5. THE HUBBLE CONSTANT

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In a uniform expanding universe, the position r and velocity v of any particle relative to another obey Hubble's relation $v = H_0 r$, where H_0 is Hubble's constant.* As cosmological distances are measured in Mpc, the natural unit for H_0 is km s^{-1} Mpc^{-1}, which has the dimensions of inverse time: $[100 \text{ km s}^{-1} \text{ Mpc}^{-1}]^{-1} = 9.78 \times 10^9$ yr.

The real universe is nonuniform on small scales, and its motion obeys the Hubble relation only as a large scale average. But as typical non-Hubble motions ("peculiar velocities") are less than about 500 km s^{-1}, on scales more than about 5,000 km s^{-1} the deviations from Hubble flow are less than about 10%, so the notion of a global Hubble constant is well defined. The value of H_0 averaged over the local 15,000 km s^{-1} volume is known to lie within 10% of its global value even if H_0 itself is not known this precisely [1–3].

The Hubble constant is only meaningful on very large scales, but very large distances can only be measured indirectly. Distance ratios are measured with selected uniform types of astronomical systems ("Standard Candles") some examples of which are given below. These are used to tie distances to an absolute scale, either the nearby one based on trigonometric parallax or to some system where a physical model is precise enough to yield a distance directly from observed properties. There are many different ways to combine these tools to calibrate large distance, some of which are reviewed here. More complete reviews can be found in Refs. [4–7].

Using stars as standard candles and the Earth's orbit as a baseline, it is possible to tie distances throughout the Galaxy directly to trigonometric parallax measurements. A good landmark point for extragalactic studies is the Large Magellanic Cloud (LMC), a satellite galaxy of our Galaxy whose distance (50 kpc) is known to about 7% and provides confirmation and calibration of other measures. Beyond that, other galaxies in the Local Group (within about 1 Mpc) and other nearby groups provide stepping stones to the Virgo cluster (about 17 Mpc distant), and finally to the Coma cluster (about 100 Mpc distant) and others where the peculiar velocities introduce only small ambiguities. Most of the effort thus lies in obtaining an accurate ratio of distances in the range between Coma (or other similarly distant clusters) and the LMC.

Table 5.1 lists several candles and calibrators with a typical range of distance accessible to each. Usually the ends of the range are not precisely defined; the near end is plagued by small numbers of accessible objects and the far end by signal to noise. The precision quoted is a typical guideline which also varies depending on the sample used; it indicates the error in a distance ratio between an object and some standard reference, not including uncertainties in the absolute calibration of the reference distance (except for the first entry, which lists the typical absolute distance uncertainty in the Cepheid distance to a galaxy.) (The units are astronomical "distance

Table 5.1: Selected extragalactic distance indicators.†

Technique	Range of distance	Precision	Verification/ calibration
Cepheids	<LMC to 17 Mpc	0.15 mag	LMC/MWG
SNIa	4 Mpc to 2 Gpc	0.1-0.2 mag	Hubble/Model, Cepheid
EPM/SNII	LMC to 200 Mpc	0.4 mag	Hubble/Model, Cepheid
PNLF	1 Mpc to 20 Mpc	0.1 mag	SBF/Cepheid
SBF	1 Mpc to 60 Mpc	0.1 mag	PNLF/Cepheid
TF	1 Mpc to 100 Mpc	0.3 mag	Hubble/Cepheid
D_n–σ	10 Mpc to 60 Mpc	0.4 mag	Hubble/SBF
BCG	50 Mpc to 1 Gpc	0.2-0.3 mag	Hubble
GCLF	<LMC to 100 Mpc	0.4 mag	SBF/MWG
SZ	100 Mpc to > 1 Gpc	—	Hubble/Model
GL	∼5 Gpc	—	Model
Hubble	20 Mpc to $\gtrsim$1Gpc	500 km s^{-1} $\div H_0 D$	BCG, SNeIa/H_0

MWG = Milky Way Galaxy
†Extracted from [4-7].

modulus," given by $\mu = 5\log_{10}(\textit{distance in parsecs}) - 5.0$; a ± 0.1 magnitude error in magnitude or distance modulus corresponds to a 5% error in distance.) The verification of this precision is made by cross-checking against some other indicator on a galaxy-by-galaxy basis. This provides a control of systematic errors, since we do not expect detailed correlations between (for example) supernova brightness and host-galaxy rotation. Some examples are given in the next column, along with options often used for absolute calibration. The Hubble relation itself is included here, as it is the most precise indication of relative distance for large distances, and is used to verify the standardization of the other candles. As velocities are easy to measure at the relevant precision, a measurement of the Hubble constant is obtained from a calibrated distance measurement at a sufficiently large distance that the Hubble relation itself is precisely defined.

5.1. Cepheid variables

The best studied and most trusted of the standard candles, Cepheids are bright stars undergoing overstable oscillations driven by the variation of helium opacity with temperature. The period of oscillation is tightly correlated with the absolute brightness of the star. The calibration of this "period-luminosity relation" ties galaxies to geometrical parallax measurements with about 0.15 mag or 7% precision [8]. There may be some indications of nonuniformity in different populations, but no evidence yet that they are significant. Cepheids have been identified in the Galaxy, the LMC, and in galaxies as distant as M100 in the Virgo cluster, at 17.1 ± 1.8 Mpc [9]. More measurements at large distances are expected from Hubble Space Telescope data. This is an important development because it allows direct absolute calibration of the best distant indicator, SNIa, as well as other methods, to better than 10% accuracy.

5.2. Type Ia Supernovae (SNIa)

A SNIa occurs when a degenerate dwarf, of the order of a solar mass and of CNO composition, undergoes explosive detonation or deflagration by nuclear burning to iron-group elements (Ni, Co, Fe). Their uniformity arises because the degenerate material only becomes unstable when it is gravitationally compressed to where the electrons become close to relativistic, which requires nearly a Chandrasekhar mass (1.4 solar masses). Theoretical models of the explosion predict approximately the right peak brightness, but cannot be relied upon for a precise calibration. SNIa are very bright, so their brightness distribution can be studied using the distant Hubble flow as a reference. Indeed, the Hubble diagram of distant SNIa (as well as cases of two SNIa in a single galaxy) shows that they can serve as remarkably precise standard candles; even though they display large variations in brightness, with detailed knowledge of the shape of the light curve, the relative intrinsic brightness of a single SNIa can be predicted to $\Delta m = 0.15$ mag or better and its distance estimated to better than 7% accuracy [10–12]. (Note that distant SNIa can even measure deviation from a linear Hubble law with precision $\Delta q_0 \simeq \Delta m/Z$.) Supernovae of all types are fairly rare events, occurring in a typical galaxy every hundred years, so it is only recently that a direct absolute calibration to SNIa host galaxies with Cepheids has been possible.

5.3. Type II Supernovae (SNII)

A SNII occurs when a massive star has accumulated 1.4 solar masses of iron group elements in its core; there is then no source of nuclear energy and the core collapses by the Chandrasekhar instability. The collapse to a neutron star releases a large gravitational binding energy, some of which powers an explosion. The large variety of envelopes around collapsing cores means that SNII are not at all uniform in their properties. However, their distances can be calibrated absolutely by the fairly reliable "expanding photosphere method" (EPM). The principle is most easily understood for an expanding spherical blackbody. Even if the disk is unresolved, the continuum spectrum yields the angular size from spectral temperature and absolute flux. Spectral lines yield the expansion velocity, which from knowledge of the elapsed time gives a physical size and hence a distance. Models of real photospheres are not trivial but yield individual distances accurate to about 20% [13]. This is in principle an independent absolute distance, but is precisely verified by comparison with Cepheids in several cases, the distant Hubble diagram and Tully Fisher distance ratios (described below) in several others, and by multiple-epoch fits of the same object.

5.4. Planetary Nebula Luminosity function (PNLF)

A planetary nebula (PN) forms when the gaseous envelope is ejected from a low-mass star as its core collapses to a white dwarf. We see bright fluorescent radiation from the ejected gas shell, excited by UV light from the hot new white dwarf. The line radiation makes PN's easy to find and measure even in far-away galaxies; a bright galaxy can have tens of thousands, of which hundreds are bright enough to use to construct a PNLF. It is found empirically that the range of PN brightnesses has a sharp upper cutoff that appears to provide a good empirical standard candle, verified by comparison with SBF distance ratios.

5.5. Surface Brightness Fluctuations (SBF)

When galaxies are farther away than the Local Group, atmospheric blurring causes stellar images to blend together. However, with modern linear detectors, it is still possible to measure the moments of the distribution of stellar brightness in a population (in particular, the brightness-weighted average stellar brightness) through spatial fluctuations in the light. Stellar populations in elliptical galaxies appear to be universal enough for this to be a remarkably good standard candle, as verified by comparison with PNLF distance ratios. Note the problem of absolute calibration: as there are no elliptical galaxies with Cepheids, instead one uses the bulge components of nearby spirals, which have similar populations.

5.6. Tully-Fisher (TF)

The TF relation refers to a correlation of the properties of whole spiral galaxies, between rotational velocity and total luminosity. In rough terms, the relation can be understood as a relation between mass and luminosity, but given the variation in structural properties and stellar populations the narrow relation is a surprisingly good standard candle. Looking at a whole galaxy gives a long range and wide applicability. The TF distance ratios and precision have been verified by cross-checking against all of the above candles, and against the Hubble flow, particularly galaxy cluster averages, which permit greater precision. The absolute calibration of TF is traditionally made by a handful of local galaxies, with Cepheid calibration, and a major thrust now is to extend Cepheid measurements to a larger, more representative, and more distant sample, especially to galaxies in the Virgo cluster.

5.7. D_n–σ

A rough equivalent to TF for elliptical galaxies, D_n–σ is a correlation between galaxy size and velocity dispersion. It has a larger dispersion than TF and less opportunity for local calibration, but it is particularly useful for verifying distance ratios of galaxy clusters, whose cores contain almost no spirals.

5.8. Brightest Cluster Galaxies (BCG)

As a result of agglomeration, rich clusters of galaxies have accumulated the largest and brightest galaxies in the universe in their centers. They are very nearly all the same brightness; when account is taken of their light profiles, they are even more uniform. These provide the best check on the approach to uniform Hubble flow on large scales. (Quasars, which are even brighter, are far too variable to be good standard candles).

5.9. Globular Cluster Luminosity Function (GCLF)

Many galaxies have systems of globular clusters orbiting them, each of which contain hundreds of thousands of stars and hence is visible at large distances. It is assumed that similar galaxies ought to have similar distributions of globular cluster luminosity, and current work is centered on verifying the precision of this assumption.

5.10. Sunyaev-Zeldovich effect (SZ)

The electron density and temperature of the hot plasma in a cluster of galaxies can be measured in two ways which depend differently on distance: the thermal x-ray emission, which is mostly bremmsstrahlung by hot electrons, and the Sunyaev-Zeldovich effect on the microwave background, caused by Compton scattering off the electrons. This provides in principle an absolute calibration. Although the model has other unconstrained parameters, such as the gas geometry, which limit the precision and reliability of distances, in the handful of cases which have been studied most recently the distances are broadly in accord with those obtained by the other techniques.

5.11. Gravitational Lenses (GL)

The time delay δt between different images of a high redshift gravitationally lensed quasar is $\delta t = C(z_Q, z_l)\delta\theta^2/H_0 \approx 1$ yr for image separations $\delta\theta$ of the order of arcseconds, with a numerical factor C of order unity determined by the specific lens geometry (the angular distribution of the lensing matter) and background cosmology. Variability of the double quasar 0957+561 has permitted measurements of δt from time series correlation, but these remain controversial and ambiguous, yielding correlation peaks at both 415 and 540 days. Although lensing does not provide a precise measurement, it is an amazing sanity check that this system, which relies on no other intermediate steps for its calibration, gives estimates on the scale of the Hubble length which are broadly consistent with local measures of H_0.

5.12. Estimates of H_0

The central idea is to find "landmark" systems whose distance is given by more than one technique. Systems are not always well defined, however. For example, the LMC size is a few percent of its distance, introducing errors of this order for any calibration based on an individual object within it. Nor are galaxy clusters as compact and well defined as individual galaxies; using galaxy clusters as calibrating systems often requires some assumptions and models about cluster membership (the most important example being the Virgo cluster, whose structure is somewhat amorphous, creating a $\pm 20\%$ or more distance ambiguity in some arguments). The best way to avoid this is to cross-correlate calibrators on a galaxy-by-galaxy basis, but this introduces problems of bias associated with sample selection that must be modeled. The basic difficulty remains that the nearby calibrators of any sort remain few and possibly anomalous.

The reason for the variable estimates of the Hubble constant lies in the many different ways to combine these techniques to obtain an absolute distance calibration in the Hubble flow, each involving several, usually individually reasonable, assumptions. Nevertheless there is broad agreement within the errors among a wide variety of independent ladders with different systematics. As examples, we cite a variety of (somewhat arbitrarily chosen) independent methods, which illustrate some of the choices and tradeoffs, summarized in Table 5.2.

1. Expanding photosphere method (EPM) distances give an absolute calibration to objects in the distant Hubble flow. A small sample of these direct distances with small flow corrections gives $H_0 = 73 \pm 6$ (statistical) $\pm$ 7 (systematic).The distance estimates and limits on the systematic error component are verified by Cepheid distances in three cases, where the Cepheid/EPM distances come out to 1.02 ± 0.08 (LMC), $1.01^{+0.23}_{-0.17}$ (M101) and 1.13 ± 0.28 (M100).
2. With HST, it is now possible to calibrate SNIa directly with Cepheid distances to host galaxies. The light from brighter SNIa decays more slowly than from faint ones, so the best fits to the distant Hubble diagram include information about the light curve shape ("LCS") rather than simply assuming uniformity; low values of H_0 arise in in the latter case. There are several options for empirical calibration, among them: (a) Three individual SNIa host galaxy distances have been calibrated directly with Cepheids. There is evidence from their light curves that two of these calibrators may indeed be unusually bright, which explains why the value of H_0 depends on whether or not the LCS correction is applied. (b) Alternatively, assuming that the mean of six well-studied SNIa in the Virgo cluster lies at the Cepheid Virgo distance of 17 Mpc yields $H_0 = 71 \pm 7$ km s^{-1} Mpc^{-1}.
3. The distance to Virgo or any other local cluster is tied to H_0 via the distant Hubble diagram for TF or D_n–σ distances for galaxies in distant clusters. This can be done with a large scale

flow model fit to many clusters. Using a Virgo distance of 17 Mpc yields $H_0 = 82 \pm 11$ km s^{-1} Mpc^{-1}. Alternatively, we can use the distance ratio to a fiducial reference such as the Coma cluster, for which such models predict almost vanishing peculiar velocity, and which is in any case distant enough for flow to be unimportant. (The flow models give its Hubble velocity as 7170 ± 125 km s^{-1}; relative to the CMBR its velocity is 7197 ± 73 km s^{-1}.) If (as estimated from TF, D_n–σ, SNeI) the Coma to Virgo ratio lies in the range 5.5 to 5.75, 17 Mpc for Virgo leads to $H_0 = 77$ to 73 km s^{-1} Mpc^{-1}, subject to uncertainty over the Virgo depth. Nearly the same TF calibration is given by six local Cepheid calibrators, and by several more in the M101 group. This avoids the Virgo depth uncertainty, but replaces it with doubts about whether all of the local calibrators might be anomalous (although the apparent uniformity of galaxies elsewhere argues against this being a large effect.)

4. TF comparison with distant field galaxies in the Hubble flow (after corrections for Malmquist bias in the samples, which is worse than in cluster samples) yield $H_0 = 80 \pm 10$ km s^{-1} Mpc^{-1}.
5. For completeness, some recent SZ and GL estimates are shown.

The central values by most reliably calibrated methods lie in the range $H_0 = 65$ to 85 km s^{-1} Mpc^{-1}, and indeed this corresponds roughly with the range of estimates expected from the internally estimated errors. Thus systematic errors are at least not dominant, although they could well be comparable to internal errors. The simplicity and apparent precision of the new Cepheid + SNIa ladder lead one to suspect a true value in the lower end of this range.

Footnote and References:

∗ To first order in v. The second-order term or "deceleration parameter" q_0 involves both the curvature and the choice of distance measure; see the Big-Bang Cosmology section (Sec. 3).

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Table 5.2: Some recent estimates of Hubble's constant

Technique	Calibration*	Ties to Hubble flow	Result* (km s^{-1} Mpc^{-1})	Ref.
EPM	Expanding photosphere model	Direct EPM Hubble Diagram + Flow model or TF	$73 \pm 6 \pm 7$	[13]
	Cepheids in 3 SNII hosts		same $\times$ [0.88, 1.26]	[14]
SNeIa		Direct SNIa Hubble Diagram		
	Cepheids (N5253 + SN1972E)	Direct + LCS correction	$62 - 67$	[11]
	Cepheids (N5253 + SN1972E)	Direct + LCS correction	67 ± 7	[12]
	Cepheids (N5253 + SN1972E)	Direct	54 ± 8	[15]
	Cepheids (IC4182 + SN1937C)	Direct + LCS correction	68–74 $\pm$ 6	[16]
	Cepheids (IC4182 + SN1937C)	Direct	52 ± 9	[17]
	Cepheids (N4536 + SN1981B)	Direct + LCS correction	67 ± 6	[18,19]
	Virgo mean (M100) + six Virgo SN hosts	Direct	$71 \pm 7^{\dagger}$	[14]
Clusters	Virgo mean (M100 Cepheids)	Virgo infall model	$81 \pm 11^{\dagger}$	[14]
	+ local + M101 Cepheids	Virgo/Coma ratio	73–77 $\pm 10^{\dagger}$	[14]
		Cluster TF + LS flow model fit	$82 \pm 11^{\dagger}$	[14]
	M96 Cepheids	LeoI to Virgo and Coma	$69 \pm 8^{\dagger}$	[20]
Field TF	Local Cepheids‡	Field TF Hubble Diagram + Malmquist bias correction	$\approx 80 \pm 10$	[21]
SZ	SZ model + X-ray maps + SZ maps	Direct single cluster velocities:		
		A2218	65 ± 25	[22]
		A2218,A665	55 ± 17	[22]
		Coma	74 ± 29	[23]
Gravitational lensing	Lens model, time delay	Direct, Q0957+561	<70	[24]

* For all methods based on Cepheids, add a common multiplicative error of ± 0.15 mag or 7% in H_0.

† plus Virgo depth uncertainty (scales with M100/Virgo ratio)

‡ TF calibration from 6 local Cepheid calibration is verified by M101 group galaxies and (less directly) by M100 and NGC 4571 distance to Virgo TF galaxies [9,14,25].