SEMILEPTONIC DECAYS OF D AND B MESONS, PART I

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I. Introduction

Weak decays of heavy mesons are classified as hadronic, leptonic, or semileptonic, according to the type of particles present in the final state. The most complicated are hadronic decays, in which strong interactions produce nonperturbative effects that are difficult to calculate. In leptonic decays, the final state contains only a charged lepton and a neutrino; if the decaying meson is a pseudoscalar, the effects of strong interactions in the initial state can be parametrized by a single constant. (See the note on "Pseudoscalar-Meson Decay Constants" in the $\pi^{\pm}$ Full Listings.) The rates for leptonic decay, however, are generally small and hard to measure.

Semileptonic decays are more tractable theoretically than hadronic decays, both because the $\ell\nu_\ell$ system produced by the virtual W boson is well understood, and because this system cannot have strong interactions with the hadronic decay products. In addition, semileptonic decay rates are reasonably large and experimentally accessible. As a consequence, semileptonic processes are the primary source of information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix (see "The Cabibbo-Kobayashi-Maskawa Mixing Matrix," in Section 31 of the Reviews, Tables, Figures, and Formulae part of this *Review*).

In this note, we discuss measurements of semileptonic decays of D and B mesons. These studies have two primary goals: first, to understand the dynamics of semileptonic decays, in particular, the effect of strong interactions on the underlying weak process; and second, to measure the magnitudes of the CKM elements V_{cb} and V_{ub}. These goals are related, because the determination of CKM elements relies on a good understanding of the decay process. Our discussion focuses on the sources of uncertainty in different types of measurements, especially the model dependence in the inclusive B semileptonic branching ratio and in the determination of $|V_{cb}|$ and $|V_{ub}|$.

The effects of strong interactions on the semileptonic decay amplitude can be expressed in terms of one or more Lorentz-invariant functions called form factors, which depend only on q^2, the square of the mass of the virtual W. The number of form factors required to describe a given exclusive decay depends on the quantum numbers of the hadrons in the decay. Information on the form factors is obtained by measuring the distributions of decay angles and q^2, as discussed in the following section.

In the charm sector, the absolute scale of theoretical predictions for decay amplitudes (expressed in terms of form factors) can be tested, because the CKM elements can be determined independently of the D semileptonic decay rate using the assumption of CKM unitarity and the smallness of the CKM elements for B decay. Thus, the study of D-meson semileptonic decays is concerned more with the size and q^2 dependence of the form factors than with the determination of CKM elements. However, studies of form factors in D semileptonic decays should provide theoretical information that will help to improve predictions for $b \to u\ \ell^- \overline{\nu}_\ell$ decays, and hence improve the determination of $|V_{ub}|$.

A development of major importance is the heavy quark effective theory (HQET). In the limit of infinite quark mass, the quark mass and spin decouple from the dynamics of the decay, leading to numerous symmetry relations among form factors. In this limit, the description of a process such as $B \to \overline{D}^* \ell^+ \nu_\ell$ is simplified considerably, because both the b and c quarks are heavy. The form factors can then be related to a single universal form factor, the Isgur-Wise function. HQET predictions permit a systematic approach to understanding the dynamics of B-meson semileptonic decays and to determining the magnitudes of CKM elements. HQET predictions for D semileptonic decays are less useful, because symmetry-breaking corrections in this case are expected to be large.

We begin with a brief review of the formalism of form factors for exclusive semileptonic decays and then turn to experimental results on D and B mesons. More detailed information may be found in review articles on semileptonic decays and HQET [1,2,3].

II. Form-factor formalism for exclusive semileptonic decays

The amplitude for an exclusive semileptonic process can be constructed from the available four-vectors in the decay and from form factors, which are Lorentz invariant functions of q^2, the square of the mass of the virtual W. Because these functions describe the effect of strong interactions, nonperturbative techniques such as lattice QCD are needed to calculate them. Form factors are generally largest at the maximum value of q^2, where the daughter meson has the smallest recoil velocity and the overlap between the parent- and daughter-meson wave functions is largest. Studies of form factors in D semileptonic decays have focused on the modes $D \to \overline{K}\ell^+\nu_\ell$ and $D \to \overline{K}^*\ell^+\nu_\ell$, which dominate the inclusive semileptonic rate. In B decays, the analogous modes, $B \to \overline{D}\ell^+\nu_\ell$ and $B \to \overline{D}^*\ell^+\nu_\ell$, account for about two-thirds of the inclusive semileptonic rate. The decay $B \to \overline{D}\ell^+\nu_\ell$ has a large background from $B \to \overline{D}^*\ell^+\nu_\ell$, so in B decays, form factors have been studied only for $B \to \overline{D}^*\ell^+\nu_\ell$. In this section, we discuss the formalism used in form-factor measurements for the decays $P \to P'\ell\nu_\ell$, where P and P' are pseudoscalar mesons, and $P \to V\ell\nu_\ell$, where V is a vector meson.

The differential decay rate for $P(Q\overline{q}) \to P'(q'\overline{q})\,\ell\nu_\ell$ is

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{q'Q}|^2 k_{P'}^3 |f_+(q^2)|^2}{24\pi^3} \; . \tag{1}$$

Here G_F is the Fermi decay constant, $V_{q'Q}$ is the relevant CKM matrix element, $k_{P'}$ is the momentum of P' in the rest frame of the parent meson, and $f_+(q^2)$ is a vector form factor. (Eq. (1) assumes massless charged leptons, which is almost exact for electrons and a very good approximation for muons, but it is not correct for τ's.) The dominant q^2 dependence comes from the p-wave factor $k_{P'}^3$, which can be written in terms of q^2 and the particle masses. This factor increases the rate at low q^2, which is opposite to the q^2 dependence of the form factor f_+.

The exclusive decay rate for $P \to V\ell\nu_\ell$ can be expressed in terms of three q^2-dependent helicity amplitudes, $H_\pm(q^2)$ and

$H_0(q^2)$, where the subscripts indicate the helicity of either the virtual W or the vector meson. The rate is given by

$$\frac{d\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos_V\;\, d\chi} = \frac{3G_F^2|V_{q'Q}|^2\, k_V\, q^2}{8(4\pi)^4 M^2}$$
$$\Big\{[(1+\eta\cos\theta_\ell)^2|H_+(q^2)|^2 + (1-\eta\cos\theta_\ell)^2|H_-(q^2)|^2]\sin^2\theta_V$$
$$+4\sin^2\theta_\ell\cos^2\theta_V|H_0(q^2)|^2 - 2\sin^2\theta_\ell\sin^2\theta_V\cos(2\chi)H_+(q^2)H_-(q^2)$$
$$-4\eta\sin\theta_\ell(1+\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi\, H_+(q^2)H_0(q^2)$$
$$+4\eta\sin\theta_\ell(1-\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi\, H_-(q^2)H_0(q^2)\Big\}\ . \quad (2)$$

Here M is the mass of the parent meson, k_V is the momentum of the vector meson and is a function of q^2, and the factor η=+1 ($\eta = -1$) applies to B (D) decays. The angles θ_ℓ, θ_V, and χ are defined in Fig. 1. The helicity amplitudes $H_\pm$ and H_0 can be expressed in terms of two axial-vector form factors, $A_1(q^2)$ and $A_2(q^2)$, and a vector form factor $V(q^2)$:

$$H_\pm(q^2) = (M+m)A_1(q^2) \mp \frac{2M\, k_V}{(M+m)}V(q^2)$$
$$H_0(q^2) = \frac{1}{2m\sqrt{q^2}}\Big[(M^2-m^2-q^2)(M+m)A_1(q^2)$$
$$- \frac{4M^2\, k_V^2}{(M+m)}A_2(q^2)\Big]\ , \quad (3)$$

where m is the mass of the daughter meson. The form factors f_+, A_1, A_2, and V are dimensionless.

The $V-A$ coupling results in a larger amplitude to produce a negative-helicity vector meson in $c\overline{q}$ or $b\overline{q}$ decay than one of positive helicity: $|H_-| > |H_+|$. This difference produces a forward-backward asymmetry for the charged lepton in the virtual-W rest frame, since the net angular momentum along the decay axis of the initial heavy meson must be zero. For $D(c\overline{q})$ decays, a positively charged (right-handed) lepton is produced in association with a left-handed daughter s or d quark, resulting in a softer energy spectrum for the charged lepton than for the neutrino after boosting the lepton energy into the D rest frame. (A similar argument shows that the

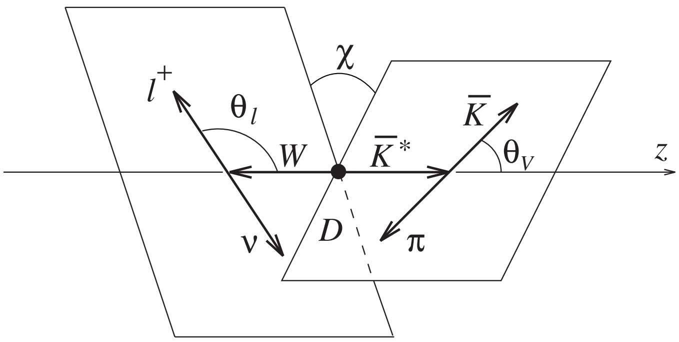

Figure 1: Definition of the angles θ_ℓ, θ_V, and χ. The decay $D \to \overline{K}^* \ell^+ \nu_\ell$ is used as an example. The polar angles θ_ℓ and θ_V are defined in the rest frames of the virtual W and the $\overline{K}^*$, respectively, and χ is the azimuthal angle between the projections of the lepton and the kaon momentum vectors in the plane perpendicular to z.

shape of the spectrum is the same for a $\overline{D}$ decay.) For $\overline{B}(b\overline{q})$ decays, a negatively charged (left-handed) lepton is produced in association with a left-handed daughter quark, giving a harder energy spectrum for the charged lepton than for the neutrino in the B rest frame. In $P \to P' \ell \nu_\ell$ decays, there is no asymmetry, since the P' meson can only have helicity zero. Thus, the effect of $V-A$ is to soften the inclusive lepton spectrum in D decays and to harden it in B decays. It is useful to define rates for decays into specific helicity states:

$$\Gamma_i = \frac{G_F^2 |V_{q'Q}|^2}{96\pi^3} \int dq^2 \; k_V \frac{q^2}{M^2} |H_i(q^2)|^2 \; . \tag{4}$$

Experiments extract various ratios of these rates, including Γ_+/Γ_-, $\Gamma_L/\Gamma_T = \Gamma_0/(\Gamma_- + \Gamma_+)$, the lepton forward-backward asymmetry $A_{FB} = (3\eta/4)(\Gamma_- - \Gamma_+)/\Gamma$, and the polarization parameter $\alpha = 2\Gamma_0/(\Gamma_+ + \Gamma_-) - 1$. The three form factors can be obtained by fitting the measured distribution of the variables q^2, θ_ℓ, θ_V, and χ.

III. D-meson semileptonic decays

In this Section, we review the main results on D and D_s semileptonic decays, including measurements of $D \to \overline{K} \ell^+ \nu_\ell$ and $D \to \overline{K}^* \ell^+ \nu_\ell$ form factors. An important question is whether the sum of the exclusive rates for these two modes, plus the corresponding Cabibbo-suppressed rates, saturate the inclusive rate. This issue can also be addressed by searching for Cabibbo-favored semileptonic decays to hadronic systems other than the K or K^*, referred to here as nonresonant states. It is also of great interest to measure the Cabibbo-suppressed semileptonic decays of the D^0 and D^+ and the semileptonic decays of the D_s^+. These decays differ from the dominant K or K^* decays by the strange quark content of either the initial meson or the final meson, or both. Although there are differences in the q^2 ranges and the pole masses for Cabibbo-favored and Cabibbo-suppressed decays, the form-factor intercepts for these modes should be similar. We review the measurements that attempt to address these questions. In our tables, averages of experimental results labeled "*PDG value*" are not necessarily an exact average of the numbers in the tables but come from making an overall fit to the branching ratios in the Full Listings that follow this note.

$D \to \overline{K}\ell^+\nu_\ell$ decays

With the new CLEO-II measurement [4], the ratio $\Gamma(D^0 \to K^- e^+\nu_e)/\Gamma(D^0 \to K^-\pi^+)$ is one of the most accurately determined quantities in charm physics. This measurement, together with the new, more precise value of the absolute $D^0 \to K^-\pi^+$ branching fraction, yields a much improved measurement of $\mathrm{B}(D^0 \to K^- e^+\nu_e)$, as shown in Table 1. Note that e–μ universality is assumed, and that muon measurements have been scaled up by a factor of 1.03 to account for the reduced phase space. Therefore, all results are expressed as electron branching fractions. The transition rate, $\Gamma(D^0 \to K^- e^+\nu_e) = (8.9 \pm 0.5) \times 10^{10}\ \mathrm{s}^{-1}$, shown in Table 10, is obtained using the PDG branching fraction of Table 1 and the PDG value of the D^0 lifetime.

Isospin symmetry requires equal transition rates for the Cabibbo-favored D^0 and D^+ decays. The D^+ branching fraction measurements are given in Table 2, using our branching fractions for the normalizing modes. The average $D^+ \to \overline{K}^0 e^+\nu_e$ transition rate is $(6.3 \pm 0.8) \times 10^{10}\ \mathrm{s}^{-1}$, significantly lower than $\Gamma(D^0 \to K^- e^+\nu_e)$, although it is less well measured.

Table 1: Measurements of the $D^0 \to K^- e^+ \nu_e$ branching fraction. The PDG branching fraction for $D^0 \to K^- \pi^+$ has been used, where needed, to normalize.

Experiment	Reference[†]	Lepton	Norm. mode	$B(D^0 \to K^- e^+ \nu_e)\%$	m_p (GeV)
E691	[5] ANJOS 89F	e	$D^0 \to K^- \pi^+$	3.65 ± 0.54	$2.1^{+0.4}_{-0.2} \pm 0.2$
CLEO-I	[6] CRAWFORD 91B	e	$D^0 \to K^- \pi^+$	3.61 ± 0.37	$2.1^{+0.4+0.3}_{-0.2-0.2}$
CLEO-I*	[6] CRAWFORD 91B	μ	$D^0 \to K^- \pi^+$	3.26 ± 0.50	
E687*	[7] FRABETTI 93I	μ	$D^0 \to K^- \pi^+$	3.39 ± 0.75	
CLEO-II*	[4] BEAN 93C	e, μ	$D^0 \to K^- \pi^+$	3.92 ± 0.25	$2.00 \pm 0.12 \pm 0.18$
MARK-III	[8] ADLER 89	e	absolute	3.4 ± 0.6	
E653*	[9] KODAMA 93B	μ	μ inclusive	2.5 ± 0.6	
PDG value				3.68 ± 0.21	

* Muon measurements have been scaled up by 1.03 to be equivalent to electrons.
† Here and in several later Tables, the reference ID used in the Data Listings is given following the reference number.

The discrepancy may be be due, in part, to poorly measured normalizing branching fractions. In any case, we use the average transition rate $\Gamma(D \to \overline{K} e^+ \nu_e) = (8.2 \pm 0.4) \times 10^{10}$ s^{-1} to compute the form factor discussed below.

Table 2: Measurements of the $D^+ \to \overline{K}^0 e^+ \nu_e$ branching fraction.

Experiment	Reference	Norm. mode	$B(D^+ \to \overline{K}^0 e^+ \nu_e)\%$
MARK-III*	[10] BAI 91	absolute	$6.8^{+1.6}_{-1.1}$
E691	[11] ANJOS 91C	$D^+ \to K^- \pi^+ \pi^+$	6.0 ± 1.6
CLEO-II*	[4] BEAN 93C	$D^+ \to \overline{K}^0 \pi^+$	7.1 ± 0.8
PDG value			6.7 ± 0.8

* Average of electrons and muons.

The decay $D^0 \to K^- e^+ \nu_e$ is the only one so far that allows a useful study of the q^2 dependence of heavy-meson form factors. Experiments to date are precise enough to determine only an intercept and a slope parameter for the function $f_+^K(q^2)$. This form factor is commonly parametrized with a pole form,

$$f_+^K(q^2) = f_+^K(0)/(1 - q^2/m_p^2) \ , \tag{5}$$

where m_p is an appropriate pole mass, expected to be approximately the mass of the D_s^*. Values of m_p derived from the experiments are given in Table 1 and are in agreement with $m_{D_s^*}$. The form-factor intercept is computed by integrating Eq. (1), assuming the pole form for $f_+^K(q^2)$. Using the average transition rate of the two charge states, the result is $|f_+^K(0)||V_{cs}| = 0.73 \pm 0.02 \pm 0.02$, where the first error is from the uncertainty in the decay rate and the second is from the uncertainty in the shape of $f_+^K(q^2)$. Using $|V_{cs}| = 0.974$ from the unitarity of the CKM matrix, the value $f_+^K(0) = 0.75 \pm 0.02 \pm 0.02$ is obtained. This result is in general agreement with predictions from models and lattice gauge calculations, as shown in Table 8.

$D \to \overline{K}^* \ell^+ \nu_\ell$ decays

The differential decay distribution for $D \to \overline{K}^* \ell^+ \nu_\ell$ is governed, as explained in Section II, by one vector and two axial-vector form factors. Experiments do not yet have enough data to measure the q^2 dependences of these form factors. The practice is to assume that they have a simple pole form, as for the pseudoscalar case, using for pole masses those of the lowest lying $c\overline{s}$ states with the appropriate quantum numbers: the D_s^* mass, 2.1 GeV, for the pole mass m_V, and the D_s^{**} mass, 2.5 GeV, for m_A. The ratios of form-factor intercepts, $R_2 = A_2(0)/A_1(0)$ and $R_V = V(0)/A_1(0)$, are then determined by fitting to the q^2 and angular decay distributions. These ratios have been measured in $D^+ \to \overline{K}^{*0} \ell^+ \nu_\ell$ decay and are given in Table 3. The corresponding ratios of longitudinal to transverse polarization, and of positive to negative transverse polarization, are also shown.

Table 3: $D^+ \to \overline{K}^{*0} \ell^+ \nu_\ell$ form factor ratios.

Experiment	Ref.	$R_2(0)$	$R_V(0)$	Γ_L/Γ_T	Γ_+/Γ_-
E691	[12]	$0.0 \pm 0.5 \pm 0.2$	$2.0 \pm 0.6 \pm 0.3$	$1.8^{+0.6}_{-0.4} \pm 0.3$	$0.15^{+0.07}_{-0.05} \pm 0.03$
E653	[13]	$0.82^{+0.22}_{-0.23} \pm 0.11$	$2.00^{+0.34}_{-0.32} \pm 0.16$	$1.18 \pm 0.18 \pm 0.08$	$0.16 \pm 0.05 \pm 0.02$
E687	[16]	$0.78 \pm 0.18 \pm 0.10$	$1.74 \pm 0.27 \pm 0.28$	$1.20 \pm 0.13 \pm 0.13$	
Ave.		0.73 ± 0.15	1.89 ± 0.25	1.23 ± 0.13	0.16 ± 0.04

The normalization of the form factors is given by the transition rate, which is proportional to $A_1(0)^2$ and is only weakly dependent on R_V and R_2. Tables 4 and 5 give measurements of the $D^+ \rightarrow \overline{K}^{*0} e^+ \nu_e$ and $D^0 \rightarrow K^{*-} e^+ \nu_e$ branching fractions with the associated average transition rates in Table 10. The D^+ measurement is the better of the two, due to the long D^+ lifetime, the high efficiency for observing the $\overline{K}^{*0}$, and the recent, more precise measurement of the normalizing $D^+ \rightarrow K^- \pi^+ \pi^+$ branching fraction.

Table 4: Measurements of the $D^+ \rightarrow \overline{K}^{*0} e^+ \nu_e$ branching fraction.

Experiment	Reference	Lepton	Norm. mode	$\mathrm{B}(D^+ \rightarrow \overline{K}^{*0} e^+ \nu_e)$%
E691	[12] ANJOS 90E	e	$D^+ \rightarrow K^- \pi^+ \pi^+$	4.4 ± 0.6
E653*	[18] KODAMA 92C	μ	$D^+ \rightarrow K^- \pi^+ \pi^+$	4.4 ± 1.0
			$D^0 \rightarrow K^- e^+ \nu_e$	3.9 ± 1.1
E687*	[16] FRABETTI 93E	μ	$D^+ \rightarrow K^- \pi^+ \pi^+$	5.4 ± 0.8
WA82	[14] ADAMOVICH 91	e	$D^+ \rightarrow K^- \pi^+ \pi^+$	5.6 ± 1.6
ARGUS	[19] ALBRECHT 91	e	$D^+ \rightarrow K^- \pi^+ \pi^+$	5.0 ± 1.2
CLEO-II*	[4] BEAN 93C	e, μ	$D^+ \rightarrow K^- \pi^+ \pi^+$	6.1 ± 1.1
PDG value				4.8 ± 0.4

*Muon measurements have been scaled up by 1.05 to be equivalent to electrons.

Table 5: Measurements of the $D^0 \rightarrow K^{*-} e^+ \nu_e$ branching fraction.

Experiment	Reference	Norm. mode	$\mathrm{B}(D^0 \rightarrow K^{*-} e^+ \nu_e)$%
CLEO-I	[6] CRAWFORD 91B	$D^0 \rightarrow K^- e^+ \nu_e$	1.9 ± 0.7
CLEO-II	[4] BEAN 93C	$D^0 \rightarrow \overline{K}^0 \pi^+ \pi^-$	2.0 ± 0.4
PDG value			2.0 ± 0.4

The new measurements confirm the old result that the transition rates for K^* decays are significantly smaller than for K decays, contrary to expectations from models. This result is also confirmed by several direct measurements of the K^*-to-K ratio, obtained without the use of normalizing branching fractions, shown in Table 6. This ratio is about 0.6, which leads to the low values for the form-factor intercepts $A_1(0)$ and $A_2(0)$ given in Table 7. These values are compared with predictions in Table 8. Note that the form-factor intercepts in these tables have been corrected for the new average value $\Gamma(D \to \overline{K}^* e^+ \nu_e) = (4.6 \pm 0.4) \times 10^{10}$ s^{-1}.

Table 6: Direct measurements of $\Gamma(D \to \overline{K}^* \ell \nu_\ell)/\Gamma(D \to \overline{K} \ell \nu_\ell)$.

Experiment	Reference	Measured ratio	Value
CLEO-I	[6] CRAWFORD 91B	$\dfrac{\Gamma(D^0 \to K^{*-} e^+ \nu_e)}{\Gamma(D^0 \to K^- e^+ \nu_e)}$	$0.51 \pm 0.18 \pm 0.06$
CLEO-II	[4] BEAN 93C	$\dfrac{\Gamma(D^0 \to K^{*-} e^+ \nu_e)}{\Gamma(D^0 \to K^- e^+ \nu_e)}$	$0.60 \pm 0.09 \pm 0.07$
CLEO-II	[4] BEAN 93C	$\dfrac{\Gamma(D^+ \to \overline{K}^{*0} e^+ \nu_e)}{\Gamma(D^+ \to \overline{K}^0 e^+ \nu_e)}$	$0.65 \pm 0.09 \pm 0.10$
E653	[18] KODAMA 92C	$\dfrac{\Gamma(D^+ \to \overline{K}^{*0} \mu^+ \nu_\mu)}{\Gamma(D^0 \to K^- \mu^+ \nu_\mu)}$	$0.43 \pm 0.09 \pm 0.09$

Table 7: Form factors for $D^+ \to \overline{K}^{*0} e^+ \nu_e$. All results are normalized to $\Gamma(D \to \overline{K}^* e^+ \nu_e) = (4.6 \pm 0.4) \times 10^{10}$ s^{-1}.

Experiment	Ref.	$A_1(0)$	$V(0)$	$A_2(0)$
E653	[18]	0.57 ± 0.08	1.2 ± 0.3	0.47 ± 0.16
E691	[12]	0.49 ± 0.07	1.0 ± 0.3	0.0 ± 0.2
E687	[15]	0.59 ± 0.05	1.0 ± 0.3	0.46 ± 0.11
Ave.		0.56 ± 0.04	1.1 ± 0.2	0.40 ± 0.08

Table 8: Form factors: comparison with theory.

		$f_+(0)$	$A_1(0)$	$V(0)$	$A_2(0)$
Model	Exp. average =	0.75 ± 0.03	0.56 ± 0.04	1.1 ± 0.2	0.40 ± 0.08
Quark models	ISGW [30]	0.8	0.8	1.1	0.8
	WSB [31]	0.76	0.88	1.3	1.2
	KS [32]	0.7	0.82	0.8	0.8
	AW/GS [33]	0.7	0.8	1.5	0.6
Lattice gauge	BKS [34]	$0.9 \pm 0.1 \pm 0.2$	$0.8 \pm 0.1 \pm 0.3$	$1.4 \pm 0.5 \pm 0.5$	$0.6 \pm 0.1 \pm 0.2$
	LMMS [35]	0.63 ± 0.08	0.53 ± 0.03	0.9 ± 0.1	0.2 ± 0.2
Sum rules	BBD [36]	0.60	0.5	1.1	0.6

Cabibbo-suppressed semileptonic decays

The rates of Cabibbo-suppressed pseudoscalar semileptonic decays determine the product of $|V_{cd}|$ times a form factor. Assuming that $|V_{cd}|$ is known precisely from unitarity of the CKM matrix, the main goal is to determine the form factor. The form factors measured in Cabibbo-suppressed charm decays are related by HQET [38] to those for $b \to u$ semileptonic decays in the q^2 region near $q^2_{\rm max}$, approximately 18–26 GeV^2. Assuming appropriate pole forms for the form factors, the ratios of branching fractions can be written as

$$R_\pi = \frac{\mathrm{B}(D^0 \to \pi^- e^+ \nu_e)}{\mathrm{B}(D^0 \to K^- e^+ \nu_e)} = 2 \times \frac{\mathrm{B}(D^+ \to \pi^0 e^+ \nu_e)}{\mathrm{B}(D^+ \to \overline{K}^0 e^+ \nu_e)}$$
$$= 1.97 \times \left|\frac{f_+^\pi(0)}{f_+^K(0)}\right|^2 \left|\frac{V_{cd}}{V_{cs}}\right|^2 . \qquad (6)$$

The factor of 2 difference between these two modes arises from the $1/\sqrt{2}$ in the coupling of $d\bar{d}$ to a π^0.

To date, measurements of Cabibbo-suppressed semileptonic decays suffer from poor statistics; better results should be forthcoming from the new high-statistics charm experiments. The MARK-III measurement [20] of $\mathrm{B}(D^0 \to \pi^- e^+ \nu_e)$, together with our value for $\mathrm{B}(D^0 \to K^- e^+ \nu_e)$ from Table 1, yields $R_\pi = 0.11^{+0.06}_{-0.03} \pm 0.1$. This can be compared with the CLEO-II measurement of $\mathrm{B}(D^+ \to \pi^0 e^+ \nu_e)$ [21], giving $R_\pi = 0.17 \pm 0.05 \pm 0.03$. From unitarity of the CKM matrix, we have $|V_{cd}/V_{cs}|^2 = 0.051 \pm 0.002$, giving values of $|f_+^\pi(0)/f_+^K(0)| = 1.0^{+0.3}_{-0.2} \pm 0.04$ and $1.3 \pm 0.2 \pm 0.1$ for the MARK-III and CLEO-II results. Model predictions, which range from 0.7 to 1.4 [39], are in agreement with these results. The E653 collaboration [22] reports a measurement, $\Gamma(D^+ \to \rho^0 \mu^+ \nu_\mu)/\Gamma(D^+ \to \overline{K}^{*0} \mu^+ \nu_\mu) = 0.044^{+0.031}_{-0.025} \pm 0.014$, in good agreement with expectations, within the large errors.

Nonresonant semileptonic decays

There is now substantial evidence that Cabibbo-favored semileptonic decays are nearly saturated by $D \to \overline{K} e^+ \nu_e$ and $D \to \overline{K}^* e^+ \nu_e$. Table 9 gives the results of searches for other decays. Early results from the hybrid bubble chamber [37] indicating large nonresonant contributions are clearly inconsistent with much more sensitive later measurements. Limits on the sum of the nonresonant (and higher-mass resonant) decays can be inferred from the comparison of the sum of exclusive rates with the inclusive rate, as discussed below.

Table 9: Limits on specific nonresonant modes.

Experiment	Reference	Mode	$\Gamma/10^{10}$ s^{-1} 90% CL upper limit
E687	[16] FRABETTI 93E	$D^+ \to (K^-\pi^+)_{\rm NR}\mu^+\nu_\mu$	< 0.4
		$D^+ \to K^-\pi^+\pi^0\mu^+\nu_\mu$	< 0.1
E653	[17] KODAMA 93B	$D^0 \to K^-\pi^+\pi^-\mu^+\nu_\mu$	< 0.3
		$D^0 \to (K^*\pi)^-\mu^+\nu_\mu$	< 0.4

Inclusive semileptonic decays

The inclusive transition rates are $\Gamma(D^+ \to Xe^+\nu_e) = (16.3 \pm 1.8) \times 10^{10}$ s^{-1} and $\Gamma(D^0 \to Xe^+\nu_e) = (18.6 \pm 2.9) \times 10^{10}$ s^{-1}. These rates are dominated by old MARK-III measurements [23]. Isospin symmetry requires that the Cabibbo-favored semileptonic rates for the two charged states be equal, but small differences can be expected for the Cabibbo-suppressed rates. We ignore these differences, which are expected to be less than 2% of the inclusive rate, and use the average for the two charge states, $\Gamma(D \to Xe^+\nu_e) = (16.9 \pm 1.5) \times 10^{10}$ s^{-1}. This inclusive rate can be compared with the sum of the rates for $D \to \overline{K}e^+\nu_e$, $D \to \overline{K}^*e^+\nu_e$, and an estimated 8% contribution for the Cabibbo-suppressed modes, as shown in Table 10. (We average the D^0 and D^+ exclusive rates here as well.) The sum of these rates implies a deficit of slightly less than two standard deviations. CLEO [4] has measured all four of the exclusive K and K^* modes, normalizing to the best measured decay, $D^0 \to K^-\pi^+$. Averaging over the charge states, CLEO obtains $\Gamma(D \to (\overline{K}+\overline{K}^*)e^+\nu_e) = (14.8 \pm 1.3) \times 10^{10}$ s^{-1}. Adding 8% for Cabibbo-suppressed decays results in a deficit of $(0.9 \pm 2.0) \times 10^{10}$ s^{-1}, consistent with zero. The rather large errors leave some room for small contributions from other exclusive channels. As seen in Table 9, the limits on individual channels are much more stringent.

Table 10: Exclusive transition rates. For comparison, $\Gamma(D \to X e^+ \nu_e) = (16.9 \pm 1.5) \times 10^{10}\ \mathrm{s}^{-1}$.

Mode	$\Gamma(D^0)/10^{10}\ \mathrm{s}^{-1}$	$\Gamma(D^+)/10^{10}\ \mathrm{s}^{-1}$	$\Gamma_{\mathrm{Ave}}/10^{10}\ \mathrm{s}^{-1}$
$D \to \overline{K} e \nu_e$	8.9 ± 0.5	6.3 ± 0.8	8.2 ± 0.4
$D \to \overline{K}^* e \nu_e$	4.8 ± 1.0	4.5 ± 0.4	4.6 ± 0.4
Estimated Cabibbo-suppressed			1.0 ± 0.3
Sum of exclusive rates			13.8 ± 0.6

D_s^+ semileptonic decays

Because ϕ mesons are easy to observe, the first D_s^+ semileptonic decay measured was the decay to a vector, $D_s^+ \to \phi \ell^+ \nu_\ell$. The ratio of decay rates, $R_\phi \equiv \Gamma(D_s^+ \to \phi \ell^+ \nu_\ell)/\Gamma(D_s^+ \to \phi \pi^+)$, has been measured by several experiments (Table 11). The average value given in the table includes both electron and muon results.

Table 11: Measurements of $R_\phi \equiv \Gamma(D_s^+ \to \phi \ell^+ \nu_\ell)/\Gamma(D_s^+ \to \phi \pi^+)$.

Experiment	Reference	R_ϕ
CLEO-I	[28] ALEXANDER 90B	$0.49 \pm 0.10^{+0.10}_{-0.14}$
ARGUS	[27] ALBRECHT 91	$0.57 \pm 0.15 \pm 0.15$
E687	[26] FRABETTI 93G	$0.58 \pm 0.17 \pm 0.07$
CLEO-II	[25] BUTLER 94	$0.54 \pm 0.05 \pm 0.04$
PDG value		0.54 ± 0.05

As yet there are no direct measurements of D_s^+ branching fractions. One method for estimating the $D_s^+ \to \phi \pi^+$ branching fraction is based on the belief that $D_s^+ \to \phi \ell^+ \nu_\ell$ and $D \to \overline{K}^* \ell^+ \nu_\ell$ decays are quite similar. Quark-model-based estimates [40] for the ratio $\kappa = \Gamma(D_s^+ \to \phi \ell^+ \nu_\ell)/\Gamma(D \to \overline{K}^* \ell^+ \nu_\ell)$ are about 1.0. The branching fraction is then

$$\mathrm{B}(D_s^+ \to \phi \pi^+) = \frac{\tau_{D_s}\, \kappa\, \Gamma(D \to \overline{K}^* \ell^+ \nu_\ell)}{R_\phi} \ . \tag{7}$$

The average over the two $\Gamma(D \to \overline{K}^* e^+ \nu_e)$ charge states is $(4.6 \pm 0.4) \times 10^{10}\ \mathrm{s}^{-1}$. Using the PDG value for the D_s^+ lifetime, the value for R_ϕ in Table 11, and $\kappa = 1.0$ gives $\mathrm{B}(D_s^+ \to \phi \pi^+) =$

(4.0±0.5)%. This value also has a theoretical uncertainty, which we conservatively estimate to be about 25%. (The PDG value of this branching fraction, which combines several estimates, is (3.5±0.4)%.)

We will have more confidence in this method for determining the absolute D_s^+ branching fraction scale, and indeed more confidence in our understanding of charmed semileptonic decays in general, if the patterns for the D_s^+ semileptonic decays are seen to be the same as those for D^0 and D^+. It is important to measure the form factor ratios for $D_s^+ \to \phi \ell^+ \nu_\ell$ decay, as has been done for $D \to \overline{K}^* \ell^+ \nu_\ell$, to shed some light on the validity of the assumption of equal transition rates. With small numbers of events, Fermilab experiments E653 and E687 have made first measurements of these ratios (Table 12). These results, to be compared with the $D \to \overline{K}^* e \nu_e$ values of Table 3, are not yet precise enough to draw conclusions about the degree of similarity of the D_s^+ and D decays.

Perhaps even more important is the ratio of the rates of semileptonic decays of the D_s^+ to vector and pseudoscalar mesons, since this ratio is not as expected in D^+ and D^0 decays. The first observation of the pseudoscalar modes is from Fermilab E653 [24], who obtained $[\Gamma(D_s^+ \to \phi \mu^+ \nu_\mu)]/[\Gamma(D_s^+ \to (\eta + \eta')\mu^+ \nu_\mu)] = 0.26 \pm 0.11$. The result is certainly consistent with a low vector-to-pseudoscalar ratio. This experiment was unable to separate the η and η' modes.

Table 12: $D_s^+ \to \phi \ell^+ \nu_\ell$ form factor ratios.

Experiment	Ref.	$R_2(0)$	$R_V(0)$	Γ_L/Γ_T
E653	[24]	$2.1^{+0.6}_{-0.5} \pm 0.2$	$2.3^{+1.1}_{-0.9} \pm 0.4$	$0.54 \pm 0.21 \pm 0.10$
E687	[29]	$1.1 \pm 0.8 \pm 0.1$	$1.8 \pm 0.9 \pm 0.2$	$1.0 \pm 0.5 \pm 0.1$
Ave.		1.8 ± 0.5	2.0 ± 0.7	0.6 ± 0.2

Conclusions on D-meson semileptonic decays

The study of semileptonic decays of D mesons has progressed enormously in the last few years. The q^2 dependence of the pseudoscalar form factor has been measured, and the intercepts of the three vector-decay form factors been determined by several experiments. New experiments confirm the old observation that the $D \to \overline{K}^{*}\ell^{+}\nu_{\ell}$ decay rate, and therefore the A_1 form factor, is low compared with model calculations. Presently, no model appears able to explain all of the form factors.

Problems remain in the precise quantitive comparisons between the transition rates of the different charge states in the $D \to \overline{K}\ell^{+}\nu_{\ell}$ exclusive decays. The difficulty may lie in the branching fractions of the normalizing modes for the D^{+} decays. Strict upper limits are observed on rates of nonresonant decays. These limits are consistent with the observation that the transition rates for $D \to \overline{K}\ell^{+}\nu_{\ell}$ and $D \to \overline{K}^{*}\ell^{+}\nu_{\ell}$, plus a reasonable estimate of the Cabibbo-suppressed rate, nearly saturates the inclusive rate.

Measurements of Cabibbo-suppressed semileptonic decays have begun. Experiments are making good progress with D_s^{+} semileptonic decays. First results have been obtained for the ratios of form-factor intercepts in $D_s^{+} \to \phi\ell^{+}\nu_{\ell}$ and for observations of the pseudoscalar decays $D_s^{+} \to \eta\ell^{+}\nu_{\ell}$ and $D_s^{+} \to \eta'\ell^{+}\nu_{\ell}$.

In the near future, we can expect further significant advances. It will be very interesting to compare the form factors for Cabibbo-favored, Cabibbo-suppressed, and D_s^{+} decays for the pseudoscalar- and vector-meson final states. Measurement of the q^2 dependences of the three vector-decay form factors may help in understanding the puzzle of the small semileptonic transition rate to vector-meson final states.

Continuation of this discussion can be found in the Listings for the $B^{\pm}$.

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