31. THE CABIBBO-KOBAYASHI-MASKAWA MIXING MATRIX

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In the Standard Model with SU(2) $\times$ U(1) as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix relating these bases was defined for six quarks and given an explicit parametrization by Kobayashi and Maskawa [1] in 1973. It generalizes the four-quark case, where the matrix is parametrized by a single angle, the Cabibbo angle [2].

By convention, the three charge $2e/3$ quarks (u, c, and t) are unmixed, and all the mixing is expressed in terms of a 3×3 unitary matrix V operating on the charge $-e/3$ quarks (d, s, and b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{31.1}$$

The values of individual matrix elements can in principle all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Using the constraints discussed below together with unitarity, and assuming only three generations, the 90% confidence limits on the magnitude of the elements of the complete matrix are:

$$\begin{pmatrix} 0.9745 \text{ to } 0.9757 & 0.219 \text{ to } 0.224 & 0.002 \text{ to } 0.005 \\ 0.218 \text{ to } 0.224 & 0.9736 \text{ to } 0.9750 & 0.036 \text{ to } 0.046 \\ 0.004 \text{ to } 0.014 & 0.034 \text{ to } 0.046 & 0.9989 \text{ to } 0.9993 \end{pmatrix} . \tag{31.2}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of others.

There are several parametrizations of the Cabibbo-Kobayashi-Maskawa matrix. In view of the need for a "standard" parametrization in the literature, we advocate:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{31.3}$$

proposed by Chau and Keung [3]. The choice of rotation angles follows earlier work of Maiani [4], and the placement of the phase follows that of Wolfenstein [5]. The notation used is that of Harari and Leurer [6] who, along with Fritzsch and Plankl [7], proposed this parametrization as a particular case of a form generalizable to an arbitrary number of "generations." The general form was also put forward by Botella and Chau [8]. Here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, with i and j being

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"generation" labels, $\{i, j = 1, 2, 3\}$. In the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified with the Cabibbo angle [2].

The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases. Then all s_{ij} and c_{ij} are positive, $|V_{us}| = s_{12}c_{13}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}c_{13}$. As c_{13} is known to deviate from unity only in the fifth decimal place, $|V_{us}| = s_{12}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}$ to an excellent approximation. The phase δ_{13} lies in the range $0 \leq \delta_{13} < 2\pi$, with non-zero values generally breaking CP invariance for the weak interactions. The generalization to the n generation case contains $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases [6,7,8]. The range of matrix elements in Eq. (31.2) corresponds to 90% CL limits on the angles of $s_{12} = 0.219$ to $0.223, s_{23} = 0.036$ to 0.046, and $s_{13} = 0.002$ to 0.005.

Kobayashi and Maskawa [1] originally chose a parametrization involving the four angles, θ_1, θ_2, θ_3, δ:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} , \qquad (31.4)$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1, 2, 3$. In the limit $\theta_2 = \theta_3 = 0$, this reduces to the usual Cabibbo mixing with θ_1 identified (up to a sign) with the Cabibbo angle [2]. Slightly different forms of the Kobayashi-Maskawa parametrization are found in the literature. The CKM matrix used in the 1982 *Review of Particle Properties* is obtained by letting $s_1 \to -s_1$ and $\delta \to \delta + \pi$ in the matrix given above. An alternative is to change Eq. (31.4) by $s_1 \to -s_1$ but leave δ unchanged. With this change in s_1, the angle θ_1 becomes the usual Cabibbo angle, with the "correct" sign (*i.e.* $d' = d\cos\theta_1 + s\sin\theta_1$) in the limit $\theta_2 = \theta_3 = 0$. The angles θ_1, θ_2, θ_3 can, as before, all be taken to lie in the first quadrant by adjusting quark field phases. Since all these parametrizations are referred to as "the" Kobayashi-Maskawa form, some care about which one is being used is needed when the quadrant in which δ lies is under discussion.

Other parametrizations, mentioned above, are due to Maiani [4] and to Wolfenstein [5]. Still other parametrizations [9] have come into the literature in connection with attempts to define "maximal CP violation". No physics can depend on which of the above parametrizations (or any other) is used as long as a single one is used consistently and care is taken to be sure that no other choice of phases is in conflict.

Our present knowledge of the matrix elements comes from the following sources:

(1) New analyses have been performed comparing nuclear beta decay to muon decay. The previous radiative corrections [10] already

included order $Z\alpha^2$ effects and more recent results [11–15] concentrate on nuclear mismatch and structure-dependent radiative corrections. The results in Ref. 15 violate CVC, and the updated [13] average ft values for superallowed 0^+ to 0^+ transitions of Refs. 11 and 12 do not agree with each other within the estimated uncertainties:

$$ft = 3150.8 \pm 1.7 \text{ sec} \qquad \text{(Refs. 11 and 13)} ,$$
$$ft = 3145.7 \pm 1.5 \text{ sec} \qquad \text{(Refs. 12 and 13)} , \tag{31.5}$$

The common experimental error is ± 0.82. We have taken an average of the above values and scaled up the error to take account of the uncertainty in the nuclear structure dependent radiative corrections and corresponding inconsistency of the theoretical results. This transforms to

$$|V_{ud}| = 0.9736 \pm 0.0010 , \tag{31.6}$$

which is almost one standard deviation smaller than the result in the previous *Review of Particle Properties*. It is consistent with the result $|V_{ud}| = 0.9734 \pm 0.0007$ from the update in Ref. 14.

(2) Analysis of K_{e3} decays yields [16]

$$|V_{us}| = 0.2196 \pm 0.0023 . \tag{31.7}$$

With isospin violation taken into account in K^+ and K^0 decays, the extracted values of $|V_{us}|$ are in agreement at the 1% level. A reanalysis [13] obtains essentially the same value, but quotes a somewhat smaller error which is only statistical. The analysis of hyperon decay data has larger theoretical uncertainties because of first order SU(3) symmetry breaking effects in the axial-vector couplings, but due account of symmetry breaking [17] applied to the WA2 data [18] gives a corrected value [19] of 0.222 ± 0.003. We average these two results to obtain:

$$|V_{us}| = 0.2205 \pm 0.0018 . \tag{31.8}$$

(3) The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. The dimuon production cross sections of the CDHS group [20] yield $\overline{B}_c\,|V_{cd}|^2 = 0.41 \pm 0.07 \times 10^{-2}$, where $\overline{B}_c$ is the semileptonic branching fraction of the charmed hadrons produced. The corresponding value from a more recent Tevatron experiment [21], where a next-to-leading-order QCD analysis has been carried out, is $0.534 \pm 0.021^{+0.025}_{-0.051} \times 10^{-2}$, where the last error is from the scale uncertainty. Assuming a similar scale error for the CDHS result and averaging these two results gives $0.49 \pm 0.05 \times 10^{-2}$. Supplementing this with data [22] on the mix of charmed particle species produced by neutrinos and PDG values for their semileptonic branching fractions to give [21] $\overline{B}_c = 0.099 \pm 0.012$, yields

$$|V_{cd}| = 0.224 \pm 0.016 \tag{31.9}$$

(4) Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange quark density in the parton-sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an SU(3) symmetric sea, leads to a lower bound [20], $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of D_{e3} decay with the expression [23] that follows from the standard weak interaction amplitude:

$$\Gamma(D \to \overline{K} e^+ \nu_e) = |f_+^D(0)|^2 \, |V_{cs}|^2 \, (1.54 \times 10^{11} \text{ s}^{-1}) \; . \tag{31.10}$$

Here $f_+^D(q^2)$, with $q = p_D - p_K$, is the form factor relevant to D_{e3} decay; its variation has been taken into account with the parametrization $f_+^D(t)/f_+^D(0) = M^2/(M^2 - t)$ and $M = 2.1$ GeV$/c^2$, a form and mass consistent with Mark III and E691 measurements [24,25]. Combining data on branching ratios for D_{e3} decays from Mark III, E691, and CLEO experiments [24–26] with accurate values [27] for τ_{D^+} and τ_{D^0}, yields $(0.762 \pm 0.055) \times 10^{11}$ s^{-1} for $\Gamma(D \to \overline{K} e^+ \nu_e)$. Therefore

$$|f_+^D(0)|^2 \, |V_{cs}|^2 = 0.495 \pm 0.036 \; . \tag{31.11}$$

A very conservative assumption is that $|f_+^D(0)| < 1$, from which it follows that $|V_{cs}| > 0.62$. Calculations of the form factor either performed [28,29] directly at $q^2 = 0$, or done [30] at the maximum value of $q^2 = (m_D - m_K)^2$ and interpreted at $q^2 = 0$ using the measured q^2 dependence, gives the value $f_+^D(0) = 0.7 \pm 0.1$. It follows that

$$|V_{cs}| = 1.01 \pm 0.18 \; . \tag{31.12}$$

The constraint of unitarity when there are only three generations gives a much tighter bound (see below).

(5) The ratio $|V_{ub}/V_{cb}|$ can be obtained from the semileptonic decay of B mesons produced on the $\Upsilon(4S)$ $b\overline{b}$ resonance by measuring the lepton energy spectrum above the endpoint of the $b \to c\ell\nu$ spectrum. There the $b \to u\ell\nu$ decay rate can be obtained by subtracting the background from nonresonant e^+e^- reactions. This continuum background is determined from auxiliary measurements off the $\Upsilon(4S)$. Both the CLEO [31] and ARGUS [32] collaborations have reported evidence for $b \to u$ transitions in semileptonic B decays. The interpretation of the result in terms of $|V_{ub}/V_{cb}|$ depends fairly strongly on the theoretical model used to generate the lepton energy spectrum, especially for $b \to u$ transitions [29,30,33]. Combining the experimental and theoretical uncertainties, we quote

$$|V_{ub}/V_{cb}| = 0.08 \pm 0.02 \; . \tag{31.13}$$

(6) The heavy quark effective theory [34](HQET) provides a nearly model-independent treatment of B semileptonic decays to charmed mesons. From measurements [35–37] of the exclusive decay $B \to \overline{D}^{*}\overline{\ell}\nu_\ell$, the value $|V_{cb}| = 0.041 \pm 0.003 \pm 0.002$ has been extracted [38] using corrections based on the HQET. A new analysis of inclusive decays [39], where the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual $V - A$ interaction, gives $|V_{cb}| \cdot (\tau_b/1.5\ \text{ps})^{1/2} = 0.041 \pm 0.002$. Using a value [40] for the b lifetime $\tau_b = 1.55 \pm 0.06$ ps and combining with the exclusive result, we obtain

$$|V_{cb}| = 0.041 \pm 0.003\ . \tag{31.14}$$

The results for three generations of quarks, from Eqs. 31.6, 31.8, 31.9, 31.12, 31.13, and 31.14 plus unitarity, are summarized in the matrix in Eq. (31.2). The ranges given there are different from those given in Eqs. (31.6)–(31.14) because of the inclusion of unitarity, but are consistent with the one-standard-deviation errors on the input matrix elements. Note in particular that the unitarity constraint has pushed $|V_{ud}|$ about one standard deviation higher than given in Eq. (31.6).

The data do not preclude there being more than three generations. Moreover, the entries deduced from unitarity might be altered when the CKM matrix is expanded to accommodate more generations. Conversely, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that any additional element in the first row have a magnitude $|V_{ub'}| < 0.08$. When there are more than three generations the allowed ranges (at 90% CL) of the matrix elements connecting the first three generations are

$$\begin{pmatrix} 0.9720 \text{ to } 0.9752 & 0.217 \text{ to } 0.223 & 0.002 \text{ to } 0.005 & \ldots \\ 0.199 \text{ to } 0.234 & 0.818 \text{ to } 0.975 & 0.036 \text{ to } 0.046 & \ldots \\ 0 \text{ to } 0.11 & 0 \text{ to } 0.52 & 0 \text{ to } 0.9993 & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}, \tag{31.15}$$

where we have used unitarity (for the expanded matrix) and Eqs. 31.6, 31.8, 31.9, 31.12, 31.13, and 31.14.

Further information, particularly on CKM matrix elements involving the top quark, can be obtained from flavor-changing processes that occur at the one-loop level. We have not used this information in the discussion above since the derivation of values for V_{td} and V_{ts} in this manner from, for example, B mixing, $b \to s\gamma$, or $K \to \pi\nu\overline{\nu}$, requires an additional assumption that the top-quark loop, rather than new physics, gives the dominant contribution to the process in question.

The measured value [41] of $\Delta M_d = 0.496 \pm 0.032\ \text{ps}^{-1}$ from $B_d^0 - \overline{B}_d^0$ mixing can be turned in this way into information on

$|V_{tb}^* V_{td}|$. Using $\widehat{B}_{B_d} f_{B_d}^2 = (1.2 \pm 0.2)(173 \pm 40\ \text{MeV})^2$ from lattice QCD calculations [42], next-to-leading-order QCD corrections [43], and $m_t = 174 \pm 16$ GeV as input,

$$|V_{tb}^*\, V_{td}| = 0.009 \pm 0.003\ , \tag{31.16}$$

where the error bar comes primarily from the theoretical uncertainty in the hadronic matrix elements.

In the ratio of B_s to B_d mass differences, many of the factors (such as the QCD correction and dependence on the t-quark mass) cancel, and we have

$$\frac{\Delta M_{B_s}}{\Delta M_{B_d}} = \frac{\widehat{B}_{B_s} f_{B_s}^2}{\widehat{B}_{B_d} f_{B_d}^2}\ \frac{|V_{tb}^* \cdot V_{ts}|^2}{|V_{tb}^* \cdot V_{td}|^2}\ . \tag{31.17}$$

With $\widehat{B}_{B_s} \approx \widehat{B}_{B_d}$ and $f_{B_s}/f_{B_d} = 1.16 \pm 0.10$ from lattice QCD [42] and the experimental limit [41] $\Delta M_{B_s}/\Delta M_{B_d} > 11.6$,

$$|V_{td}|/|V_{ts}| < 0.37\ . \tag{31.18}$$

The CLEO observation [44] of $b \to s\gamma$ can be translated [45] similarly into $|V_{ts}|/|V_{cb}| = 1.1 \pm 0.43$, where the large uncertainty is again dominantly theoretical. Ultimately $K \to \pi\nu\overline{\nu}$ decays offer high precision because the matrix elements can be directly measured, but experiment is presently several orders of magnitude away from the requisite sensitivity. All these additional indirect constraints are consistent with the matrix elements obtained from the direct measurements plus unitarity, assuming three generations; adding the indirect constraints to the fit leaves the ranges of CKM matrix elements in Eq. (31.2) essentially unchanged.

Direct and indirect information on the CKM matrix is neatly summarized in terms of the "unitarity triangle." The name arises since unitarity of the 3×3 CKM matrix applied to the first and third columns yields

$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td} V_{tb}^* = 0\ . \tag{31.19}$$

The unitarity triangle is just a geometrical presentation of this equation in the complex plane [46]. We can always choose to orient the triangle so that $V_{cd}\, V_{cb}^*$ lies along the horizontal; in the parametrization we have chosen, V_{cb} is real, and V_{cd} is real to a very good approximation in any case. Setting cosines of small angles to unity, Eq. (31.19) becomes

$$V_{ub}^* + V_{td} = s_{12}\, V_{cb}^*\ , \tag{31.20}$$

which is shown as the unitarity triangle in Fig. 31.1(a). Rescaling the triangle by a factor $[1/|s_{12}\, V_{cb}|]$, the coordinates of the vertices become

$$A\big(\text{Re}(V_{ub})/|s_{12}\, V_{cb}|, -\text{Im}(V_{ub})/|s_{12}\, V_{cb}|\big)\ ,\ B(1,0)\ ,\ C(0,0)\ . \tag{31.21}$$

In the approximation of the Wolfenstein parametrization [5], with matrix elements expressed in powers of the Cabibbo angle, $\lambda \sim s_{12}$:

$$
\begin{aligned}
V_{us} &\sim \lambda \\
V_{ub} &\sim \lambda^3 A(\rho - i\eta) \\
V_{cb} &\sim \lambda^2 A \\
V_{td} &\sim \lambda^3 A(1 - \rho - i\eta) \ ,
\end{aligned} \tag{31.22}
$$

the coordinates of the vertex A of the unitarity triangle are simply (ρ, η), as shown in Fig. 31.1(b).

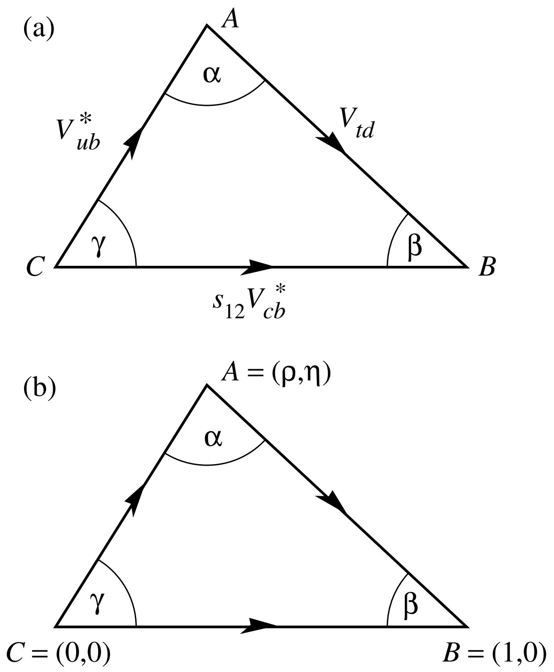

Figure 31.1: (a) Representation in the complex plane of the triangle formed by the CKM matrix elements V_{ub}^*, V_{td}, and $s_{12}\,V_{cb}^*$. (b) Rescaled triangle with vertices $A(\rho, \eta)$, $B(1, 0)$, and $C(0, 0)$.

CP-violating processes will involve the phase in the CKM matrix, assuming that the observed CP violation is solely related to a nonzero value of this phase. This allows additional constraints to be brought to bear. More specifically, a necessary and sufficient condition for CP violation with three generations can be formulated in a parametrization-independent manner in terms of the non-vanishing of the determinant of the commutator of the mass matrices for the charge $2e/3$ and charge $-e/3$ quarks [47]. CP violating amplitudes or differences of rates all are proportional to the CKM factor in this quantity. This is the product of factors $s_{12} s_{13} s_{23} c_{12} c_{13}^2 c_{23} s_{\delta_{13}}$

in the parametrization adopted above, and is $s_1^2 s_2 s_3 c_1 c_2 c_3 s_\delta$ in that of Ref. 1. With the approximation of setting cosines to unity, this is just twice the area of the unitarity triangle. While hadronic matrix elements whose values are imprecisely known generally now enter, the constraints from CP violation in the neutral kaon system are tight enough to very much restrict the range of angles and the phase of the CKM matrix. For example, the constraint obtained from the CP-violating parameter ϵ in the neutral K system corresponds to the vertex A of the unitarity triangle lying on a hyperbola for fixed values of the hadronic matrix elements. [48] For CP-violating asymmetries of neutral B mesons decaying to CP eigenstates, there is a direct relationship between the magnitude of the asymmetry in a given decay and $\sin 2\phi$, where $\phi = \alpha,\ \beta,\ \gamma$ is an appropriate angle of the unitarity triangle [46].

The combination of all the direct and indirect information can be used to find the overall constraints on the CKM matrix and thence the implications for future measurements of CP violation in the B system [48].

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