32. CP VIOLATION

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The symmetries C (particle-antiparticle interchange) and P (space inversion) hold for strong and electromagnetic interactions. After the discovery of large C and P violation in the weak interactions, it appeared that the product CP was a good symmetry. Then CP violation was observed in K^0 decays at a level given by the parameter $\epsilon = 2.3 \times 10^{-3}$. Larger CP-violation effects are anticipated in B^0 decays.

The eigenstates of the K^0–$\overline{K}^0$ system can be written

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle \ , \qquad |K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle \ . \tag{32.1}$$

If CP invariance held, we would have $q = p$ so that K_S would be CP even and K_L CP odd. (We define $|\overline{K}^0\rangle$ as CP $|K^0\rangle$). CP violation in K^0–$\overline{K}^0$ mixing gives

$$\frac{p}{q} = \frac{(1+\widetilde{\epsilon})}{(1-\widetilde{\epsilon})} \ . \tag{32.2}$$

CP violation can also occur in the decay amplitudes

$$A\big(K^0 \to \pi\pi(I)\big) = A_I e^{i\delta_I} \ , \qquad A\big(\overline{K}^0 \to \pi\pi(I)\big) = A_I^* e^{i\delta_I} \ , \tag{32.3}$$

where I is the isospin of $\pi\pi$, δ_I is the final-state phase shift, and A_I would be real if CP invariance held. The ratios of CP-violating to CP-conserving amplitudes $\eta_{+-} = A(K_L^0 \to \pi^+\pi^-)/A(K_S^0 \to \pi^+\pi^-)$ and $\eta_{00} = A(K_L^0 \to \pi^0\pi^0)/A(K_S^0 \to \pi^0\pi^0)$ can be written as

$$\eta_{+-} = \epsilon + \epsilon' \ , \qquad \eta_{00} = \epsilon - 2\epsilon' \ , \tag{32.4a}$$

$$\epsilon = \widetilde{\epsilon} + i\ \big(\mathrm{Im}\, A_0/\mathrm{Re}\, A_0\big) \ , \tag{32.4b}$$

$$|\sqrt{2}\epsilon'| = (\mathrm{Re}\, A_2/\mathrm{Re}\, A_0)\ \big(\mathrm{Im}\, A_2/\mathrm{Re}\, A_2 - \mathrm{Im}\, A_0/\mathrm{Re}\, A_0\big) \ . \tag{32.4c}$$

If CP violation is confined to the mass matrix, as in a superweak theory, ϵ' is zero and $\eta_{+-} = \eta_{00} = \epsilon = \widetilde{\epsilon}$. The measurement of ϵ'/ϵ has as its goal finding an effect that requires CP violation in the decay amplitude; this corresponds to a relative phase between A_2 and A_0 as seen in Eq. (32.4c).

In the Standard Model, CP violation arises as a result of a single phase entering the CKM matrix (q.v.). As a result in what is now the standard phase convention, two elements have large phases, $V_{ub} \sim e^{-i\gamma}$, $V_{td} \sim e^{-i\beta}$. Because these elements have small magnitudes and involve the third generation, CP violation in the K^0 system is small. A definite nonzero value for ϵ'/ϵ is expected but hadronic uncertainties allow theoretical values between 10^{-4} and 3×10^{-3}. On the other hand, large effects are expected in the B^0 system, which is a major motivation for B factories.

The most clearcut experiments would be those that measure asymmetries between B^0 and $\overline{B}^0$ decays. The time-dependent rate to a CP eigenstate a is given by

$$\Gamma_a \sim e^{-\Gamma t}\Big(\left[1+|r_a|^2\right] \pm \left[1-|r_a|^2\right] \cos(\Delta M t) \mp 2\eta_a \mathrm{Im}\, r_a \sin(\Delta M t)\Big) \ , \tag{32.5}$$

where the top sign is for B^0 and the bottom for $\overline{B}^0$, η_a is the CP eigenvalue and

$$r_a = (q_B/p_B)\, \overline{A}_a/A_a \ . \tag{32.6}$$

The quantity (q_B/p_B) comes from the analogue for B^0 of Eq. (32.1); however, for B^0 the eigenstates have a negligible lifetime difference and are distinguished only by the mass difference ΔM; also as a result $|q_B/p_B| \approx 1$ so that $\widetilde{\epsilon}_B$ is purely imaginary. A_a ($\overline{A}_a$) are the decay amplitudes to a for B^0 ($\overline{B}^0$). If only one quark weak transition contributes to the decay $|\overline{A}_a/A_a| = 1$ so that $|r_a| = 1$ and the $\cos(\Delta M t)$ term vanishes. The basic goal of the B factories is to observe the asymmetric $\sin(\Delta M t)$ term. For B^0 ($\overline{B}^0$) $\to \psi K_s$ from the transition $b \to c\overline{c}s$, one finds in the Standard Model the asymmetry parameter

$$-2\mathrm{Im}\, r_a = \sin 2\beta \ . \tag{32.7}$$

The asymmetry is given directly in terms of a CKM phase with no hadronic uncertainty and is expected to be between 0.2 and 0.8. For B^0 ($\overline{B}^0$) $\to \pi^+\pi^-$ from the transition $b \to u\overline{u}d$

$$-2\mathrm{Im}\, r_a = \sin 2(\beta+\gamma) \ . \tag{32.8}$$

(This result has some hadronic uncertainty due to penguin contributions, but these should be able to be estimated from other observations.) While either of these asymmetries could be ascribed to $B^0 - \overline{B}^0$ mixing (q_B/p_B or $\widetilde{\epsilon}_B$), the difference between the two asymmetries is evidence for direct CP violation. From Eq. (32.6) (with $\overline{A}_a/A_a = 1$) it is seen this corresponds to a phase difference between $A_{\psi K_S}$ and $A_{\pi^+\pi^-}$. Thus this is analogous to ϵ'. In the standard phase convention 2β in Eq. (32.7) and (32.8) arises from $B^0 - \overline{B}^0$ mixing whereas the 2γ comes from V_{bu} in the transition $b \to u\overline{u}d$.

CP violation in the decay amplitude is also revealed by the $\cos(\Delta M t)$ in Eq. (32.5) or by a difference in rates of B^+ and B^- to charge-conjugate states. These effects, however, require two contributing amplitudes to the decay (such as a tree amplitude plus a penguin) and also require final-state interaction phases. Predicted effects are very uncertain and are generally small.

For further details, see the notes on CP violation in the K^0_L, K^0_S, and B^0 Full Data Listings of this *Review*.