beta = -4\,\mathrm{Re}\left[g^V_{RR}(g^S_{LL})^* + g^V_{LL}(g^S_{RR})^*\right] \ ,$$
$$\beta' = 4\,\mathrm{Im}\left[g^V_{RR}(g^S_{LL})^* - g^V_{LL}(g^S_{RR})^*\right] \ ,$$
$$c = \tfrac{1}{2}\left[|g^S_{RL} - 2g^T_{RL}|^2 + |g^S_{LR} - 2g^T_{LR}|^2\right] \ ,$$
$$c' = \tfrac{1}{2}\left[|g^S_{RL} - 2g^T_{RL}|^2 - |g^S_{LR} - 2g^T_{LR}|^2\right] \ .$$

If also the electron mass is neglected, the energy and angular distribution of the electron in the rest frame of a polarized muon ($\mu^{\mp}$) is given by the Michel spectrum:

$$d^2\Gamma \sim \left\{3(1-x) + \frac{2\rho}{3}(4x-3) \right.$$
$$\left. \mp\xi \cos\theta[1-x+\frac{2\delta}{3}(4x-3)]\right\} x^2 dx\, d(\cos\theta) \ .$$

Here θ is the angle between the electron momentum and the muon spin, and $x \equiv 2E_e/m_\mu$. For pure V–A coupling, we obtain $\rho = \xi\delta = 3/4$, $\xi = 1$, and the differential decay rate is

$$d^2\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[3-2x \pm \cos\theta(1-2x)\right] x^2 dx\, d(\cos\theta) \ .$$

Here the coefficient in front of the square bracket is the total decay rate.

In order to determine the amplitudes $g^\gamma_{\varepsilon\mu}$ uniquely from experiment, Fetscher *et al.* [2] introduced four probabilities $Q_{\varepsilon\mu}(\varepsilon, \mu = R, L)$ for the decay of a μ-handed muon into an ε-handed electron and showed that there exist upper bounds on Q_{RR}, Q_{LR}, and Q_{RL}, and a lower bound on Q_{LL}. These probabilities are given in terms of the $g^\gamma_{\varepsilon\mu}$'s by

$$Q_{\varepsilon\mu} = \frac{1}{4}|g^S_{\varepsilon\mu}|^2 + |g^V_{\varepsilon\mu}|^2 + 3(1-\delta_{\varepsilon\mu})|g^T_{\varepsilon\mu}|^2 \ , \qquad (2)$$

where $\delta_{\varepsilon\mu} = 1$ for $\varepsilon = \mu$ and $\delta_{\varepsilon\mu} = 0$ for $\varepsilon \neq \mu$. They are related to the parameters a, b, c, a', b', and c' by

$$Q_{RR} = 2(b+b')/A \ ,$$
$$Q_{LR} = [(a-a') + 6(c-c')]/2A \ ,$$
$$Q_{RL} = [(a+a') + 6(c+c')]/2A \ ,$$
$$Q_{LL} = 2(b-b')/A \ ,$$

with $A = 16$. In the pure V–A theory, $Q_{LL} = 1$ and the others are zero.

Since the upper bounds on Q_{RR}, Q_{LR}, and Q_{RL} are found to be small, and since the helicity of the ν_μ in pion decay is known from experiment [9,10] to very high precision to be

-1 [11], the cross section S of *inverse* muon decay, normalized to the V–A value, yields [2]

$$|g^S_{LL}|^2 \leq 4(1-S) \tag{3}$$

and

$$|g^V_{LL}|^2 = S\ . \tag{4}$$

Thus the Standard Model assumption of a pure V–A leptonic charged weak interaction for e and μ is confirmed (within errors) by experiments at energies far below the mass of the $W^{\pm}$: Eq. (4) gives a lower limit for V–A, and Eqs. (2) and (3) give upper limits for the other four-fermion interactions. The existence of such upper limits may also be seen from $Q_{RR}+Q_{RL} = (1-\xi')/2$ and $Q_{RR}+Q_{LR} = \frac{1}{2}(1+\xi/3-16\xi\delta/9)$. Table 1 gives the current experimental limits on the magnitudes of the $g^{\gamma}_{\epsilon\mu}$'s.

Table 1. Ninety-percent confidence level experimental limits for the coupling constants $g^{\gamma}_{\epsilon\mu}$. The limits on $|g^S_{LL}|$ and $|g^V_{LL}|$ are from Ref. 12, and the others are from Ref. 13. The experimental uncertainty on the muon polarization in pion decay is included.

$\lvert g^S_{RR}\rvert < 0.066$	$\lvert g^V_{RR}\rvert < 0.033$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.125$	$\lvert g^V_{LR}\rvert < 0.060$	$\lvert g^T_{LR}\rvert < 0.036$
$\lvert g^S_{RL}\rvert < 0.424$	$\lvert g^V_{RL}\rvert < 0.110$	$\lvert g^T_{RL}\rvert < 0.122$
$\lvert g^S_{LL}\rvert < 0.55$	$\lvert g^V_{LL}\rvert > 0.96$	$\lvert g^T_{LL}\rvert \equiv 0$

Limits on the "charge retention" coordinates, as used in the older literature (*e.g.*, Ref. 14), are given by Burkard *et al.* [15].

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